Hadronic contributions to electroweak observables

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in collaboration with

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earlier work with Xu Feng, Dru Renner

- **Introduction**
- **Lepton anomalous magnetic moments**
- **Conclusion**

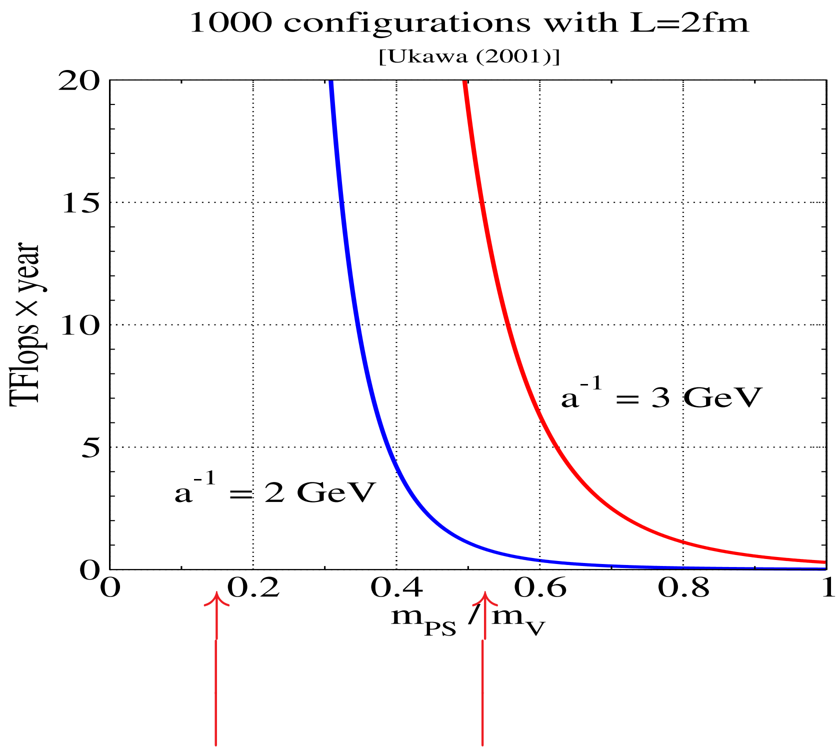
Progress in perturbation theory

Some examples

- *The relation between the QED charge renormalized in* $\overline{\mathrm{MS}}$ *and on-shell schemes at four loops, the QED on-shell beta-function at five loops and asymptotic contributions to the muon anomaly at five and six loops*
  (Baikov, Chetyrkin, Kühn, Sturm)
- *Complete Tenth-Order QED Contribution to the Muon g-2*
  (Aoyama, Hayakawa, Kinoshita, Nio)
- *Higher-order QCD corrections to supersymmetric particle production and decay at the LHC*
  (Krämer, Mühlleitner)

What about progress in lattice field theory?

see panel discussion in Lattice2001, Berlin, 2001



physical point

contact to χPT (?)

formula $C \propto \left(\frac{m_\pi}{m_\rho}\right)^{-z_\pi} (L)^{z_L} (a)^{-z_a}$

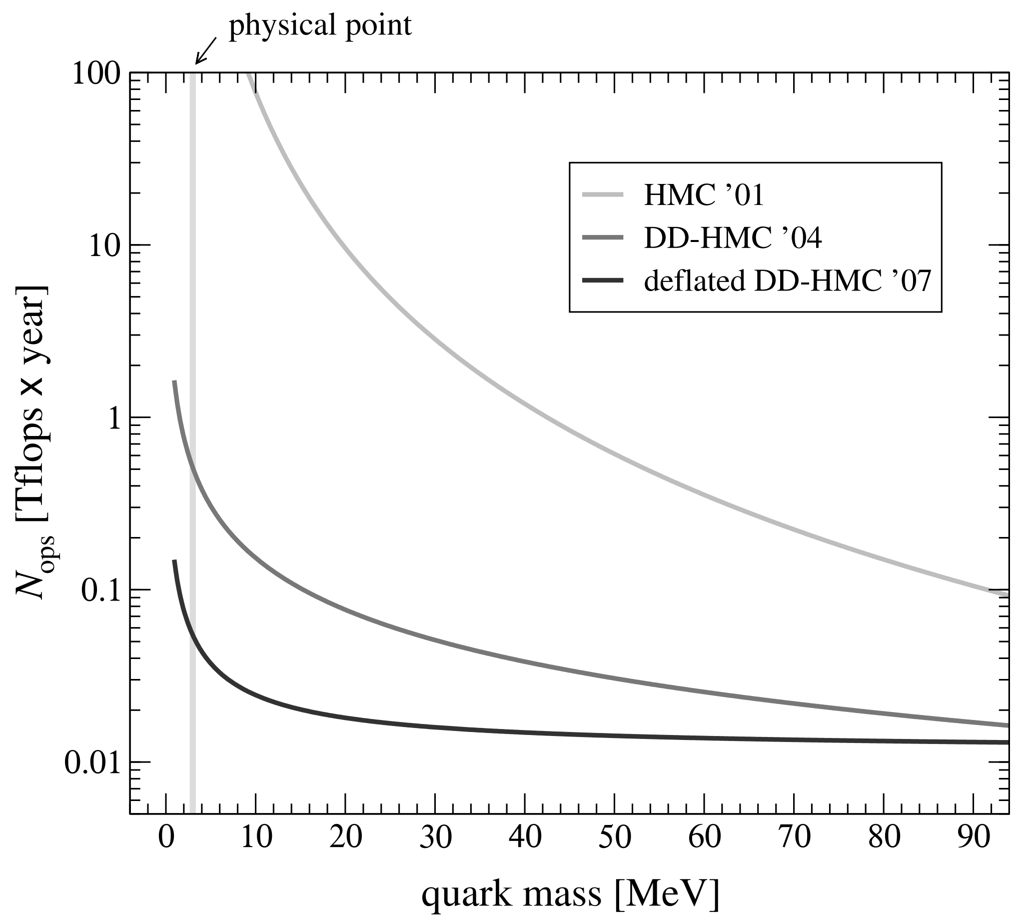
$$z_\pi = 6, \ z_L = 5, \ z_a = 7$$

"both a 10^8 increase in computing power AND spectacular algorithmic advances before a useful interaction with experiments starts taking place."
(Wilson, 1989)

