Higgs boson masses in SUSY models

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Two SUSY models: (Next-to) Minimal Supersymmetric Standard Model

MSSM	NMSSM $\hat{=}$ MSSM + Higgs superfield singlet
3 neutral Higgs bosons 2 charged Higgs bosons 4 neutralinos	5 neutral Higgs bosons 2 charged Higgs bosons 5 neutralinos
lightest CP-even Higgs boson: upper theoretical mass bound: at tree-level: $M_h \leq M_Z$ at loop-level: $M_h < 140$ GeV	extra contributions $\sim \lambda^2$ $\Rightarrow$ already at tree-level larger masses possible + loop corrections possibly smaller as in MSSM
no CP-violation in the Higgs sector at tree-level	CP-violation already possible at tree-level: one physical phase (at tree-level)

Higgs-boson masses and mixings in SUSY models

Why predictions?:

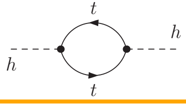
- needed for constraining the parameter space
  ($M_h^{\text{exp}} = 125.09 \pm 0.21\ (\text{stat}) \pm 0.11\ (\text{syst})\ \text{GeV}$)
- important input in calculations of
  cross sections, partial decay widths and branching ratios

How?

Determine zeroes of the determinant of the 2-point vertex function:

$$\det[p^2 - M(p^2)] = 0$$

$$M_{ij}(p^2) = M^2_{\text{tree}_i}\delta_{ij} - \hat{\Sigma}_{ij}(p^2):$$ loop-corrected mass matrix



⇒ loop-corrected Higgs masses

Higgs-boson self energies

Renormalized Higgs-boson self energy $\hat{\Sigma}_{ij}$:

$$\underbrace{\Sigma_{ij}(p^2)}_{\text{unrenormalized self energy}} + \left[\mathcal{R}\left[p^2 \underbrace{\delta\mathcal{Z}_H}_{\text{Z factors}} - \frac{1}{2}\left[\delta\mathcal{Z}_H \underbrace{M_{\text{tree}}}_{\text{mass matrix at tree-level}} + M_{\text{tree}}\,\delta\mathcal{Z}_H\right]\right]\mathcal{R}^\dagger\right]_{ij} - \underbrace{(\mathcal{R}\delta M\mathcal{R}^\dagger)_{ij}}_{\text{mass matrix counterterm}}$$

Higgs-boson self energies in the MSSM

	real parameters				complex parameters	
	$\overline{\text{DR}}$ scheme		OS/mixed schemes		OS/mixed schemes	
	$p^2 = 0$	$p^2 \neq 0$	$p^2 = 0$	$p^2 \neq 0$	$p^2 = 0$	$p^2 \neq 0$
one-loop				✓		✓
two-loop						
$\mathcal{O}(\alpha_t\alpha_s)$	✓	✓	✓	✓	✓	
$\mathcal{O}(\alpha_t^2)$			✓		✓	
$\mathcal{O}(\alpha_b\alpha_s)$			✓			
$\mathcal{O}(\alpha_t\alpha_b, \alpha_b^2, \alpha_\tau^2)$			✓			
$\mathcal{O}(\alpha_\tau\alpha_b)$			✓			
full	✓					
1st 5 rows above		✓				
three-loop						
$O(\alpha_t\alpha_s^2)$			✓			

Higher orders: "only" logarithmic contributions known

Higgs-boson self energies in the MSSM

[Chankowski, Pokorski, Rosiek, 92; hep-ph/9303309; Dabelstein hep-ph/9409375]

[Frank, Hahn, Heinemeyer, Hollik, HR, Weiglein, hep-ph/0611326]

[Zhang, hep-ph/9808299; Degrassi, Slavich, Zwirner, hep-ph/0105096]

[Degrassi, Di Vita, Slavich, arXiv:1410.3432]

[Heinemeyer, Hollik, Weiglein, hep-ph/9812472; Degrassi, Slavich, Zwirner, hep-ph/0105096]

[Borowka, Hahn, Heinemeyer, Heinrich, Hollik, arXiv:1404.7074, arXiv:1505.03133;
Degrassi, Di Vita, Slavich, arXiv:1410.3432]

[Heinemeyer, Hollik, HR, Weiglein, arXiv:0705.0746]

[Brignole, Degrassi, Slavich, Zwirner, hep-ph/0112177]

[Hollik, Paßher, arXiv:1401.8275; arXiv:1409.1687]

