On the Trilinear Higgs Self-Couplings in the complex NMSSM

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in coll. with Margarete Mühlleitner, Juraj Streicher, Kathrin Walz and Hanna Ziesche

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Cám ơn Maggie rất nhiều!

Overview

Why higher order trilinear Higgs couplings?

- Hints for the Higgs potential
  - SM: $V_{SM} = -\mu^2 h^\dagger h + \lambda(h^\dagger h)^2$,
    Higgs self couplings: $\lambda_{HHH} = \frac{3M_H^2}{v}, \lambda_{HHHH} = \frac{3M_H^2}{v^2}$
  - MSSM:

$$
\begin{aligned}
V_{MSSM} &= (\mu^2 + m_{H_d}^2)H_{d,i}^* H_{d,i} + (\mu^2 + m_{H_u}^2)H_{u,i}^* H_{u,i} + \epsilon^{ij}(m_{12}^2 H_{u,i}H_{d,j} + \text{ H.c}) \\
&+ \frac{g_1^2 + g_2^2}{8}(H_{u,i}^* H_{u,i} - H_{d,i}^* H_{d,i})^2 + \frac{g_2^2}{2}|H_{u,i}^* H_{d,i}|^2
\end{aligned}
$$

    Decoupling limit: $(M_{H^\pm} \gg M_h)$ then $\lambda_{hhh} \to \frac{3M_h^2}{v}$, h is the lightest Higgs boson, also at higher order
  - NMSSM:

$$
\begin{aligned}
V_{NMSSM} &= (|\lambda S|^2 + m_{H_d}^2)H_{d,i}^* H_{d,i} + (|\lambda S|^2 + m_{H_u}^2)H_{u,i}^* H_{u,i} + m_S^2|S|^2 \\
&+ \frac{1}{8}(g_2^2 + g_1^2)(H_{d,i}^* H_{d,i} - H_{u,i}^* H_{u,i})^2 + \frac{1}{2}g_2^2|H_{d,i}^* H_{u,i}|^2 \\
&+ |-\epsilon^{ij}\lambda H_{d,i}H_{u,j} + \kappa S^2|^2 + \left[-\epsilon^{ij}\lambda A_\lambda S H_{d,i}H_{u,j} + \frac{1}{3}\kappa A_\kappa S^3 + \text{ H.c}\right],
\end{aligned}
$$

    Decoupling limit: $(M_{H^\pm} \gg M_h)$ then $\lambda_{hhh} \neq \frac{3M_h^2}{v}$, h is the SM-like Higgs boson depend on the mixing with singlet component.
- Match the accuracy of Higgs mass calculation
- Increase the accuracy of Higgs to Higgs decays

What have been done?

- MSSM Trilinear Higgs Self-Couplings
  - Full one-loop correction in Feynman diagram approach (complex MSSM)
    [Williams, Rzehak, Weiglein]
  - Two-loop correction in effective potential approach (real MSSM)
    [Brucherseifer, Gavin, Spira]
- (complex) NMSSM Trilinear Higgs Self-Couplings
  - Full one-loop correction in Feynman diagram approach
    [Mühlleitner, DTN, Streicher, Walz]
  - Two-loop $\mathcal{O}(\alpha_t\alpha_s)$ correction in Feynman diagram approach
    [Mühlleitner, DTN, Ziesche]

What is the complex NMSSM?

1. Superpotential

$$W_{NMSSM} = \epsilon_{ij}[y_e \hat{H}_d^i \hat{L}^j \hat{E}^c + y_d \hat{H}_d^i \hat{Q}^j \hat{D}^c - y_u \hat{H}_u^i \hat{Q}^j \hat{U}^c] - \epsilon_{ij} \lambda \hat{S} \hat{H}_d^i \hat{H}_u^j + \frac{1}{3} \kappa \hat{S}^3$$

2. Two complex Higgs doublets and one complex Higgs singlet

$$H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + h_d + i a_d) \\ h_d^- \end{pmatrix}, \quad H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{1}{\sqrt{2}}(v_u + h_u + i a_u) \end{pmatrix}, \quad S = \frac{e^{i\varphi_s}}{\sqrt{2}}(v_s + h_s + i a_s).$$

3. Soft SUSY breaking terms

$$L_{soft} = \mathcal{L}_{soft,\, MSSM} - m_S^2 |S|^2 + (\epsilon_{ij} \lambda A_\lambda S H_d^i H_u^j - \frac{1}{3} \kappa A_\kappa S^3 + h.c.)$$

$\lambda, \kappa, A_\lambda, A_\kappa$ are in general complex.

