Non-global logarithms and BFKL beyond the leading order

Simon Caron-Huot

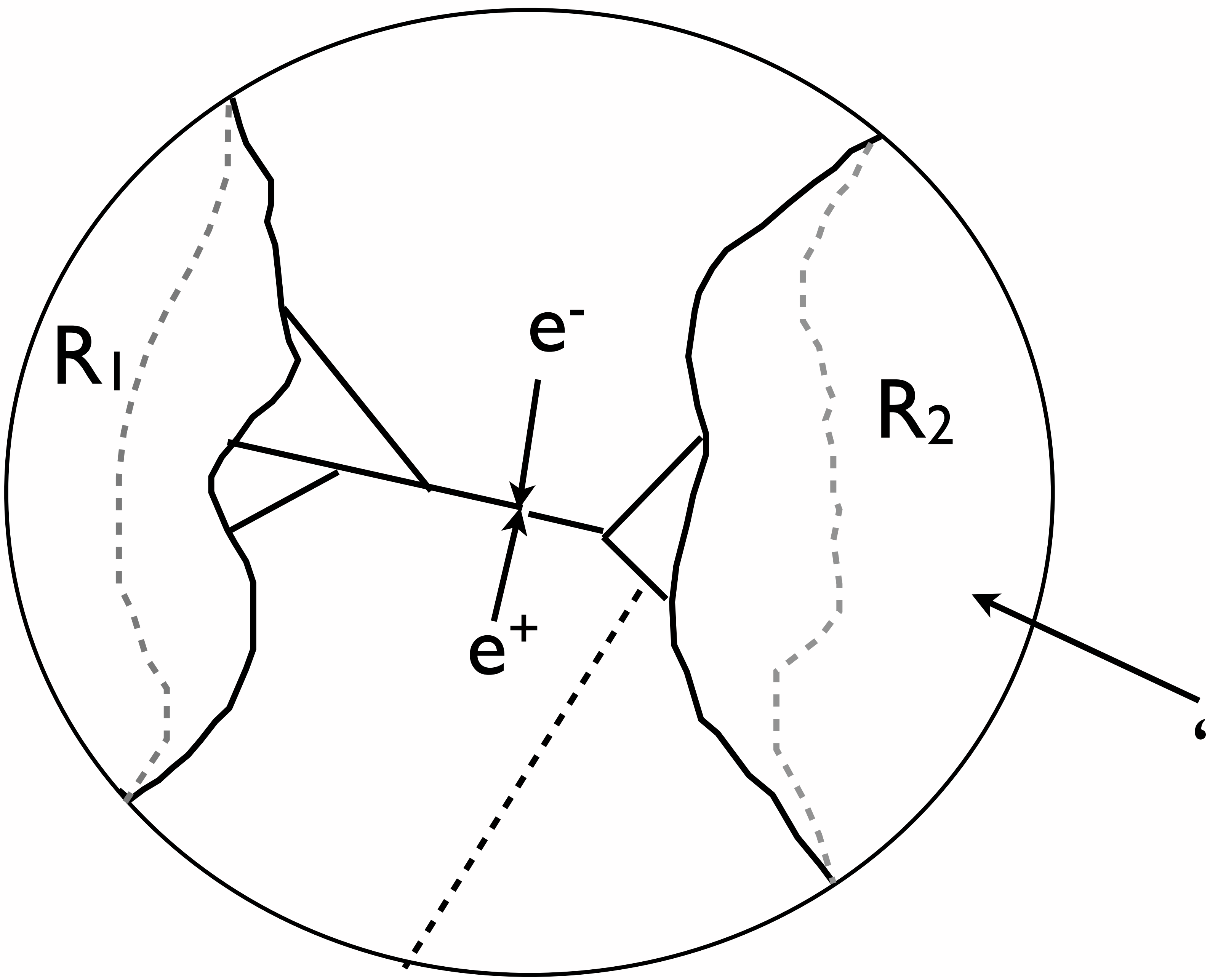
(Niels Bohr International Academy, Copenhagen)

(based on: ph/1501.03754)

['Parton Showers and Resummation 2015', Krakòw, May 28th, 2015]

Setup: a simple question

What is the cross-section for $e^+e^- \to X$,
with energy $<Q_0$ outside some region R ($R=R_1 \cup R_2$)



Allowed 'potato-shaped' regions

Motivation

- Study how exclusion boundaries affect soft gluons
- Same physics affects many ‘non-global’ event shapes:
  -`Away from jet energy flow’/`Gaps between jets’
  -Hemisphere mass function
  -Possibly: Cone jets (see: T.~ Becher’s talk)
- Learn to deal with color coherence at finite Nc
- Soft gluons are interesting

Simplifying features:

- IR and collinear safe observable
- Only two scales ($\rightarrow$one log: Q/Q_0)
- No collinear logs, only soft (wide angle) logs
- e^+e^- : No ISR [avoids nonabelian Coulomb phases]

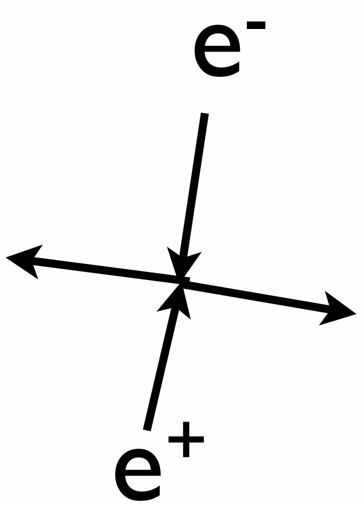
Simplifying features:

- IR and collinear safe observable
- Only two scales (→one log: Q/Q_0)
- No collinear logs, only soft (wide angle) logs
- e^+e^- : No ISR [avoids nonabelian Coulomb phases]

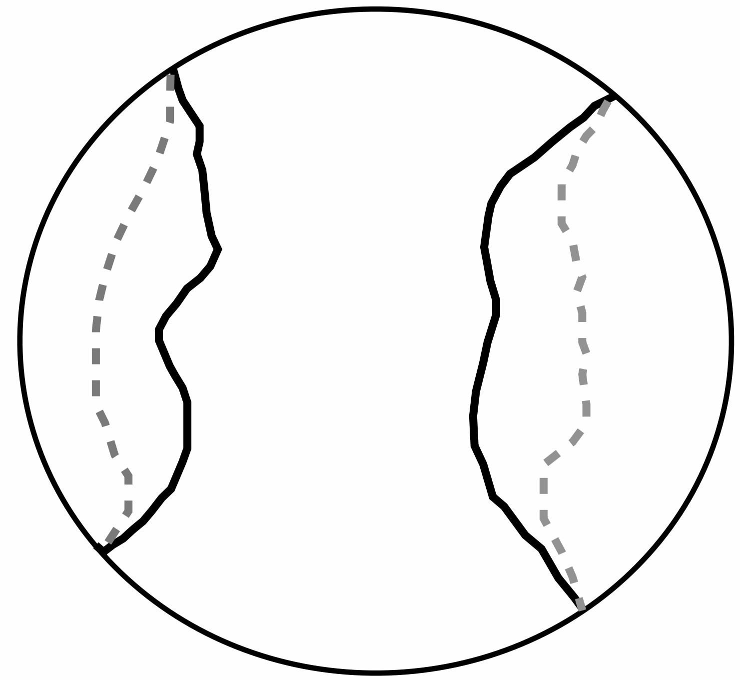
Complicating feature(s):