$\Rightarrow$ need of **Exaflops Computers**

Recent pricture

- many algorithmic improvements by various groups



German Supercomputer Infrastructure

- JUQUEEN (IBM BG/Q)
  at Supercompter center Jülich
  5 Petaflops

- HLRN (Hannover-Berlin)
  Gottfried and Konrad
  (CRAY XC30)
  2.6 Petaflops

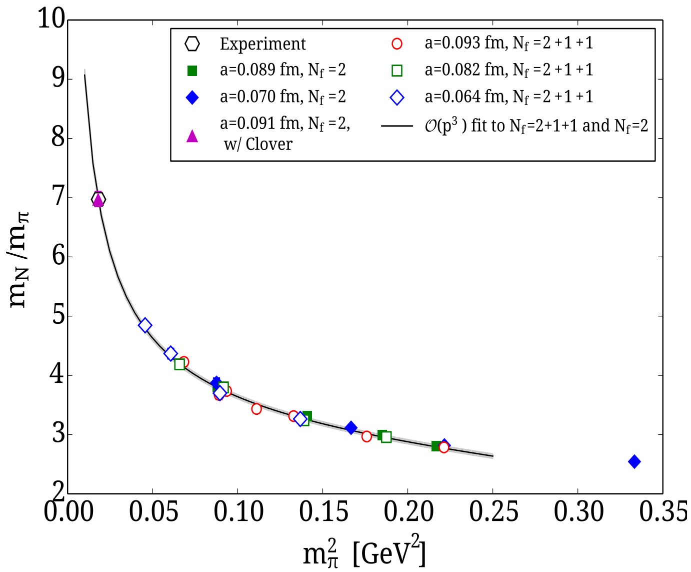
- Leibniz Supercomputer center Munich
  combined IBM/Intel system SuperMUC
  3 Petaflops

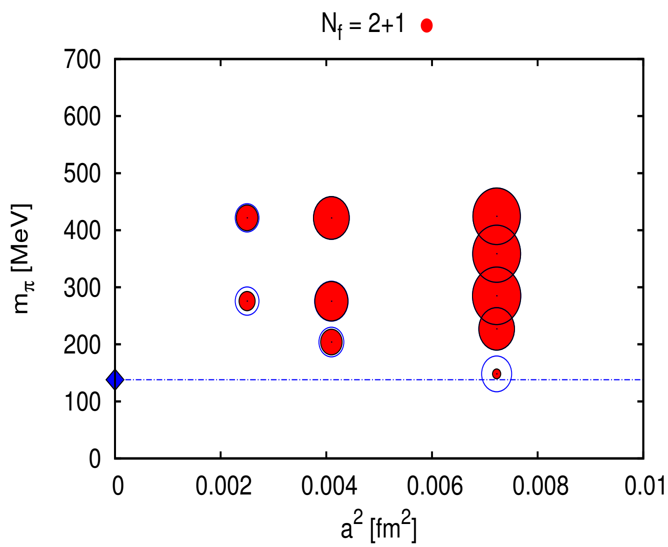
Computertime: through local calls, e.g. NIC, or Europe wide: PRACE → peer reviewed

Progress in lattice simulations

	M_{Ds}/M_K	M_{Ds}/M_D	f_K/f_π	f_{Ds}/f_D
lat.	3.96(2)	1.049(4)	1.197(6)	1.19(2)
PDG	3.988	1.0556(02)	1.197(06)	1.26(6)

reach the physical hadron masses **reach continuum limit**



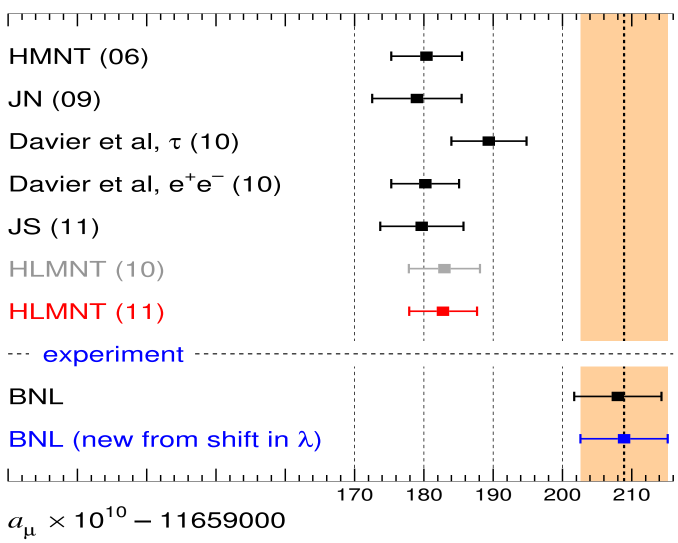


- ready to compute other quantities:
  - better understanding of QCD (e.g. hadron structure, flavour physics)
  - obtain precision results (e.g. strong coupling, quark masses, decay constants)
  - help to uncover BSM physics (e.g. B-system, lepton moments)

The muon anomalous magnetic moment

$$a_{\text{lepton}}^{\text{NP}} \propto \frac{m_{\text{lepton}}^2}{\Lambda_{\text{NP}}^2}$$

$a_\mu^{\text{exp}} = 116592089(63) \times 10^{-11}$ [0.54 ppm] [B. Lee Roberts, Chinese Phys. C 34, 2010]
$a_\mu^{\text{SM}} = 116591828(49) \times 10^{-11}$ [0.42 ppm] [Hagiwara et al., J. Phys. G38, 2011]



There is a $\approx 3\sigma$ discrepancy between a_μ^{exp} and a_μ^{SM}:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 261(80) \times 10^{-11}$$

[Hagiwara et al., J. Phys. G38, 2011]

- new proposed experiments at Fermilab and J-PARC
  $\rightarrow$ reduce experimental uncertainty by factor 4

The contributions to the muon anomalous magnetic moment

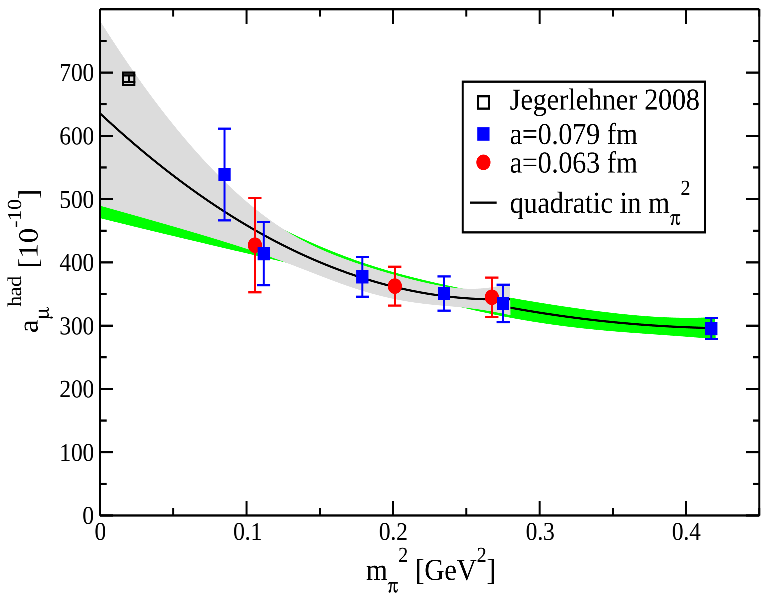
$$a_\mu^{\mathrm{QCD}} = a_\mu^{\mathrm{lo,hvp}}(\alpha^2) + a_\mu^{\mathrm{ho,hvp}}(\alpha^3) + a_\mu^{\mathrm{lbl}}(\alpha^3)$$

