Cosmological constraints from ultracompact mini-halos

Ingredients & assumptions

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based on:

TB, Scott & Akrami, PRD'12 [1110.2484]

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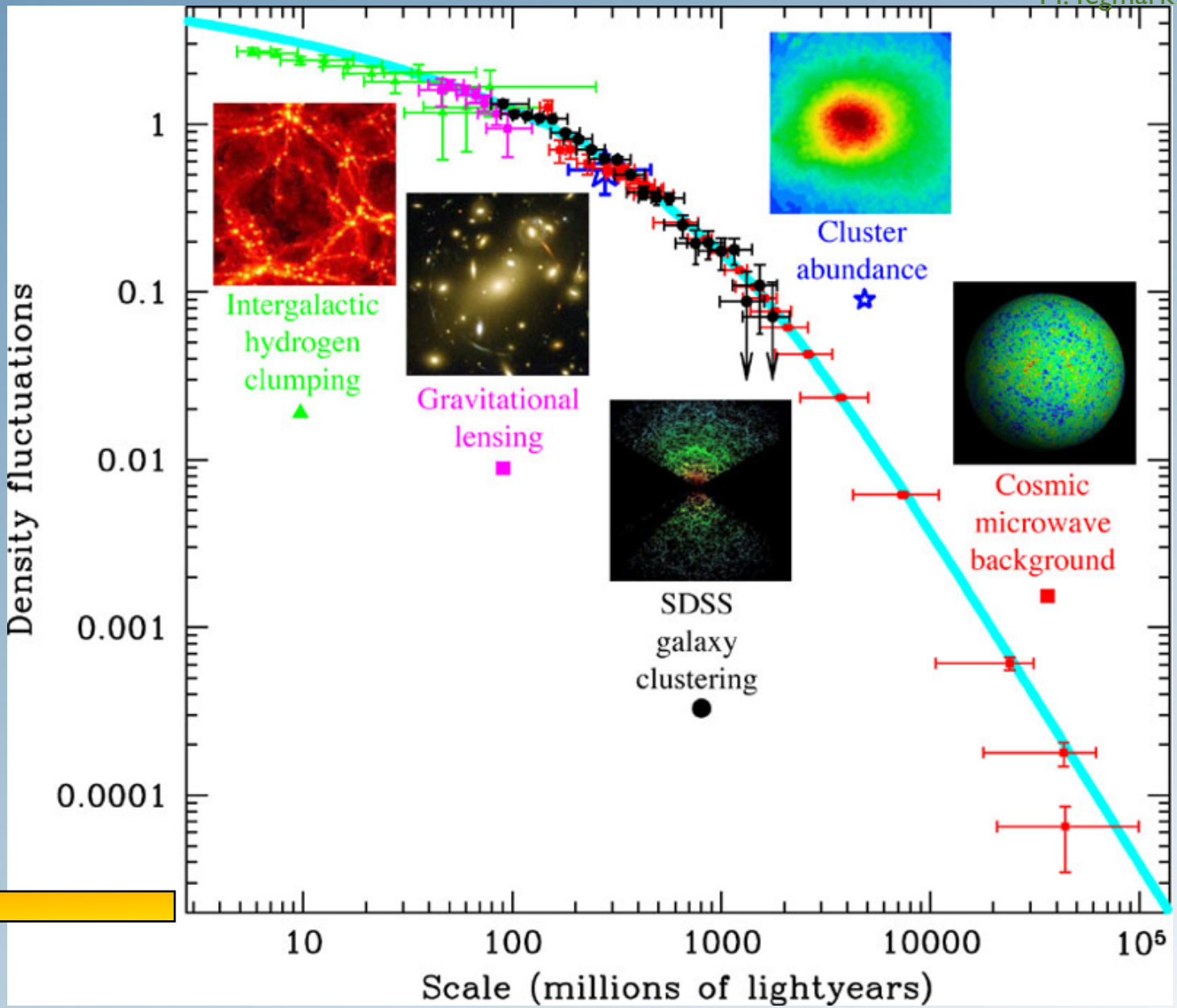
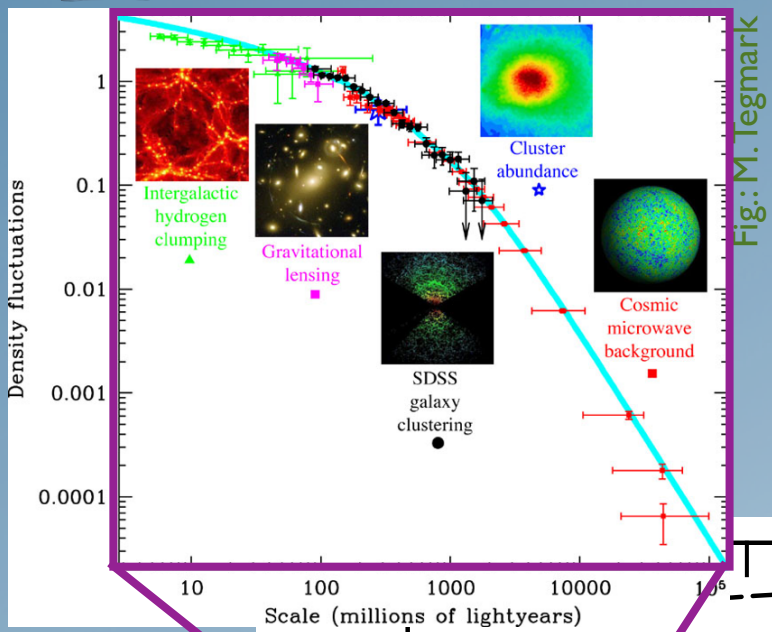
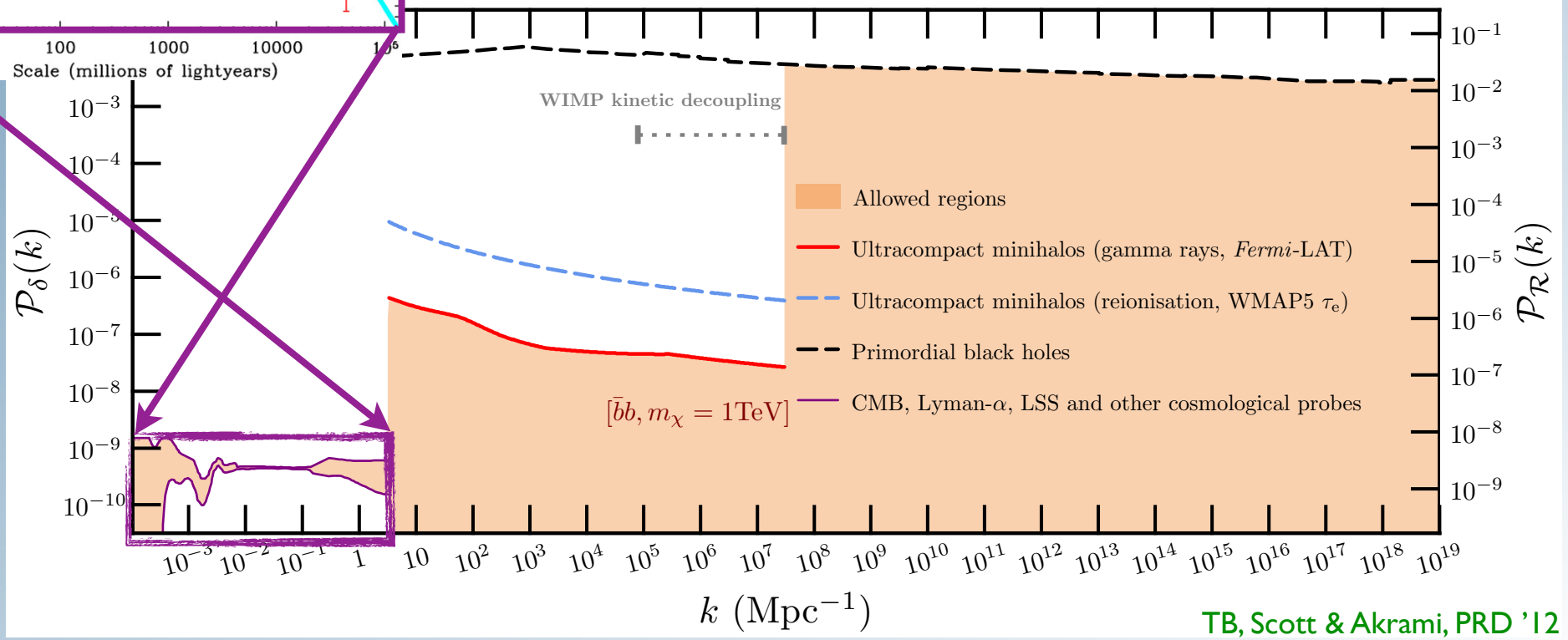
Motivation