- Will work exactly in $1/N_c$ [and beyond LL]

What we're looking for:



$$\otimes \left[\mathcal{P} e^{-\int_{Q_0}^{Q} \frac{d\lambda}{\lambda} K(\lambda)} \right] \otimes$$

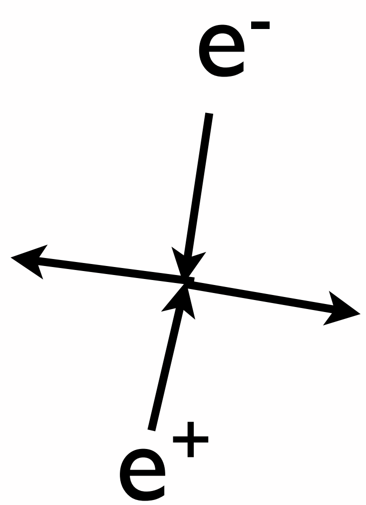


Hard process
@Q

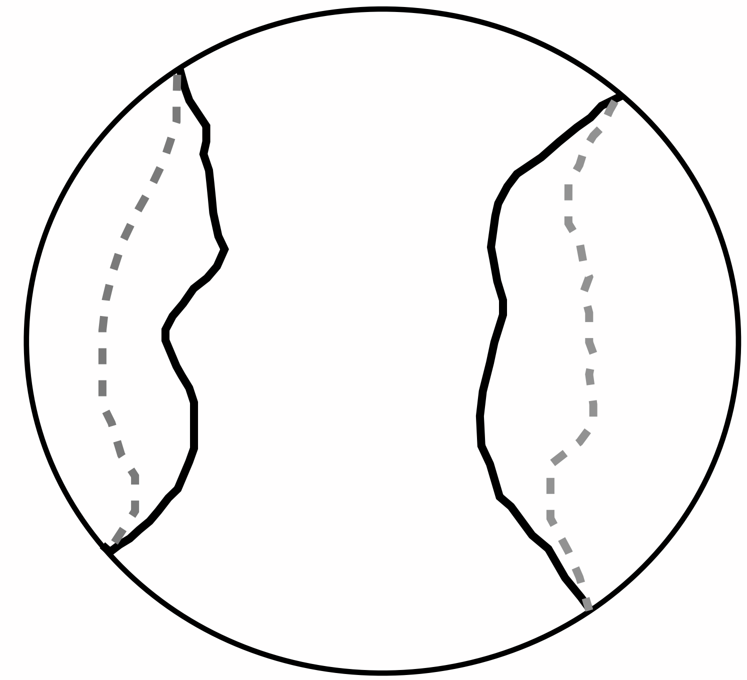
Universal (RG)
evolution

Veto
@ IR scale Q_0
[cf Z. Nagy's talk]

What we're looking for:



$$\otimes \left[\mathcal{P} e^{-\int_{Q_0}^{Q} \frac{d\lambda}{\lambda} K(\lambda)} \right] \otimes$$



Hard process
@Q

Universal (RG)
evolution

Veto
@ IR scale Q_0
[cf Z. Nagy's talk]

What is K? What data must it act on?

Key tools:

- Factorization of soft emissions (@amplitude level):

$$\lim_{k\to 0} H_{n+1}(k, \{p_1\}) = \mathcal{S}(k) H_n(\{p_1\})$$

$$\mathcal{S}(k) = g\epsilon_\mu \sum_i \frac{p_i^\mu T_i^a}{p_i \cdot k} + O(g^3)$$

- Bookkeeping device for multicolor states:

'color density matrix;'
depends on SU(N_c) matrices

Key tools:

- Factorization of soft emissions (@amplitude level):

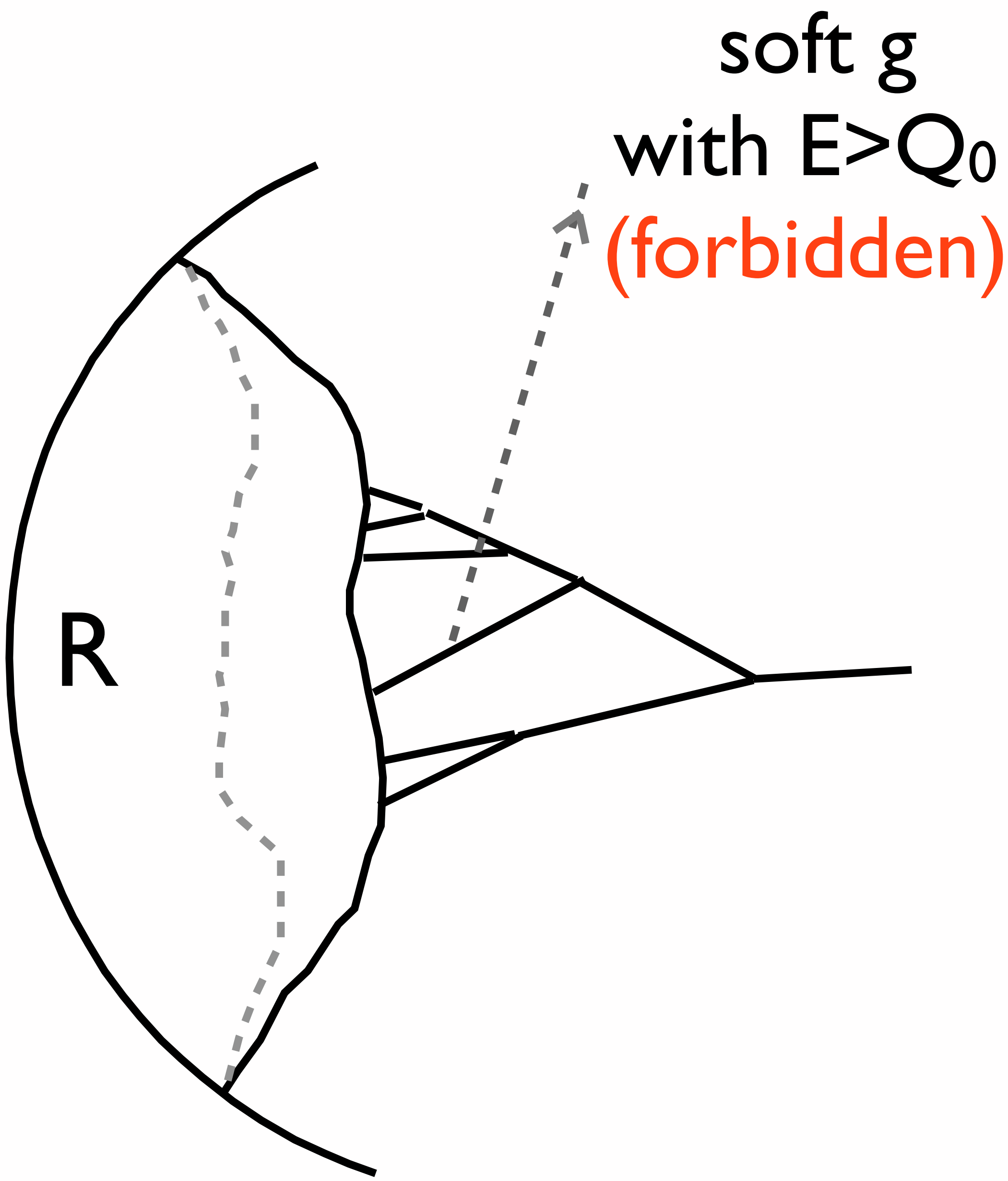
$$\lim_{k\to 0} H_{n+1}(k,\{p_1\}) = \mathcal{S}(k) H_n(\{p_1\})$$

$$\mathcal{S}(k) = g\epsilon_\mu \sum_i \frac{p_i^\mu T_i^a}{p_i \cdot k} + O(g^3)$$