4. dynamic μ term

$$\mu_{eff} H_d . H_u \quad \text{with} \quad \mu_{eff} = \frac{\lambda v_s e^{i\varphi_s}}{\sqrt{2}}$$

5. CP-odd and CP-even Higgs bosons can already mix at tree-level.

$$(h_d, h_u, h_s, a_d, a_u, a_s) \rightarrow (h_1, h_2, h_3, h_4, h_5, G)$$

one CP-violating phase: $\phi_y = \phi_u - 2\phi_s - \phi_\kappa + \phi_\lambda$

The loop-corected Higgs masses and trilinear couplings

- Loop-corrected Higgs mass matrices

$$M^2(p^2) = \begin{pmatrix} m_{h_1}^2 - \hat{\Sigma}_{h_1 h_1} & -\hat{\Sigma}_{h_1 h_2} & -\hat{\Sigma}_{h_1 h_3} & -\hat{\Sigma}_{h_1 h_4} & -\hat{\Sigma}_{h_1 h_5} \\ -\hat{\Sigma}_{h_2 h_1} & m_{h_2}^2 - \hat{\Sigma}_{h_2 h_2} & -\hat{\Sigma}_{h_2 h_3} & -\hat{\Sigma}_{h_2 h_4} & -\hat{\Sigma}_{h_2 , h_5} \\ -\hat{\Sigma}_{h_3 h_1} & -\hat{\Sigma}_{h_3 h_2} & m_{h_3}^2 - \hat{\Sigma}_{h_3 h_3} & -\hat{\Sigma}_{h_3 h_4} & -\hat{\Sigma}_{h_3 h_5} \\ -\hat{\Sigma}_{h_4 h_1} & -\hat{\Sigma}_{h_4 h_2} & -\hat{\Sigma}_{h_4 h_3} & m_{h_4}^2 - \hat{\Sigma}_{h_4 h_4} & -\hat{\Sigma}_{h_4 h_5} \\ -\hat{\Sigma}_{h_5 h_1} & -\hat{\Sigma}_{h_5 h_2} & -\hat{\Sigma}_{h_5 h_3} & -\hat{\Sigma}_{h_5 h_4} & m_{h_5}^2 - \hat{\Sigma}_{h_5 h_5} \end{pmatrix},$$

$\hat{\Sigma}_{h_i h_j}(p^2)$ is renormalized self-energy of $h_i \to h_j$ transition

$$\hat{\Sigma}_{h_i h_j}(p^2) = \hat{\Sigma}^{(\alpha)}_{h_i h_j}(p^2) + \hat{\Sigma}^{(\alpha_s \alpha_t)}_{h_i h_j}(0)$$

loop-corrected Higgs masseigenstates

$$(h_1, h_2, h_3, h_4, h_5) \to (H_1, H_2, H_3, H_4, H_5), \quad H_i = \mathbf{Z}_{ij} h_j$$

Z: wave function renormalization factor

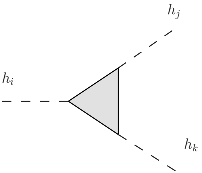
- Loop-corrected trilinear Higgs couplings

$$\mathcal{M}_{H_i \to H_j H_k} = \sum_{i',j',k'=1}^{5} \mathbf{Z}_{ii'} \mathbf{Z}_{jj'} \mathbf{Z}_{kk'} (\lambda_{h_{i'} h_{j'} h_{k'}} + \Delta^{(\alpha)} \lambda_{h_{i'} h_{j'} h_{k'}} + \Delta^{(\alpha_s \alpha_t)} \lambda_{h_{i'} h_{j'} h_{k'}}) + \Delta^{(\alpha)} M^{G,Z}_{h_i \to h_j h_k} .$$

The full one-loop correction to the trilinear couplings

[Mühlleitner, DTN, Streicher, Walz]