[Brignole, Degrassi, Slavich, Zwirner, hep-ph/0206101;
Heinemeyer, Hollik, HR, Weiglein, hep-ph/0411114]

[Dedes, Degrassi, Slavich, hep-ph/0305127]

[Allanach, Djouadi, Kneur, Porod, Slavich, hep-ph/0406166]

[Martin, hep-ph/0211366]

[Martin, hep-ph/0405022]

[Harlander, Kant, Mihaila, Steinhauser, arXiv:0803.0672; arXiv:1005.5709]

Higgs-boson self energies in the NMSSM

	real parameters				complex parameters	
	$\overline{\text{DR}}$ scheme		OS/mixed schemes		OS/mixed schemes	
	$p^2 = 0$	$p^2 \neq 0$	$p^2 = 0$	$p^2 \neq 0$	$p^2 = 0$	$p^2 \neq 0$
one-loop		✓		✓		✓
two-loop						
$\mathcal{O}(\alpha_t\alpha_s)$	✓		✓		✓	
$\mathcal{O}(\alpha_t^2)$						
$\mathcal{O}(\alpha_b\alpha_s)$	✓					
$\mathcal{O}(\alpha_t\alpha_b, \alpha_b^2, \alpha_\tau^2)$						
$\mathcal{O}(\alpha_\tau\alpha_b)$						
all Yukawa type	✓					

[Degrassi, Slavich, arXiv:0907.4682; Staub, Porod, Herrmann, arXiv:1007.4049 (also complex para.)]

[Ender, Graf, Mühlleitner, HR, arXiv:1108.1110]

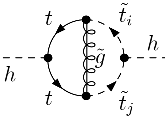
[Graf, Gröber, Mühlleitner, HR, Walz, arXiv:1206.6806]

[Degrassi, Slavich, arXiv:0907.4682]

[Mühlleitner, Nhung, HR, Walz, arXiv:1412.0918] ← in this talk

[Goodsell, Nickel, Staub, arXiv:1411.4665]

Dominant 2-loop contributions



$\mathcal{O}(\alpha_t\alpha_s)$: $(\alpha_t = \frac{y_t^2}{4\pi}$ with y_t: top Yukawa coupling)

- largest contribution $\propto m_t^2\alpha_t\alpha_s$: ($m_t$: top quark mass)

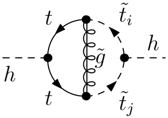
  Approximations:

  $\star$ vanishing gauge couplings and bottom Yukawa coupling:

  $e,\ M_W,\ M_Z \to 0$ but $\sin\theta_W,\ v = \frac{2M_W\sin\theta_W}{e}$ fixed and $m_b \to 0$

$$\hat{\Sigma}_{ij}(p^2) = \Sigma_{ij}(p^2) + \Big[\mathcal{R}\big[p^2\delta\mathcal{Z}_H - \frac{1}{2}(\delta\mathcal{Z}_H M_{\text{tree}} + M_{\text{tree}}\,\delta\mathcal{Z}_H)\big]\mathcal{R}^\dagger\Big]_{ij} - (\mathcal{R}\delta M\mathcal{R}^\dagger)_{ij}$$

Dominant 2-loop contributions



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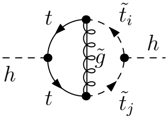
  ⋆ vanishing external momenta and charged Higgs boson mass:

  $p,\ M_{H^\pm} \to 0$

$$\hat{\Sigma}_{ij}(0) = \Sigma_{ij}(0) + \Big[\mathcal{R}\big[-\frac{1}{2}(\delta\mathcal{Z}_H M_{\text{tree}} + M_{\text{tree}}\,\delta\mathcal{Z}_H)\big]\mathcal{R}^\dagger\Big]_{ij} - (\mathcal{R}\delta M\mathcal{R}^\dagger)_{ij}$$

Dominant 2-loop contributions



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  ⋆ vanishing external momenta and charged Higgs boson mass:

  $p,\ M_{H^\pm} \to 0$

  ⋆ vanishing λ and κ coupling, $\lambda,\ \kappa \to 0$, while $\mu_{\text{eff}} = \lambda v_S/\sqrt{2}$ fixed

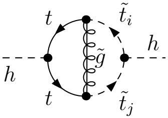
  ⇒ MSSM limit – check with MSSM result ✓ $\varphi_\mu^{\text{MSSM}} = \varphi_\lambda + \varphi_S + \varphi_u$