- Bookkeeping device for multicolor states:

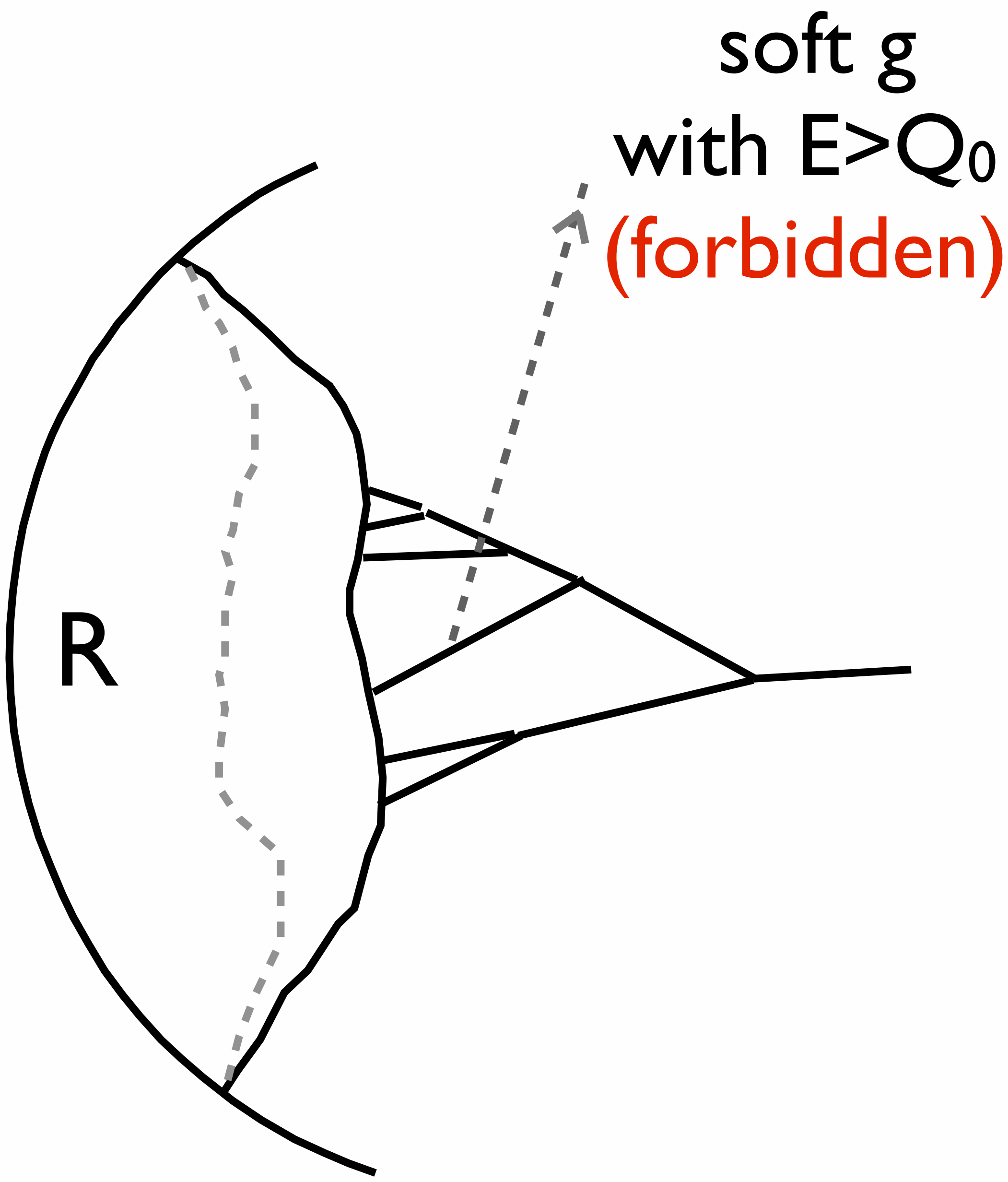
'color density matrix;'
depends on SU(N_c) matrices $U(\theta)$

The physics



Probability of
not radiating g outside R,
depends on
all gluons inside R!

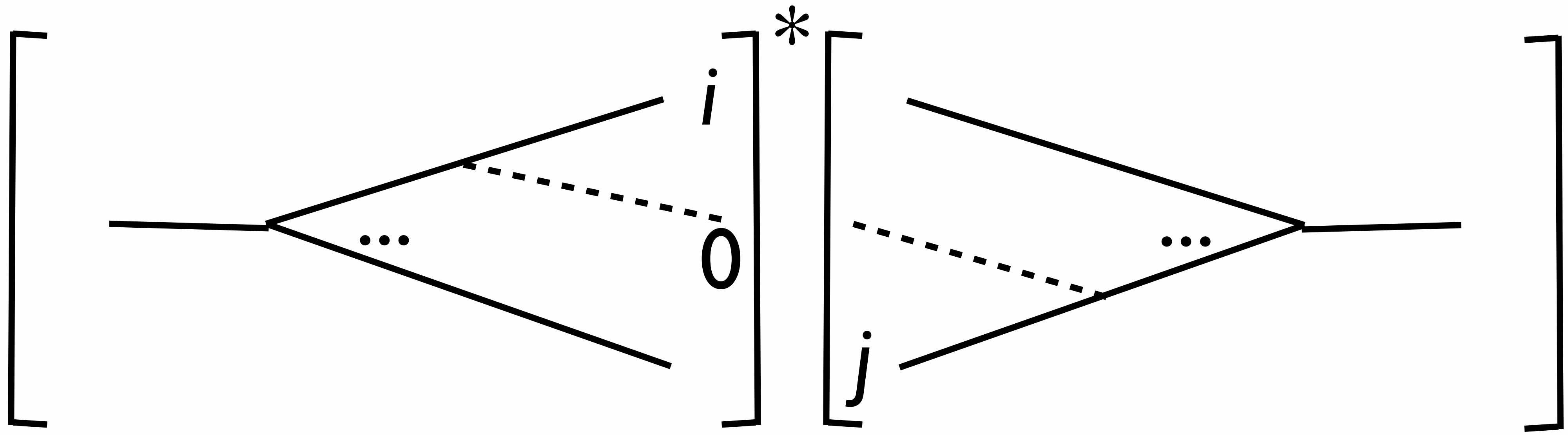
The physics



Probability of
not radiating g outside R,
depends on
all gluons inside R!

Need to keep track of
angle and color of all
harder gluons
(that's a minimum!)

