Fast integration program for spin tracking

Straight Forward Integrators Up To Now

Two Integrators*:
- 4^{th} order Runge-Kutta (Selcuk)
- 5^{th} order Predictor-Corrector (Yannis)

Both are:
- Accurate for short times
- Slow
- Not very flexible
- Expressed in cartesian lab-frame coordinates
- Hard to parallelize (well)
- Written in old style/language

=> decide to write a new code in C++

* http://arxiv.org/abs/1503.02247

Goals For New Development

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New code should be:

- Fast
- Parallel, possibly on GPUs
- Very accurate
- Flexible
- Able to use different integration algorithms
- Able to give indications about precision
- Able to track particles in 3 dimensional lattices

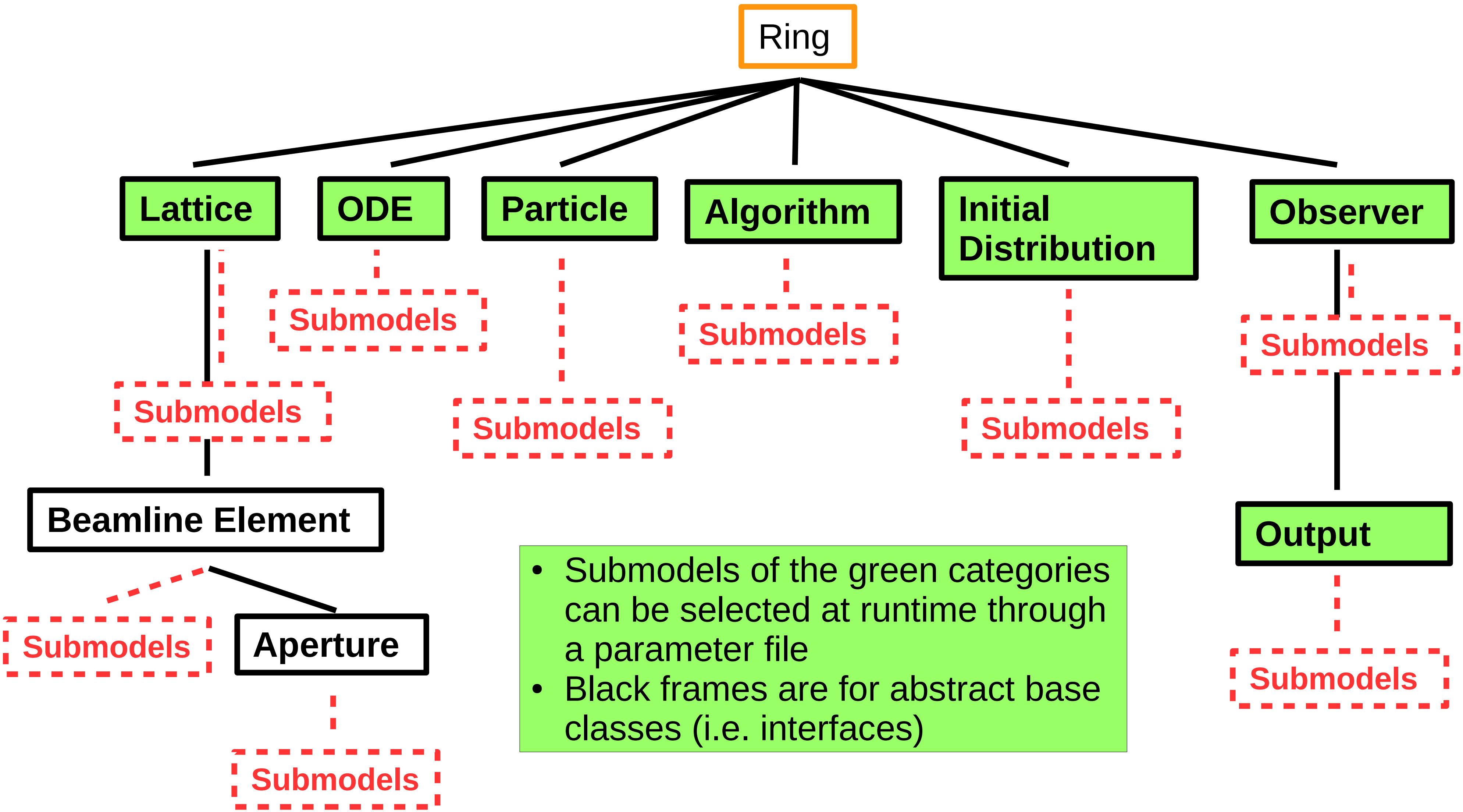
Design Decisions

Use:

- Boost library, especially package odeint (different integrators)
- VexCL* for parallelizing code on CPUs/GPUs
- ROOT and/or Gnuplot for postprocessing
- Curvilinear coordinate system with s as independent parameter
- Easy to extend modular design with fixed interfaces
- Virtual constructors to select different models at runtime

* **V**ector **EX**pression template library for Open**CL**/CUDA,
https://github.com/ddemidov/vexcl,
http://arxiv.org/abs/1212.6326

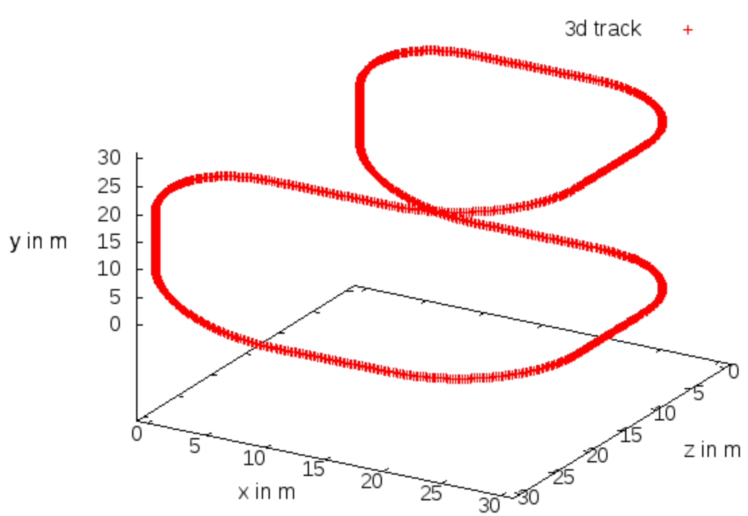
Modular Design



- Submodels of the green categories can be selected at runtime through a parameter file
- Black frames are for abstract base classes (i.e. interfaces)

Current Status

- **Complete framework is available**
- Only a few submodels are implemented
- Program is currently only serial
- First benchmarking done for electric/magnetic rings with/without rf
- Program is much faster than old Runge-Kutta integrator
- Can deal with 3D lattices
- Can use arbitrary precision numbers
- Can output to text- or ROOT-file



Data type	Significant digits	Runtime/s
double	15	2.06
long double	18	2.35
float128	34	9.84
mpfr<15>	15	25.39
mpfr<18>	18	22.51
mpfr<30>	30	23.56
mpfr<40>	40	24.65
mpfr<50>	50	27.49
mpfr<100>	100	28.55
mpfr<1000>	1000	433.25

Lattice: 3D electric, Runge-Kutta 4, dt=10^{-8}s, T=1ms

Benchmarking: Magnetic Ring

- All benchmarking is done with **Bulirsch-Stoer algorithm** at required relative and absolute accuracies of 10^{-8} (10^{-10} with rf cavity)
- **Data type: long double**

Pitch Effect: Small correction C of spin precession frequency due to vertical pitch, $\omega = \omega_a(1 - C)$

Without focusing: ω is matched to the 10^{-11} level, same accuracy as frequency fit

With focusing: ω is matched to the 10^{-10} level, frequency is still determined to the 10^{-11} level
→ analytic formula also makes approximations
→ can express analytic results in terms of tune → tune is correct to at least 10^{-10} level