M. Tegmark
Density fluctuations
1
0.1
0.01
0.001
0.0001
10
100
1000
10000
10^5
Scale (millions of lightyears)
Intergalactic hydrogen clumping
Gravitational lensing
Cluster abundance
SDSS galaxy clustering
Cosmic microwave background

New constraints on $\mathcal{P}(k)$:

- Primordial (linear) power spectrum well measured at 'large' scales
- Below ~Mpc scales, only upper bounds available...





TB, Scott & Akrami, PRD '12

UCMHs

- Ultracompact Minihalos are DM halos that form shortly after matter-radiation equality Ricotti & Gould, ApJ '09
  - isolated collapse
  - formation by radial infall (Bertschinger, ApJS '95)

$$\longrightarrow \quad \rho \propto r^{-9/4}$$

- Excellent targets for indirect detection with gamma rays

Scott & Sivertsson, PRL '09
Lacki & Beacom, ApJ '10

- Required density contrast at horizon entry:

$$\delta \equiv \frac{\Delta\rho}{\rho} \sim 10^{-3} \quad @ \quad z \gg z_{\rm eq}$$

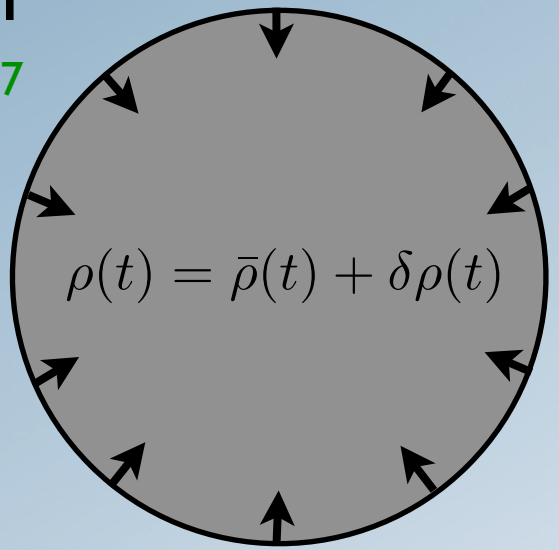
  - PBH: $\delta \gtrsim 0.3$
  - typical observed value: $\delta \sim 10^{-5}$ at 'large' scales

1st step: necessary density contrast

Spherical collapse

- Simplified, but widely used description of gravitational collapse

Peebles, ApJ '67



- Full (non-linear) evolution exactly solvable during matter domination
  - overdense region stops growing after finite $\frac{t_c}{2}$
  - afterwards, collapse start: $\rho(t \to t_c) \to \infty$
  - matching with linear solution gives:

$$\delta_c \equiv \frac{\delta\rho(t_c)_{\rm lin}}{\bar{\rho}(t_c)} = \frac{3}{5}\left(\frac{3\pi}{2}\right)^{2/3} \approx 1.686$$

➡ Linear theory sufficient to calculate *time* of collapse!

Required density contrast

- Growth of linear dark matter density perturbations:
  - logarithmically during radiation domination
  - linearly with scale factor during matter domination

- Exact solution for $z < z_{\rm eq}$ in terms of primordial curvature perturbation: [e.g. Weinberg '08]

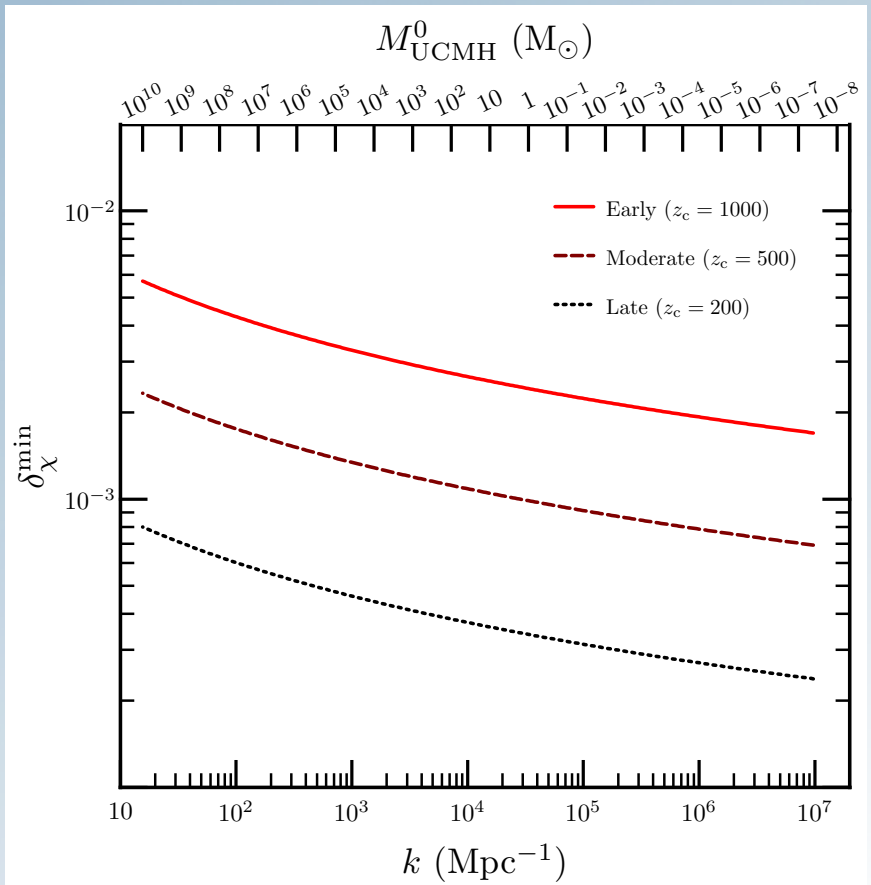
$$\delta_\chi(k,t) = \frac{9k^2t^2}{10a^2}\mathcal{T}\left(\frac{\sqrt{2}k}{a_{\rm eq}H_{\rm eq}}\right)\mathcal{R}^0(k)$$