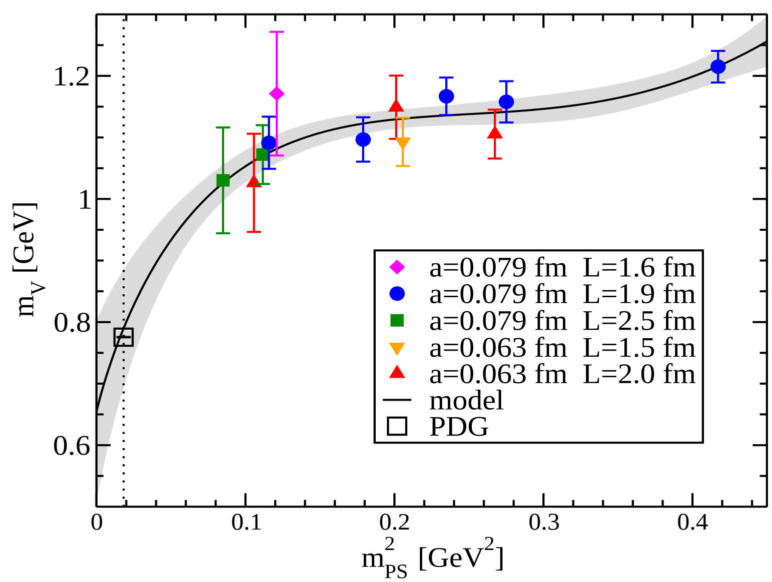
type	value $[10^{-10}]$	error $[10^{-10}]$
QCD-LO	694.9	4.3
QCD-NLO	1.8	2.6
QED/EW	11658487.3	0.2
Total	11659184.0	5.9
Experiment	11659208.9	6.3
Discrepancy	24.9	8.7

First (2011) lattice results for muon anomalous magnetic moment

$$a_\mu^{\rm hvp} = \alpha^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{m_\mu^2}\right) \Pi_{\rm R}(Q^2)$$

where $\Pi_{\rm R}(Q^2) = \Pi(Q^2) - \Pi(0)$, $w(x)$ known function





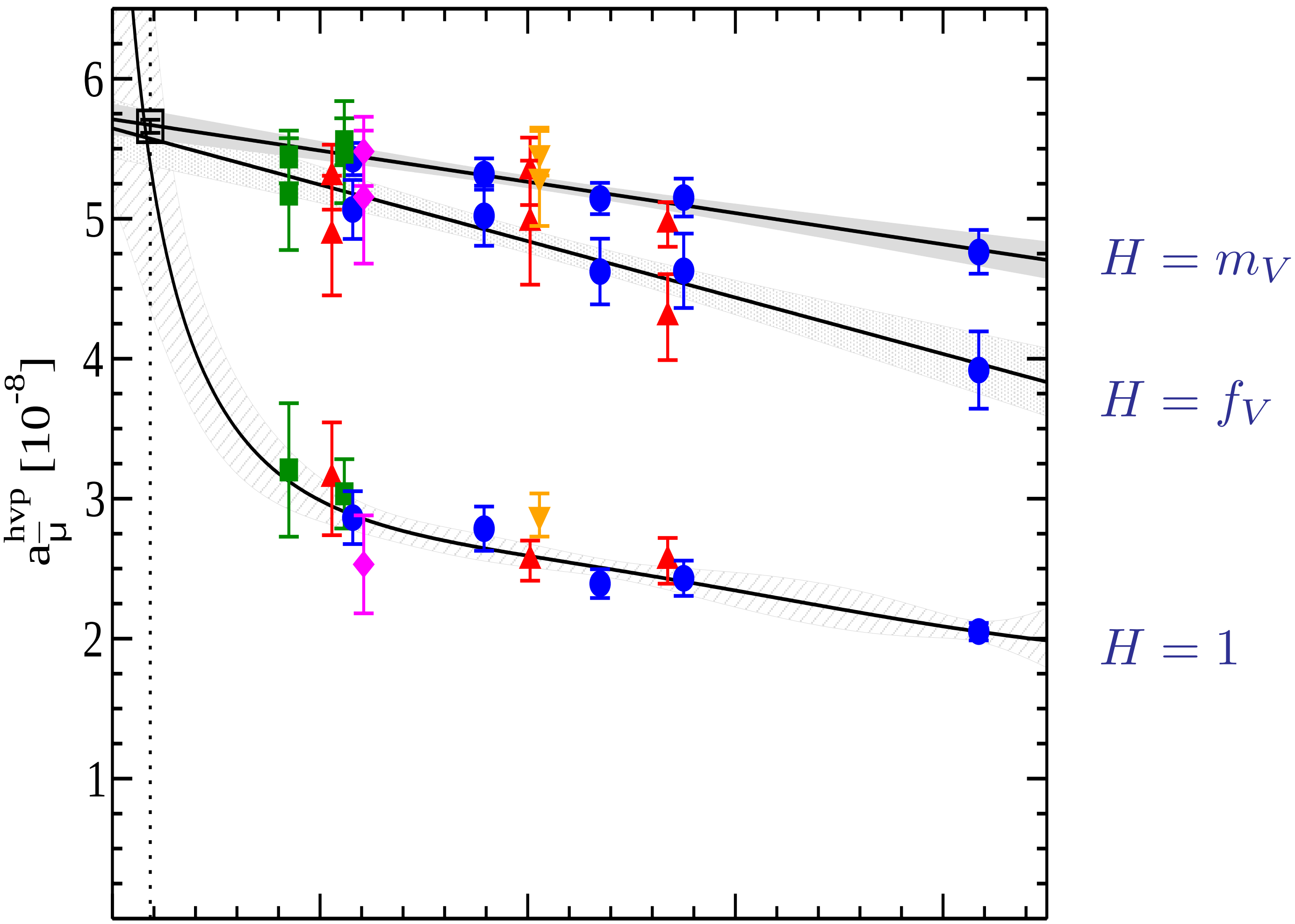
$$a_\mu^{\rm hvp} \propto \alpha^2 g_V^2 \frac{m_\mu^2}{m_V^2}$$

$$m_{\rm V}$$

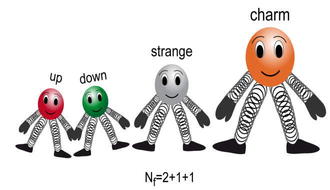
Redefinition

$$a_{\bar{\mu}}^{\rm hvp} = \alpha^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{H^2}\frac{H_{\rm phys}^2}{m_\mu^2}\right) \Pi_{\rm R}(Q^2)$$

physical $a_\mu^{\rm hvp}$ for $m_{PS} \to m_\pi$, i.e. when $H \to H_{\rm phys}$



