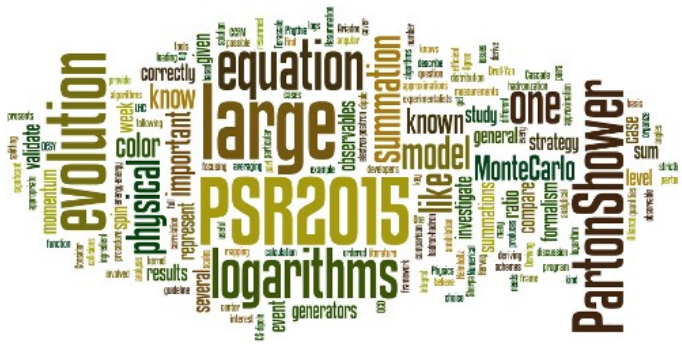
N-jettiness Subtractions for NNLO QCD Calculations.

Jonathan Gaunt, DESY

Parton Shower and Resummation 2015, Krakow, Poland, 26th May 2015



Based on arXiv:1505.04794

Work done together with Maximilian Stahlhofen, Frank Tackmann and Jonathan Walsh

Higher order computations in QCD

Higher order computations in QCD plagued by IR divergences:
- implicit divergences in phase space integration for real emission diagrams
- explicit poles from loop integration in virtual diagrams

These IR divergences eventually cancel between real and virtual when calculating an IR-safe observable (KLN theorem), but cause problems when computing the individual pieces.

One well-known solution to this problem is SUBTRACTIONS.

Idea is to subtract terms from real emission graphs that reproduce IR soft and collinear behaviour of reals $\rightarrow$ reals can be integrated in d=4 using MC.

The subtraction terms must be simple enough that they can be integrated in $d=4-2\varepsilon$ and added back to the virtuals, cancelling their explicit $1/\varepsilon$ poles.

q_T subtractions

For production of high-mass colourless systems V, a possible subtraction scheme was devised in 2007 by Catani and Grazzini - the q_T subtraction scheme.

Phys. Rev. Lett. 98 (2007) 222002

[of course many other NNLO schemes – see talks by S. Moch and M. Czakon]

This scheme has been very successful, producing a large number of NNLO results

H:	Catani, Grazzini, Phys. Rev. Lett. 98 (2007) 222002
W/Z:	Catani, Cieri, Ferrera, de Florian, and Grazzini, , Phys. Rev. Lett. 103 (2009) 082001
$\gamma\gamma$:	Catani, Cieri, Ferrera, de Florian, and Grazzini, Phys. Rev. Lett. 108 (2012) 072001
WH:	Ferrera, Grazzini, and Tramontano, Phys. Rev. Lett. 107 (2011) 152003
ZH:	Ferrera, Grazzini, and Tramontano, Phys. Lett. B 740 (2015) 51–55
ZZ:	Cascioli et al., Phys. Lett. B 735 (2014) 311–313
W^+W^-:	Gehrmann et al., Phys. Rev. Lett. 113 (2014), 21 212001
$Z\gamma/W\gamma$:	Grazzini, Kallweit, Rathlev, and Torre, Phys. Lett. B 731 (2014) 204–207
	Grazzini, Kallweit, Rathlev, arXiv:1504.01330

Also a proposal to use the method to calculate $t\bar{t}$ production at NNLO

Zhu, Li, Li, Shao, Yang, Phys. Rev. Lett. 110 (2013), 8 082001
Catani, Grazzini, Torre, Nucl.Phys. B890 (2014) 518–538

q_T subtractions

How does this method work?

Express NNLO cross section as an integral over q_T, and divide into two pieces:

$$\sigma(X) = \int_0 dq_T \frac{d\sigma(X)}{dq_T} = \int_0^{q_{T,cut}} dq_T \frac{d\sigma(X)}{dq_T} + \int_{q_{T,cut}} dq_T \frac{d\sigma(X)}{dq_T}$$

Measurement

At least one real emission required – this can be computed entirely from V+j at NLO

q_T subtractions

How does this method work?