$\propto a(t)$

from fit to numerical calculation

analytical relation to $\delta_\chi(t,t_k)$ during radiation domination

- Equate with $\delta_c = 1.69$ and solve for $\delta_\chi^{\rm min}$ *at horizon crossing*



2nd step: growth by accretion and abundance

Radial infall

- Tiny initial velocities and formation in isolation lead to almost pure radial infall, resulting in an extremely steep density profile:

Fillmore & Goldreich, ApJ '84

Bertschinger, ApJ '85

$$\rho_\chi(r,z) = \frac{3\Omega_\chi}{16\pi\Omega_m} \frac{M_{\rm UCMH}(z)}{R_{\rm UCMH}(z)^{\frac{3}{4}}\, r^{\frac{9}{4}}}$$

Vogelsberger *et al.*, MNRAS '09

$$\left[R_{\rm UCMH}(z) = 0.019 \left(\frac{1000}{z+1}\right)\left(\frac{M_{\rm UCMH}(z)}{M_\odot}\right)^{\frac{1}{3}} {\rm pc} \quad \text{defined by} \quad \delta_R = 2\right]$$

- Breakdown of radial infall due to angular momentum conservation (non-zero velocity dispersion!):

[Obtained by equating mean tangential velocity of infalling gas with Keplerian velocity]

$$\frac{r_{\rm min}}{R^0_{\rm UCMH}} \approx 2.9\times10^{-7}\left(\frac{1000}{z_c+1}\right)^{2.43}\left(\frac{M^0_{\rm UCMH}}{M_\odot}\right)^{-0.06}$$

⇨ conservatively assume that $\rho_\chi(r \le r_{\rm min}) = const.$

Properties (2)

- If dark matter consists of WIMPs, self-annihilations also deplete the central regions:
  Berezinsky, Gurevich & Zybin, PLB '92

$$\rho_\chi(r \leq r_{\rm cut}) \equiv \rho_{\chi,\rm max} = \frac{m_\chi}{\langle\sigma v\rangle(t - t_{\rm i})}$$

← usually slightly more important than angular momentum conservation

- A small central core as expected for UCMHs
  - is important for the DM annihilation signal, but does not change the mass of the UCMH
  - implies survival probabilities close to unity

Berezinsky, Dokuchaev & Eroshenko, PRD '06 +'08
Berezeinsky *et al.*, PRD '10

Expected abundance today

- Probability that an overdense region of size $R = 1/(aH)$ collapses into a UCMH:

$$\beta(R) = \frac{1}{\sqrt{2\pi}\sigma_{\chi,\mathrm{H}}(R)} \int_{\delta_\chi^{\min}}^{\delta_\chi^{\max}} \exp\left[-\frac{\delta_\chi^2}{2\sigma_{\chi,\mathrm{H}}^2(R)}\right] \mathrm{d}\delta_\chi$$

- During radiation domination, the UCMH 'seed' simply contains the DM mass inside the horizon (at $aH = R$)

- $10 \lesssim z \lesssim z_{\mathrm{eq}} : M_{UCMH} \propto a(t)$ via accretion

$$\Rightarrow \quad M^0_{\mathrm{UCMH}} \approx 4 \times 10^{13} \left(\frac{R}{\mathrm{Mpc}}\right)^3 M_\odot$$