Simulations with up, down, strange and charm quarks

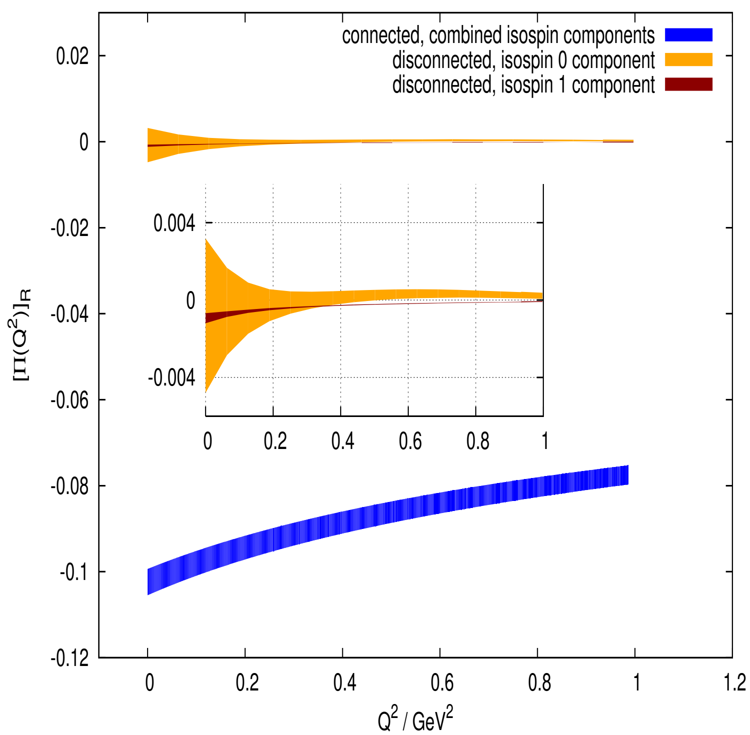


Ensemble	β	a[fm]	$L^3 \times T$	m_{PS}[MeV]	L[fm]
D15.48	2.10	0.061	$48^3 \times 96$	227	2.9
D30.48	2.10	0.061	$48^3 \times 96$	318	2.9
D45.32sc	2.10	0.061	$32^3 \times 64$	387	1.9
B25.32t	1.95	0.078	$32^3 \times 64$	274	2.5
B35.32	1.95	0.078	$32^3 \times 64$	319	2.5
B35.48	1.95	0.078	$48^3 \times 96$	314	3.7
B55.32	1.95	0.078	$32^3 \times 64$	393	2.5
B75.32	1.95	0.078	$32^3 \times 64$	456	2.5
B85.24	1.95	0.078	$24^3 \times 48$	491	1.9
A30.32	1.90	0.086	$32^3 \times 64$	283	2.8
A40.32	1.90	0.086	$32^3 \times 64$	323	2.8
A50.32	1.90	0.086	$32^3 \times 64$	361	2.8
Phys	2.10	0.094	$48^3 \times 96$	128	4.6

Disconnected (singlet) contribution

- local current correlator $m_{\mathrm{PS}} \approx 390\,\mathrm{MeV}$

- 5000 configurations with 50 stochastic volume sources $\rightarrow$ 250.000 measurments

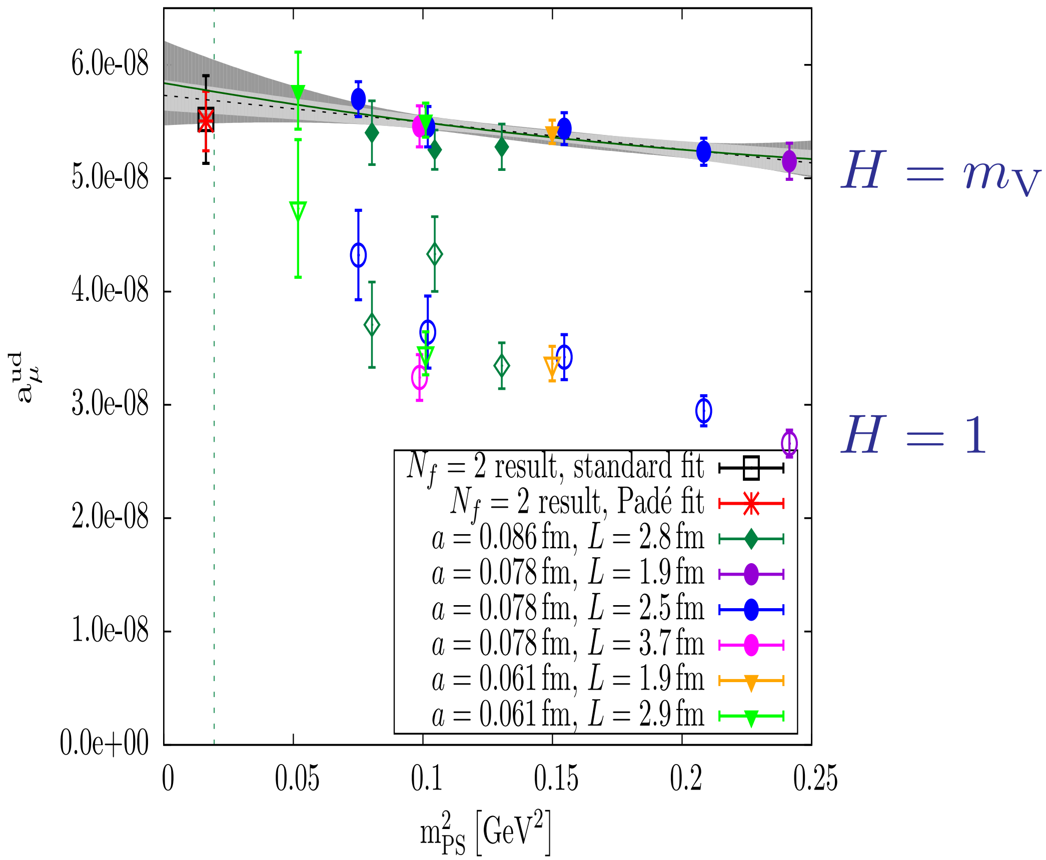


- with disconnected: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.42(14) \cdot 10^{-8}$

- without disconnected: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.26(25) \cdot 10^{-8}$

[Burger, Pientka, Petschlies, KJ, arXiv:1501.05110 , 2015]

Light quark contribution for $N_f = 2+1+1$



- $N_f = 2+1+1$ result: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.69(12) \cdot 10^{-8}$
- new $N_f = 2$ result at physical point: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.52(39) \cdot 10^{-8}$
- correlated [1,1] Padé fit gives: $a_{\mu,\mathrm{ud}}^{\mathrm{hvp}} = 5.50(26) \cdot 10^{-8}$