U's: a bookkeeping device

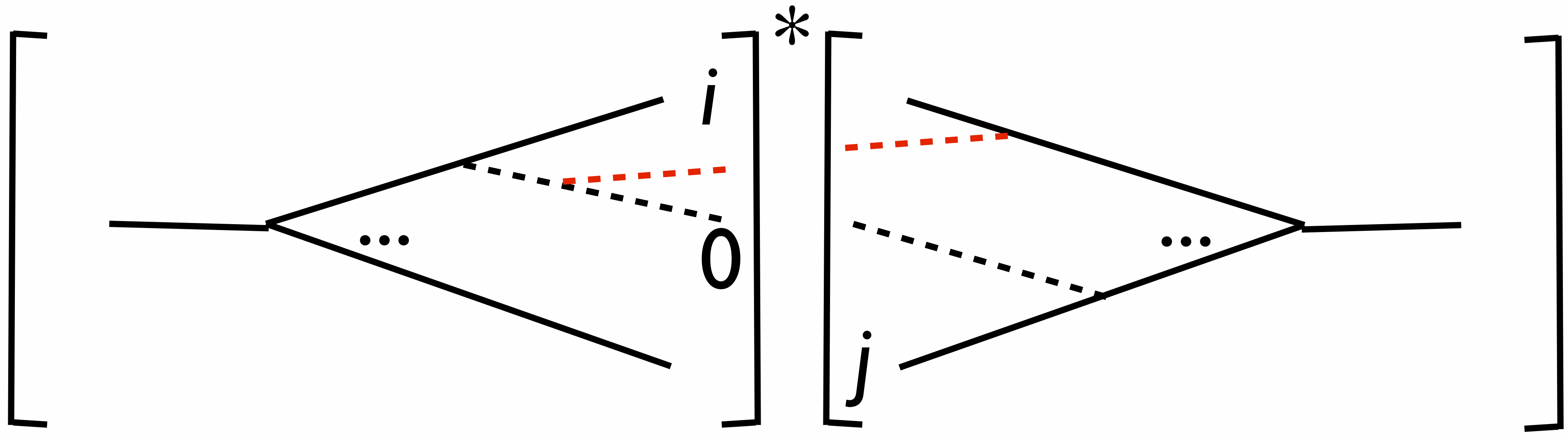


Squared amplitude for soft gluon '0' is proportional to:

$$\langle \text{left}| T_i^a \qquad T_j^a |\text{right}\rangle$$

Color rotation of harder partons i and j

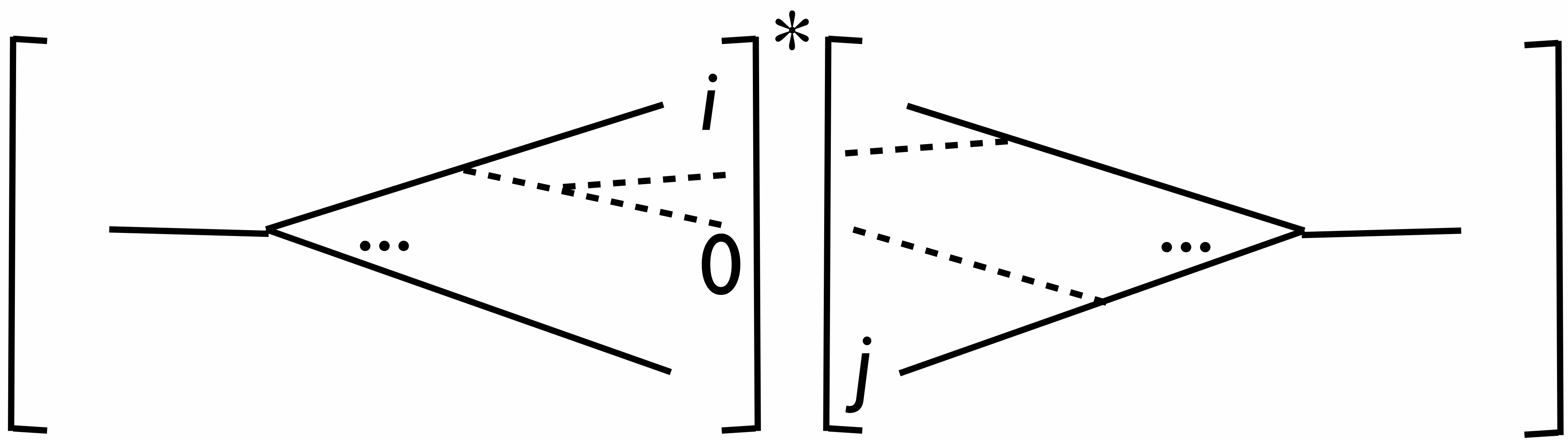
U's: a bookkeeping device



Subsequent emissions can rotate '0' itself:

$$\langle \mathrm{left}| T_i^a T_0^b \qquad T_i^b T_j^a |\mathrm{right}\rangle$$

U's: a bookkeeping device

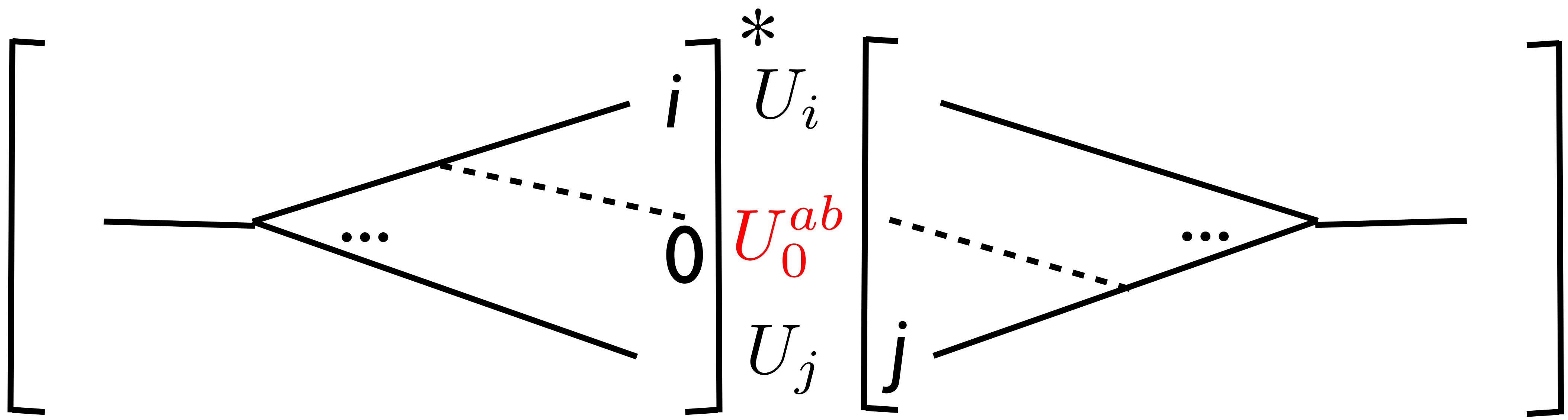


Subsequent emissions can rotate '0' itself:

$$\langle \text{left} | T_i^a T_0^b \qquad T_i^b T_j^a | \text{right} \rangle$$

Let: U = Cumulative rotation from all subsequent emissions

U's: a bookkeeping device



U_i U_0^{ab} U_j

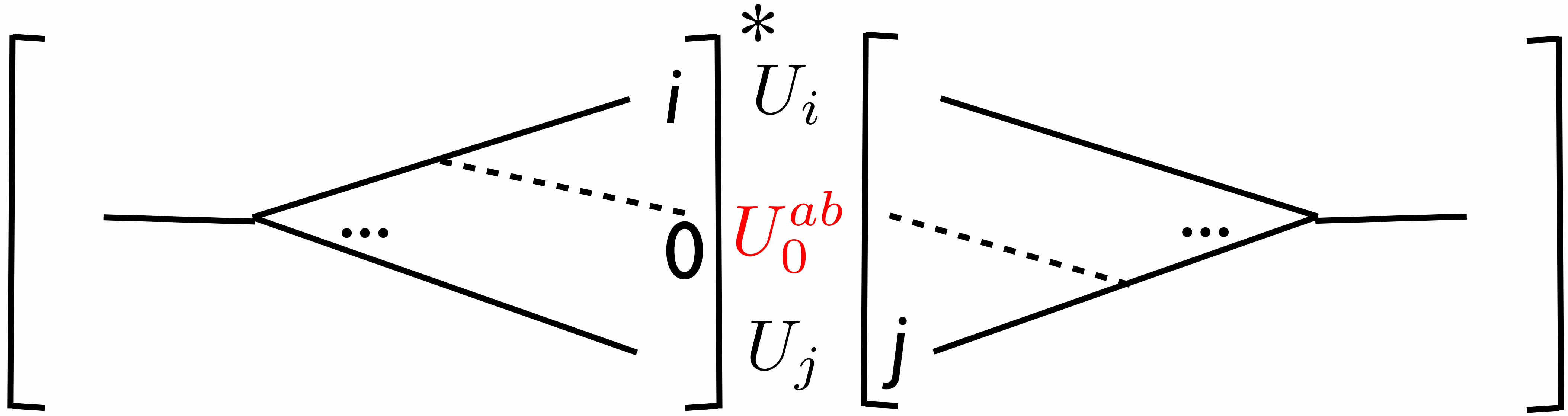
Subsequent emissions can rotate '0' itself:

$$\langle \text{left}| T_i^a T_0^b \qquad T_i^b T_j^a |\text{right}\rangle$$

Let: U = Cumulative rotation from all subsequent emissions

Emission of '0': $U_i U_j \to (T^a U_i) U_0^{ab} (U_j T^b)$

U's: a bookkeeping device



$$\left[\cdots \right]^{*} \begin{matrix} U_i \\ U_0^{ab} \\ U_j \end{matrix} \left[\cdots \right]$$

Subsequent emissions can rotate '0' itself:

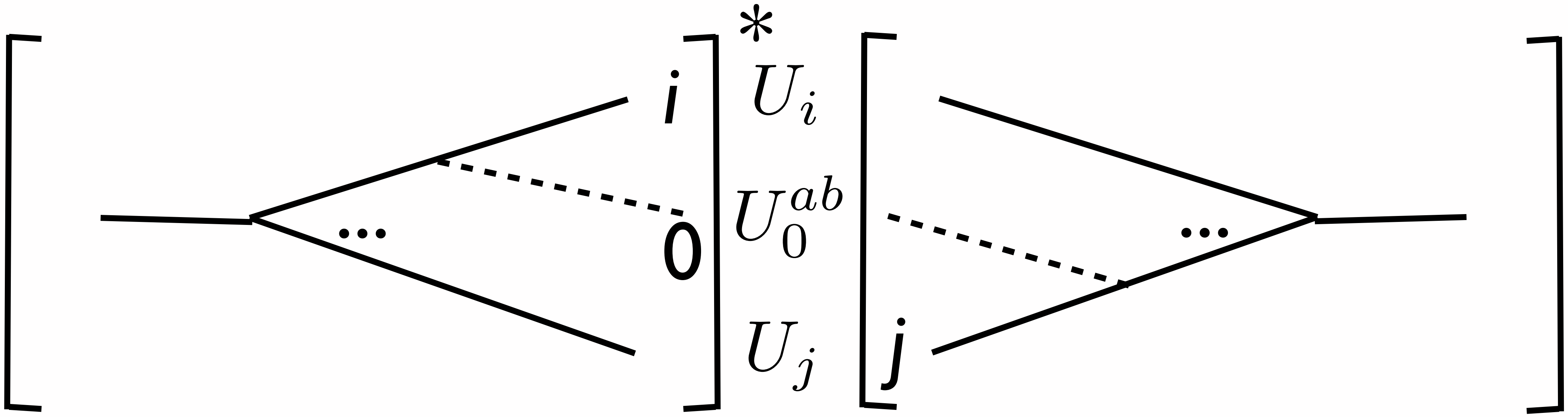
$$\langle \text{left} | T_i^a T_0^b \qquad T_i^b T_j^a | \text{right} \rangle$$

Let: U = Cumulative rotation from all subsequent emissions

Emission of '0': $U_i U_j \to (T^a U_i) U_0^{ab} (U_j T^b)$

Now subsequent emissions can act on U_0!

U's: a bookkeeping device



Subsequent emissions can rotate '0' itself:

$$\langle \text{left}| L_i^a \;\; U_0^{ab} \;\; R_j^a |\text{right}\rangle$$

Let: U = Cumulative rotation from all subsequent emissions

Emission of '0': $U_i U_j \to (U_i T^a) U_0^{ab} (U_j T^b) \equiv L_i^a U_0^{ab} R_j^b \, (U_i U_j)$

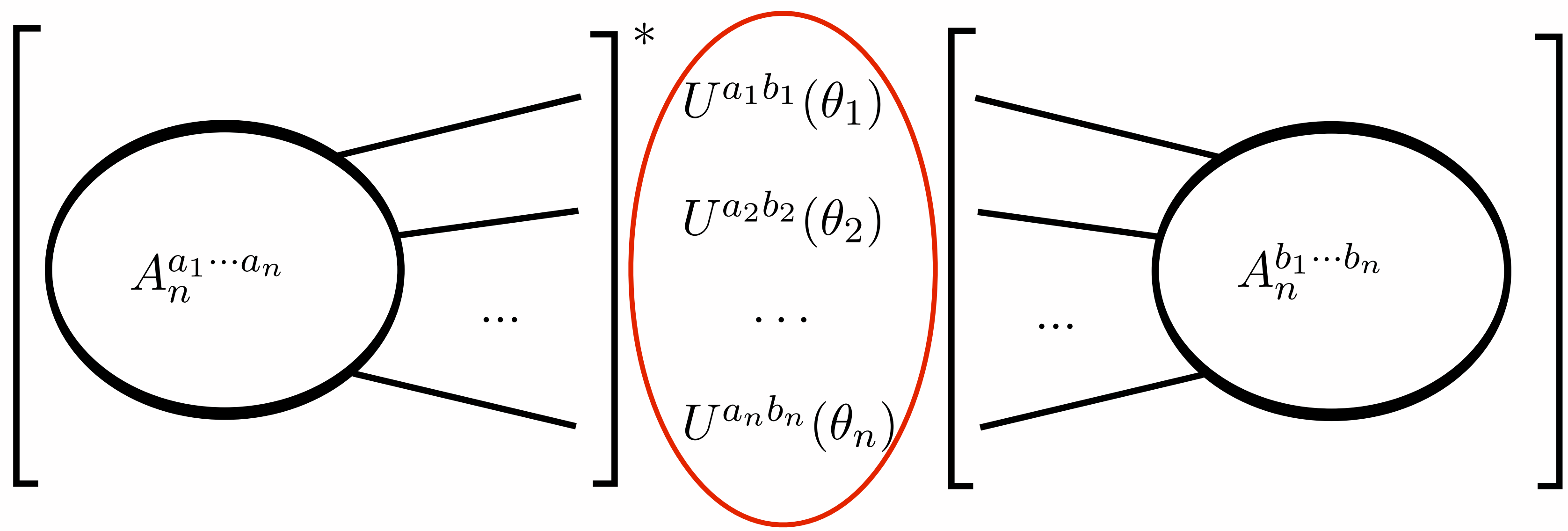
Efficient notation:

R acts on matrix element, *L* on conjugate

Formal definition

- Physically, U_i should represent color rotation from all softer, but resolved partons with E>μ
- Cutoffs make life hard. Standard technique: take cutoff to zero and renormalize with MSbar.
- Bare $\sigma[U]$: dress each final state with a color rotation:

$$\sigma[U] \equiv \sum$$



$$\sigma[U] \equiv \sum_n \int d\Pi_n \Big[A_n^{a_1 \cdots a_n}(\{p_i\})\Big]^* U^{a_1 b_1}(\theta_1) \cdots U^{a_n b_n}(\theta_n) \Big[A_n^{b_1 \cdots b_n}(\{p_i\})\Big] u(\{p_i\}) \,.$$

- Bare color density matrix IR divergent. Assuming RG eq., can renormalize:

$$\sigma^{\text{ren}}[U;\mu] = \mathcal{P}\exp\left[\int_0^\mu \frac{d\lambda}{\lambda} K(\lambda, \alpha_s(\lambda))\right] \sigma[U]$$

Then σ^{ren} will be finite.

- Precisely, U[Θ_i] is the color rotation experienced by a resolved parton at Θ_i if all radiation below μ is unresolved (so only modes > μ contribute)
- RG equation is expected physically, due to decoupling of radiation at vastly different scales (direct combinatorial proof in paper)

LL example:

- 1. Start from a dijet at scale Q

$$\sigma[U;Q] = \mathrm{Tr}\,[U^\dagger(\theta_1)U(\theta_2)] + O(\alpha_s)$$

- 2. Evolve down to scale Q_0 using RG:

$$[\mu\partial_\mu + \beta\partial_{\alpha_s}]\,\sigma[U;\mu] = K\sigma[U;\mu]$$

With each action of *K* one new resolved gluon U can be added

- 3. Kill all resolved gluons outside R by averaging procedure:

$$\langle U^{ab}(\theta)\rangle = \delta^{ab}\chi_{\theta\in R}$$

Evolution operator

- Extract from 1/eps pole in one-loop fixed-order
- Real: $$\sigma^{(1)}[U] \supset \sum_{i,j} \int \frac{d^3 p_0}{2E_0} \frac{p_i \cdot p_j}{p_0 \cdot p_i \, p_0 \cdot p_j} \times L_i^a U_0^{ab} R_j^b$$ $$\to \frac{1}{\epsilon_{\mathrm{IR}}} \int d^2\Omega_0 \frac{\alpha_{ij}}{\alpha_{0i}\alpha_{0j}}$$

Evolution operator

- Extract from 1/eps pole in one-loop fixed-order
- Real: $$\sigma^{(1)}[U] \supset \sum_{i,j} \int \frac{d^3 p_0}{2E_0} \frac{p_i \cdot p_j}{p_0 \cdot p_i p_0 \cdot p_j} \times L_i^a U_0^{ab} R_j^b$$ $$\to \frac{1}{\epsilon_{\mathrm{IR}}} \int d^2\Omega_0 \frac{\alpha_{ij}}{\alpha_{0i}\alpha_{0j}}$$
- Virtual: $$\frac{1}{\epsilon_{\mathrm{IR}}} \sum_{i,j} R_i^a R_i^b \left[\log(2p_i \cdot p_j - i0) + c.c.\right]$$ $$+ (\text{collinear div.}) - (\text{collinear subtraction})$$

Evolution operator

- Extract from 1/eps pole in one-loop fixed-order
- Real: $$\sigma^{(1)}[U] \supset \sum_{i,j} \int \frac{d^3p_0}{2E_0} \frac{p_i{\cdot}p_j}{p_0{\cdot}p_i p_0{\cdot}p_j} \times L_i^a U_0^{ab} R_j^b$$ $$\to \frac{1}{\epsilon_{\rm IR}} \int d^2\Omega_0 \frac{\alpha_{ij}}{\alpha_{0i}\alpha_{0j}}$$
- Virtual: $$\frac{1}{\epsilon_{\rm IR}} \sum_{i,j} R_i^a R_i^b \left[\log(2p_i{\cdot}p_j - i0) + c.c.\right]$$ $$+ \text{(collinear div.)} - \text{(collinear subtraction)}$$

Determined by KLN theorem

Evolution operator

- Extract from 1/eps pole in one-loop fixed-order
- Real: $$\sigma^{(1)}[U] \supset \sum_{i,j} \int \frac{d^3p_0}{2E_0} \frac{p_i \cdot p_j}{p_0 \cdot p_i p_0 \cdot p_j} \times L_i^a U_0^{ab} R_j^b$$ $$\to \frac{1}{\epsilon_{\text{IR}}} \int d^2\Omega_0 \frac{\alpha_{ij}}{\alpha_{0i}\alpha_{0j}}$$
- Real+Virtual: $$\alpha_{ij} = \frac{1-\cos(\theta_{ij})}{2}$$

$$K^{(1)} = \sum_{i,j} \int \frac{d^2\Omega_0}{4\pi} \frac{\alpha_{ij}}{\alpha_{0i}\alpha_{0j}} \left(U_0^{aa'} (L_i^a R_j^{a'} + R_i^{a'} L_j^a) - L_i^a L_j^a - R_i^a R_j^a \right)$$

Combination is collinear safe

Finite N_c one-loop evolution equation for non-global logs!

[Weigert '02]

- Planar limit: products in terms of traces

$$K\,U_{12} = \frac{\lambda}{8\pi^2}\int \frac{d^2\Omega_0}{4\pi}\,\frac{\alpha_{12}}{\alpha_{01}\alpha_{02}}\,(U_{12} - U_{10}U_{02})$$

- Closed nonlinear equation for U_{ij}

[Banfi,Marchesini,Smye '02]

- $1/N_c^2$ expansion can be automated
(next order involves quadrupoles, etc.)

