Sommerfeld enhancements in the pair annihilation of neutralinos and charginos: an EFT approach

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arXiv:1210.7928, 1303.0200, 1411.6924, 1411.6930

Dark Matter:

Astrophysical probes,

Laboratory tests, and

Theory aspects

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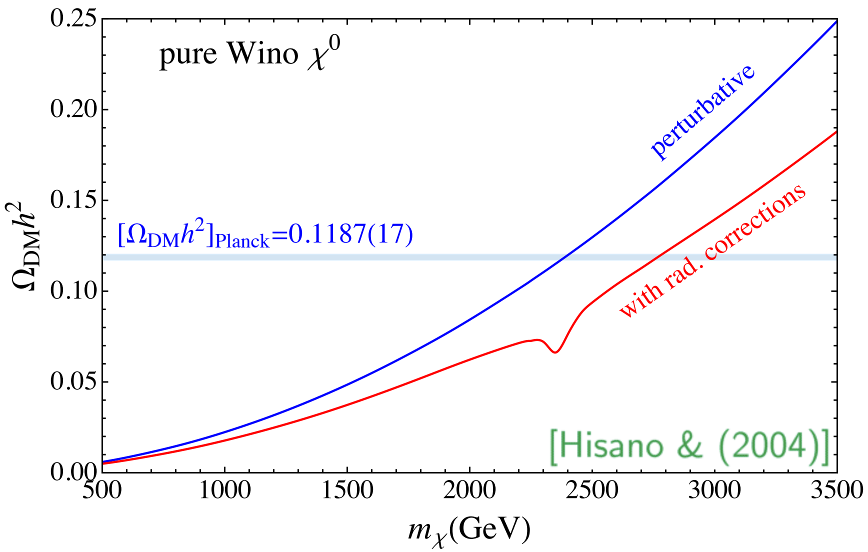
Munich Institute for Astro- and Particle Physics

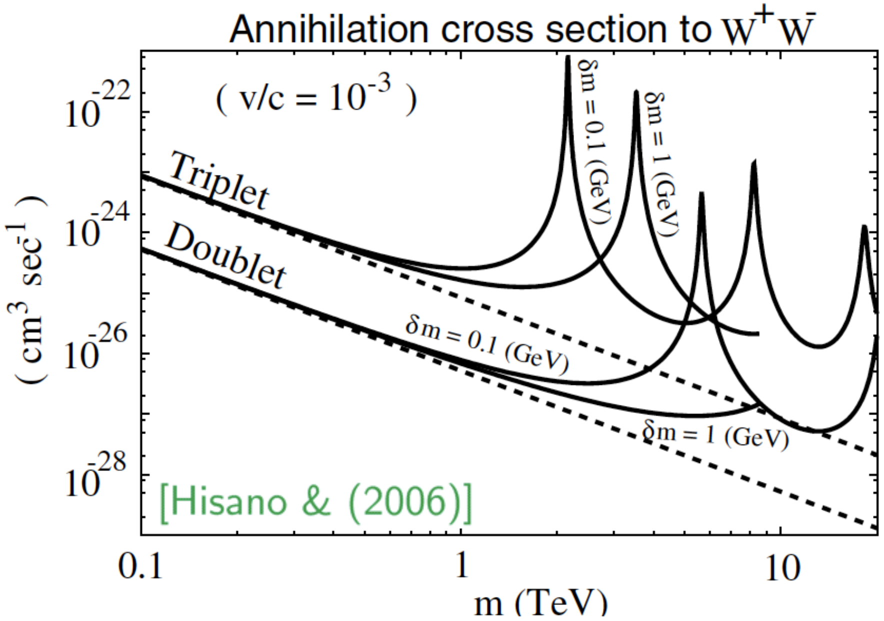
Outline

Motivation

- Increasing precision in the experimental measurement of the DM relic abundance and in indirect searches can be used to place strong bounds on WIMP models

- Sommerfeld enhancement on the ann. cross section can significantly shift m_{χ^0} consistent with observed $\Omega_{\rm DM}h^2$





- resonances can boost the ann. cross section by several orders of magnitude in today's DM annihilation and explain cosmic rays signals

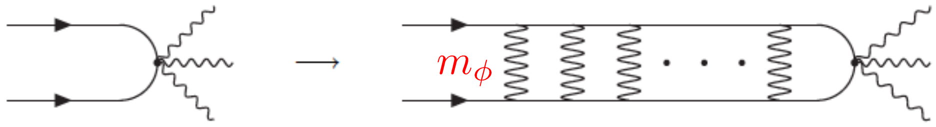
- Compute SF for neutralinos away from the wino and higgsino limits, considering all relevant effects in a controlled approximation

Sommerfeld enhancement for neutralinos in the MSSM

Intrinsic in non-relativistic pair-annihilation or production
(e.g. threshold production of heavy particles at colliders) [Sommerfeld (1931)]

In DM annihilation this happens when the Coulomb (Yukawa) force generated by massless (massive) particle exchange between the DM particles becomes strong at small relative velocities

$\hookrightarrow v \lesssim \alpha$ and $1/m_\phi \gtrsim 1/\alpha\, m_\chi$



Hisano & 2004; Cirelli & 2007; Arkani-Hamed & 2009; Iengo 2009; Cassel 2010; Slatyer 2010; Hryczuk & 2010;. . .

MSSM with lightest χ^0, $m_{\chi^0} \gtrsim 1$ TeV $\quad \chi_1^0 = \begin{pmatrix} \kappa_1^0 \\ \bar{\kappa}_1^0 \end{pmatrix} \quad [\kappa_1^0 = Z^*_{N\,i1}\,(-i\tilde{B}^0, -i\tilde{W}^0, \tilde{H}^1, \tilde{H}^2)_i]$

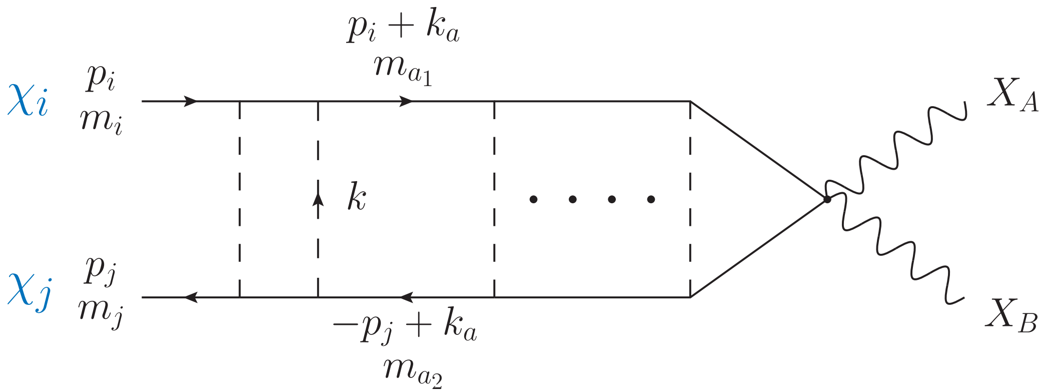
- Wino- or Higgsino-like $\tilde{\chi}^0$ must be relatively heavy to produce the observed dark matter density: $m_{\tilde{\chi}^0} \sim \mathcal{O}(\text{TeV}) \rightarrow m_{\tilde{\chi}^0} \gg m_W, m_Z$
- $\tilde{\chi}^0$ is non-relativistic during thermal decoupling in the early universe: $v \sim 0.3\,c$
- Mass degeneracies with slightly heavier particles in $\tilde{\chi}^0/\tilde{\chi}^\pm$ sector are generic for heavy SUSY $\rightarrow$ Co-annihilations in the relic abundance calculation