- Abundance today: $\Omega_{UCMH}(M^0_{UCMH}) \sim 10^2 \beta(R)$

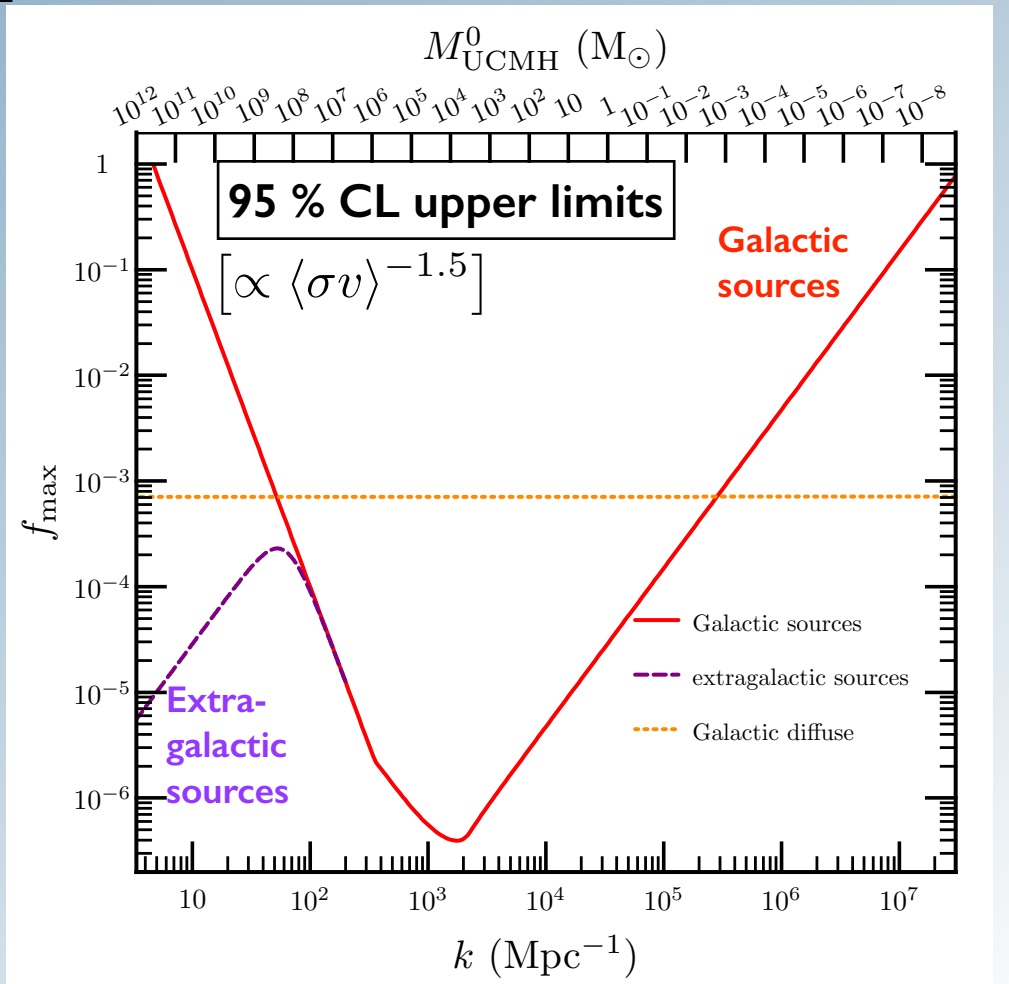
3rd step: limits on abundance

Present-day limits

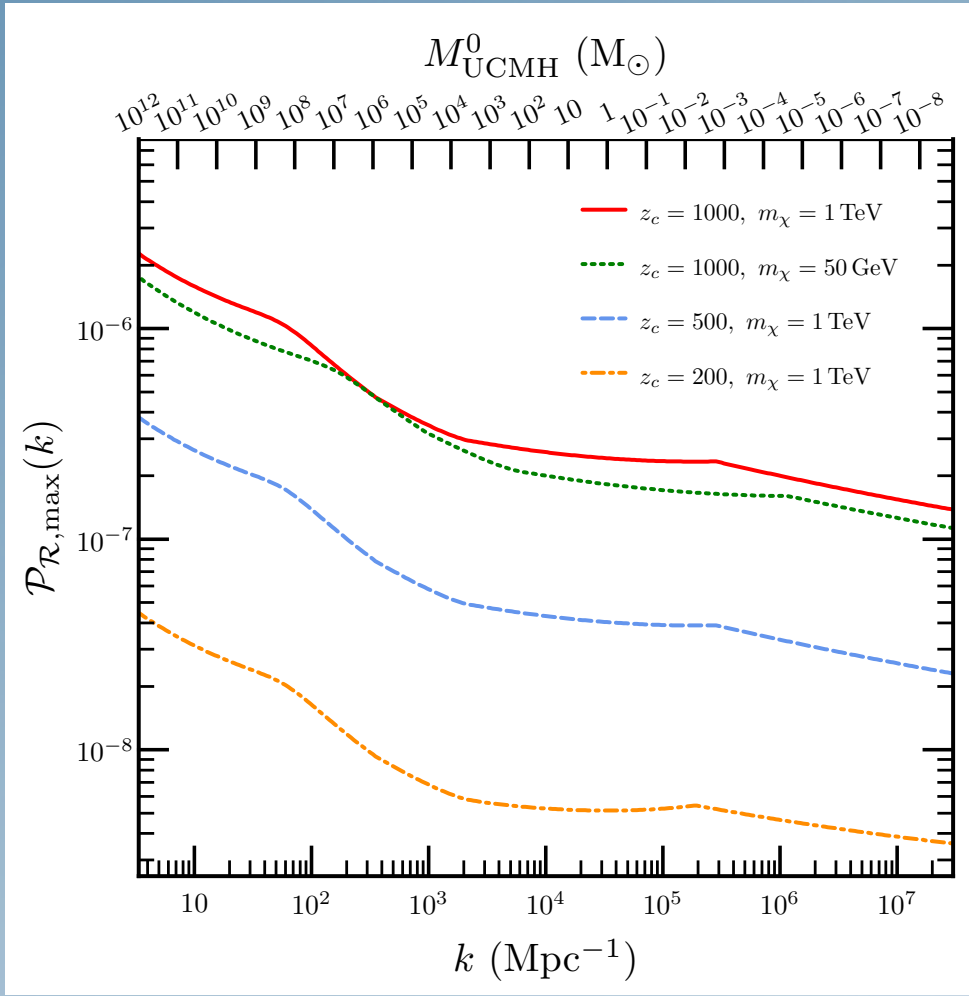
- Assume in the following that
  - Dark matter consists of WIMPs with $m_\chi = 1\,\mathrm{TeV}, \quad \langle\sigma v\rangle = 3\cdot 10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}$
  - [UCMHs collapse before $z_c = 1000$]
  - UCMHs track the bulk dark matter