$\Delta^{(\alpha)}\lambda_{h_{i'} h_{j'} h_{k'}}$

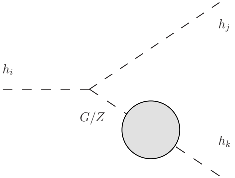


- $\mathcal{O}(\alpha)$ including full momentum dependence
- Renormalization scheme:

$$\underbrace{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M_Z, M_W, M_{H^\pm}, e}_{\text{on-shell scheme}}, \underbrace{\tan\beta, \lambda, v_s, \kappa, A_\kappa}_{\overline{\text{DR}}\text{ scheme}} .$$

Higgs fields are renormalized in $\overline{\text{DR}}$ scheme
complex phases need not to be renormalized

$\Delta^{(\alpha)} M^{G,Z}_{h_i \to h_j h_k}$
using tree-level masses to maintain gauge invariance

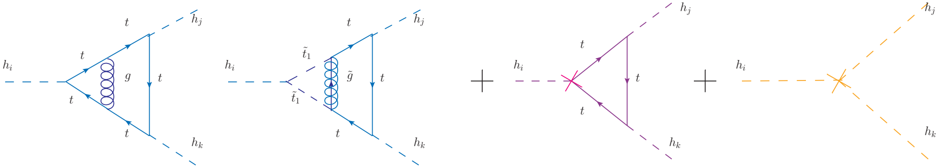


- Using FeynArt, FormCalc (D=4), LoopTools

The two-loop correction to the trilinear couplings

[Mühlleitner, DTN, Ziesche]

$$\Delta^{(\alpha_s \alpha_t)} \lambda_{h_i h_j h_k} = \Delta^{(2)} \lambda^{\text{UR}}_{h_i h_j h_k} + \Delta^{(2)} \lambda^{\text{CT1L}}_{h_i h_j h_k} + \Delta^{(2)} \lambda^{\text{CT2L}}_{h_i h_j h_k}$$



- Zero external momentum approximation
- Dimensional reduction (DRED)
  - SUSY is preserved in two-loop Yukawa corrections to Higgs Masses
    Stoekinger, Hollik' 2005
  - In practice: $\text{Tr}[1] = 4, [\gamma_5, \gamma^\mu] = 0, (g^4)^\mu_\nu (g^d)^\mu_\rho = (g^d)^\mu_\rho$
    Loop momentum in d dimension $\rightarrow$ tensor deduction in d dimension
- Tensor reduction: TARCER using Tarasov's Algorithm

$$c_1 A^d(m_i^2) A^d(m_j^2) + c_2 I(m_1^2, m_2^2, m_3^2)$$

A: one-loop tadpole integral, I: two-loop tadpole integral

- Using FeynArt, SARAH (model file), FeynCalc

Renormalization

- The following parameters need to be renormalized

$$\begin{aligned} t_\phi &\to t_\phi + \delta^{(1)} t_\phi + \delta^{(2)} t_\phi \qquad\qquad \text{with} \quad \phi = h_d, h_u, h_s, a_d, a_s \ , \\ M^2_{H^\pm} &\to M^2_{H^\pm} + \delta^{(1)} M^2_{H^\pm} + \delta^{(2)} M^2_{H^\pm} \ , \\ v &\to v + \delta^{(1)} v + \delta^{(2)} v \ , \\ \tan\beta &\to \tan\beta + \delta^{(1)} \tan\beta + \delta^{(2)} \tan\beta \ , \\ |\lambda| &\to |\lambda| + \delta^{(1)} |\lambda| + \delta^{(2)} |\lambda| \ . \end{aligned}$$

- Chose the renormalization scheme as

$$\underbrace{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M^2_{H^\pm}, v}_{\text{on-shell scheme}}, \ \underbrace{\tan\beta, |\lambda|}_{\overline{\text{DR}} \text{ scheme}} \ .$$

- Higgs wave function need also to be renormalized. Chose the $\overline{\text{DR}}$ scheme

$$H_u \to \left(1 + \frac{1}{2}\delta^{(1)} Z_{H_u} + \frac{1}{2}\delta^{(2)} Z_{H_u}\right) H_u \ , \quad \delta Z_{H_d} = \delta Z_{H_s} = 0$$

For $\overline{\text{DR}}$ scheme of top mass:

$$\delta^{(2)} Z_{H_u} = \frac{\alpha_s (m_t^2)^{\overline{\text{DR}}}}{8\pi^2 v^2 \sin^2\beta} \left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right), \quad \delta^{(2)} \tan\beta = \frac{1}{2} \tan\beta \, \delta^{(2)} Z_{H_u}, \quad \delta^{(2)} |\lambda| = \frac{-|\lambda|}{2} \delta^{(2)} Z_{H_u}$$

