Genetic Optimization of Nonlinearities in SLS-2

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Purpose of the Research

- SLS-2 is being designed around the principle of strong fields and low dispersion, necessitating a strong chromaticity correction scheme.
- Perturbation theory & local optimizer techniques have been yielding schemes with only marginally acceptable DA and lifetime.
- Genetic algorithms are global and not limited to a certain order of correction, and so are an appealing way to develop the correction scheme.

Features of this Optimizer

1. Requires only modest computing resources
   - About 800 CPU-hours to converge on modern Intel cores.
2. Does not use perturbation theory beyond Twiss calculations.
3. Does not require seeding.
4. Constrains chromatic and amplitude-dependent tune footprints.

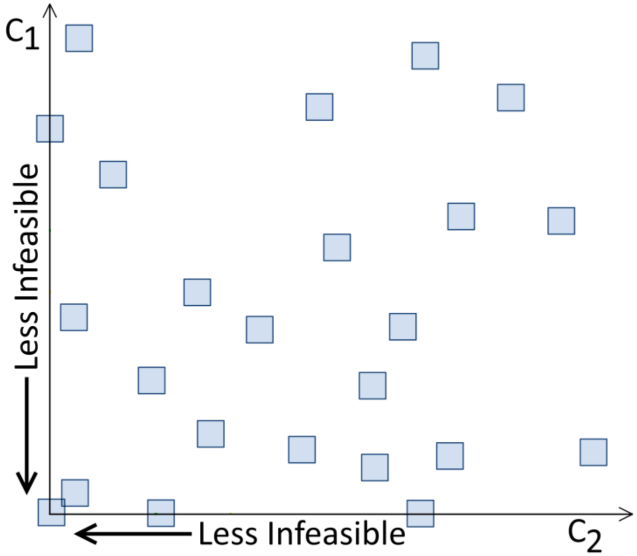
But First: Dominance Constraints

- Physics simulation outputs not only objective values, but also constraint values.
- The constraints are enforced as part of the sorting algorithm

A solution is called infeasible if any one of its constraints are not met, otherwise it is called feasible.

When comparing two individuals to determine dominance:

1. If both solutions are infeasible, determine dominance by comparing **only constraints**.
2. Feasible solutions **always** dominate infeasible solutions.
3. If both solutions are feasible, determine dominance by comparing **only objectives**.



Effect of Dominance Constraints

1. If both solutions are infeasible, determine dominance by comparing **only constraints**.
2. Feasible solutions **always** dominate infeasible solutions.
3. If both solutions are feasible, determine dominance by comparing **only objectives**.

By applying dominance-constraint criteria …

… the behavior of the optimizer becomes:

1. The initial population is a flat random distribution over the variable space.
2. The initial population evolves into a feasible population, which should span the feasible variable space.
3. The feasible population evolves to optimize the objective functions.

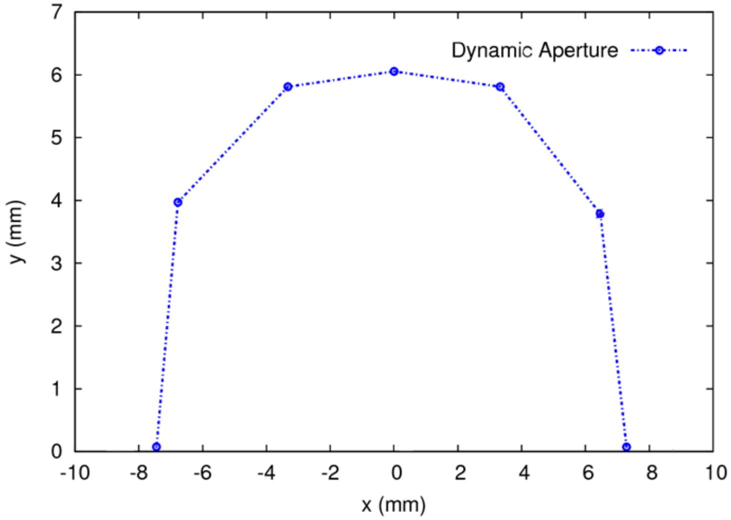
RANDOM POPULATION → **FEASIBLE POPULATION** → **OBJECTIVE-OPTIMIZED POPULATION**

Optimization Problem Preview

- 3 Objective Functions
  - On-energy DA to maximize injection efficiency
  - ±3% DA as proxy to maximize momentum aperture
- 7 Constraints
  - (Implicit: calculable linear optics at ±3%, ±5%)
  - Chromaticity
  - Magnet Strength
  - Nonlinear dispersion
  - Chromatic tune footprint
  - ADTS footprint
  - Linear aperture size
- Variables
  - Chromatic sextupoles
  - Harmonic sextupoles
  - Octupoles