Express NNLO cross section as an integral over q_T, and divide into two pieces:

$$\sigma(X) = \int_0 dq_T \frac{d\sigma(X)}{dq_T} = \int_0^{q_{T,cut}} dq_T \frac{d\sigma(X)}{dq_T} + \int_{q_{T,cut}} dq_T \frac{d\sigma(X)}{dq_T}$$

Measurement

This contribution probes the q_T spectrum at small q_T

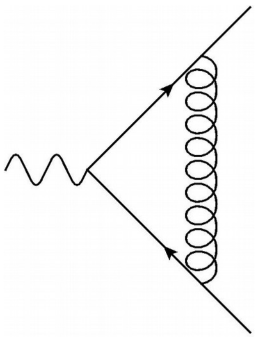
But this is the RESUMMATION region for q_T – very extensively studied!

Factorization formula for q_T

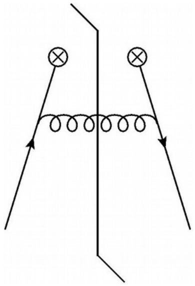
In fact, we know that at small q_T there exists an all-order factorization formula for the differential cross section:

$$\frac{d\sigma}{dq_T} = H \times [B_a \otimes B_b \otimes S](q_T) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

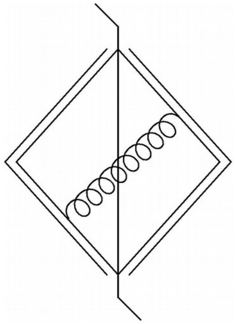
Hard function: Finite part of $e^+e^- \to 2$ jet virtuals



Beam functions: hadronic matrix elements of a bilocal operator. Summarize collinear ISR.



Soft function: vacuum matrix elements of Wilson lines. Summarize soft radiation.



All individually IR finite and straightforward to compute analytically
(some kind of rapidity regulator required in computation of beam and soft functions)

q_T slicing and subtraction formulae

Substitute the fixed order NNLO expansion of this into the cross section formula:

$$\sigma(X) = \sigma^{sing}(X, q_{T,\delta}) + \int_{q_{T,\delta}} dq_T \frac{d\sigma(X)}{dq_T} + \mathcal{O}(q_{T,\delta}^2/Q^2)$$

Singular cumulant ≡ integrated result from factorization formula

This is a PHASE SPACE SLICING METHOD. Expect numerical cancellations for this method to be maximally bad. Can rephrase as a (global) subtraction by adding and subtracting singular cumulant between $q_{T\delta}$ and some q_{Toff}

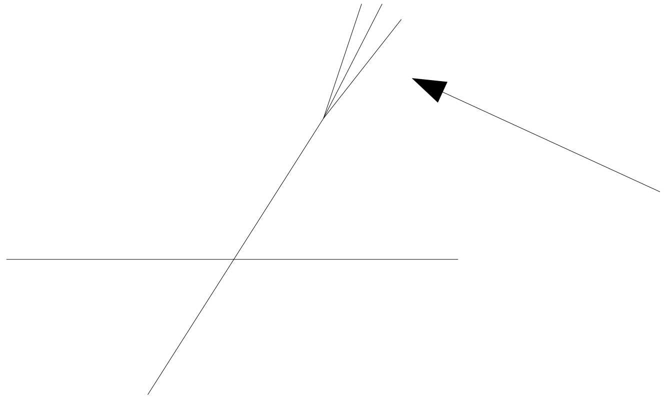
$$\sigma(X) = \sigma^{sing}(X, q_{T,off}) + \int_{q_{T,\delta}} dq_T \left[\frac{d\sigma(X)}{dq_T} - \frac{d\sigma^{sing}(X)}{dq_T}\right] + \mathcal{O}(q_{T,\delta}^2/Q^2)$$

Now point-by-point SUBTRACTION in q_T, and $q_{T\delta}$ essentially becomes a technical cut-off

Limitations of q_T subtraction and N-jettiness

What is the limitation of the q_T subtraction method?