Including the charm quark

- charm quark contribution from perturbation theory

  [Bodenstein, Dominguez, Schilcher, Phys. Rev. D85, 2012]

$$a_\mu^{\mathrm{hvp,c}} = 144(1) \times 10^{-11}$$

- comparable to hadronic light-by-light scattering contribution

  [Prades et al., Adv. Ser. Direct. High Energy Phys. 20, 2009]

$$a_\mu^{\mathrm{lbl}} = 105(26) \times 10^{-11}$$

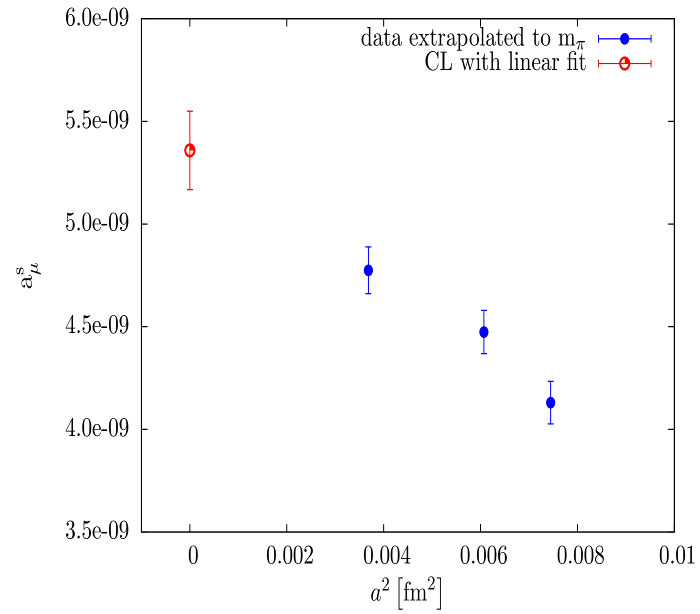
- and also to electroweak contribution

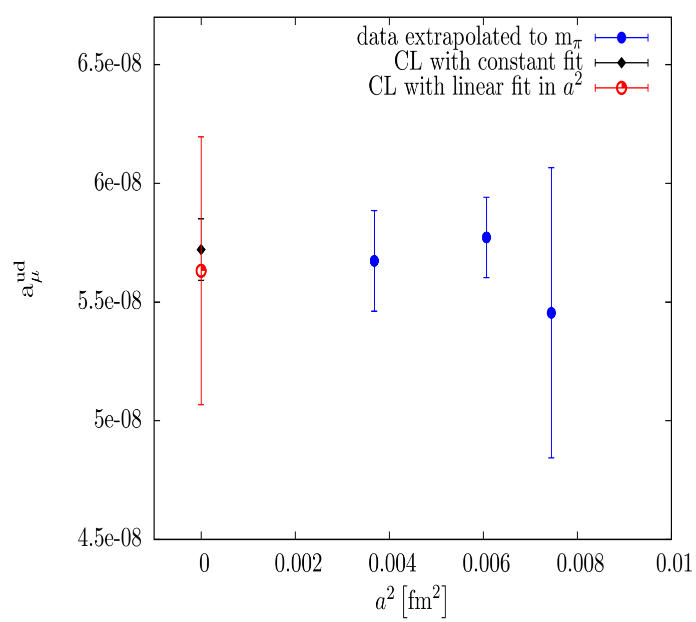
  [Jegerlehner, Nyffeler, Phys. Rept. 477 , 2009]

$$a_\mu^{\mathrm{EW}} = 153(2) \times 10^{-11}$$

- unambiguous flavour separation

The strange and charm contributions

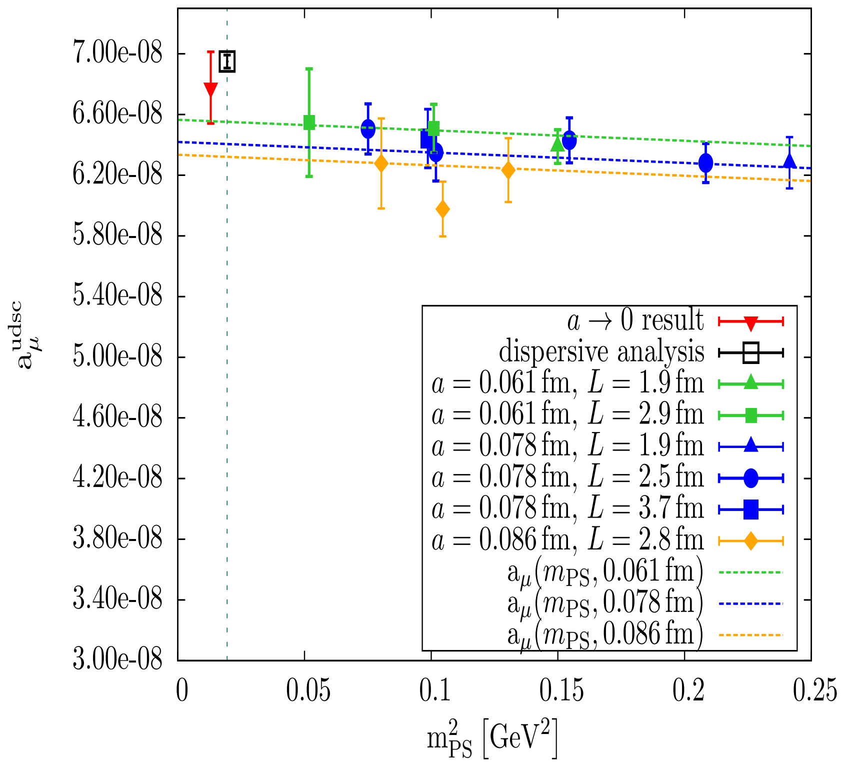




- need to take lattice artefacts into account
- discretization errors $O(a^2) \leftarrow$ analytical proof [Burger, Pientka, Petschlies, KJ, 2015]
- fit formula $a_\mu^{\rm hvp}(m_{\rm PS}, a) = A + B\ m_{\rm PS}^2 + C\ a^2$

	$a_\mu^{\rm s}$	$a_\mu^{\rm c}$
this work	$53.6(1.9) \cdot 10^{-10}$	$14.18(61) \cdot 10^{-10}$
HPQCD 2014	$53.41(59) \cdot 10^{-10}$	$14.42(39) \cdot 10^{-10}$

The total 4-flavour contribution



Fit:

$$a_\mu^{\mathrm{hvp}}(m_{\mathrm{PS}}, a) = A + B\ m_{\mathrm{PS}}^2 + C\ a^2$$

- $N_f = 2 + 1 + 1$ result: $a_\mu^{\mathrm{hvp}} = 6.78(24) \cdot 10^{-8}$

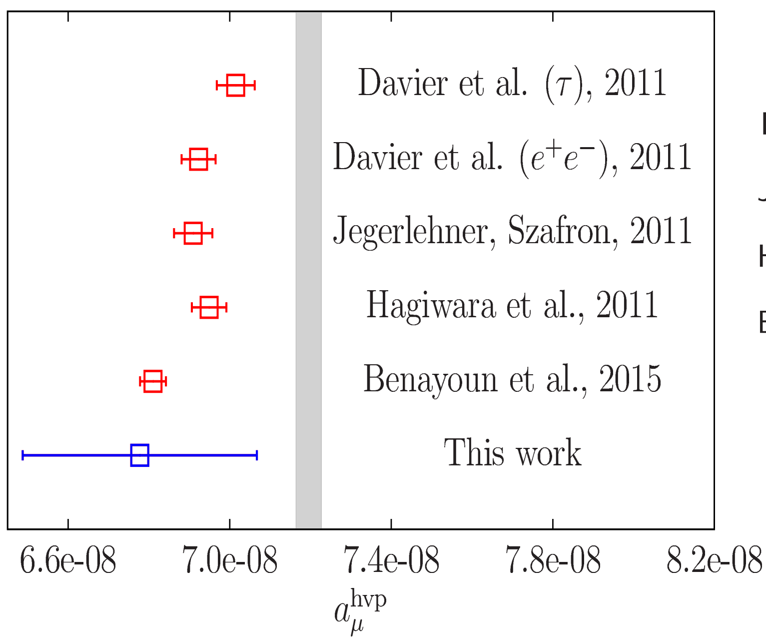
  [Burger, Feng, Pientka, Petschlies, Renner, KJ, JHEP 1402, 2013]