Objectives

1. On-energy DA obj. is straightforward.



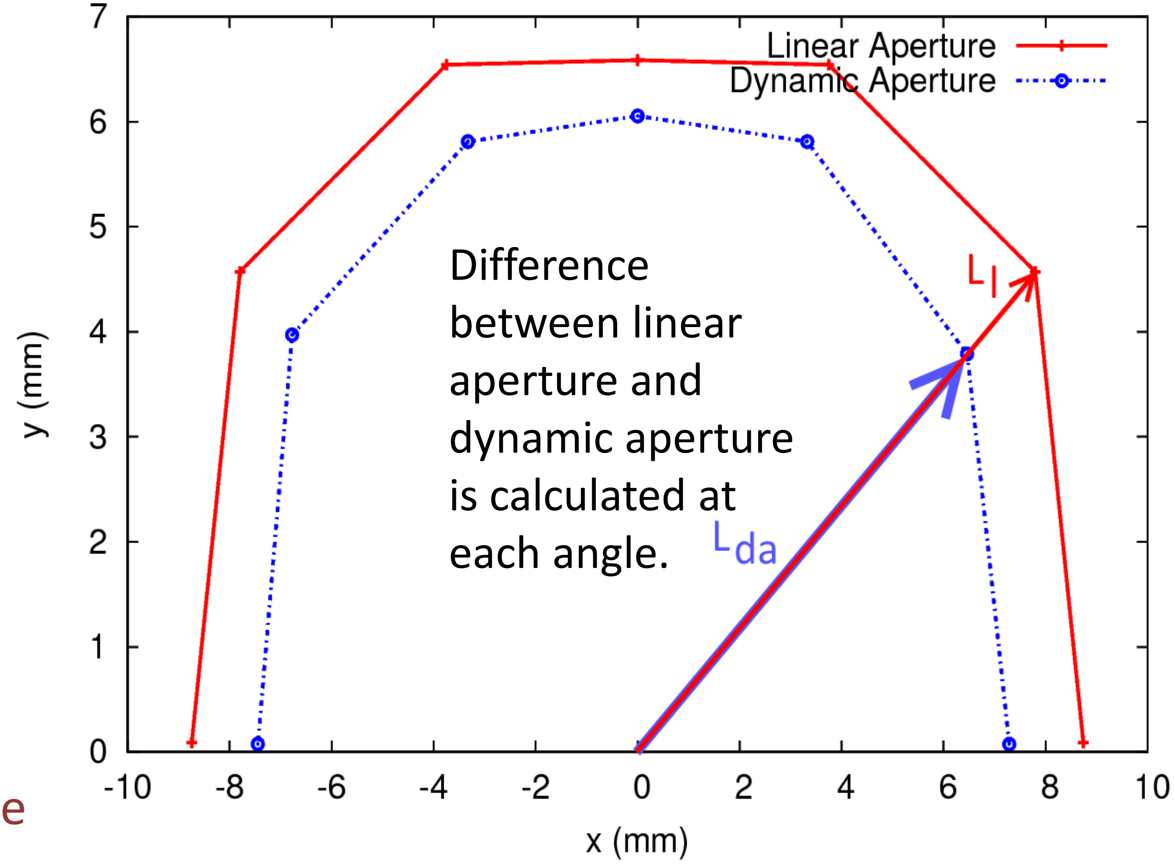
- Make this area as large as possible at the injection point, so as to maximize the phase space volume of particles the ring can capture and store.
- This also allows for more versatility in designing injection system.

2. Two off-momentum DA objectives as a proxy for MA.
    1. What we really want is maximize element-by-element momentum aperture, but that is expensive to calculate.
    2. Off-momentum DA objective along with chromatic tune footprint constraint used to indirectly maximize the MA.

Dynamic Aperture Objective

$$\min\ f(x) = \frac{1}{N_{angle}} \sum_{N_{angle}} \begin{cases} \left(\frac{L_l - L_{da}}{L_l}\right)^2, & \text{if } L_{da} < L_l \\ 0, & \text{otherwise} \end{cases}$$

- Comparison of DA to LA.
- Linear aperture is obtained analytically from transfer matrices.
  - LA is energy dependent!
- DA calculated using ele-by-ele tracking.
- No gain in merit for DA exceeding linear aperture dimension.
- Hard penalty on linear aperture size. Constrains nonlinear β.
  - typically 2.5 mm



Difference between linear aperture and dynamic aperture is calculated at each angle.

Particle called stable if tracked OK for 200 turns.

Constraints and Types

- Various types of constraints:
  1. Corrected chromaticity. }- Constrained variable space
  2. Magnet strength.
  3. Nonlinear dispersion. - Dominance constraints
  4. Chromatic tune footprint.
  5. Amplitude dependent tune shift.
  6. Linear aperture size. - Modified objective function

Chromaticity Constraint

- With $N_{sext} > 2$, There are infinitely many sextupole strengths with chromaticity set to particular χ_x, χ_y.
- Constrained variable space: Rather than act on $K_{2,chrom}$, the optimizer acts on the N_{sext}-2 subspace ω.
- All trial solutions generated by optimizer guaranteed to have the desired chromaticity.

Chromaticity response matrix:

$$\mathbf{A} = \begin{pmatrix} \frac{d\chi_x}{dK_1} & \frac{d\chi_y}{dK_1} \\ \frac{d\chi_x}{dK_2} & \frac{d\chi_y}{dK_2} \\ \vdots & \vdots \\ \frac{d\chi_x}{dK_N} & \frac{d\chi_y}{dK_N} \end{pmatrix}$$

Pseudoinverse of **A**

$$\vec{K} = A_p \begin{pmatrix} \chi_x \\ \chi_y \end{pmatrix} + Q_1 \omega$$

N-2 Vector.

Thin-QR decomposition of: $\mathbf{I} - \mathbf{A}_p\mathbf{A}$

Magnet Strength and Nonlinear η

- These two are implemented as dominance constraints.

1. Sextupole strength constraint is straightforward. Set by limited due to practical magnet limitations.
   - $K_2 < 500\ \mathrm{m}^{-3}$ (Integrated strength $50\ \mathrm{m}^{-2}$)

2. Nonlinear dispersion constraint
   - Keeps off-momentum DA centered

$$\max\left(x_i - \eta_{i,x}\Delta E\right)$$

maximum over all elements *i*

closed orbit *x* at element *i*

dispersion at element *i*

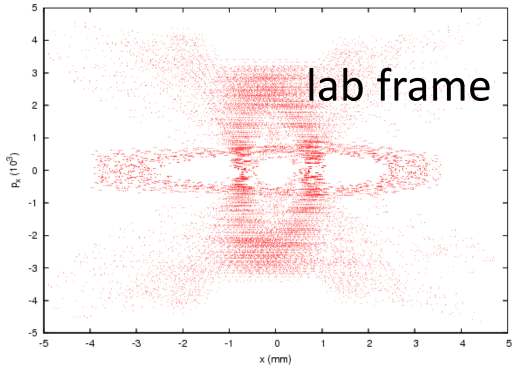
Energy deviation. Typically ±3%.

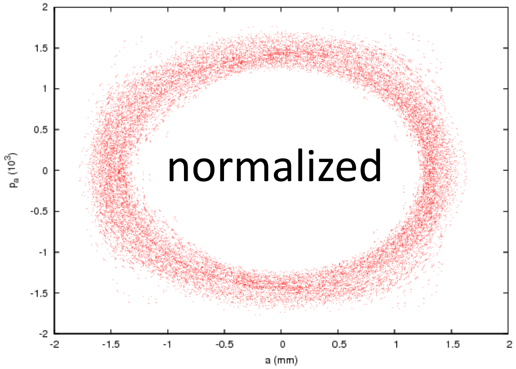
Constrain Chromatic Tune Footprint

- Also implemented as dominance constraint.
- Quick & easy to calculate: off-momentum tune footprint obtained from linear off-momentum optics.
  - Calculated in ~1% steps.
- Neatly confined +/- 5% tune footprint necessary to obtain good +/- 5% momentum aperture.
- Typically constrained to a half-integer box.
- Chromatic tune also often is constrained indirectly by other constraints and objectives.