Runs into trouble when we want to compute NNLO cross sections for processes with jets in the final state. e.g. pp → jj at NNLO:



q_T for dijet system is not zero even when emissions become triply collinear – this is not regulated by NLO pp → jjj subtraction.

Can we get a workable q_T-style subtraction for pp → V + N jets? We need to use a different IR safe observable that resolves any additional real radiation from the Born configuration, such that the IR singularities sit at the origin.

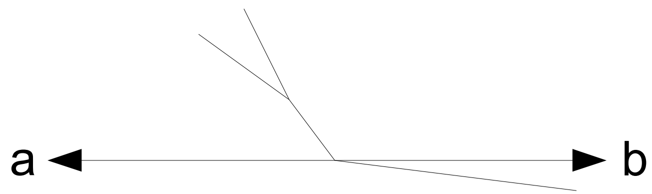
Such an observable exists – N-jettiness $\mathcal{T}_N$ Stewart, Tackmann, Waalewijn, Phys. Rev. Lett. 105 (2010) 092002,

Definition of N-jettiness

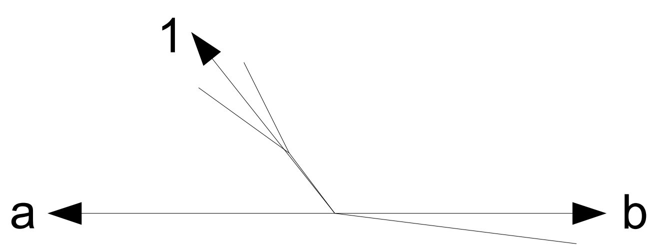
N-jettiness is defined as follows:

First we construct N+2 massless jet axes $q_i, \quad i = a, b, 1 \ldots N$

The beam axes q_a and q_b are always set to lie along the two beam directions.



To construct the remaining q_i it suffices to use any IR-safe projection onto the Born phase space – e.g. one can run any IR-safe jet algorithm to cluster the M final state partons into N jets (this will of course contain some exclusion around beam regions).



DESY

Definition of N-jettiness

Then the N-jettiness observable is defined according to:

Final-state partons

$$\mathcal{T}_N(\Phi_M) = \sum_{k=1}^{M} \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

Normalisation factors. Some freedom to choose these

e.g. $Q_i = Q$ 'Invariant mass' measure

$Q_i = \rho_i E_i$ 'Geometric' measure

NOTE: one can also define N-jettiness for e^+e^- and DIS, such that one can construct NNLO subtractions with arbitrary final state jets here too. For these processes one has zero or one initial state q vectors respectively.

Factorization Formula for N-jettiness

To construct the NNLO subtraction we need the factorization formula for N-jettiness – this has been derived in SCET: Stewart, Tackmann, Waalewijn, Phys. Rev. D 81 (2010) 094035, Phys. Rev. Lett. 105 (2010) 092002, Jouttenus, Stewart, Tackmann, Waalewijn, Phys. Rev. D 83 (2011) 114030

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(X)}{\mathrm{d}\mathcal{T}_N} = \int \mathrm{d}\Phi_N \, \frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\mathcal{T}_N} \, X(\Phi_N)$$

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\mathcal{T}_N} = \int \Big[\prod_i \mathrm{d}\mathcal{T}_N^i\Big] \frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\mathcal{T}_N^a \, \mathrm{d}\mathcal{T}_N^b \cdots \mathrm{d}\mathcal{T}_N^N} \, \delta\Big(\mathcal{T}_N - \sum_i \mathcal{T}_N^i\Big)$$

Beam functions – collinear ISR

Jet functions (as for thrust)