Renormalization of (s)top sector

The parameters need to be renormalized at $\mathcal{O}(\alpha_s)$

$$m_t,\ m_{\tilde{Q}_3},\ m_{\tilde{t}_R} \quad \text{and} \quad A_t$$

- On-shell renomalization scheme.

$$\delta X^{\rm OS} = \frac{1}{\epsilon}\delta X_{\rm pole} + \delta X_{\rm fin}\ ,$$

  Note: the terms which are proportional to ϵ are not taken into account
- $\overline{\rm DR}$ renomalization scheme

$$\delta X^{\overline{\rm DR}} = \frac{1}{\epsilon}\delta X_{\rm pole}\ .$$

- Translation of the parameters from two schemes if needed
  Rough treatment

$$A_t^{\rm (OS)} = A_t^{(\overline{\rm DR})} - \delta A_t^{\rm fin}\ ,$$
$$(m^2_{\tilde{Q}_L})^{\rm (OS)} = (m^2_{\tilde{Q}_L})^{(\overline{\rm DR})} - \delta(m^2_{\tilde{Q}_L})^{\rm fin}\ ,$$
$$(m^2_{\tilde{t}_R})^{\rm (OS)} = (m^2_{\tilde{t}_R})^{(\overline{\rm DR})} - \delta(m^2_{\tilde{t}_R})^{\rm fin}\ .$$

Delicate treatment for top mass
$M_t \to m_t^{\overline{\rm MS}}(M_t) \to m_t^{\overline{\rm MS}}(M_{\rm SUSY})$
$\to m_t^{\overline{\rm DR},\rm SM}(M_{\rm SUSY}) \to m_t^{\overline{\rm DR},\rm NMSSM}$

Checks

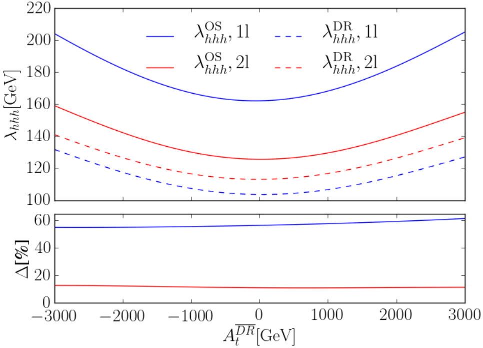
- Check UV finite
- Two independent calculations are in agreement
- Compared to the real MSSM, found a good agreement (Thanks Spira for this comparison)

Numerical analysis: constrains

- Using NMSSMCALC to compute effective couplings of the Higgs bosons, normalized to the corresponding SM values, as well as the masses, the widths and the branching ratios of the Higgs bosons.
- We chose the scenarios which are accordance with the LHC Higgs data by using the programs HiggsBounds and HiggsSignals
- The resulting supersymmetric particle spectrum is in accordance with present LHC searches for SUSY particles

The SM-like trilinear coupling: OS vs $\overline{\text{DR}}$ schemes

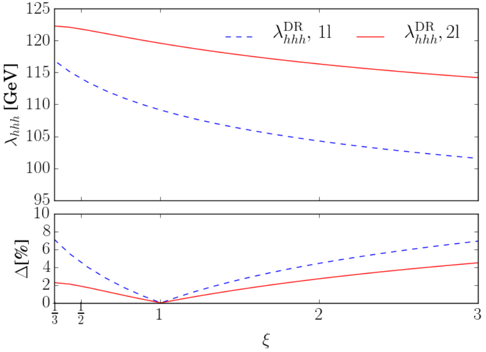
[Mühlleitner, DTN, Ziesche]



- h is dominated by h_u
- Including $\mathcal{O}(\alpha_t + \alpha_t\alpha_s)$, $p2 = 0$
- In OS scheme: one-loop correction 140%, two-loop correction −24%
- In $\overline{\text{DR}}$ scheme: one-loop correction 74%, two-loop correction 9%
- Difference between $\mathcal{O}(\alpha_t)$ and $\mathcal{O}(\alpha)$ less then 4%
- $\Delta = \frac{|\lambda^{m_t(\overline{\text{DR}})}_{HHH} - \lambda^{m_t(\text{OS})}_{HHH}|}{\lambda^{m_t(\overline{\text{DR}})}_{HHH}}$
- Theoretical uncertainty decreases substantially