  [Heinemeyer, Hollik, HR, Weiglein, arXiv:0705.0746]

  $\hat{\Sigma}_{ij}(0) = \Sigma_{ij}(0) - (\mathcal{R}\delta M\mathcal{R}^\dagger)_{ij}$

Dominant 2-loop contributions

$\mathcal{O}(\alpha_t \alpha_s)$: ($\alpha_t = \frac{y_t^2}{4\pi}$ with y_t: top Yukawa coupling)



- further contributions $\propto m_t \lambda \alpha_t \alpha_s$, $\propto \lambda^2 \alpha_t \alpha_s, \ldots$:

  important for NMSSM specific behaviour

Keep:

⋆ vanishing gauge couplings and bottom Yukawa coupling

⋆ vanishing external momenta

Note: tree mass matrix: not vanishing

→ contribution of Z factors to $\hat{\Sigma}$

$$\hat{\Sigma}_{ij}(0) = \Sigma_{ij}(0) + \left[\mathcal{R}\left[-\frac{1}{2}(\delta\mathcal{Z}_H M_{\text{tree}} + M_{\text{tree}}\,\delta\mathcal{Z}_H)\right]\mathcal{R}^\dagger\right]_{ij} - (\mathcal{R}\delta M \mathcal{R}^\dagger)_{ij}$$

Parameters: Higgs/gauge sector in general

Original parameter set:

$$\underbrace{m^2_{H_d}\ m^2_{H_u}\ m^2_S\ \varphi_{A_\kappa}\ \varphi_{A_\lambda}\ |A_\lambda|}_{\text{soft SUSY-breaking parameters}}\ \underbrace{g\ g'}_{\text{gauge couplings}}\ \underbrace{v_u\ v_d\ v_s\ \varphi_s\ \varphi_u}_{\text{Higgs vacuum expectation values/phases}}\ \underbrace{|\lambda|\ \varphi_\lambda\ |\kappa|\ \varphi_\kappa}_{\text{singlet Higgs couplings}}\ \underbrace{|A_\kappa|}$$

New parameter set:

$$\underbrace{t_{h_d}\ t_{h_u}\ t_{h_s}\ t_{a_d}\ t_{a_s}}_{\text{tadpole parameters}}\ \underbrace{M^2_{H^\pm}}_{\text{charged Higgs boson mass}}\ \underbrace{M^2_W\ M^2_Z}_{\text{gauge boson masses}}\ \underbrace{e}_{\text{electric charge}}\ \underbrace{\tan\beta}_{\tan\beta = \frac{v_u}{v_d}}\ v_s\ \varphi_s\ \varphi_u\ |\lambda|\ \varphi_\lambda\ |\kappa|\ \varphi_\kappa\ |A_\kappa|$$

Parameters relevant at $\mathcal{O}(\alpha_t \alpha_s)$

Parameter set from before:

$$\underbrace{t_{h_d}\ t_{h_u}\ t_{h_s}\ t_{a_d}\ t_{a_s}}_{\text{tadpole parameters}}\ \underbrace{M^2_{H^\pm}}_{\text{charged Higgs boson mass}}\ \underbrace{M^2_W\ M^2_Z}_{\text{gauge boson masses}}\ \underbrace{e}_{\text{electric charge}}\ \underbrace{\tan\beta}_{\tan\beta = \frac{v_u}{v_d}}\ v_s\ \varphi_s\ \varphi_u\ |\lambda|\ \varphi_\lambda\ |\kappa|\ \varphi_\kappa\ |A_\kappa|$$

Relevant at $O(\alpha_t \alpha_s)$:

$$t_{h_d}\ t_{h_u}\ t_{h_s}\ t_{a_d}\ t_{a_s}\ M^2_{H^\pm}\ \underbrace{v\ \sin\theta_W}_{\substack{M_W \to 0 \\ M_Z \to 0 \\ e \to 0}}\ \tan\beta\ v_s\ \varphi_s\ \varphi_u\ |\lambda|\ \varphi_\lambda\ |\kappa|\ \varphi_\kappa\ |A_\kappa|$$