- result from dispersive analysis: $a_\mu^{\mathrm{hvp}} = 6.91(05) \cdot 10^{-8}$

  [Jegerlehner, Szafron, Eur. Phys. J C71, 2011]

Comparison with phenomenological analysis

- our (first) $N_f = 2+1+1$ lattice calculation of $a_\mu^{\rm hvp}$:
  $\rightarrow$ compatible results with dispersive analyses
- still larger uncertainty



Davier et al.: [Davier, Hoecker, Malaescu, Zhang, Eur. Phys. J. C71, 2011]

Jegerlehner, Szafron: [Jegerlehner, Szafron, Eur. Phys. J. C71, 2011]

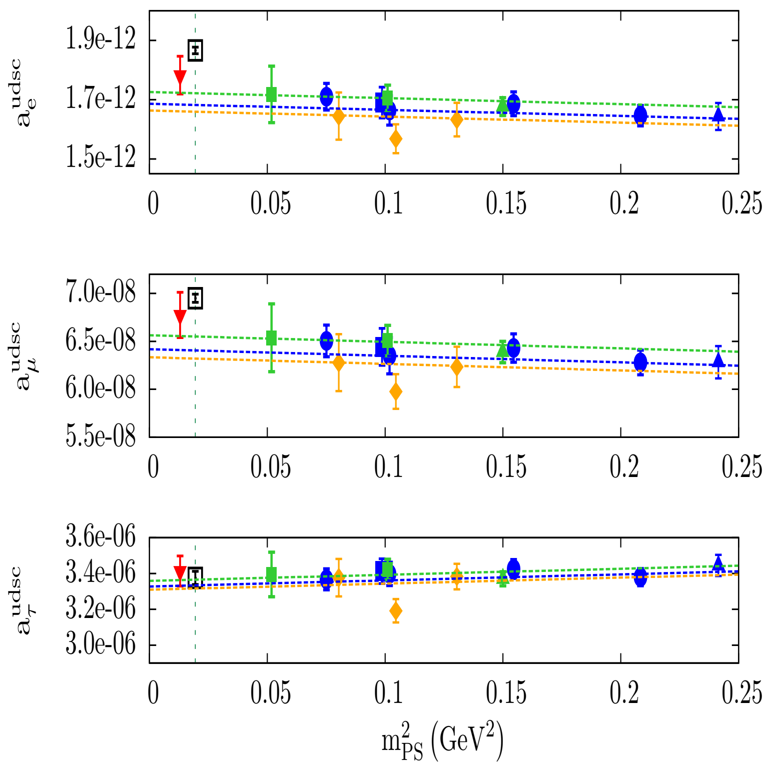
Hagiwara et al.: [Hagiwara, Liao, Martin, Nomura, Teubner, J. Phys. G38, 2011]

Benayoun et al.: [Benayoun, David, DelBuono, Jegerlehner, arXiv:1507.02943, 2015]

- grey band: tentative "experimental" value for $a_\mu^{\rm hvp}$
  $\leftarrow$ subtract all other known SM contributions from measured value (including higher-order QCD effects)

Analysis for electron and tau magnetic moments

- lepton anomalous magnetic moments sensitive to $\mathcal{O}(m_l^2)$ momentum region
- electron probes region $Q^2 \approx 0$ where no lattice data available
- τ-lepton sensitive to $Q^2 \approx 1\,\mathrm{GeV}^2$



$1.78(06)(05) \cdot 10^{-12}$ tmLQCD
$1.87(01)(01) \cdot 10^{-12}$ N & T

$6.78(24)(18) \cdot 10^{-8}$ tmLQCD
$6.95(04)(02) \cdot 10^{-8}$ Hagiwara

$3.41(08)(05) \cdot 10^{-6}$ tmLQCD
$3.38(04) \cdot 10^{-6}$ E & P

N & T: [Nomura, Teubner, Nucl. Phys. B 867, 2013]

Hagiwara: [Hagiwara, Liao, Martin, Nomura, Teubner, J. Phys. G 38, 2011]

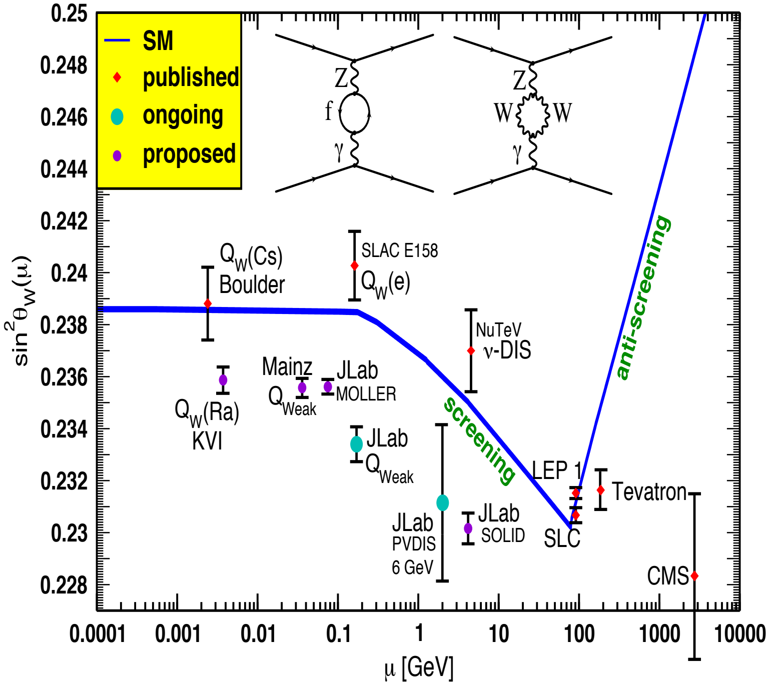
E & P: [Eidelman, Passera, Mod. Phys. Lett. A 22, 2007]

The weak mixing angle

- fundamental parameter of electroweak SM

$$\sin^2\theta_W = \frac{g'^2}{g'^2+g^2} = \frac{\alpha_{\text{QED}}}{\alpha_2}$$

- g: $SU(2)_L$ coupling constant, g': $U(1)_Y$ coupling constant



[Erler, J. Phys. Conf. Ser. 485, 2012]

- several ongoing and proposed experiments
  - only envisioned uncertainties shown
  - central values arbitrary

Relation to vacuum polarization function

- running of $\sin^2\theta_W$ in leading logarithmic approximation [Jegerlehner, Z. Phys. C 32, 1986]

$$\sin^2(\theta)(q^2) = \sin^2(\theta^0)\frac{1-\Delta\alpha_2(q^2)}{1-\Delta\alpha_{\mathrm{QED}}(q^2)} = \sin^2(\theta^0)(1+\Delta(q^2))$$