$\rightarrow$ see this morning's talk by Hamaguchi

Non-relativistic scattering

- Enhancement of NR scattering from potential loop momentum in ladder diagrams $\quad k_a^0 \sim m_{\rm LSP} v^2 \ll |\vec{k}_a| \sim m_{\rm LSP} v \ll \mu_{a_1 a_2} \sim m_{\rm LSP}/2$

$\sqrt{s} = 2\, m_{\rm LSP} + E$



$$\int \frac{dk_a^0}{2\pi} \frac{2m_{a_1}}{(p_i + k_a)^2 - m_{a_1}^2 + i\epsilon} \frac{2m_{a_2}}{(-p_j + k_a)^2 - m_{a_2}^2 + i\epsilon} = \frac{-i}{E - [M_a - 2m_{\rm LSP}] - \frac{(\vec{p}+\vec{k}_a)^2}{2\mu_a}} + \ldots$$

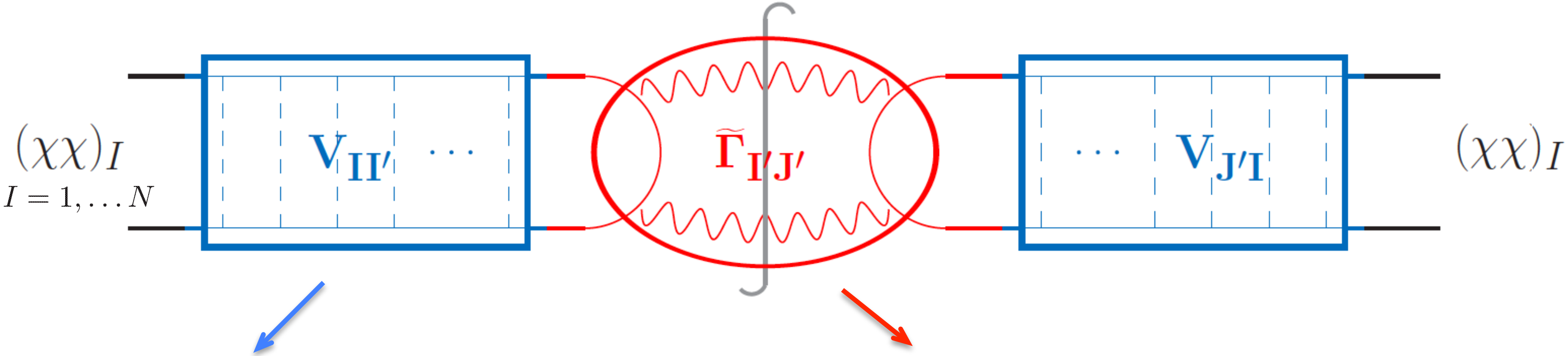
$$\frac{1}{k^2 - m_\phi^2} = -\frac{1}{\vec{k}^2 + m_\phi^2} + \ldots \quad \rightarrow \text{instantaneous interaction: } V(r)$$

- each additional ladder contributes $\propto \int d^3\vec{k} \left(\frac{m_{\rm LSP}}{\vec{k}^2} \times \frac{\alpha_{\rm EW}}{\vec{k}^2} \right) \sim \frac{\alpha_{\rm EW}}{v} \sim \mathcal{O}(1)$

- NR power-counting: No other region requires resummation, potential region enhancement only in ladder diagrams

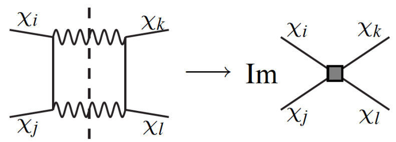
Non-relativistic effective theory approach

DM annihilation cross sections are proportional to the absorptive part of the forward scattering amplitudes



- Long-range effects described by potential interactions $V_{II'}$
  $\rightarrow$ t- and u-channel exchange of the MSSM gauge and Higgs bosons

- Short-distance annihilation described by absorptive part of $(\chi\chi)_{I'} \rightarrow X_A X_B \rightarrow (\chi\chi)_{J'}$



e.g. $\mathcal{O}_{JI}\left({}^3S_1\right) = (\chi^\dagger \sigma^i \chi^c)_J \, (\chi^{c\dagger} \sigma^i \chi)_I$

$$\delta\mathcal{L}_{\rm ann} = \sum_{L,s} \Gamma[{}^{2s+1}L_{\mathcal{J}}]_{IJ} \, \mathcal{O}_{JI}({}^{2s+1}L_{\mathcal{J}})$$

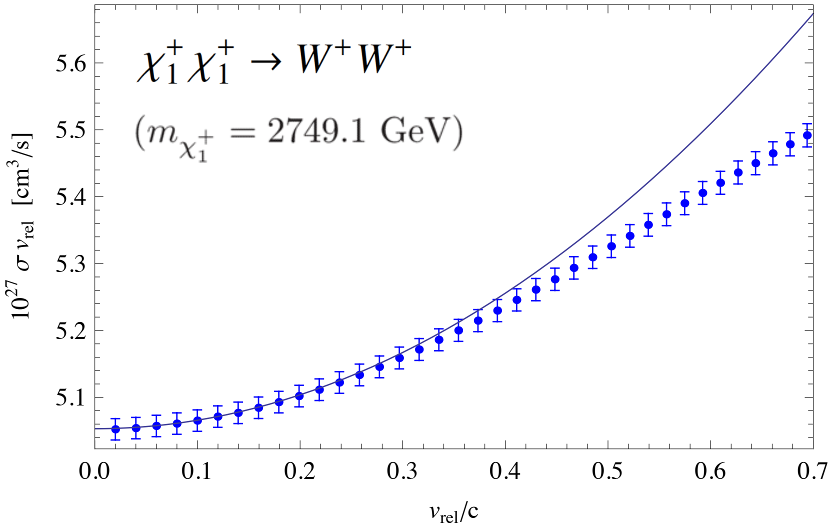
Analogous to heavy quarkonium annihilation [Bodwin, Braaten, Lepage (1995)]