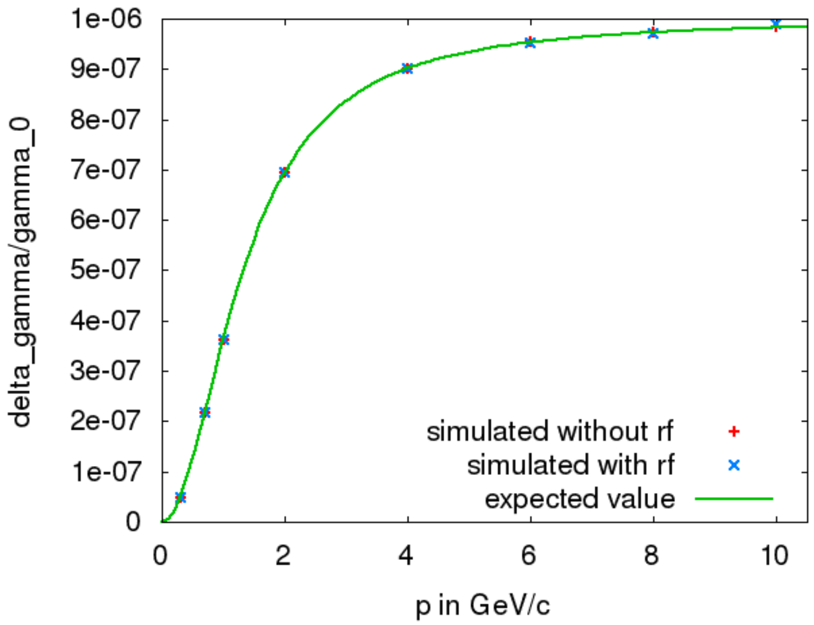
Benchmarking: Electric Ring 1

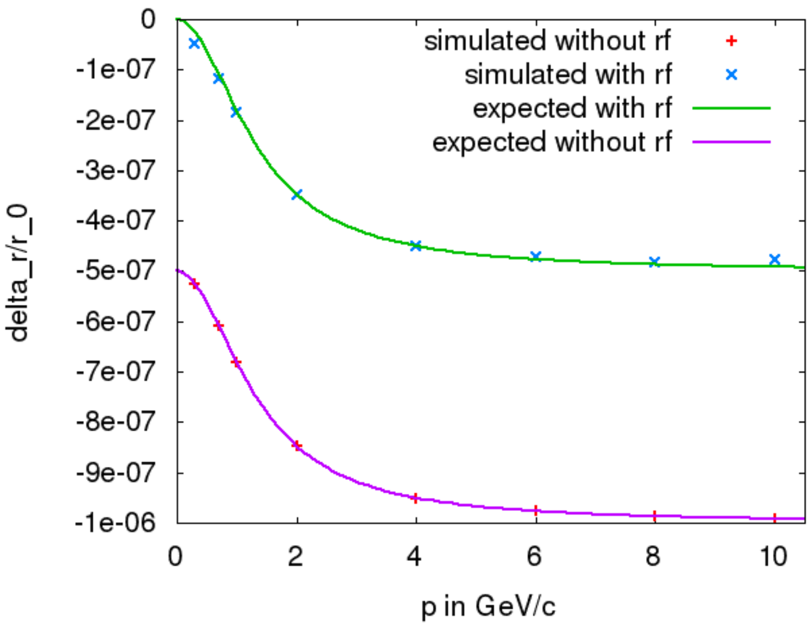
All-electric ring, ideal particle but with vertical pitch, no focusing, expect:

$$\left\langle \frac{\Delta\gamma}{\gamma_0} \right\rangle = \langle\theta_y^2\rangle \frac{\gamma_0^2 - 1}{\gamma_0^2 + 1}$$

$$\left\langle \frac{\Delta r}{r_0} \right\rangle = \begin{cases} -\frac{1}{2}\langle\theta_y^2\rangle \frac{\gamma_0^2 - 1}{\gamma_0^2 + 1} & \text{with rf} \\ -\langle\theta_y^2\rangle \frac{\gamma_0^2}{\gamma_0^2 + 1} & \text{without rf} \end{cases}$$

- Y. Orlov at ECT Trento workshop, Oct. 1-5, 2012
- http://arxiv.org/abs/1504.07304
- http://arxiv.org/abs/1503.06660