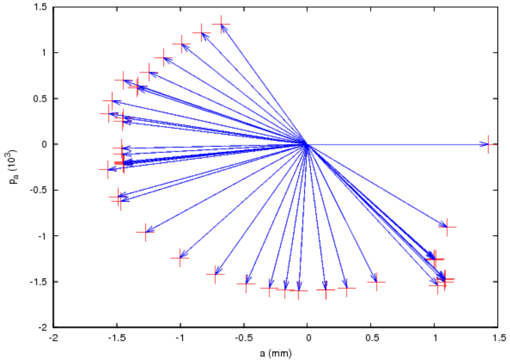
Amplitude Dependent Tune Shift

- Particle tune changes for large action particles, due to large amplitude oscillations.
- Tracking simulations on an ideal lattice fail to capture this source of particle loss.
- Particle tune at DA is calculated by normalizing trajectory and totaling ele-by-ele phase advance per turn, averaged over many turns.
  - Results agrees with DFT to arbitrary precision. Captures integer part and seems to be more robust against false peak detection.





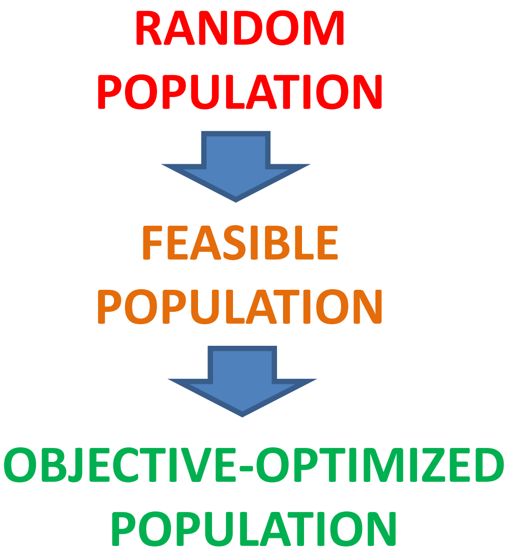
- ADTS at DA is typically constrained to a half-integer box.



Example phase advance per element, for a few elements.

Optimization Problem Review

- Variables:
  1. N-2 subspace of chromatic sextupole strengths that gives desired chromaticity.
  2. Harmonic sextupoles.
  3. Strength of octupole families



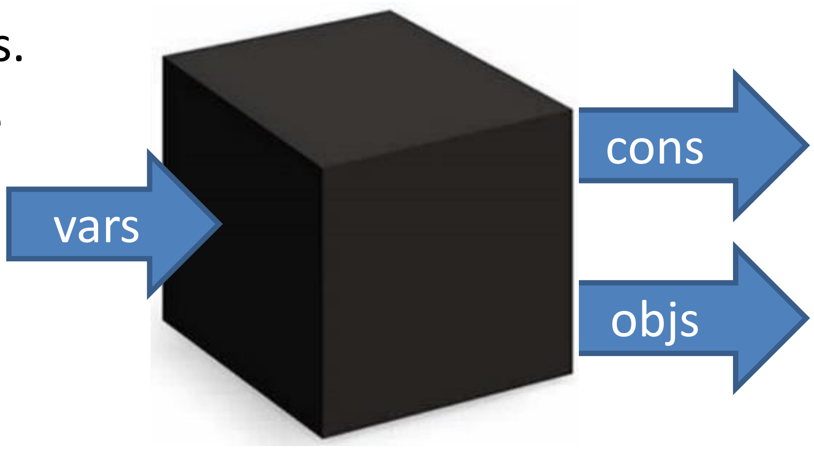
RANDOM POPULATION

FEASIBLE POPULATION

OBJECTIVE-OPTIMIZED POPULATION

Dominance Constraints (7 in total)

1. Magnet strength.
2. Nonlinear dispersion x2
   - + ΔE and - ΔE
3. Chromatic tune footprint x4
   - Qx and Qy at + ΔE and - ΔE