➡ Non-observation of DM halos in the Fermi 1y data allows to place stringent limits on $f \equiv \frac{\Omega_{UCMH}}{\Omega_m}$

Further limit by demanding that the flux from unresolved UCMHs, perpendicular to the galactic plane, does not exceed the observed diffuse flux



Dependence on assumptions



much stronger constraints possible if $z_c < 1000$!

What about changing $f_{\rm max}$?

$$\beta(R) \simeq \frac{\sigma_{\chi,\rm H}(R)}{\sqrt{2\pi}\delta_\chi^{\rm min}} \exp\left[-\frac{\delta_\chi^{\rm min\,2}}{2\sigma^2_{\chi,\rm H}(R)}\right]$$

$$\sigma^2_{\chi,H}(R) \sim \mathcal{P}_\mathcal{R}(k_R)$$

Constraints: $\beta \propto f \overset{!}{<} f_{\rm max}$

As long as $f_{\rm max} < \mathcal{O}(0.1)$, UCMH abundance constraints only have minor impact on $\mathcal{P}_{\mathcal{R},\rm max}$!

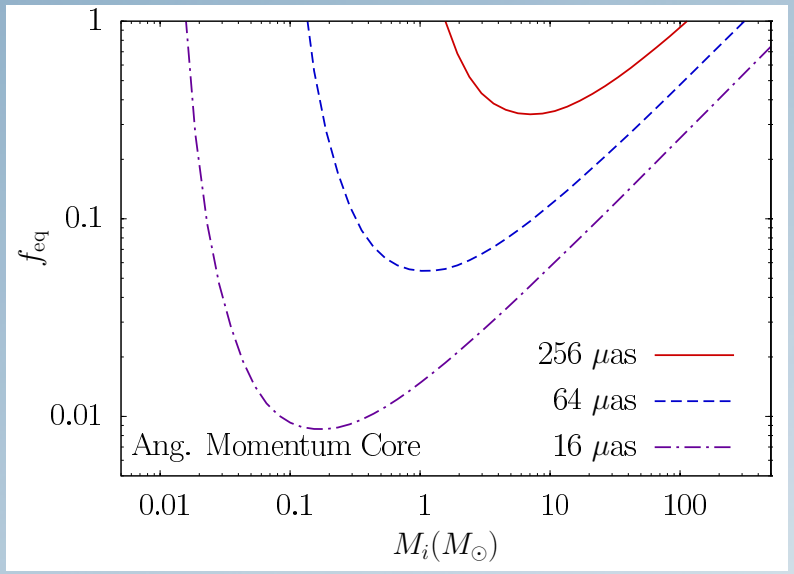
no WIMPs?