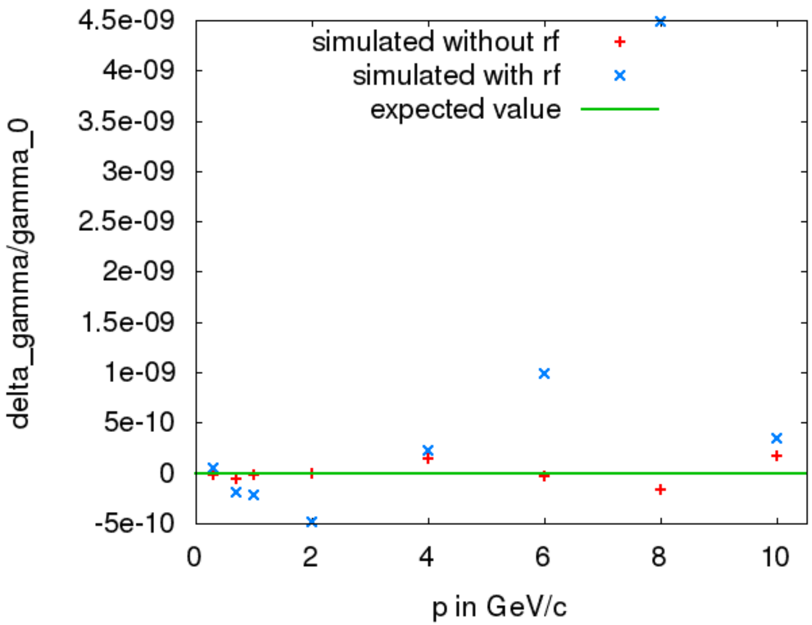


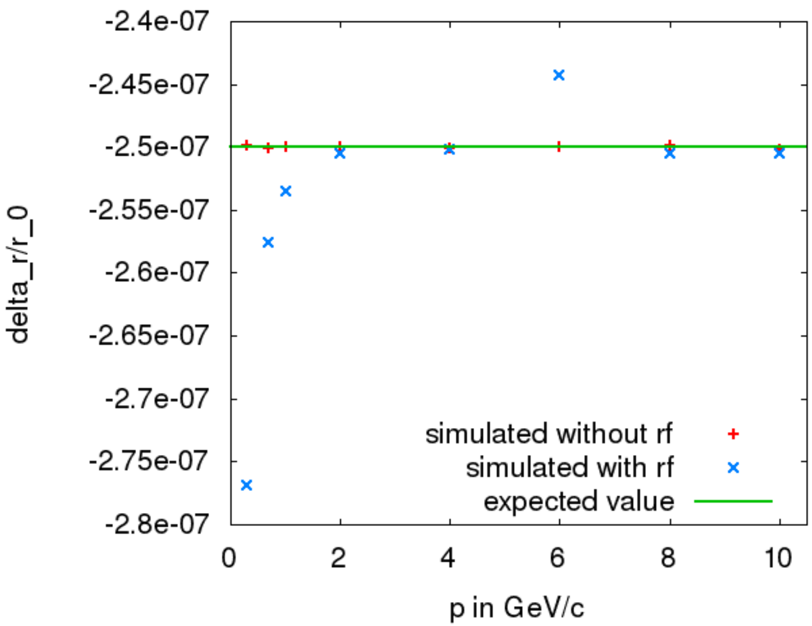
Benchmarking: Electric Ring 2

All-electric ring, ideal particle with vertical pitch, weak focusing expect:

$$\left\langle \frac{\Delta\gamma}{\gamma_0} \right\rangle = 0$$

$$\left\langle \frac{\Delta r}{r_0} \right\rangle = -\frac{1}{2}\langle\theta_y^2\rangle = -\frac{1}{4}\langle\theta_0^2\rangle = -2.5\cdot 10^{-7}$$





→ rf results not that great, probably because of extended cavity and slow oscillations

Some (not surprising) Observations

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- Errors due to machine accuracy can be cut by higher precision numbers, algorithm errors and simplification errors remain
- Algorithm errors can be cut by using shorter time steps
  → **Precision needs time!**
- Time step seems to be limited by oscillation frequencies, **dt should be much smaller than shortest oscillation period**
- Magnitude of spin is not conserved during simulation → loss rate can be good accuracy measure
- Total energy is conserved due to formulation of the problem, ratio of potential energy to kinetic energy changes over time, i.e. **radial drift is observed instead of energy loss due to non-symplectic algorithms**

Plans For Future

- **Parallelize!**
- Calculate beam parameters from many particle simulations
- Implement new submodels, i.e. new beamline elements, etc
- Possibly use Hamiltonian approach together with new geometric integrators in order to respect evolution on Lie groups
- **Use the program!**