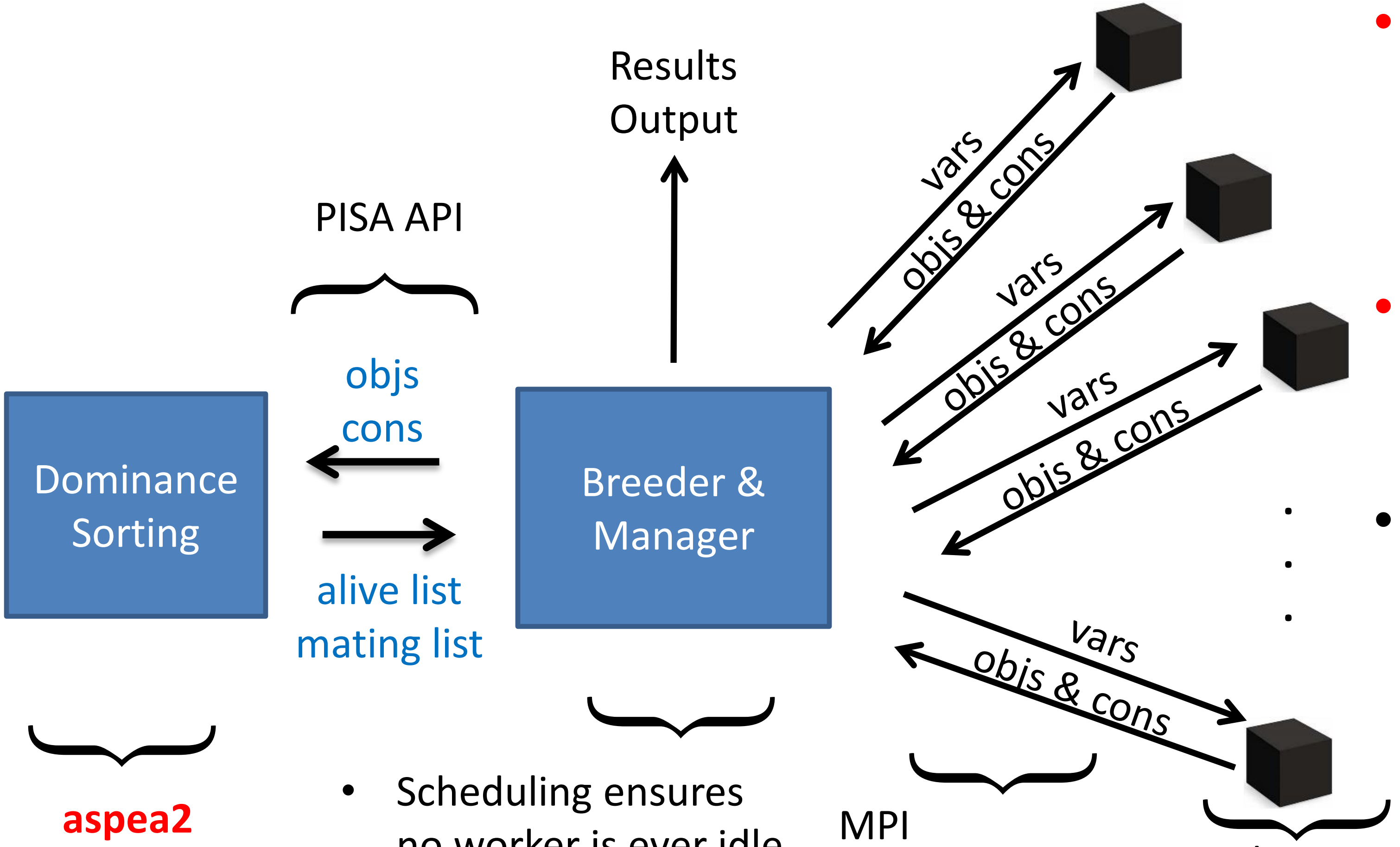


Objectives (try to make DA no smaller than LA):

(all 3 modified by hard linear aperture constraint)

1. On-momentum DA.
   - Confined by ADTS to half-integer box
2. +3% ΔE DA.
3. -3% ΔE DA.

System & Data Flow



Results Output

PISA API

objs
cons

alive list
mating list

Dominance Sorting

Breeder & Manager

vars

objs & cons

aspea2 sorting algorithm

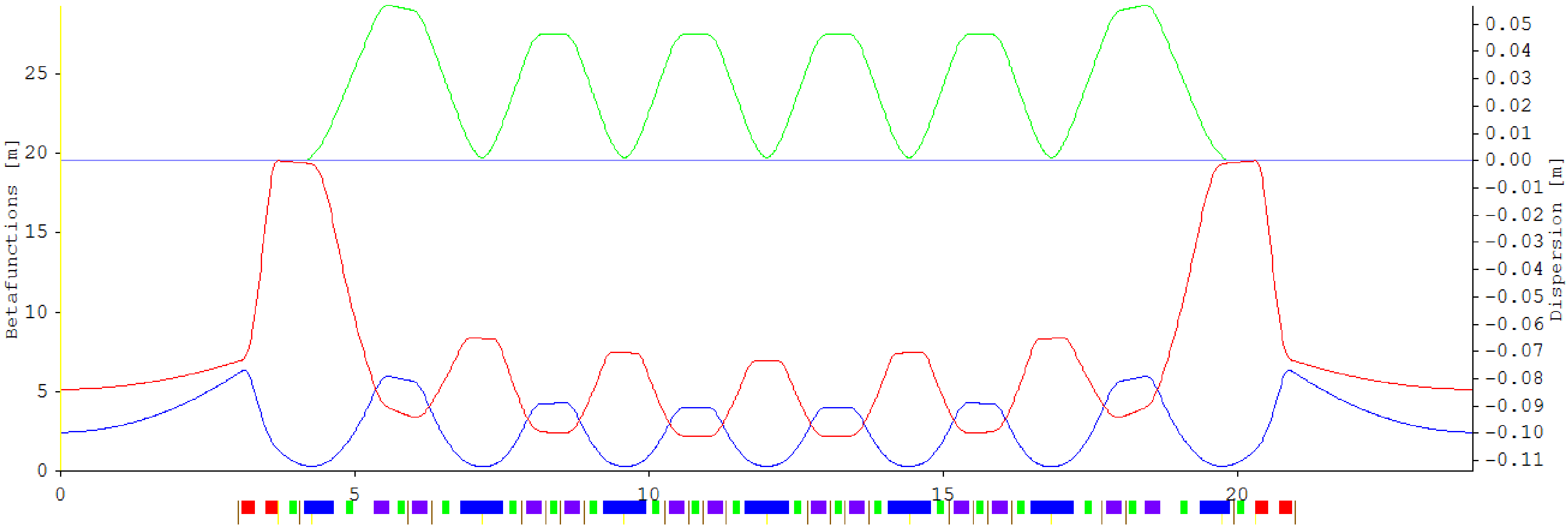
- Scheduling ensures no worker is ever idle.
- Inherent load balancing, heterogeneous environments OK.

MPI Parallelization.

Accelerator calculations coded in Fortran 90 using **Bmad subroutine library**.

- **64 off-the-shelf Intel cores** on Linux.
- **8 to 18 hours to converge**
- ~23,000 genes evaluated.

ca05s Prototype



- “The periodicity 12 prototype”
- $Q_x, Q_y = 37.28, 9.12$
- $\chi_x, \chi_y = -61.08, -46.15$
- Horizontal Emittance 126 pm
- Momentum Compaction $-9.6\ 10^{-5}$
- 5 chromatic sextupole families
- 1 harmonic sextupole family
- 4 octupole families