Renormalization Scheme

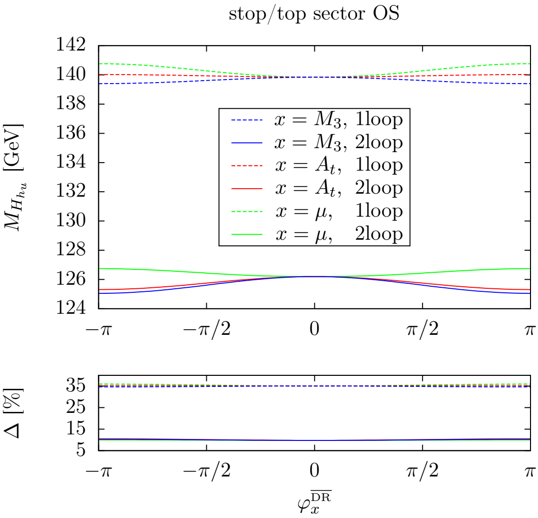
t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}	vanishing tadpole contr.
$M_{H^\pm}$	at vanishing momenta: $\hat{\Sigma}_{H^+H^-}(0) = 0$ $\Rightarrow \delta^{(2)} M^2_{H^\pm} = \Sigma_{H^+H^-}(0) - \cos^2\beta M^2_{H^\pm}\delta Z_{H_u}$
v	"on-shell" via gauge-boson self energies: $\frac{\delta^{(2)} v}{v} = \frac{\Sigma^T_{WW}(0)}{2M_W^2} + \frac{\cos^2\theta_W}{2\sin^2\theta_W}\left[\frac{\Sigma^T_{ZZ}(0)}{M_Z^2} - \frac{\Sigma^T_{WW}(0)}{M_W^2}\right]$
δZ_{H_u}	$\delta^{(2)} Z_{H_u} = -\frac{\partial\Sigma_{H_uH_u}(p^2)}{\partial p^2}\Big\vert_{\text{div.}}$: only divergent parts
$\tan\beta$	$\delta^{(2)}\tan\beta = \frac{1}{2}\tan\beta\,\delta^{(2)} Z_{H_u}$: only divergent parts
λ	$\delta^{(2)}\lambda = -\frac{\vert\lambda\vert}{2}\delta^{(2)} Z_{H_u}$: only divergent parts
φ_u, κ, $\vert A_\kappa\vert$, v_s, φ_s	0: only divergent parts

Top/Stop sector at 1-loop

	similar to [Heinemeyer, Hollik, HR, Weiglein, arXiv:0705.0746]	
top quark mass m_t	via pole mass	terms up to $\mathcal{O}(\epsilon^0)$
top squark masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$	via pole masses	"
squark mixing	$\delta Y_t = \frac{1}{2}\left[\tilde{\text{Re}}\Sigma_{12}(m^2_{\tilde{t}_1}) + \tilde{\text{Re}}\Sigma_{12}(m^2_{\tilde{t}_2})\right]$	"
bottom squark mass $m_{\tilde{b}_L}$	dependent	
top trilinear coupling A_t	dependent	

- Alternatively: only divergent parts in counterterms of the top/stop sector
- only divergent parts in any counterterm:
  check with [Degrassi, Slavich, arXiv:0907.4682] ✓

Results: Non-vanishing phases



Scenario:

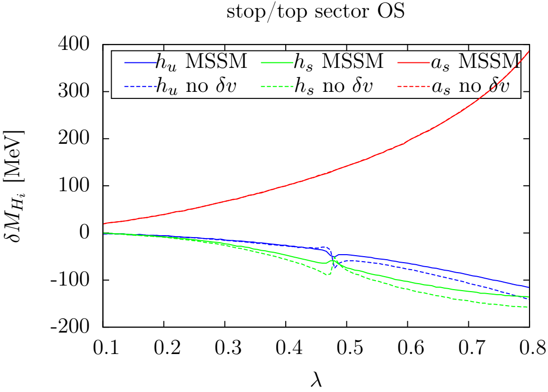
no CP-violation at tree-level

- obvious for φ_{A_t}, φ_{M_3}
- choice for $\varphi_\mu = \varphi_\lambda + \varphi_S$:
  - $\varphi_\lambda = 2\varphi_s$

$\rightarrow$ stronger dependence on phases at two-loop

$$\Delta = [M_{H_u}^{(n+1)} - M_{H_u}^{(n)}]/M_{H_u}^{(n)}$$

Results: Deviations from full result



solid:

$M_{H_i}^{\text{full}} - M_{H_i}^{\text{tree+1-loop+}\mathcal{O}(m_t^2\alpha_t\alpha_s)}$

effect of MSSM-type corrections at 2-loop

dashed:

$M_{H_i}^{\text{full}} - M_{H_i}^{\text{tree+1-loop+2-loop}|_{\delta^{(2)}v=0}}$

effect of neglection of finite contribution of $\frac{\delta^{(2)}v}{v}$

Conclusion

Higgs mass calculations in SUSY models:

⋆ an active field

⋆ many contributions in the MSSM already available

⋆ increasing number of contributions to beyond MSSM Higgs mass spectra

NMSSM Higgs boson mass spectrum: Two-loop corrections of $\mathcal{O}(\alpha_t \alpha_s)$:

- largest contribution: MSSM-like
  further contributions for NMSSM specific behaviour
- different renormalization schemes
- also for complex parameters
- is included in `NMSSMCALC` [Baglio, Gröber, Mühlleitner, Nhung, HR, Spira, Streicher, Walz, arXiv:1312.4788]

Parameters:

$|\lambda| = 0.629$, $\varphi_\lambda = 0$, $|\kappa| = 0.208$, $\varphi_\kappa = \pi$, $\tan\beta = 4.02$, $M_{H^\pm} = 788$ GeV,

$|A_\kappa| = 179.7$ GeV, $|v_s| = \sqrt{2}\frac{173.7}{|\lambda|}$ GeV, $\varphi_S = \varphi_u = 0$,

$M_{Q_3} = 1336$ GeV, $M_{t_R} = 1170$ GeV, $M_{b_R} = 1029$ GeV, $M_{\tilde{f}\neq\{\tilde{b},\tilde{t},\tilde{\tau}\}} = 3$ TeV,

$M_{L_3} = 2465$ GeV, $M_{\tau_R} = 300 - 5$ GeV

$|A_{d,s,b}| = 1539$ GeV, $|A_{e,\mu,\tau}| = 1503$ GeV, $\varphi_{A_{d,s,b}} = \varphi_{A_{e,\mu,\tau}} = \pi$,

$|A_{u,c,t}| = 1824$ GeV, $\varphi_{A_{u,c,t}} = 0$,

$M_1 = 862.3$ GeV, $M_2 = 201.5$ GeV, $M_3 = 2285$ GeV,

$\varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0$,

$m_t = 173.5$ GeV, $\mu_{\text{ren}} = \sqrt{M_{Q_3} M_{t_R}}$

Top/Stop sector at 1-loop

Properties of the "modified" on-shell scheme:

- δZ_{H_u} is purely divergent
- $\tan\beta$ and λ are $\overline{\mathrm{DR}}$ values
  $\rightarrow$ no additional conversion of parameters is needed
- conversion from "on-shell"-type to $\overline{\mathrm{DR}}$ top sector parameters possible without diagram-by-diagram conversion

Open Question:

- Experimental value for the top pole mass?
  What definition is used?

Conversions

bare masses:

$$m_t^{\overline{\mathrm{DR}}} + \delta^{(1)} m_t^{\overline{\mathrm{DR}}}|_{\frac{1}{\epsilon}} = m_t^{\mathrm{OS}} + \delta^{(1)} m_t^{\mathrm{OS}}|_{\frac{1}{\epsilon}} + \delta^{(1)} m_t^{\mathrm{OS}}|_{\epsilon^0} + \delta^{(1)} m_t^{\mathrm{OS}}|_{\epsilon^1}$$

Z-factors:

$$\delta Z_{H_u}^{\overline{\mathrm{DR}}} = \underbrace{-\frac{3(m_t^2)^{\overline{\mathrm{DR}}}}{8\pi^2 v^2 \sin^2\beta}\frac{1}{\epsilon}}_{\text{one-loop}} + \underbrace{\frac{\alpha_s (m_t^2)^{\overline{\mathrm{DR}}}}{8\pi^2 v^2 \sin^2\beta}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right)}_{\text{two-loop}},$$

$$\delta Z_{H_u}^{\mathrm{OS}} = \underbrace{-\frac{3(m_t^2)^{\mathrm{OS}}}{8\pi^2 v^2 \sin^2\beta}\frac{1}{\epsilon}}_{\text{one-loop}} + \underbrace{\frac{\alpha_s (m_t^2)^{\mathrm{OS}}}{8\pi^2 v^2 \sin^2\beta}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right) - \frac{3}{4\pi^2}\frac{m_t^{\mathrm{OS}}(\delta m_t)_{\mathrm{fin}}}{v^2 \sin^2\beta}\frac{1}{\epsilon}}_{\text{two-loop}}.$$