- formulae for lattice computation [Burger, Petschlies, Pientka, KJ, arXiv:1505.03283, 2015]

$$\Delta^{ud}(q^2) = -\tfrac{1}{4}g^2\Pi^{uu}(q^2) + \tfrac{5}{9}e^2\Pi^{uu}(q^2)$$

$$\Delta^{s}(q^2) = -\tfrac{1}{12}g^2\Pi^{ss}(q^2) + \tfrac{1}{9}e^2\Pi^{ss}(q^2)$$

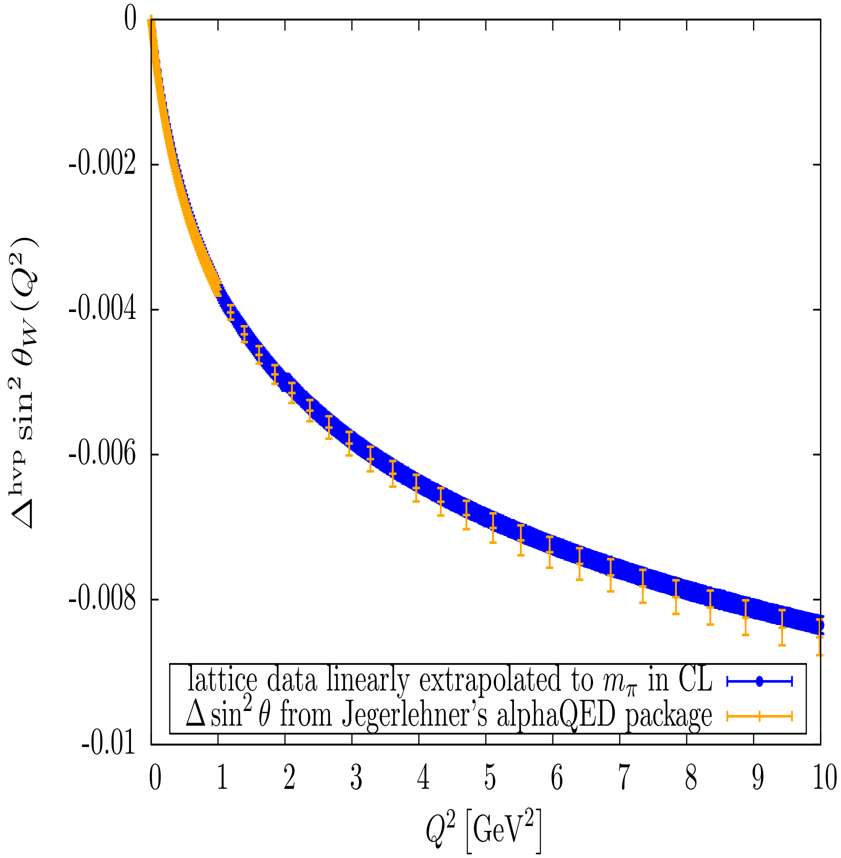
$$\Delta^{c}(q^2) = -\tfrac{1}{6}g^2\Pi^{cc}(q^2) + \tfrac{4}{9}e^2\Pi^{cc}(q^2)$$

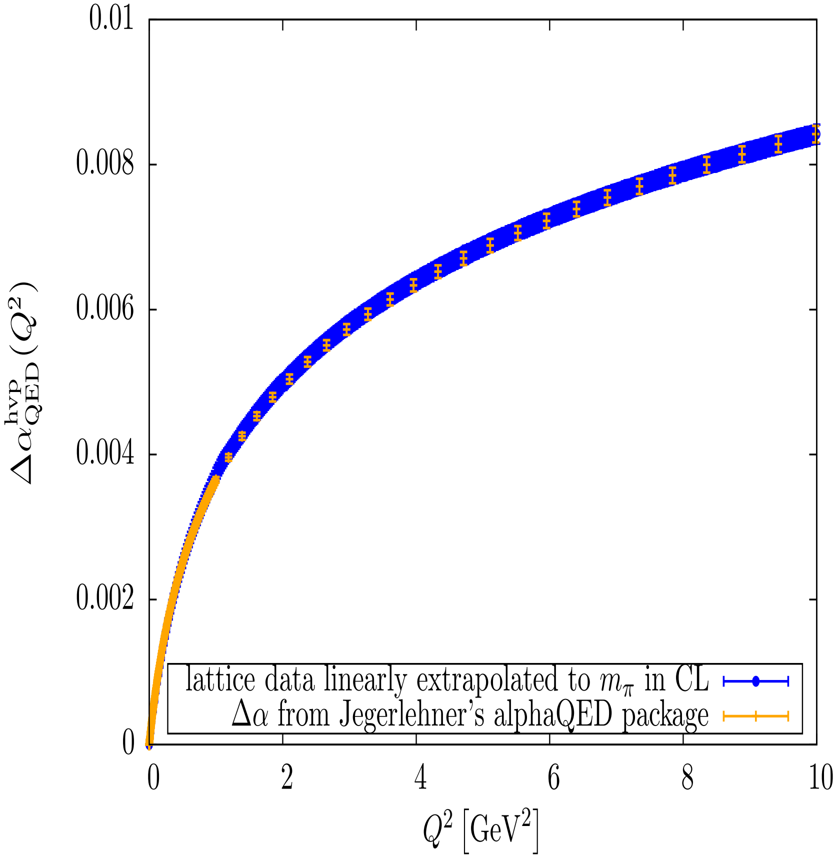
$$\alpha_{\mathrm{QED}}(Q^2) = \frac{\alpha_0}{1-\Delta\alpha_{\mathrm{QED}}(Q^2)}\ ;\quad \Delta\alpha^{\mathrm{hvp}}_{\mathrm{QED}}(Q^2) = -4\pi\alpha_0\Pi_{\mathrm{R}}\left(Q^2\right)$$

- need only HVP (with redefinition)

$$\Pi_{\mathrm{R}}(Q^2) = \Pi_{\mathrm{R}}\left(Q^2\frac{m_V^2}{m_\rho^2}\right)$$

Running of $\sin^2\theta_W$ and α_{QED}





alphaQED: [Jegerlehner, Nuovo Cim. 034C, 2011]

$\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}$ and $\sin^2(\theta_{\mathrm{W}})$ for selected Q^2

- selected Q^2, continuum and chiral extrapolation
- momentum Q^2 in GeV

$\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}$

Q^2	this work	dispersive analysis
0.02	$0.163(05)(09) \cdot 10^{-3}$	$0.190(02) \cdot 10^{-3}$
1.00	$3.721(96)(145) \cdot 10^{-3}$	$3.651(40) \cdot 10^{-3}$
2.00	$4.993(102)(144) \cdot 10^{-3}$	$4.916(61) \cdot 10^{-3}$
3.00	$5.800(111)(151) \cdot 10^{-3}$	$5.725(74) \cdot 10^{-3}$
4.00	$6.396(108)(156) \cdot 10^{-3}$	$6.333(84) \cdot 10^{-3}$
6.00	$7.264(114)(159) \cdot 10^{-3}$	$7.223(98) \cdot 10^{-3}$
8.00	$7.906(124)(151) \cdot 10^{-3}$	$7.850(107) \cdot 10^{-3}$
10.0	$8.419(130)(159) \cdot 10^{-3}$	$8.420(114) \cdot 10^{-3}$