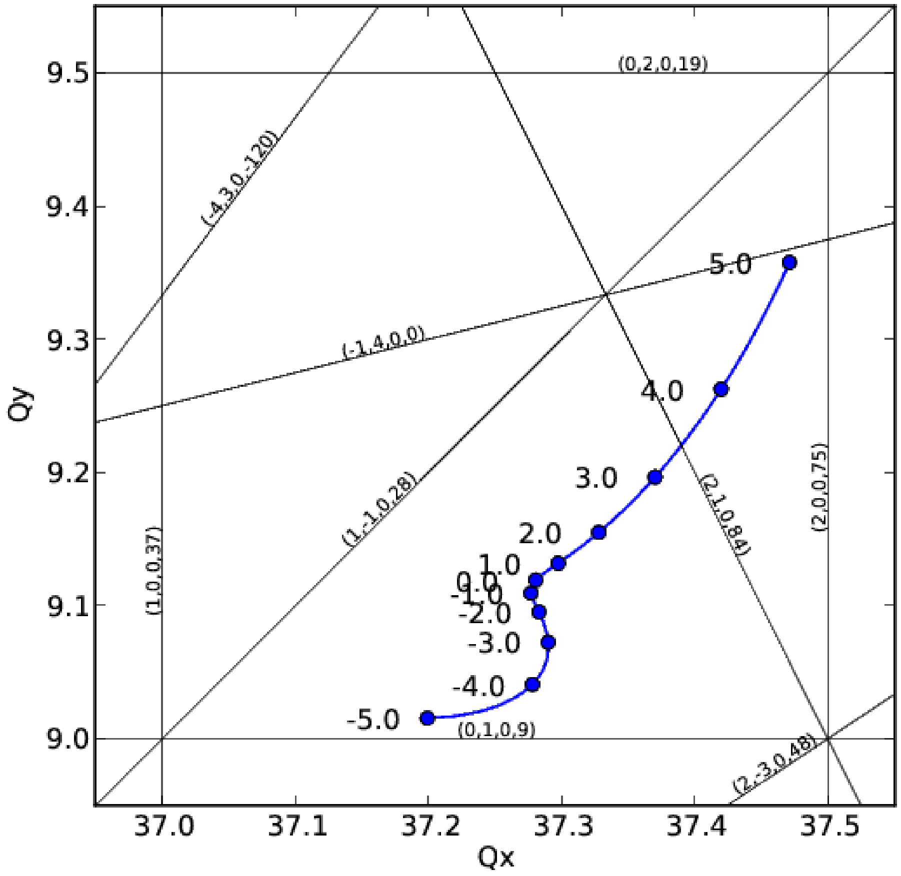
What makes ca05s special

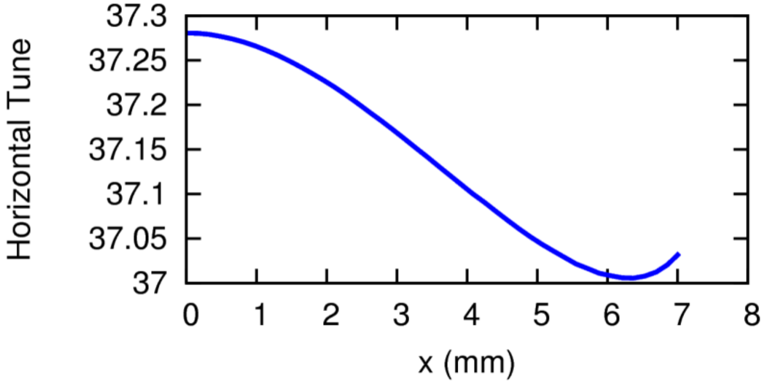
- 12-fold periodicity
  - quieter tune plane
- Smaller natural chromaticity to correct.

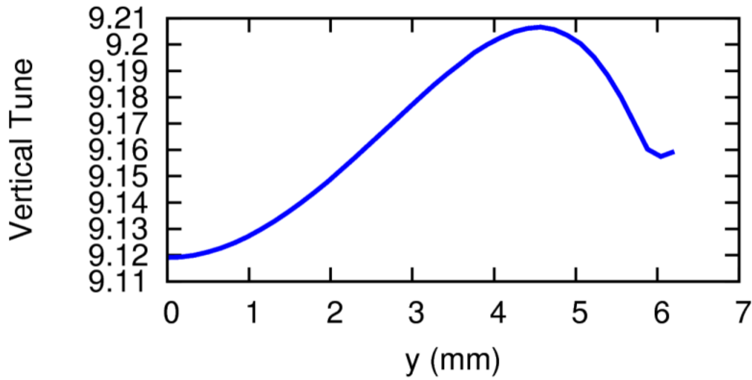
ca05s Tune Footprint Results

- Example chromatic footprint and ADTS footprint from completed optimizer run.
- Result is typical of the final population.
- Important point: Tunes are neatly steered away from constraints.

+-5% Chromatic Footprint with low order and allowed higher order resonance lines.





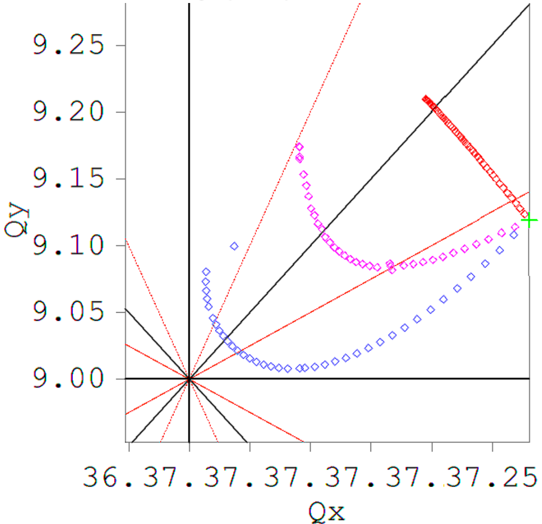


ADTS:

Blue along +x

Magenta along +x,+y

Red along +y



ca05s Results

- On- and off-momentum DA improvement.