- Lattice Monte-Carlo techniques allow to solve finite N_c equation

[Weigert '02,
Hatta&Ueda '13]

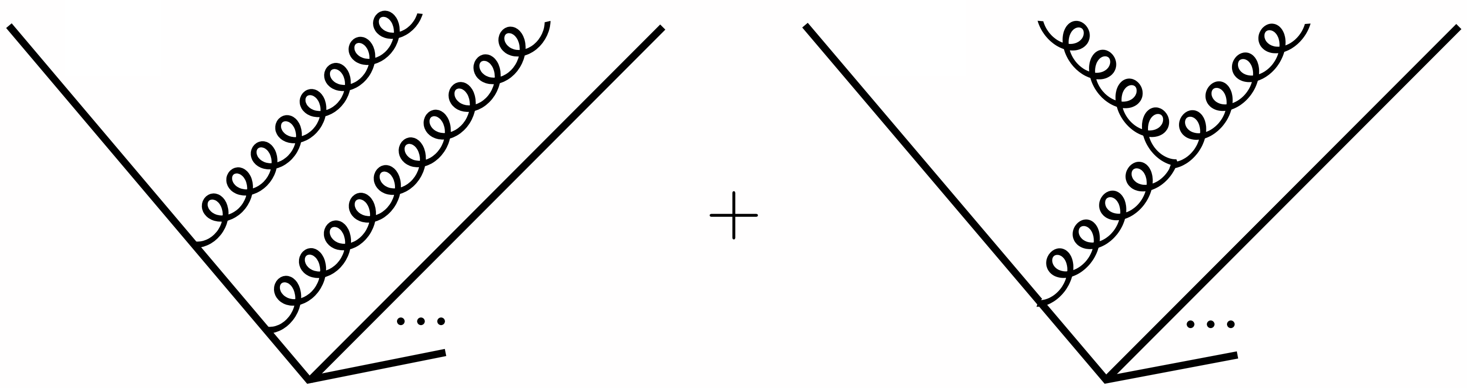
[Larkoski,Moult&Neill '05]

- Other strategy:
  'dressed gluon expansion'

$$K^{(1)} = \sum_{i,j} \int \frac{d^2\Omega_0}{4\pi} \frac{\alpha_{ij}}{\alpha_{0i}\alpha_{0j}} \Big(\underbrace{U_0^{aa'}(L_i^a R_j^{a'} + R_i^{a'} L_j^a)}_{\delta K} \underbrace{- L_i^a L_j^a - R_i^a R_j^a}_{K_0} \Big)$$

- Integrate exactly K_0 → Sudakov exponents
- Work perturbatively in δK : inserts 'dressed' gluon
- Since collinear safety of K, not preserved, must include: collinear cutoff, 'zero-bin subtraction', etc. For details (and a nice derivation), see their paper!
- U-matrix calculus: an efficient way to generate and organize the terms

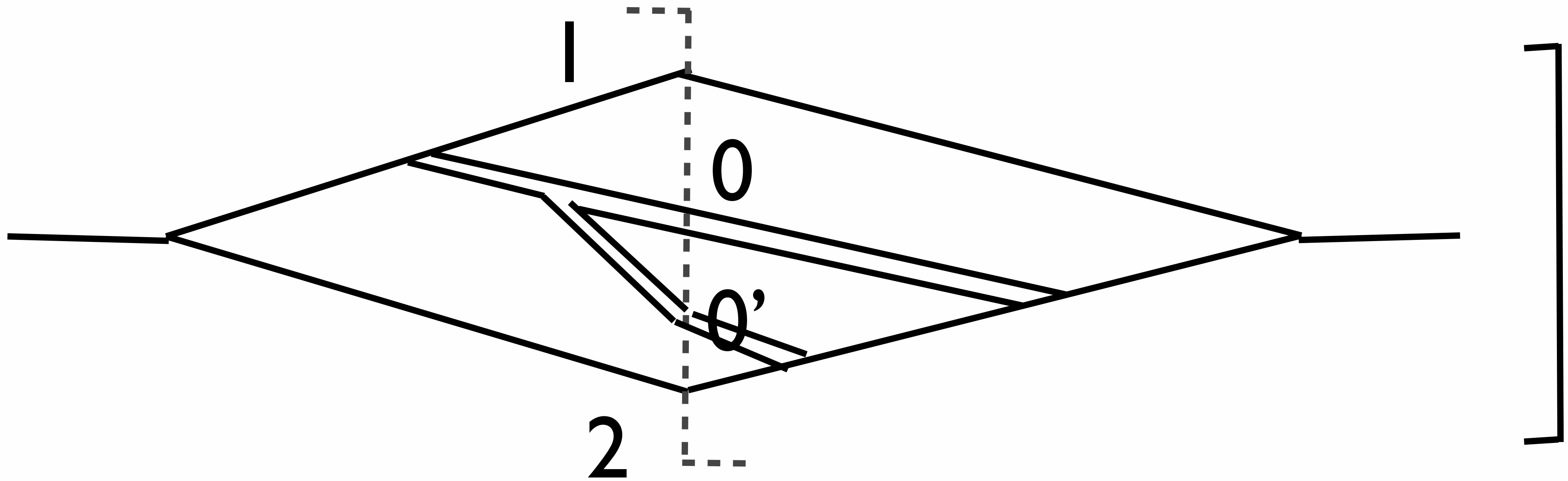
- To understand RG structure, let's now look at NLL
- Main ingredient: soft current for two soft partons



$$\mathcal{S}(k_1, k_2) = \sum_{i,j} \frac{1}{2}\{R_i^a, R_j^b\} S_i(k_1) S_j(k_2) + i f^{abc} \sum_i R_i^c S_i(k_1, k_2)$$

[...,Catani&Grazzini,...]

- For this talk, let just take planar limit.
- Get only strings of dipoles:



$$K U_{12} \supset \int \frac{d^2\Omega_0}{4\pi} \frac{d^2\Omega_{0'}}{4\pi} K_{[1\,00'\,2]} U_{10} U_{00'} U_{0'2}$$

Naively:

$$K_{[1\,00'\,2]} = \int_0^\infty a da \left[\left| \mathcal{S}(a\beta_0, \beta_{0'}) \right|^2 - \Big|_{a\to 0} \theta(1-a) - \Big|_{a\to\infty} \theta(a-1) \right]$$

a=relative energy

subtraction of 1-loop

Naively:

$$K_{[1\,00'\,2]} = \int_0^\infty \color{blue}{a da} \left[\left| \mathcal{S}(\color{blue}{a}\beta_0, \beta_{0'}) \right|^2 - \Big|_{a\to 0} \theta(1-a) - \Big|_{a\to\infty} \theta(a-1) \right]$$

a=relative energy

subtraction of 1-loop

planar soft current squared is relatively simple:

$$|\mathcal{S}|^2 = \frac{s_{10'}s_{20} + 2s_{10}s_{20} + 3s_{10}s_{10'} + 2s_{10'}s_{20'} + s_{12}s_{00'}}{s_{10}s_{00'}s_{0'2}(s_{10}+s_{10'})(s_{20}+s_{20'})}$$

$$+ (n_F - 4)\frac{s_{12}}{s_{00'}(s_{10}+s_{10'})(s_{20}+s_{20'})}$$

$$+ (\frac{1}{2}n_s - n_F + 1)\frac{(s_{10}s_{20'} - s_{10'}s_{20'})^2}{s_{00'}^2(s_{10}+s_{10'})^2(s_{20}+s_{20'})^2}$$

N=4SYM

general gauge thy

Lorentz invariance and ordering variable

- One-loop was not written in Lorentz-invariant form

$$\int d^{2-2\epsilon}\Omega_0 \frac{p_i \cdot p_j}{p_0 \cdot p_i p_0 \cdot p_j}(\cdots)$$

Lorentz invariance and ordering variable

- One-loop was not written in Lorentz-invariant form

$$\int d^{2-2\epsilon}\Omega_0 \left(\frac{p_i{\cdot}p_j}{p_0{\cdot}p_i p_0{\cdot}p_j}\right)^{1-\epsilon} (\cdots)$$