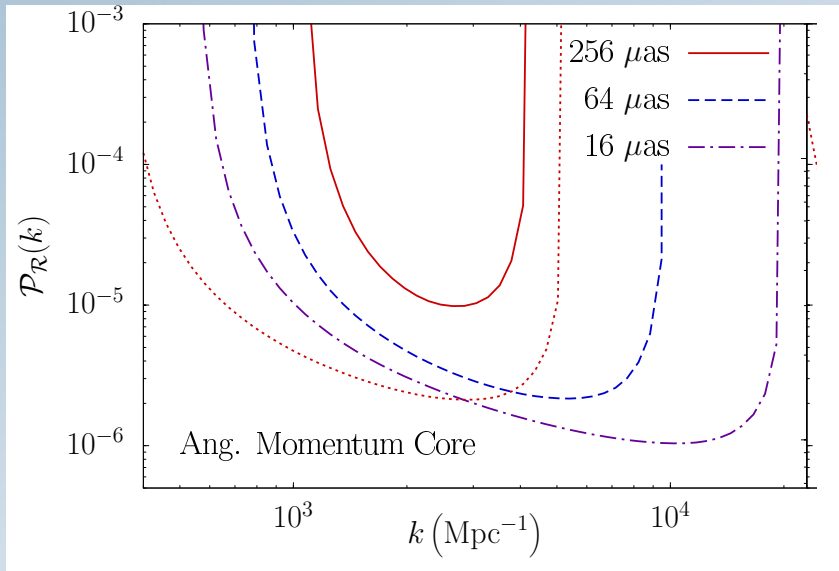
NB: this list is NOT updated: apologies…

- Gravitational lensing
  - independent of DM nature!
  - Existing EROS/OGLE bounds (on point-like MACHOs) not applicable
  - Astrometric lensing is a promising alternative: Gaia could probe $f \lesssim 0.01 - 0.1$ for ~solar mass UCMHs

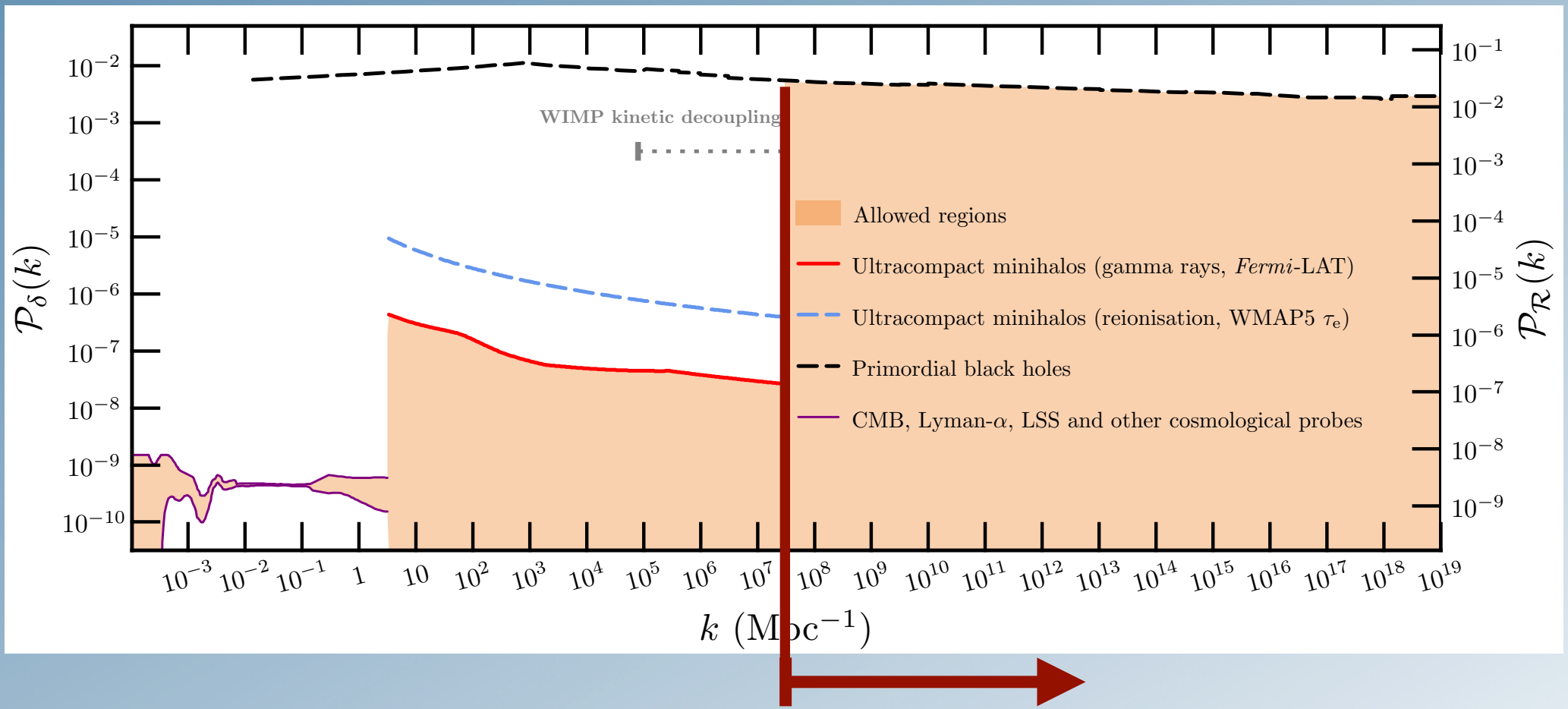
Li, Erickek & Law, PRD '12

- …





Summary



- NB: 'Obviously' *no* constraint if primordial $\mathcal{P}_\mathcal{R}$ is suppressed right after horizon entry, e.g. via
  - collisional damping (DM heat bath)
  - free streaming (WDM)