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\mathcal{T}_N^a \, \mathrm{d}\mathcal{T}_N^b \cdots \mathrm{d}\mathcal{T}_N^N} = \int \mathrm{d}t_a \, B_a(t_a, x_a, \mu) \int \mathrm{d}t_b \, B_b(t_b, x_b, \mu) \Big[\prod_{i=1}^{N} \int \mathrm{d}s_i \, J_i(s_i, \mu)\Big]$$
$$\times \, \vec{C}^\dagger(\Phi_N, \mu) \, \widehat{S}_\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i\}, \mu\Big) \vec{C}(\Phi_N, \mu) \, .$$

Soft function – in general this is a matrix in colour space

Hard Wilson coefficient amplitude and conjugate. Also matrices in colour space.

Advantages of N-jettiness subtractions

$$\sigma(X) = \sigma^{\text{sing}}(X, \mathcal{T}_{\text{off}}) + \int_{\mathcal{T}_\delta} \mathrm{d}\mathcal{T}_N \left[\frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} - \frac{\mathrm{d}\sigma^{\text{sing}}(X)}{\mathrm{d}\mathcal{T}_N} \theta(\mathcal{T}_N < \mathcal{T}_{\text{off}}) \right] + \mathcal{O}(\delta_{\text{IR}})$$

Note that resulting NNLO computation is automatically fully differential.

Also in a form suitable to be combined with higher-order resummation and parton showers, using GENEVA methodology:

Alioli, Bauer, Berggren, Hornig, Tackmann, Vermilion, Walsh, and Zuberi, JHEP 1309 (2013) 120
Alioli, Bauer, Berggren, Tackmann, Walsh, and Zuberi, JHEP 1406 (2014) 089

In fact, GENEVA already uses N-jettiness as an N-jet resolution variable.

→ S. Alioli's talk

Beam functions

With initial state colour the new ingredient is the beam function $B_i(t, x, \mu)$

Note that this function depends on partonic momentum fraction *x* as well as the N-jettiness measurement *t*.

It can be computed as:

$$B_i(t, x, \mu) = \sum_j \int \frac{\mathrm{d}z}{z} \mathcal{I}_{ij}(t, z, \mu, \mu_F) \, f_j\left(\frac{x}{z}, \mu_F\right)$$

Matching coefficients

Stewart, Tackmann, Waalewijn, Phys.Rev.D81:094035,2010

Matching coefficients have been computed analytically up to NNLO.

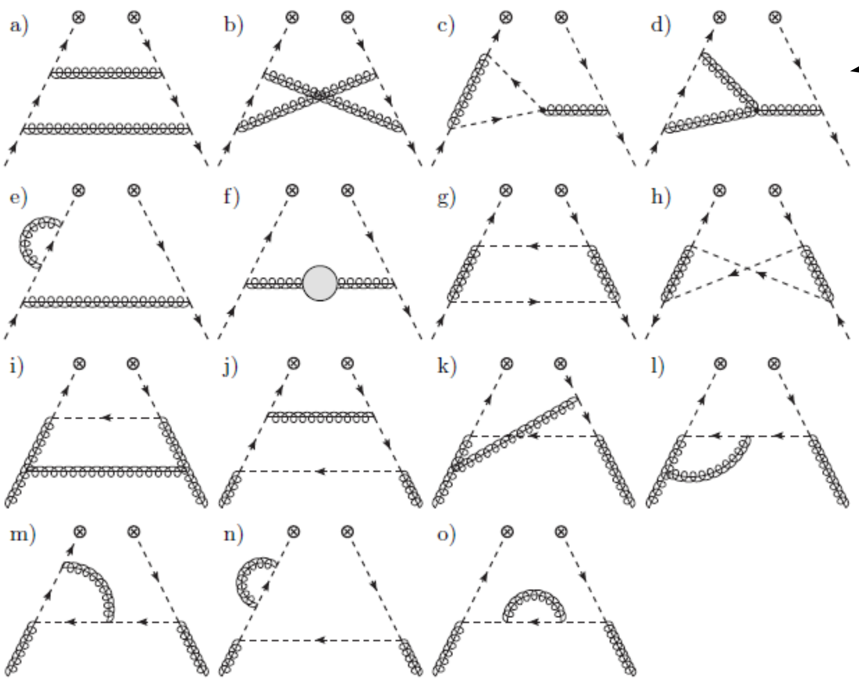
JG, Stahlhofen, Tackmann, JHEP 1404 (2014) 113, JHEP 1408 (2014) 020,

Let's look at this as an example of how ingredients in the factorization formula are computed.