[Homogeneity in p_0 required]

- Equivalent to changing ordering variable:

$$E^2 \to \frac{p_1{\cdot}p_0 p_0{\cdot}p_2}{p_1{\cdot}p_2} = \text{`}p_t^2\text{'}$$

Less naive two-loop kernel (Lorentz-invariant scheme):

$$K_{[1\,00'\,2]} = \int_0^\infty a da \left[\left| \mathcal{S}(a\beta_0, \beta_{0'}) \right|^2 - \Big|_{a\to 0} \theta(1 - a r_1) - \Big|_{a\to\infty} \theta(a r_2 - 1) \right]$$

with: $\dfrac{r_2}{r_1} = \dfrac{\alpha_{12}\alpha_{00'}}{\alpha_{0'1}\alpha_{02}}$ from mismatching E scales in (1-loop)^2

Less naive two-loop kernel (Lorentz-invariant scheme):

$$K_{[1\,00'\,2]} = \int_0^\infty a da \left[\left| \mathcal{S}(a\beta_0, \beta_{0'}) \right|^2 - \Big|_{a\to 0} \theta(1 - a r_1) - \Big|_{a\to\infty} \theta(a r_2 - 1) \right]$$

Full two-loop planar:

$$K^{(2)} U_{12} = \int_{0,0'} \frac{-2 U_{10} U_{00'} U_{0'2}}{\alpha_{10}\alpha_{00'}\alpha_{0'2}}$$

$$\times \left\{ 2\log\frac{\alpha_{12}\alpha_{00'}}{\alpha_{10'}\alpha_{02}} + \left[1 + \frac{\alpha_{12}\alpha_{00'}}{\alpha_{10}\alpha_{20'} - \alpha_{10'}\alpha_{20}} \right] \log\frac{\alpha_{10}\alpha_{20'}}{\alpha_{10}\alpha_{20'}} \right.$$

$$\left. + (n_F - 4)(\cdots) + (\frac{1}{2} n_s - n_F + 1)(\cdots) \right\}$$

$$+ \ldots$$

One-loop single-real (w/ hard-collinear subtraction); double-virtual

jeudi 28 mai 15

Less naive two-loop kernel (Lorentz-invariant scheme):

$$K_{[1\,00'\,2]} = \int_0^\infty a da \left[\left| \mathcal{S}(a\beta_0, \beta_{0'}) \right|^2 - \Big|_{a\to 0} \theta(1 - a r_1) - \Big|_{a\to\infty} \theta(a r_2 - 1) \right]$$

Full two-loop planar:

make it collinear-finite

$$K^{(2)} U_{12} = \int_{0,0'} \frac{-2U_{10}U_{00'}U_{0'2} + U_{10}U_{02} + U_{10'}U_{0'2}}{\alpha_{10}\alpha_{00'}\alpha_{0'2}}$$

$$\times \left\{ 2\log\frac{\alpha_{12}\alpha_{00'}}{\alpha_{10'}\alpha_{02}} + \left[1 + \frac{\alpha_{12}\alpha_{00'}}{\alpha_{10}\alpha_{20'} - \alpha_{10'}\alpha_{20}}\right] \log\frac{\alpha_{10}\alpha_{20'}}{\alpha_{10}\alpha_{20'}} \right.$$

$$\left. + (n_F - 4)(\cdots) + (\frac{1}{2}n_s - n_F + 1)(\cdots) \right\}$$

$\# K^{(1)}$

only possible structure

by Lorentz invariance and KLN theorem

Less naive two-loop kernel (Lorentz-invariant scheme):

$$K_{[1\,00'\,2]} = \int_0^\infty a da \left[\left| \mathcal{S}(a\beta_0, \beta_{0'}) \right|^2 - \Big|_{a\to 0} \theta(1 - a r_1) - \Big|_{a\to\infty} \theta(a r_2 - 1) \right]$$

Full two-loop planar:

$$K^{(2)} U_{12} = \int_{0,0'} \frac{-2U_{10}U_{00'}U_{0'2} + U_{10}U_{02} + U_{10'}U_{0'2}}{\alpha_{10}\alpha_{00'}\alpha_{0'2}}$$

$$\times \left\{ 2\log\frac{\alpha_{12}\alpha_{00'}}{\alpha_{10'}\alpha_{02}} + \left[1 + \frac{\alpha_{12}\alpha_{00'}}{\alpha_{10}\alpha_{20'} - \alpha_{10'}\alpha_{20}}\right] \log\frac{\alpha_{10}\alpha_{20'}}{\alpha_{10}\alpha_{20'}} \right.$$

$$\left. + (n_F - 4)(\cdots) + (\frac{1}{2}n_s - n_F + 1)(\cdots) \right\}$$

$$\gamma_K^{(2)} K^{(1)} \,. \qquad \square$$

Sudakov double logs for narrow cones

Check: two-loop BFKL/BK

- Conformal transformation maps 2-sphere to transverse plane in BFKL: [Hofman&Maldacena; Hatta '08]

$$\int \frac{d^2\Omega_i}{4\pi} \to \frac{d^2 z}{\pi}, \quad \alpha_{ij} \to (z_i - z_j)^2$$

- Fact: pQCD is close to a CFT. In a CFT there is nothing special about 'infinity'.

- Broken by QCD β-function. Check:

$$K^{(2)} - H^{(2)}_{\mathrm{BFKL}} \propto b_0 = \frac{11C_A - 4n_F T_F - n_S T_S}{3}$$

Highly nontrivial!

Upshots

- Agreement with BFKL: evidence result is correct
- Calculations readily automated
  (products of U's; large N_c expansion; squares of Catani-Grazzini soft currents)
- Finite N_c case worked out in paper and matched with (recent!) Balitsky-JIMWLK.
- Universal two-loop
- Three-loops will require only 3- and 2-real
  (+ Lorentz + KLN+Υ_K (+Hermiticity of H_{BFKL}))

[Herranen+SCH, in progress]

Proof of RG equation

- Input: soft factorization of (loop) amplitudes

$$\lim_{\{k_i\}\to 0} H(\{k_i\},\{p_j\}) = \mathcal{S}(\{k_i\})H(\{p_j\})$$

- Output:

$$\mathcal{P}e^{\int_0^\mu \frac{d\lambda}{\lambda}(K^{(1)}+\ldots+K^{(L)})}\sigma[U]^{(L+1)}$$

has no subdivergences at (L+1) loops.
→ $\mathcal{P}e^{\int_0^\mu \frac{d\lambda}{\lambda}K}\sigma[U]$ is finite!

- Proof constructive: explicit formal expressions for *K*:

$$K = \int_{\text{energies}} \left[|\mathcal{S}|^2 - (\text{subtractions})\right]$$ at all loops

Conclusion

- Two-scale problems involving soft wide-angle gluons can be solved: color density matrix
- Efficient U-matrix calculus: can be used by others!
- Proven: all logs generated by RG equation in e^+e^-; exponent now known to NLL
- Next: collinear logs when R becomes small
  -Compare with existing 2-loop fixed-order NGL's
  -Make contact with Soper&Nagy's density matrix?
- Next: Coulomb phases in hadron collisions
  [Superleading logs: $K^{(2)}$~log[μ]; Martinez'talk]