Touschek Lifetime

Linear: 3.686 h

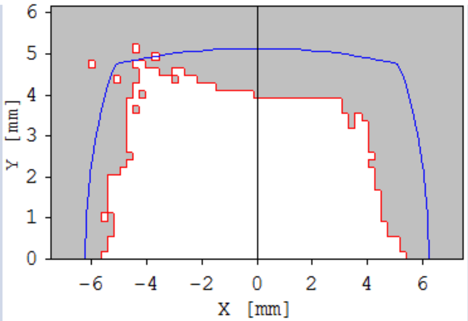
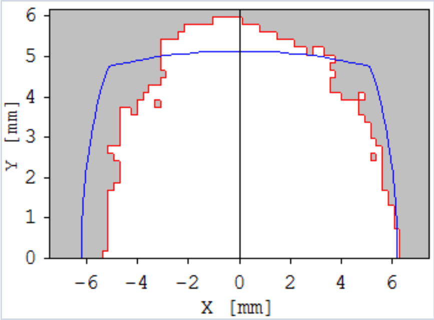
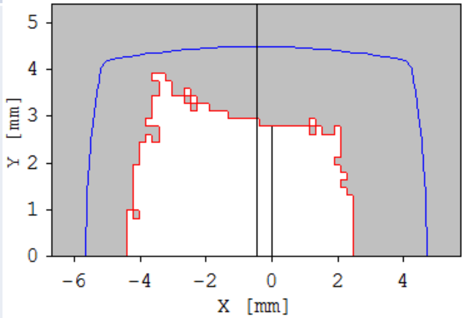
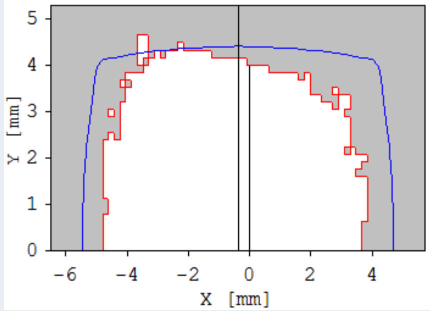
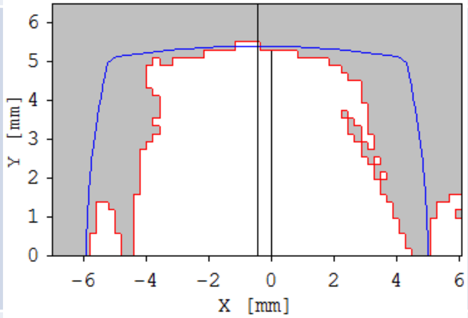
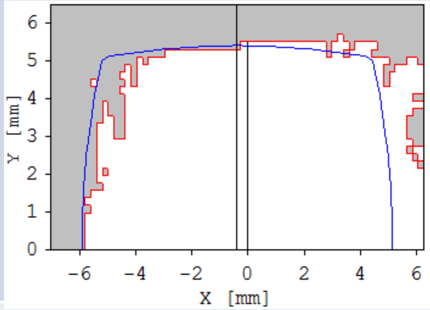
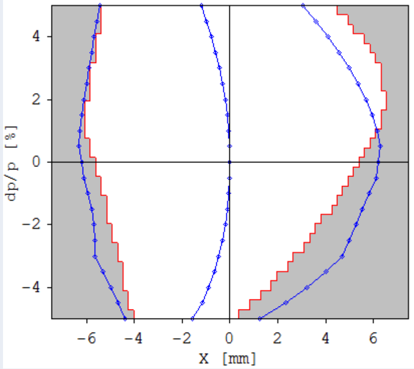
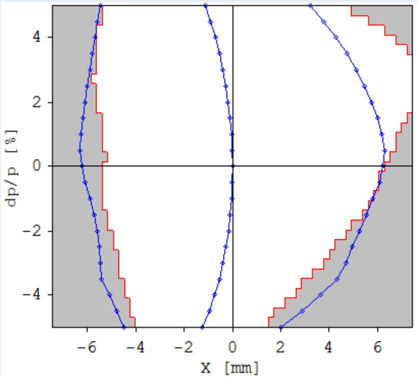
PT: 3.475 h

GO: 3.636 h

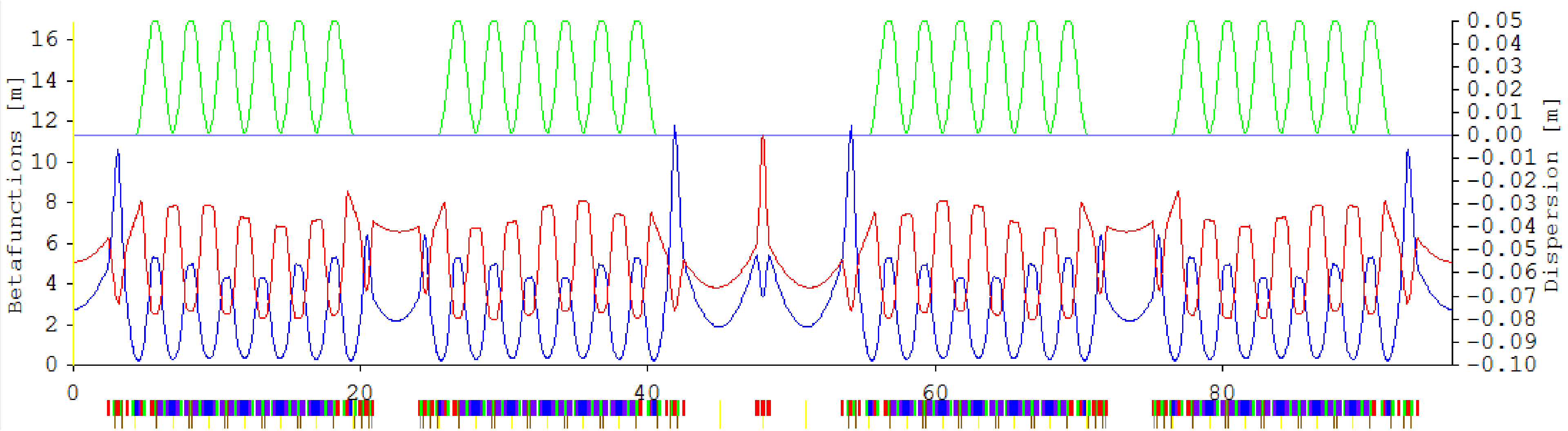
Good recovery of linear Touschek lifetime for this lattice.

- ±ΔE DA + FP cons.

- Note: 3HC cavity left out of lifetime calculation.
- Constraint on non-linear dispersion prevents too much curvature in DA center.