The SM-like trilinear coupling: $\overline{\text{DR}}$ scheme, scale uncertainty

[Mühlleitner, DTN, Ziesche]



- h is dominated by h_u
- Including $\mathcal{O}(\alpha_t + \alpha_t\alpha_s)$, $p2 = 0$
- In $\overline{\text{DR}}$ scheme: one-loop correction 74%, two-loop correction 9%
- $\Delta = [\lambda_{hhh}(\mu_R) - \lambda_{hhh}(\mu_0)]/\lambda_{hhh}(\mu_0)$ $\mu_0 = 2097$ GeV
- $\overline{\text{DR}}$ parameters at different scales are estimated roughly by

$$p^{\rm OS} + \delta p^{\rm OS}(\mu) = p^{\overline{\rm DR}}(\mu) + \delta p^{\overline{\rm DR}}(\mu)$$

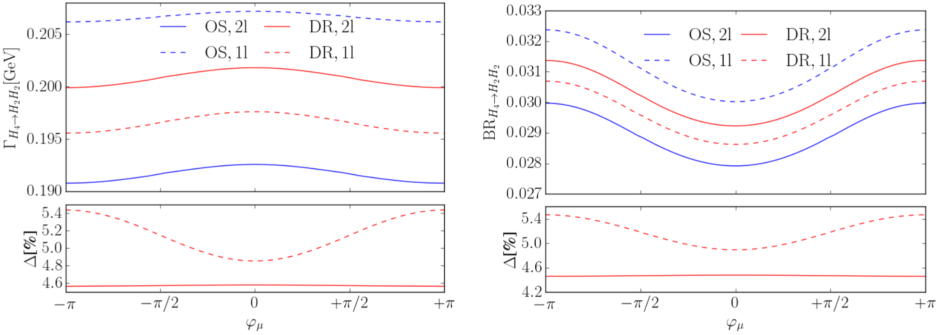
$$\begin{aligned}\tan\beta^{\rm pure\overline{DR}}(\mu_1) \; - \; \tan\beta^{\rm pure\overline{DR}}(\mu_2) &= a_1(m_t^{\rm OS}) \ln\frac{\mu_1^2}{\mu_2^2} \\ &+2\, a_2(m_t^{\rm OS}, \alpha_s^{\overline{\rm DR}}(\mu_1)) \ln \mu_1^2 - 2\, a_2(m_t^{\rm OS}, \alpha_s^{\overline{\rm DR}}(\mu_2)) \ln \mu_2^2 \\ &+2\, b_2(m_t^{\rm OS}, \alpha_s^{\overline{\rm DR}}(\mu_1)) \ln^2 \mu_1^2 - 2\, b_2(m_t^{\rm OS}, \alpha_s^{\overline{\rm DR}}(\mu_2)) \ln^2 \mu_2^2 \,.\end{aligned}$$

a_1 coef of 1-loop UV part. a_2, b_2 coef of double pole and single pole of 2-loop UV part

The Higgs to Higgs decays

[Mühlleitner, DTN, Ziesche]

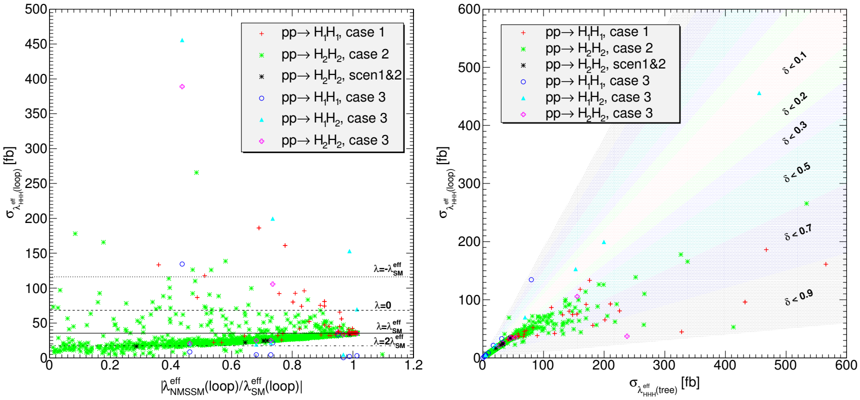
$H_4 \to H_2 H_2$
H_4 is mainly h_d, H_2 is dominated by h_u