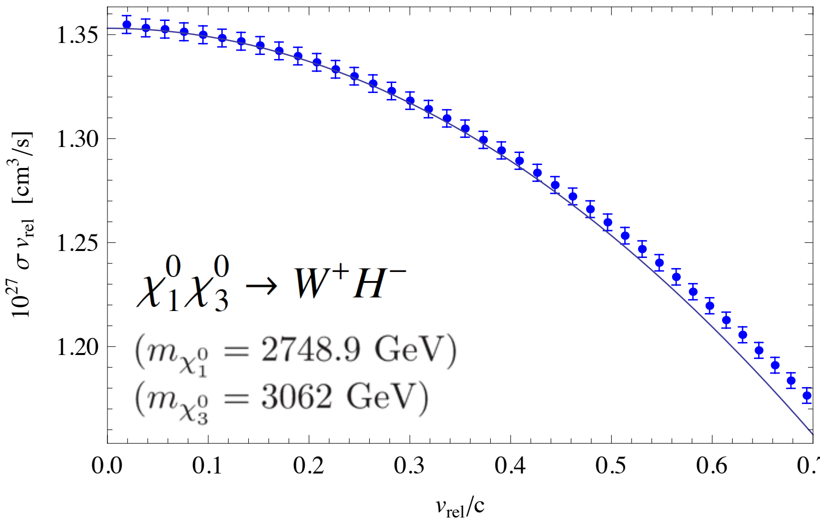
$$\sigma_{(\chi\chi)_I \to \text{light}} \, v_{\rm rel} = \left(\frac{1}{4}\sum_{s_I}\right) 2\,\mathrm{Im}\,\langle(\chi\chi)_I|\,\delta\mathcal{L}_{\rm ann}\,|(\chi\chi)_I\rangle$$

Non-relativistic effective theory approach

- Short-distance part: $\delta\mathcal{L}_{\rm ann} = \sum_{L,s} \Gamma[^{2s+1}L_{\mathcal{J}}]_{IJ}\, \mathcal{O}_{JI}(^{2s+1}L_{\mathcal{J}})$

$\Gamma[^{2s+1}L_{\mathcal{J}}]_{IJ}$: imaginary part of Wilson Coefficients, determined analytically in the **general MSSM**, also off-diagonal and P- and $\mathcal{O}(v^2)$ S-wave contributions [Beneke, Hellman, PRF (2012); Hellman, PRF (2013)]





EFT expansion still a good approximation up to $v_{\rm rel} \sim 0.6$ (accuracy $\sim$ few %)

$\longrightarrow$ can be used for calculations in the early universe ($v_{\rm rel} \sim 0.4 - 0.6$ at χ^0 decoupling)

Non-relativistic effective theory approach

- Long-range effects encoded in matrix-elements of four-fermion operators

[Beneke, Hellman, PRF (2014)]

e.g. $\langle (\chi\chi)_I |\, \mathcal{O}_{JJ'}(^{2s+1}S_s)\, |(\chi\chi)_I\rangle \propto \psi_J^{I\,*}(0)\, \psi_{J'}^I(0)$

$\psi_J^{(I)}$: scattering wave-function of initial state I, obtained by solving a multi-state Schrödinger equation

$$\left(-\frac{\vec{\partial}^2}{m_\chi}\, \delta_{JJ'} + V_{JJ'}(r) \right) \psi_{J'}^{(I)}(\vec{r}) = m_\chi v^2\, \psi_J^{(I)}(\vec{r})$$

$V_{JJ'}$: calculated at leading order from potential $W^\pm$, Z, γ and h^0, H^0, A, $H^\pm$ exchange [$\to$ analytic expression in arXiv:1411.6924]

Sommerfeld enhancement factors for the $(\chi\chi)_I$ annihilation rates

$$S_I[^{2s+1}S_s] = \frac{\vec{\psi}^{(I)*}(0) \cdot \Gamma[^{2s+1}S_s] \cdot \vec{\psi}^{(I)}(0)}{\Gamma[^{2s+1}S_s]_{II}}$$

[Slatyer (2010)]

- one SF for each channel $[I = (\chi_i^0\chi_j^0,\ \chi_i^+\chi_j^-),\ (\chi_i^0\chi_j^\pm),\ (\chi_i^\pm\chi_j^\pm)]$ and each partial wave!
- $S_I = S_I(v)$ in a non-trivial way!

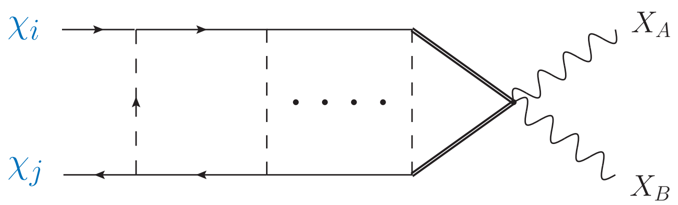
Sommerfeld enhanced cross sections

Sommerfeld enhanced $\chi^0/\chi^\pm$ co-annihilation cross sections

$$\sigma_{(\chi\chi)_I}\, v_{\rm rel} \;=\; \sum_{{}^1S_0,\,{}^3S_1} S_I\,\Gamma_{II} + \vec{p}_I^{\,2}\left(\sum_{{}^1P_1,\,{}^3P_{\mathcal{J}}} S_I\,\Gamma_{II} + \sum_{{}^1S_0,\,{}^3S_1} S_I^{(p^2)}\,\Gamma_{II}^{(p^2)}\right)$$

- All N states can be included in the Schrödinger eq. $\rightarrow$ new method!
  Only practical limitation is CPU time needed
- Active $(\chi\chi)_I$ channels: $M_I - 2m_{\rm LSP} \leq m_{\rm LSP} v_{\rm f.o}^2$, $v_{\rm f.o} \sim 1/3$
- heavier two-particle states have little effect on the SE of the lighter states but a heavy state could couple more strongly to the annihilation process than the lighter states

$\rightarrow$ Allow heavy channels to appear in the last loop before annihilation vertex



local effect if $\frac{m_{\rm LSP}\, v}{M_H - 2m_{\rm LSP}} \ll 1$

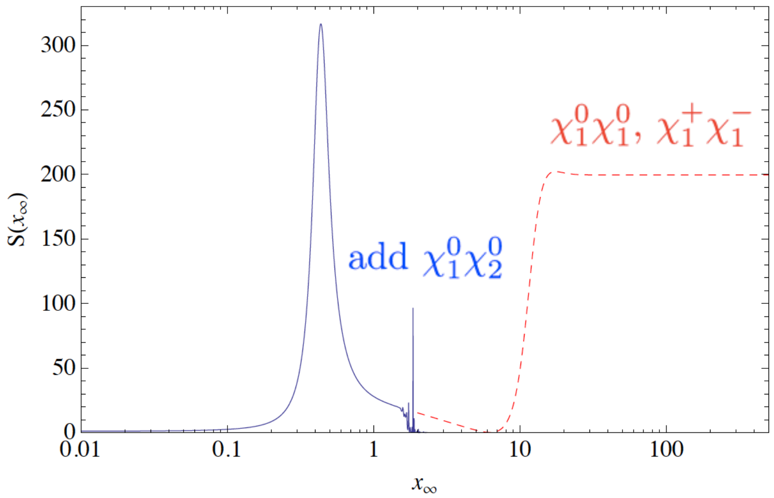
$$\Gamma[{}^{2s+1}L_{\mathcal{J}}] \rightarrow \Gamma[{}^{2s+1}L_{\mathcal{J}}] + \delta\Gamma$$