→ *model-dependent!*

BACKUP: notation and further details

Density fluctuations

- Introduce tiny primordial perturbations:

$$\rho(t) = \bar{\rho}(t)[1 + \delta(\mathbf{x})]$$

- Spectrum usually assumed to be uncorrelated and isotropic:

$$\langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}'} \rangle \equiv \frac{2\pi^2}{k^3} \mathcal{P}_\delta(k)\, \delta(\mathbf{k} - \mathbf{k}')$$

- Density contrast smoothed over some scale R :

$$\delta_R(\mathbf{x}) \equiv V^{-1} \int d^3x\, W\left(\frac{|\mathbf{x} - \mathbf{x}'|}{R}\right) \delta(\mathbf{x}')$$

window function

Gaussian Statistics

Probability to find an average density contrast δ in a spherical region of size R :

$$p_R(\delta) = \frac{1}{\sqrt{2\pi}\sigma(R)} \exp\left[-\frac{\delta^2}{2\sigma^2(R)}\right]$$

mass variance:

$$\sigma^2(R) = \left\langle |\delta_R(\mathbf{x})|^2 \right\rangle = \int_0^\infty W_{\rm TH}^2(kR)\mathcal{P}_\delta(k)\frac{dk}{k}$$

$$W_{\rm TH}(x) = 3j_1(x)/x = 3\,x^{-3}\,(\sin x - x\cos x)$$

Time evolution

- Time-independent on super-horizon scales $k \ll aH$:
  [in total matter gauge]

$$\delta^2_{\rm H}(k,t) \equiv \frac{(aH)^4}{k^4}\mathcal{P}_\delta(k) = \mathcal{P}_\mathcal{R}(k) \times \begin{cases} 16/81 & \text{radiation dom.} \\ 4/25 & \text{matter dom.} \end{cases}$$

- Time-evolution on sub-horizon scales $k \gtrsim aH$ encoded in transfer function:

$$T^2(k,t) \equiv \delta^2_{\rm H}(k,t)/\delta^2_{\rm H}(k,t_i)$$

➡ Mass variance in DM fluctuations only $(k \gg k_{\rm eq})$:

$$\sigma^2_{\chi,{\rm H}}(R) = \delta^2_{\rm H}(t_{k_R}) \times \frac{9}{16}\int_0^\infty dx\, x^3 W^2_{\rm TH}(x) \frac{\mathcal{P}_\mathcal{R}(xk)}{\mathcal{P}_\mathcal{R}(k)} \frac{T^2_\chi\left(\theta = x/\sqrt{3}\right)}{T^2_{\rm r}\left(\theta = 1/\sqrt{3}\right)}$$

at horizon entry

Normalization depends on scale and assumed spectrum!

[see also TB, Kiefer & Polarski, PRD'02]

Example spectra

- Scale-free spectrum:

$$\delta^2_{\rm H}(t_k) = \left(\frac{10}{9}\right)^2 \delta^2_{\rm H}(t_{k_{\rm WMAP}}) \left(\frac{k}{k_{\rm WMAP}}\right)^{n-1} \qquad k_{\rm WMAP} = 0.05\,{\rm Mpc}^{-1}$$

normalization from CMB observations

- Phenomenological step in spectrum

$$\delta^2_{\rm H,step}(t_k) = \delta^2_{\rm H}(t_k) \times \begin{cases} 1 & \text{for } k < k_s \\ p^2 & \text{for } k \geq k_s \end{cases}$$

- Generalized curvature spectrum: assume *locally*

$$\mathcal{P}_\mathcal{R}(k) = \mathcal{P}_\mathcal{R}(k_{\rm R}) \left(\frac{k}{k_{\rm R}}\right)^{n_{\rm R}(k_{\rm R})-1} \qquad \text{for} \quad k_{\rm R} \equiv 1/R$$

$$\Rightarrow \sigma^2_{\chi,{\rm H}}(R) = 0.91\,\mathcal{P}_\mathcal{R}(k_{\rm R}) \times \begin{cases} 0.43 & \text{for } n_R = 0 \\ 1 & \text{for } n_R = 1 \\ 3.2 & \text{for } n_R = 2 \end{cases}$$