- Include $\mathcal{O}(\alpha + \alpha_t \alpha_s)$, $\mathcal{O}(\alpha)$ with full momemtum dependence
- tree-level CP violating phase $\phi_y = 0$
- OS: 21% at one-loop, -7% at two-loop
- $\overline{\text{DR}}$: 6.5% at one-loop, 2% at two-loop

Effect of trilinear Higgs self-coupling in Higgs boson pair production

[Mühlleitner, DTN, Streicher, Walz]



- Using HPAIR which include NLO QCD k-factor in the infinite top mass limit [Spira]
- Here Loop means using $\mathcal{O}(\alpha)$ effective Higgs Self-Coupling
- possible enhancement or detraction in the NMSSM

Conclusions

For the (c)NMSSM

- Full one-loop with full momentum dependence has been computed.
- Correction at one-loop level is of 75% in $\overline{\text{DR}}$ scheme, and 140% in OS scheme.
- The Leading two-loop correction of $\mathcal{O}(\alpha_t \alpha_s)$ in zero momentum approximation has been done
- The two-loop correction is of 9% in $\overline{\text{DR}}$ scheme and of -24% in OS scheme
- Theoretical uncertainty is significantly reduced for the SM-like trilinear couplings.
- Loop corrections to Higgs Self-coupling effect significantly the total crossection of Higgs pair production

Conclusions

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- Full one-loop with full momentum dependence has been computed.
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THANK YOU FOR YOUR ATTENTION

The one-loop top Yukawa ($\mathcal{O}(\alpha_t)$) correction to the trilinear couplings

$\Delta^{(\alpha_t)}\lambda_{h_{i'}h_{j'}h_{k'}}$

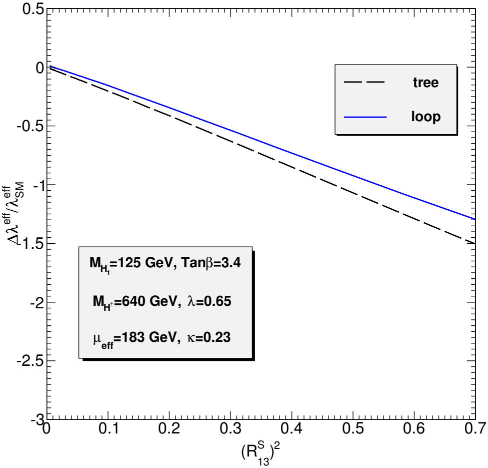
- Only one-loop diagrams with top/stop in loops
- Zero external momentum approximation is used
- This is the main contribution of the one-loop order, especially for SM-like trilinear coupling
- Useful computation to understand the two-loop $\mathcal{O}(\alpha_t\alpha_S)$ correction
- Renormalization of parameters (like the $\mathcal{O}(\alpha_t\alpha_S)$ case)

$$\underbrace{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M^2_{H^\pm}, v}_{\text{on-shell scheme}}, \underbrace{\tan\beta, |\lambda|}_{\overline{\text{DR}}\text{ scheme}} .$$

- Do not include $\Delta^{(\alpha)}M^{G,Z}_{h_i\to h_jh_k}$, violate gauge invariance with zero momentum approximation

Effective coupling at decoupling Limit

[Mühlleitner, DTN, Streicher, Walz]



- Decoupling Limit: $M_{H\pm} \gg M_h$
- Effective: $p^2 = 0$
- Loop: $\mathcal{O}(\alpha)$
- R^S_{13}: mixing with singlet component

Parameters - Scenario 1

The SM input parameters

$$\alpha(M_Z) = 1/128.962\,, \qquad \alpha_s^{\overline{\text{MS}}}(M_Z) = 0.1184\,, \qquad M_Z = 91.1876\,\text{GeV}\,,$$
$$M_W = 80.385\,\text{GeV}\,, \qquad m_t = 173.5\,\text{GeV}\,, \qquad m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}}) = 4.18\,\text{GeV}\,.$$