Solving the Schrödinger equation: closed channels

$$\psi_J^{(I)}(0) = \lim_{r\to\infty} [U^{-1}(r)]_{JI} \qquad U_{JI}(r) = e^{ik_J r} \left([u'_L(r)]_{JI} - ik_J [u_L(r)]_{JI}\right)$$

$[u_L(r)]_{JI}$: regular solutions of the radial Schrödinger equation, $I = 1 \ldots N$

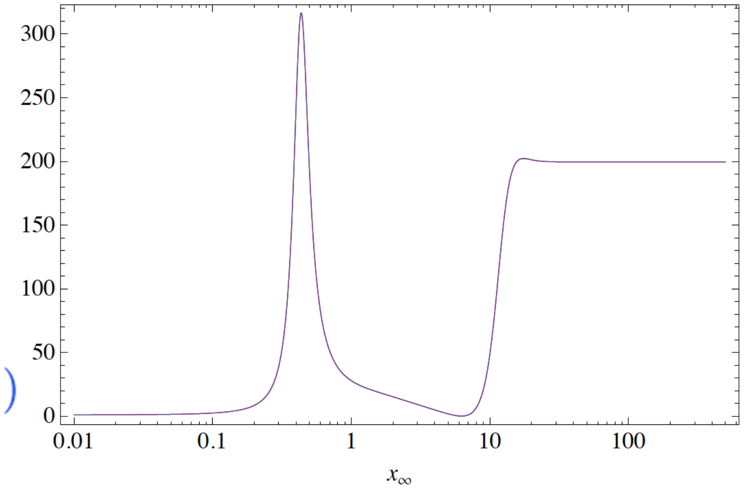
$$k_J = \sqrt{m_{\rm LSP}(E + 2m_{\rm LSP} - M_J)} \quad \to \quad \text{channel } J \text{ is closed if } M_J > E + 2m_{\rm LSP}$$



- closed channel J: $[u_L(r_\infty)_{JI}] \propto e^{|k_J| r_\infty}$
- open-channel solutions inherit the exponential growth due to off-diagonal potentials if $M_J - [2m_{\rm LSP} + m_{\rm LSP} v^2] > \frac{M_{\rm EW}^2}{m_{\rm LSP}}$
- solutions for $u_L(r)$ become linearly dependent, matrix U_{JI} gets ill-conditioned

- **improved method:** obtain inverse of U_{JI} by direct solution of a differential equation system
- variable phase method:
  $[u_L(r)]_{JI} = f_J(r)\alpha_{JI}(r) - g_J(r)\beta_{JI}(r)$

$$\frac{d\alpha_{JI}}{dr} = Z_{JK}\,\alpha_{KI} \quad , \quad [U^{-1}]_{JI}(r_\infty) \propto e^{-i\hat{k}_J r_\infty}\,\alpha^{-1}_{JI}(r_\infty)$$



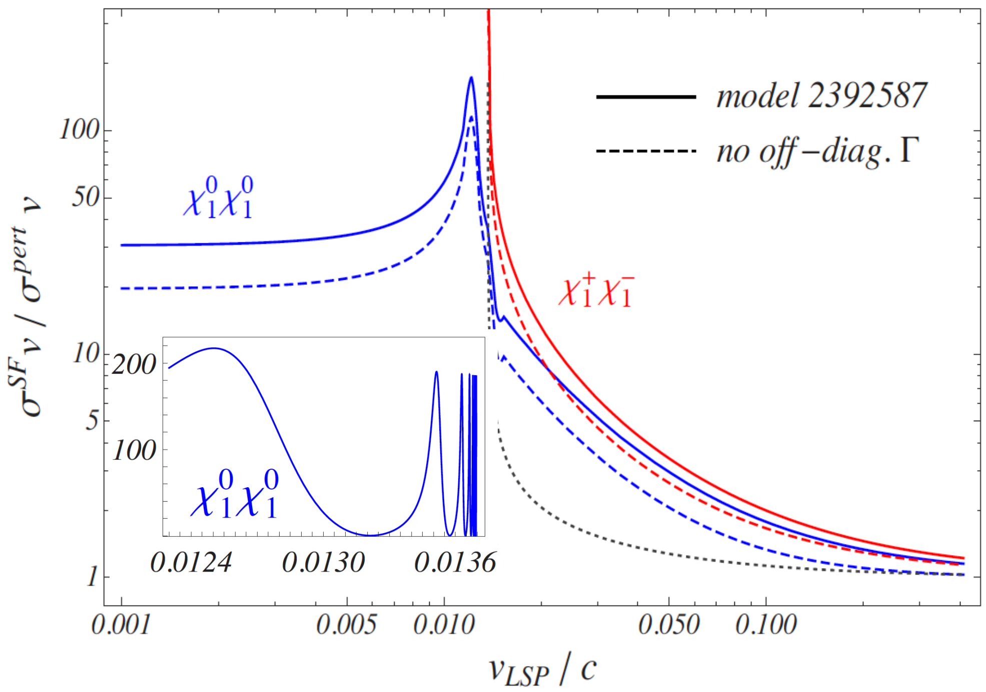
Relic abundance calculation in benchmark models

Wino-like neutralino LSP

pMSSM benchmark model:

2392587 Cahill-Rowley et al. '13 [arXiv:1305.2419]

- $m_{\chi_1^0} = 1650$ GeV
- $|Z_{N\,21}|^2 = 0.999$
- $\delta m_{\chi_1^+} = 0.155$ GeV



- (co-)annihilation sectors:

neutral	$\chi_1^0\chi_1^0,\ \chi_1^+\chi_1^-$
single charged	$\chi_1^0\chi_1^+\ (\chi_1^0\chi_1^-)$
double charged	$\chi_1^+\chi_1^+\ (\chi_1^-\chi_1^-)$

pure wino:

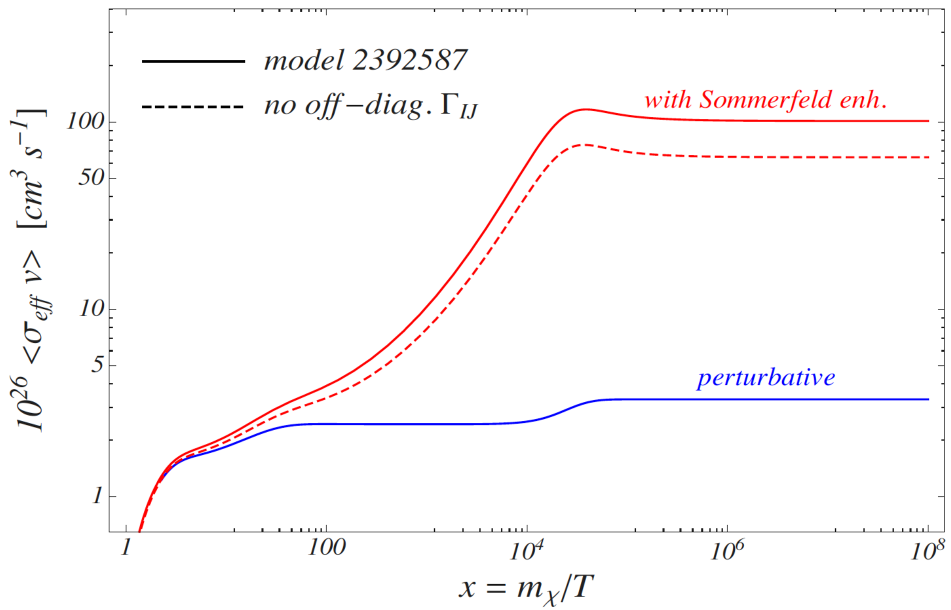
$$V_{\mathrm{even}\,L+S}(r) = \begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2\,\frac{e^{-M_W r}}{r} \\ -\sqrt{2}\,\alpha_2\,\frac{e^{-M_W r}}{r} & -\frac{\alpha}{r} - \alpha_2\,c_W^2\,\frac{e^{-M_Z r}}{r} \end{pmatrix}$$

$$V_{\mathrm{odd}\,L+S}(r) = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{\alpha}{r} - \alpha_2\,c_W^2\,\frac{e^{-M_Z r}}{r} \end{pmatrix}$$

$$\Gamma[{}^1S_0] = \frac{\pi\alpha_2^2}{m_\chi^2}\begin{pmatrix} 1 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{3}{2} \end{pmatrix} \qquad \Gamma[{}^3S_1] = \frac{25}{24}\frac{\pi\alpha_2^2}{m_\chi^2}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Gamma[{}^1P_1] = \frac{1}{6}\frac{\pi\alpha_2^2}{m_\chi^2}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad \Gamma[{}^3P_{\mathcal{J}}] = \frac{7}{3}\frac{\pi\alpha_2^2}{2m_\chi^2}\begin{pmatrix} 1 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{3}{2} \end{pmatrix}$$

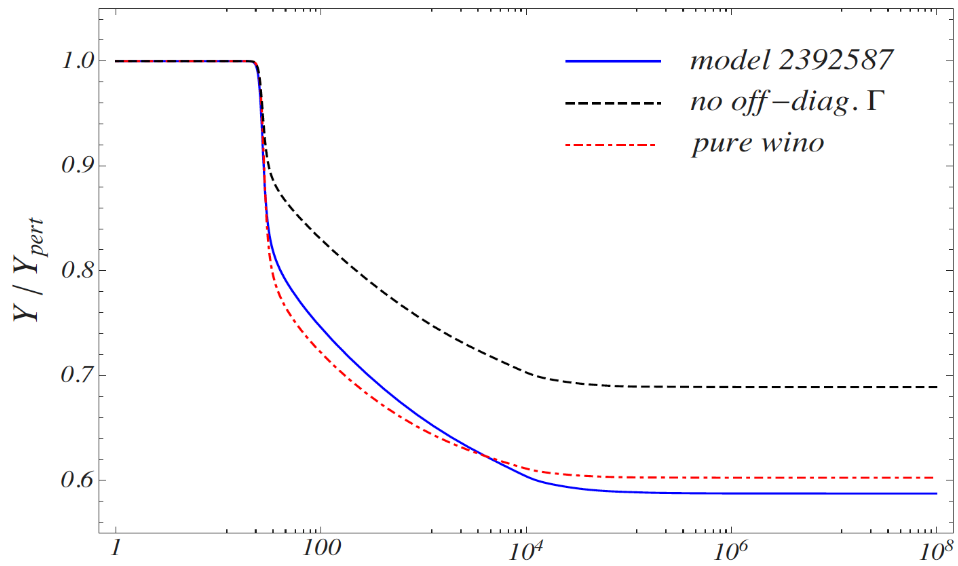
Wino-like neutralino LSP: SE and relic density



$\langle \sigma_{\text{eff}}\, v_{\text{rel}} \rangle$

- enhancement factors $\mathcal{O}(100)$
- departure from chemical eq.: $x \simeq 20$

$\rightarrow$ freeze-out delayed for $\sigma_{\text{eff}}^{\text{SF}} v_{\text{rel}}$



Yield Y and $\Omega_\chi\, h^2$:

- $\Omega_\chi^{\text{pert}} h^2 = 0.112$
- $\Omega_\chi^{\text{SF}} h^2 = 0.066 \rightarrow$ 40% reduction

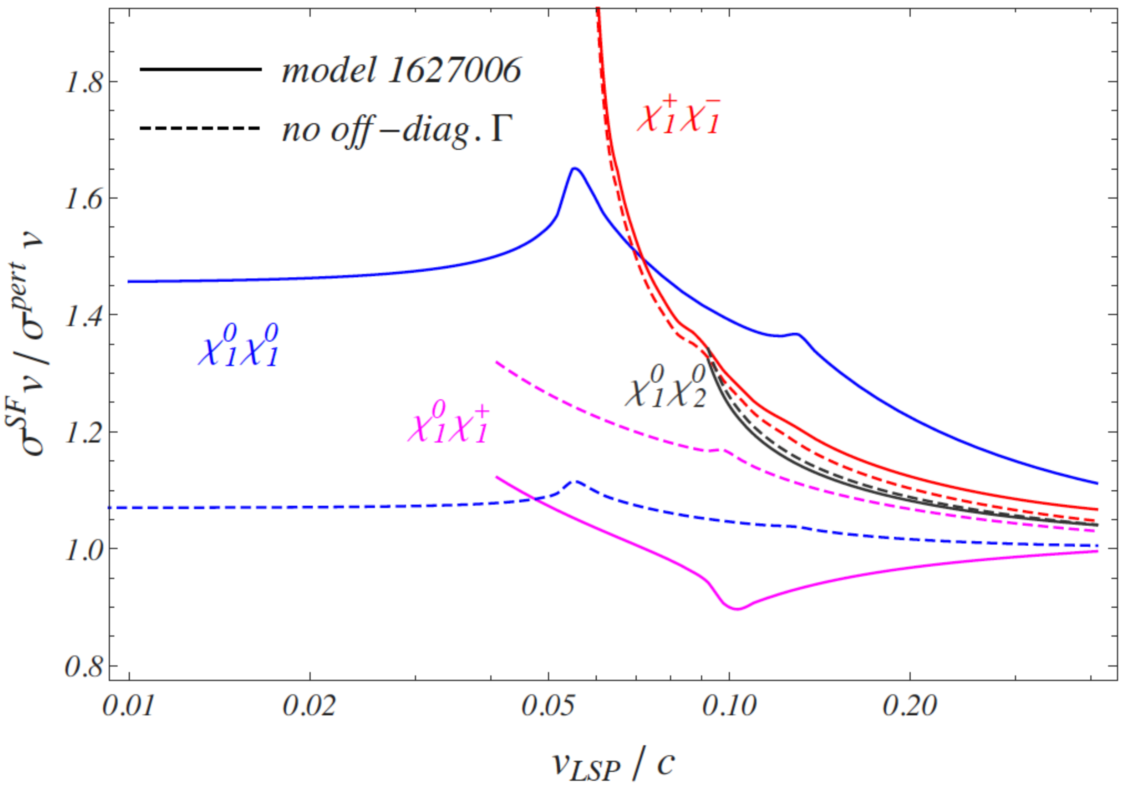
$\rightarrow$ 15% difference on $\Omega_\chi^{\text{SF}} h^2$ if the off-diagonal Γ entries are neglected