$\sin^2(\theta_{\mathrm{W}})$

Q^2	this work	experiment
0.02	$-0.158(05)(08) \cdot 10^{-3}$	Qweak
1.00	$-3.706(83)(127) \cdot 10^{-3}$	PVDIS
2.00	$-5.021(96)(135) \cdot 10^{-3}$	PVDIS
3.00	$-5.801(104)(135) \cdot 10^{-3}$	SoLID
4.00	$-6.398(102)(135) \cdot 10^{-3}$	SoLID
6.00	$-7.251(111)(136) \cdot 10^{-3}$	SoLID
8.00	$-7.867(112)(137) \cdot 10^{-3}$	SoLID
10.0	$-8.352(119)(138) \cdot 10^{-3}$	SoLID

- $\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}$: nice agreement with dispersive analysis
- $\sin^2(\theta_{\mathrm{W}})$: input for experiment

CP3-Origins-2013-040 DNRF90, DIAS-2013-40
FERMILAB-PUB-13-484-T, FTUAM-13-28
IFIC/13-76, IFT-UAM/CSIC-13-106
MITP/13-067, YITP-13-114

Review of lattice results concerning low energy particle physics

August 26, 2014

FLAG Working Group

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Abstract

We review lattice results related to pion, kaon, D- and B-meson physics with the aim of making them easily accessible to the particle physics community. More specifically, we report on the determination of the light-quark masses, the form factor $f_+(0)$, arising in semileptonic $K \to \pi$ transition at zero momentum transfer, as well as the decay constant ratio f_K/f_π of decay constants and its consequences for the CKM matrix elements V_{us} and V_{ud}. Furthermore, we describe the results obtained on the lattice for some of the low-energy constants of $SU(2)_L \times SU(2)_R$ and $SU(3)_L \times SU(3)_R$ Chiral Perturbation Theory and review the determination of the B_K parameter of neutral kaon mixing. The inclusion of heavy-quark quantities significantly expands the FLAG scope with respect to the previous review. Therefore, we focus here on D- and B-meson decay constants, form factors, and mixing parameters, since these are most relevant for the determination of CKM matrix elements and the global CKM unitarity-triangle fit. In addition we review the status of lattice determinations of the strong coupling constant α_s.

Quantity	Sect.	■	$N_f = 2+1+1$	■	$N_f = 2+1$	■	$N_f = 2$
m_s(MeV)	3.3			3	93.8(1.5)(1.9)	2	101(3)
m_{ud}(MeV)	3.3			3	3.42(6)(7)	1	3.6(2)
m_s/m_{ud}	3.3			3	27.46(15)(41)	1	28.1(1.2)
m_d(MeV)	3.4				4.68(14)(7)		4.80(23)
m_u(MeV)	3.4				2.16(9)(7)		2.40(23)
m_u/m_d	3.4				0.46(2)(2)		0.50(4)
$f_+^{K\pi}(0)$	4.3			2	0.9661(32)	1	0.9560(57)(62)
f_{K^+}/f_{π^+}	4.3	2	1.194(5)	4	1.192(5)	1	1.205(6)(17)
f_K(MeV)	4.6			3	156.3(0.9)	1	158.1(2.5)
f_π(MeV)	4.6			3	130.2(1.4)		
Σ(MeV)	5.1			2	271(15)	1	269(8)
F_π/F	5.1	1	1.0760(28)	2	1.0624(21)	1	1.0744(67)
$\bar{\ell}_3$	5.1	1	3.70(27)	3	3.05(99)	1	3.41(41)
$\bar{\ell}_4$	5.1	1	4.67(10)	3	4.02(28)	1	4.62(22)
$\hat{B}_K$	6.2			4	0.766(10)	1	0.729(25)(17)
$B_K^{\overline{\rm MS}}(2\ {\rm GeV})$	6.2			4	0.560(7)	1	0.533(18)(12)
f_D(MeV)	7.1			2	209.2(3.3)	1	208(7)
f_{D_s}(MeV)	7.1			2	248.6(2.7)	1	250(7)
f_{D_s}/f_D	7.1			2	1.187(12)	1	1.20(2)
$f_+^{D\pi}(0)$	7.2			1	0.666(29)		
$f_+^{DK}(0)$	7.2			1	0.747(19)		
f_B(MeV)	8.1	1	186(4)	3	190.5(4.2)	1	189(8)
f_{B_s}(MeV)	8.1	1	224(5)	3	227.7(4.5)	1	228(8)
f_{B_s}/f_B	8.1	1	1.205(7)	2	1.202(22)	1	1.206(24)
$f_{B_d}\sqrt{\hat{B}_{B_d}}$(MeV)	8.2			1	216(15)		
$f_{B_s}\sqrt{\hat{B}_{B_s}}$(MeV)	8.2			1	266(18)		
$\hat{B}_{B_d}$	8.2			1	1.27(10)		
$\hat{B}_{B_s}$	8.2			1	1.33(6)		
ξ	8.2			1	1.268(63)		
$\hat{B}_{B_s}/\hat{B}_{B_d}$	8.2			1	1.06(11)		
$\Delta\zeta^{B\pi}({\rm ps}^{-1})$	8.3			2	2.16(50)		
$f_+^{B\pi}(q^2):\ a_0^{\rm BCL}$	8.3			2	0.453(33)		
$a_1^{\rm BCL}$				2	−0.43(33)		
$a_2^{\rm BCL}$				2	0.9(3.9)		
$\mathcal{F}^{B\to D^*}(1)$	8.4			1	0.906(4)(12)		
$R(D)$	8.4			1	0.316(12)(7)		
$\alpha_{\overline{\rm MS}}^{(5)}(M_Z)$	9.9			4	0.1184(12)		

Table 1: Summary of the main results of this review, grouped in terms of N_f, the number of dynamical quark flavours in lattice simulations. Quark masses and the quark condensate are given in the $\overline{\rm MS}$ scheme at running scale $\mu = 2\,{\rm GeV}$; the other quantities listed are specified in the quoted sections. The columns marked ■ indicate the number of results that enter our averages for each quantity. We emphasize that these numbers only give a very rough indication of how thoroughly the quantity in question has been explored on the lattice and recommend to consult the detailed tables and figures in the relevant section for more significant information and for explanations on the source of the quoted errors.

Summary

- discussed hadronic contributions to electroweak observables
- anomalous magnetic moment of muon
  - improved precision through new lattice observable
  - lattice result: still larger error than dispersive analysis
  - lattice precision can be systematically improved
  - first results for light-by-light contribution are appearing
- electron and tau anomalous magnetic moments
  - nice agreement with phenomenology
  - lattice precision comparable with dispersive analysis
- running of $\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}(Q^2)$ and $\sin^2(\theta_{\mathrm{W}})(Q^2)$
  - first lattice calculations of these quantities
  - input/benchmark for experiment/phenomenology
- worldwide lattice community activity:
  prospect for accurate determination of these electroweak observables