Using $\alpha_s^{\overline{\text{DR}}}$

Senario 1:

$$m_{\tilde{u}_R,\tilde{c}_R} = m_{\tilde{d}_R,\tilde{s}_R} = m_{\tilde{Q}_{1,2}} = m_{\tilde{L}_{1,2}} = m_{\tilde{e}_R,\tilde{\mu}_R} = 3\ \text{TeV}\,,\ m_{\tilde{t}_R} = 1909\,\text{GeV}\,,$$
$$m_{\tilde{Q}_3} = 2764\,\text{GeV}\,,\ m_{\tilde{b}_R} = 1108\,\text{GeV}\,,\ m_{\tilde{L}_3} = 472\,\text{GeV}\,,\ m_{\tilde{\tau}_R} = 1855\,\text{GeV}\,,$$
$$|A_{u,c,t}| = 1283\,\text{GeV}\,,\ |A_{d,s,b}| = 1020\,\text{GeV}\,,\ |A_{e,\mu,\tau}| = 751\,\text{GeV}\,,$$
$$|M_1| = 908\,\text{GeV},\ |M_2| = 237\,\text{GeV}\,,\ |M_3| = 1966\,\text{GeV}\,,$$
$$\varphi_{A_{d,s,b}} = \varphi_{A_{e,\mu,\tau}} = \varphi_{A_{u,c,t}} = \pi\,,\ \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0\,.$$

$$|\lambda| = 0.374\,, \quad |\kappa| = 0.162\,, \quad |A_\kappa| = 178\,\text{GeV}\,, \quad |\mu_{\text{eff}}| = 184\,\text{GeV}\,,$$
$$\varphi_\lambda = \varphi_\kappa = \varphi_{\mu_{\text{eff}}} = \varphi_u = 0\,, \quad \varphi_{A_\kappa} = \pi\,, \quad \tan\beta = 7.52\,, \quad M_{H^\pm} = 1491\,\text{GeV}\,.$$

Renormalization scale = SUSY scale

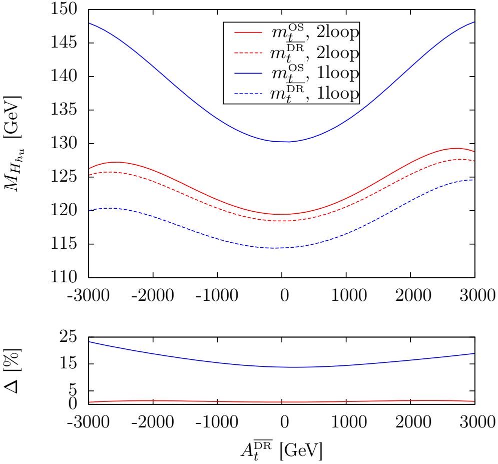
$$M_s = \sqrt{m_{\tilde{Q}_3} m_{\tilde{t}_R}}\,.$$

Senario 1: Mass spectrum

OS	H_1	H_2	H_3	H_4	H_5
mass tree [GeV]	71.14	117.49	211.12	1491	1492
main component	h_u	h_s	a_s	a	h_d
mass one-loop [GeV]	98.65	139.17	217.27	1490	1491
main component	h_s	h_u	a_s	a	h_d
mass two-loop [GeV]	94.68	125.06	217.32	1490	1491
main component	h_s	h_u	a_s	a	h_d
$\overline{\text{DR}}$	H_1	H_2	H_3	H_4	H_5
mass tree [GeV]	71.14	117.49	211.12	1491	1492
main component	h_u	h_s	a_s	a	h_d
mass one-loop [GeV]	91.60	120.00	217.36	1491	1491
main component	h_s	h_u	a_s	a	h_d
mass two-loop [GeV]	94.41	124.24	217.33	1490	1491
main component	h_s	h_u	a_s	a	h_d

Higgs Masses: OS scheme vs $\overline{\text{DR}}$ scheme

[Mühlleitner, DTN, Rzehak, Walz]



- $\hat{\Sigma}_{h_i h_j}(p^2) = \hat{\Sigma}^{(\alpha)}_{h_i h_j}(p^2) + \hat{\Sigma}^{(\alpha_s \alpha_t)}_{h_i h_j}(0)$
- $\Delta = |M_{H_{h_u}}^{m_t(\overline{\text{DR}})} - M_{H_{h_u}}^{m_t(\text{OS})}| / M_{H_{h_u}}^{m_t(\overline{\text{DR}})}$