Higgsino-like neutralino LSP

- Four states $\chi^0_{1,2}$, $\chi^\pm_1$ related to two $SU(2)_L$ doublets
- (tree-level) mass splittings due to sym. breaking: $\delta m_{\chi_1^+}$, $\delta m_{\chi_2^0} = \mathcal{O}(1\text{Gev})$
- co-annihilation sectors:

neutral	$\chi_1^0\chi_1^0$, $\chi_1^0\chi_2^0$, $\chi_2^0\chi_2^0$, $\chi_1^+\chi_1^-$
single charged	$\chi_1^0\chi_1^+$, $\chi_2^0\chi_1^+$ $(\chi_1^0\chi_1^-$, $\chi_2^0\chi_1^-)$

$\chi^\pm\chi^\pm$ annihilation rates suppressed by $\mathcal{O}(M_W/m_{\chi^0})$
(Y conservation in the symmetric limit)



pMSSM benchmark model:

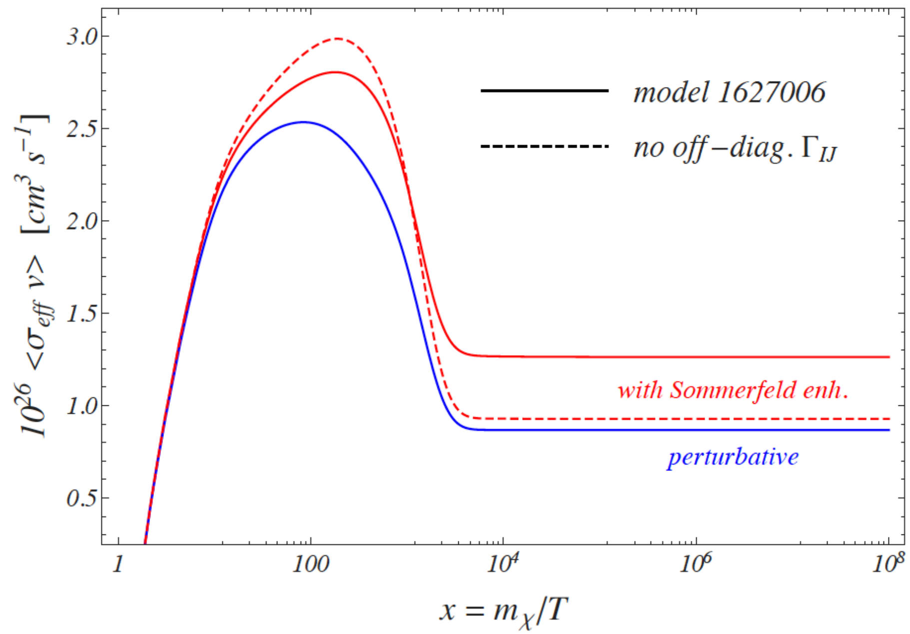
1627006
- $m_{\chi_1^0} = 1172$ GeV
- $|Z_{31}|^2 + |Z_{41}|^2 = 0.98$

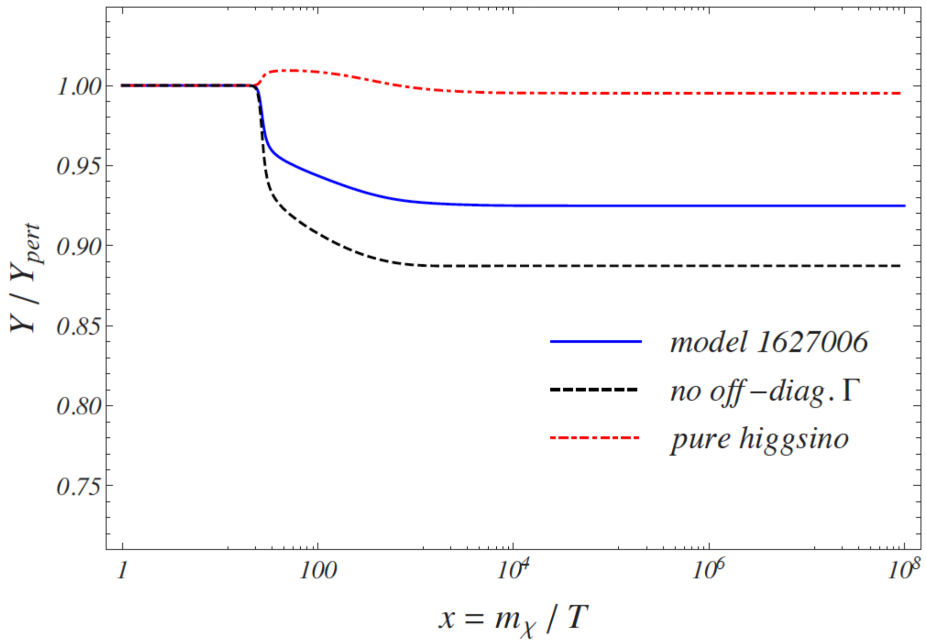
- enhancement factors $\mathcal{O}(1)$

→ due to larger $\delta m_{\chi_1^+}$, $\delta m_{\chi_2^0}$ and smaller couplings to gauge bosons

- destructive interference effect in $\chi_1^0\chi_1^+$, due to off-diagonal terms

Higgsino-like neutralino LSP: SE and relic density





$\langle \sigma_{\rm eff}\, v_{\rm rel} \rangle$

- calculation without off-diagonals misses the negative interference term in $\chi_1^0\chi_1^+$

Yield Y and $\Omega_\chi\, h^2$:

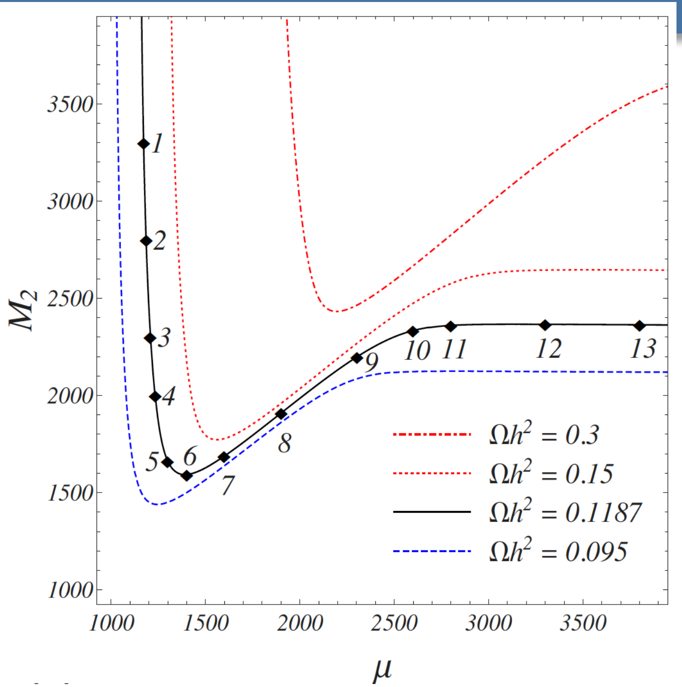
- $\Omega_\chi^{\rm pert} h^2 = 0.108$
- $\Omega_\chi^{\rm SF} h^2 = 0.100 \rightarrow$ 8% reduction

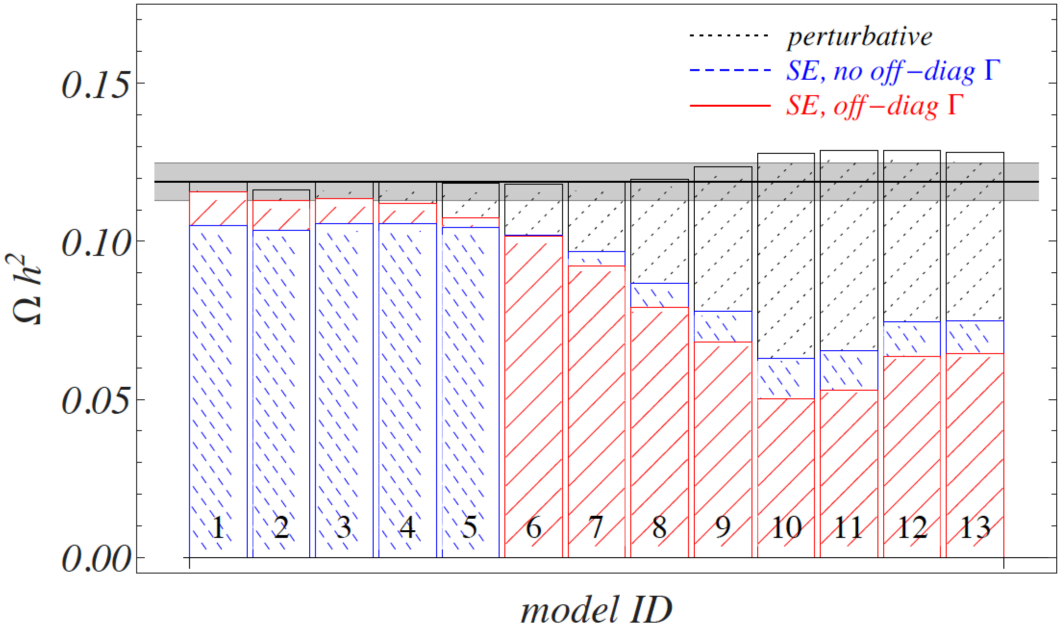
$\rightarrow$ -4% difference on $\Omega_\chi^{\rm SF} h^2$ if the off-diagonal Γ entries are neglected

- pure-higgsino: Strong cancellation between enhancement in neutral channels and suppression in charged ones

Higgsino-to-wino trajectory

- 13 models, $\Omega^{\text{DarkSUSY}} h^2 = 0.1187$
  [PLANCK+WMAP central value]
- Trajectory in $\mu - M_2$ plane:
  - Model $1-6$: higgsino-like χ_1^0
  - Model $7-9$: mixed higgsino-wino χ_1^0
  - Model $10-13$: wino-like χ_1^0





Models 1-6:

- $\Omega_\chi^{\text{SF}}/\Omega^{\text{pert}} \sim 0.97 - 0.86$
- up to 9% underestimation if no off-diag.

Models 7-9:

- $\Omega_\chi^{\text{SF}}/\Omega^{\text{pert}} \sim 0.78 - 0.55$
- 5-14% overestimation if no off-diag.

Models 10-14:

- $\Omega_\chi^{\text{SF}}/\Omega^{\text{pert}} \sim 0.39 - 0.50$
- 25-16% overestimation if no off-diag.

Bino-like neutralino LSP with wino-like enhanced co-annihilations

- Bino-like χ_1^0 is a $SU(2)_L$ singlet state (no interactions with EW bosons)

$\rightarrow$ long-range interactions suppressed, no Sommerfeld enhancements in $\chi_1^0\chi_1^0$ annihilations

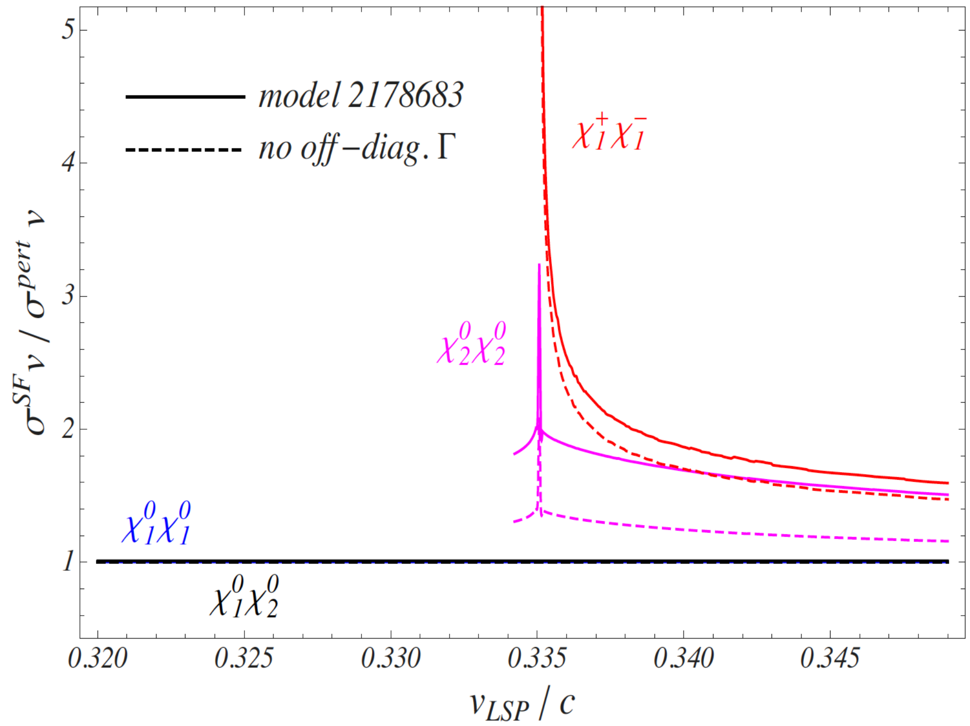
- BUT enhancements can arise in co-annihilations of heavier $\chi^0/\chi^\pm$ states

pMSSM benchmark model:

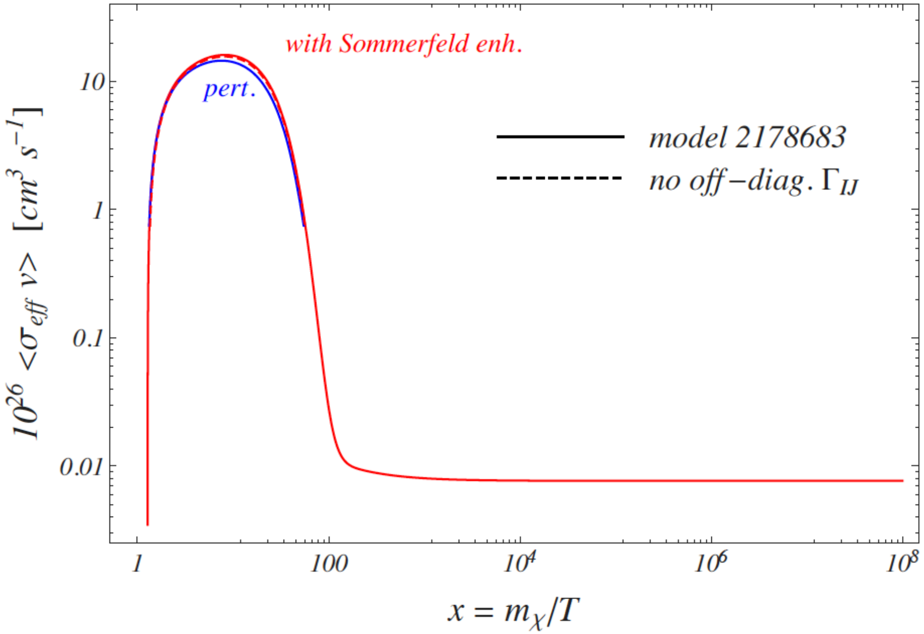
2178683 Cahill-Rowley et al. '13

- $m_{\chi_1^0} = 489$ GeV
- Wino-like $\chi_2^0, \chi_1^\pm$ at 516 GeV

[Absolute χ_1^0 rates are $\mathcal{O}(10^{-3})$ smaller than those of the wino-like states]

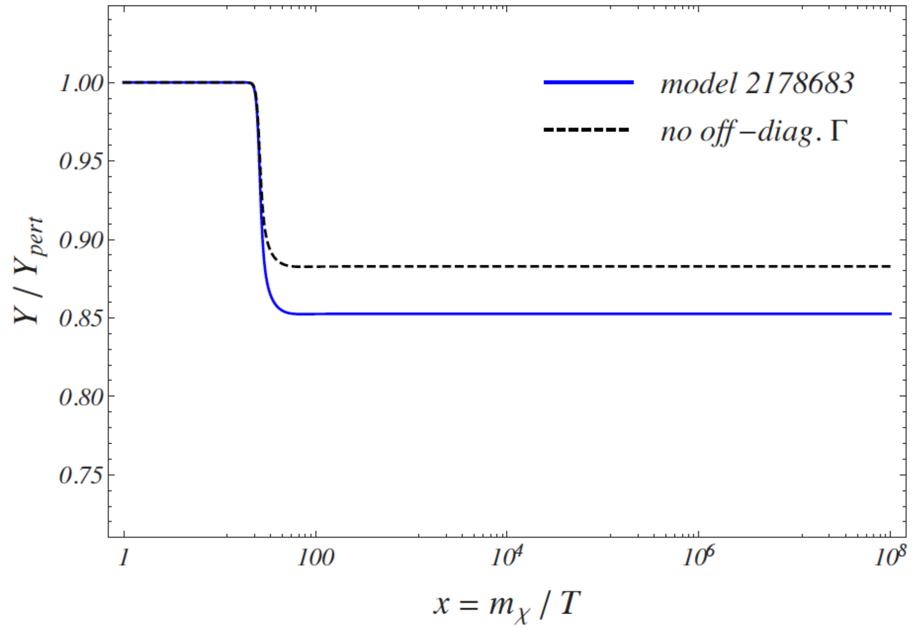


Bino-like neutralino LSP: SE and relic density



$\langle \sigma_{\rm eff}\, v_{\rm rel} \rangle$

- $x \simeq 50$: wino-like particles χ_2^0, $\chi_1^\pm$ decouple
- SF enhancement occurs in the x-range most relevant for freeze-out



Yield Y and $\Omega_\chi\, h^2$:

- $\Omega_\chi^{\rm pert} h^2 = 0.102$
- $\Omega_\chi^{\rm SF} h^2 = 0.120 \to$ 15% reduction

$\to$ 3.5% difference on $\Omega_\chi^{\rm SF} h^2$ if the off-diagonal Γ entries are neglected

Detailed investigation of the SE in the pMSSM

[Beneke, Barucha, Dighera, Hellmann, Hryczuk, Recksiegel, PRF]

Goal: scan of MSSM parameter space with neutralino LSP to identify regions where the SE is the dominant radiative correction for the relic abundance

✓ Sommerfeld effects between all nearly mass-degenerate $(\chi\chi)_I$ states with EFT framework

✓ 1-loop corrections to on-shell neutralino and chargino masses

✓ experimental constraints (ρ parameter, $g-2$, flavour...)

✓ direct detection constraints

✓ thermal corrections on fermion and gauge boson masses (small effect)

...

→ Trajectory in parameter space with correct relic abundance

- Indirect detection: strong constraints on pure-wino DM

[Cohen & (2013), Fan & Reece (2013), Hryczuk & (2014)]

→ How are these modified when we depart from the pure-wino limit?

Summary

- Given the increasing exp. accuracy of $\Omega\, h^2$, radiative corrections to $\sigma_{\rm ann}$ have to be taken into account (Sommerfeld correction can be the dominant one!)

$\rightarrow$ Also relevant in dark matter annihilation in the present universe

- Non-relativistic EFT setup for $\Omega_\chi h^2$ calculation for heavy neutralino DM

✓ Sommerfeld effects between many nearly mass-degenerate $(\chi\chi)_I$ states

✓ off-diagonal reactions $(\chi\chi)_I \rightarrow X_A X_B \rightarrow (\chi\chi)_J$ with $I \neq J$

✓ separation of $S-$ and $P-$wave annihilation rates at $\mathcal{O}(v^2)$

✓ new method to solve the Schrödinger eq. to obtain the SFs, avoids numerical instabilities due to closed channels

✓ approximate treatment of heavy channel contributions to the annihilation of lighter channels

for a generic MSSM parameter-space point

Thank you!