ΔE	Pertubation Theory	Genetic Optimizer
0.0%		
-3.0%		
3.0%		
vs. dE		

db02b Prototype



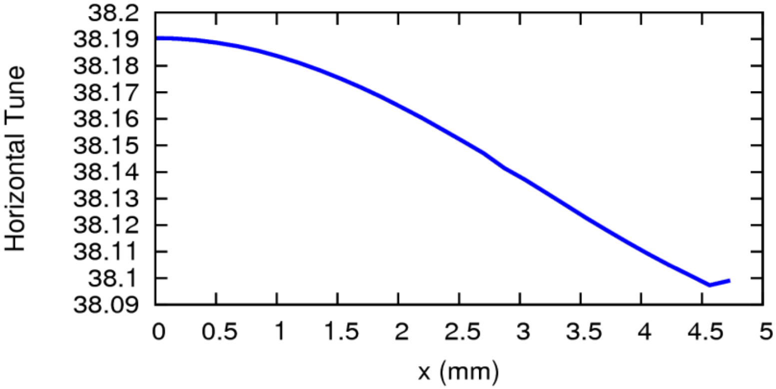
- Periodicity 3
- Q_x, Q_y = 38.19, 10.33
- χ_x, χ_y = -69.67, -34.16
- Horizontal Emittance 132 pm
- Momentum Compaction -1 10^{-5}
- 4 chromatic sextupole families
- 3 harmonic sextupole families
- 4 octupole families

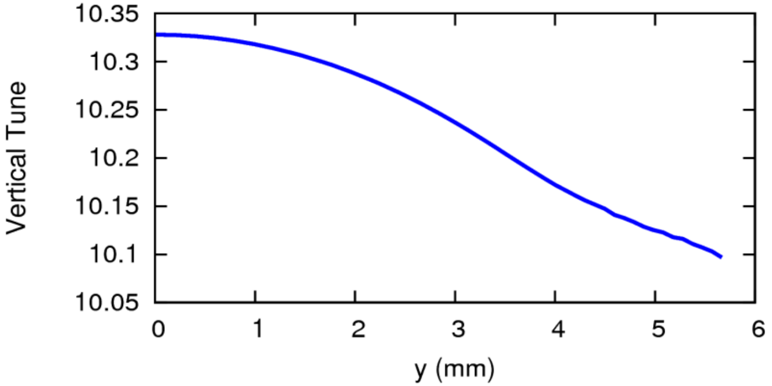
<u>What makes db02b special</u>

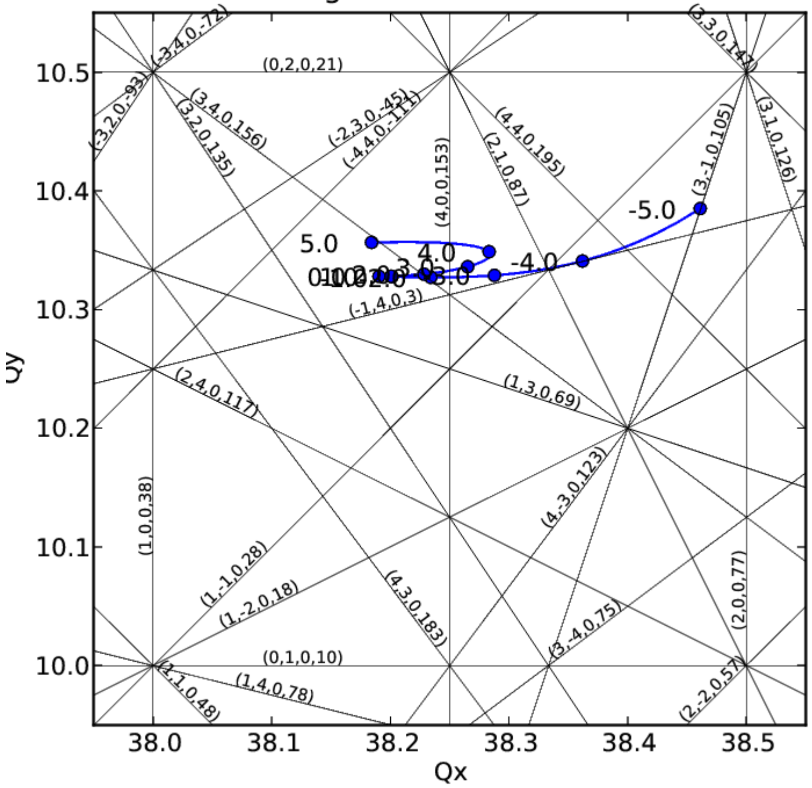
- Nonlinear optics more difficult than with periodicity 12 lattice.
- db02b lattice fits neatly into existing hall, whereas ca05s would require jackhammers.

db02b Tune Footprint Results

- Tune footprints do not have the same pronounced high order corrections.
- Fewer resonances are excluded at periodicity 3.
- Good location in tune plane difficult to find.







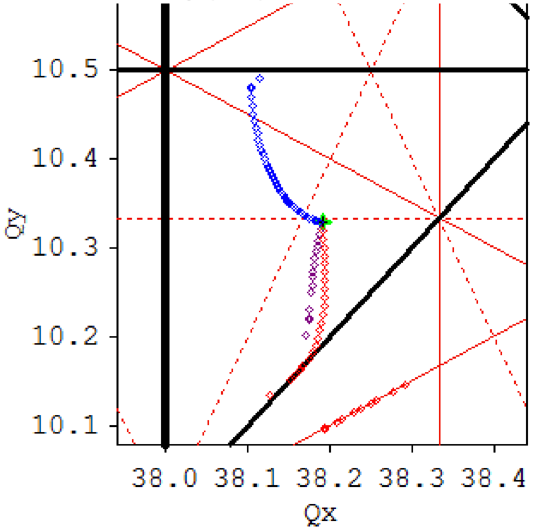
+-5% Chromatic Footprint with low order and allowed higher order resonance lines.

ADTS:

Blue along +x

Magenta along +x,+y

Red along +y



db02b DA Results

- Perturbation theory result not available.