Scenario 2

$m_{\tilde{t}_R} = 1170$ GeV , $m_{\tilde{Q}_3} = 1336$ GeV , $m_{\tilde{b}_R} = 1029$ GeV , $m_{\tilde{L}_3} = 2465$ GeV , $m_{\tilde{\tau}_R} = 301$ GeV
$|A_{u,c,t}| = 1824$ GeV , $|A_{d,s,b}| = 1539$ GeV , $|A_{e,\mu,\tau}| = 1503$ GeV , $|M_1| = 862.4$ GeV, $|\kappa| = 0.208$
$|M_2| = 201.5$ GeV , $|M_3| = 2285$ GeV , $|\lambda| = 0.629$, $|A_\kappa| = 179.7$ GeV , $|\mu_{\rm eff}| = 173.7$ GeV ,
$\tan\beta = 4.02$, $M_{H^\pm} = 788$ GeV , $\varphi_{A_{u,c,t}} = \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = \varphi_\lambda = \varphi_{\mu_{\rm eff}} = \varphi_u = \varphi_{A_\kappa} = 0$,
$\varphi_{A_{d,s,b}} = \varphi_{A_{e,\mu,\tau}} = \varphi_\kappa = \pi$.

OS	H_1	H_2	H_3	H_4	H_5
mass tree [GeV]	79.15	103.55	146.78	796.62	803.86
main component	h_s	h_u	a_s	h_d	a
mass one-loop [GeV]	103.45	129.15	139.83	796.53	802.94
main component	h_s	a_s	h_u	h_d	a
mass two-loop [GeV]	102.99	126.09	128.94	796.45	803.07
main component	h_s	h_u	a_s	h_d	a

$\overline{\rm DR}$	H_1	H_2	H_3	H_4	H_5
mass tree [GeV]	79.15	103.55	146.78	796.62	803.86
main component	h_s	h_u	a_s	h_d	a
mass one-loop [GeV]	102.80	120.52	128.80	796.36	803.09
main component	h_s	h_u	a_s	h_d	a
mass two-loop [GeV]	103.09	124.55	128.91	796.36	803.03
main component	h_s	h_u	a_s	h_d	a

Scenario 3

$m_{\tilde{t}_R} = 1940$ GeV , $m_{\tilde{Q}_3} = 2480$ GeV , $m_{\tilde{b}_R} = 1979$ GeV , $m_{\tilde{L}_3} = 2667$ GeV , $m_{\tilde{\tau}_R} = 1689$ GeV ,
$|A_{u,c,t}| = 1192$ GeV , $|A_{d,s,b}| = 685$ GeV , $|A_{e,\mu,\tau}| = 778$ GeV , $|M_1| = 517$ GeV, $|M_2| = 239$ GeV
$|M_3| = 1544$ GeV , $|\lambda| = 0.267$, $|\kappa| = 0.539$, $|A_\kappa| = 810$ GeV , $|\mu_{\text{eff}}| = 104$ GeV , $\tan\beta = 8.97$,
$M_{H^\pm} = 613$ GeV , $\varphi_{A_{d,s,b}} = \varphi_{A_{e,\mu,\tau}} = \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = \varphi_\lambda = \varphi_\kappa = \varphi_{\mu_{\text{eff}}} = \varphi_u = 0$,
$\varphi_{A_{u,c,t}} = \varphi_{A_\kappa} = \pi$.

OS	H_1	H_2	H_3	H_4	H_5
mass tree [GeV]	49.17	99.83	609.21	611.77	715.92
main component	h_s	h_u	a	h_d	a_s
mass one-loop [GeV]	87.36	139.10	608.71	611.37	694.73
main component	h_s	h_u	a	h_d	a_s
mass two-loop [GeV]	83.66	124.95	608.73	611.37	694.76
main component	h_s	h_u	a	h_d	a_s

$\overline{\text{DR}}$	H_1	H_2	H_3	H_4	H_5
mass tree [GeV]	49.17	99.83	609.21	611.77	715.92
main component	h_s	h_u	a	h_d	a_s
mass one-loop [GeV]	80.66	119.68	608.72	611.37	694.79
main component	h_s	h_u	a	h_d	a_s
mass two-loop [GeV]	83.03	124.34	608.71	611.36	694.78
main component	h_s	h_u	a	h_d	a_s