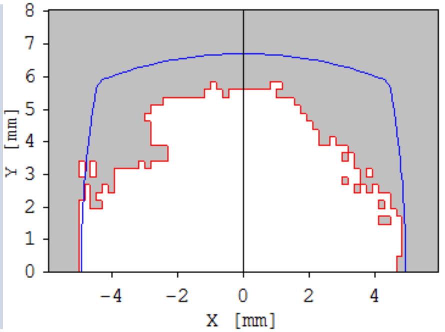
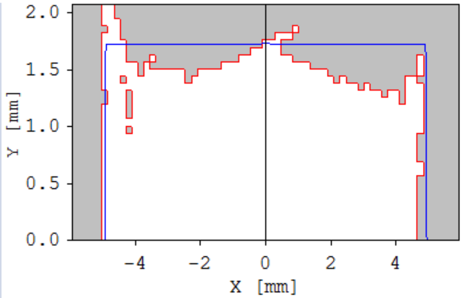
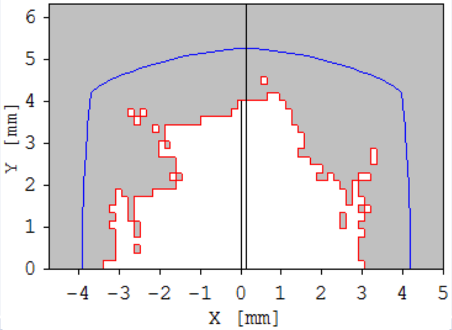
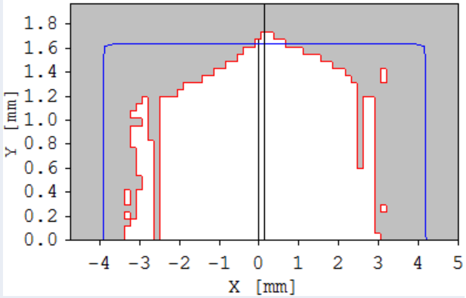
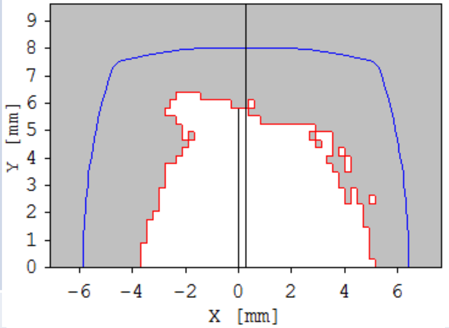
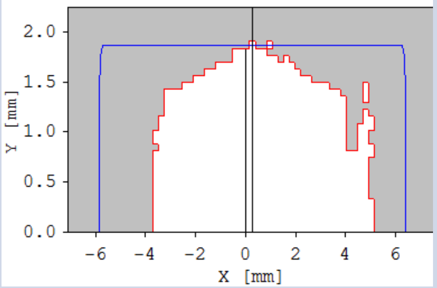
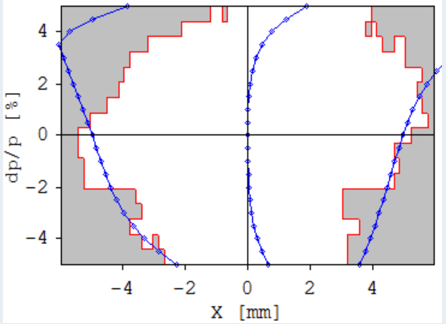
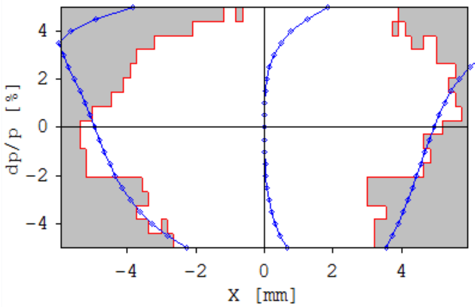
- Period 3 lattices have, in general, been more difficult to optimize.

Touschek Lifetime

Linear: 4.654 h

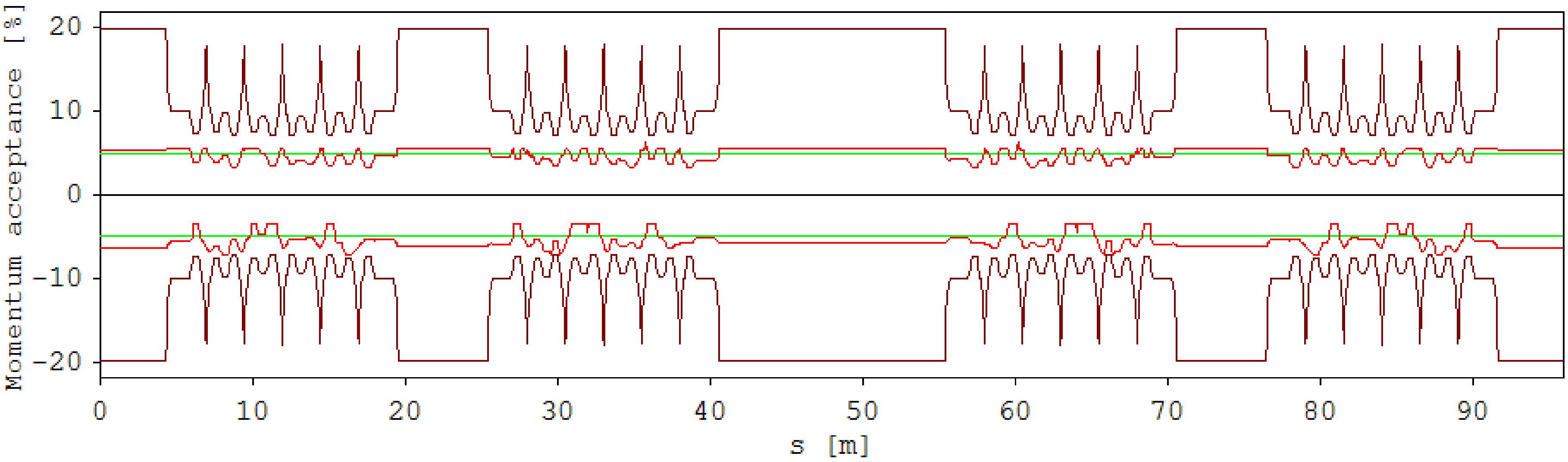
GO: 4.056 h

- Recovery of Touschek lifetime is not as good, however, possibly ok if minigap undulators taken account.

ΔE	Round (10 mm rad)	With Undulator Aper.
0.0%		
-3.0%		
3.0%		
vs. dE		

db02b Momentum ApertureResults

- Optimization of off-energy DA, along with constrained tune footprint, allows chromaticity correction with momentum aperture close to that imposed by RF.



<u>Touschek Lifetime</u>

Linear: 4.654 h

GO: 4.056 h

Conclusions

- A Multi-Objective Genetic Optimizer has been implemented for optimizing SLS2 nonlinearities.
  - It finds an improvement in dynamic aperture and Touschek lifetime vs. perturbation theory.
  - Direct constraint of chrom & ADTS tune footprints.
  - Requires only modest computing resources.
    - ~12 hour development cycle on known lattices
  - Appears to consistently locate global minimum.
- Off-momentum DA objective, along with off-momentum tune footprint constraint, can be an efficient & effective way to optimize Touschek lifetime.