Beam functions

Matching coefficients can be computed from partonic matrix elements of a bilocal operator.

Since we are considering only one collinear sector, SCET Lagrangian is just a boosted copy of QCD one → can use QCD Feynman rules. Also because we only have one direction, it's very convenient to use light-cone gauge – reduces number of diagrams to compute significantly.

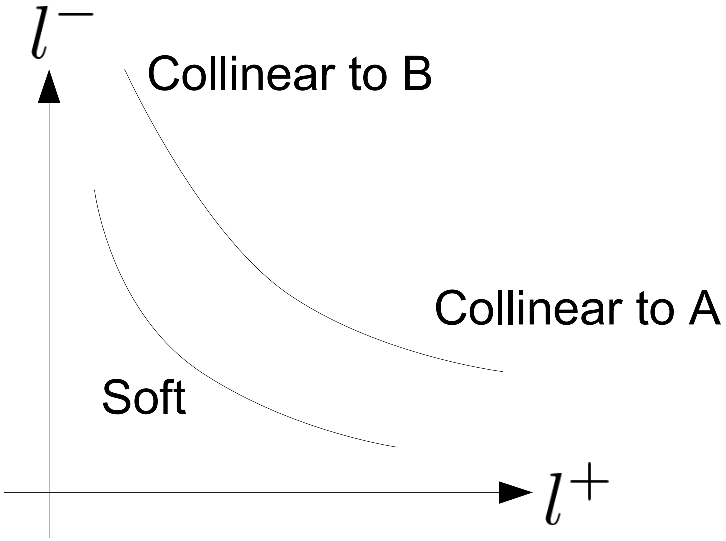


e.g. for computation of quark matching coefficients $\mathcal{I}_{qj}$

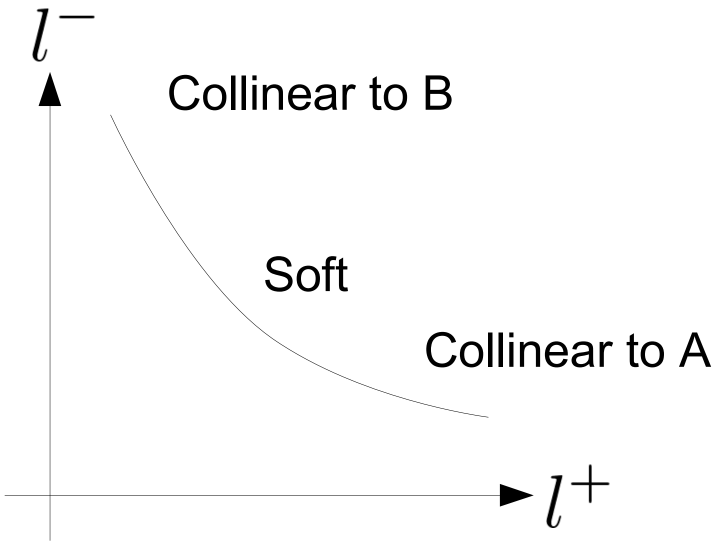
N-jettiness as a SCET_I observable

With the N-jettiness measurement *t* (essentially) all IR and UV divergences in the computation are regulated by dimensional regularisation. For k_T measurement one needs a rapidity regulator.

In SCET, related to the fact that N-jettiness is a SCET_I observable, whilst k_T belongs to SCET_II



SCET_I

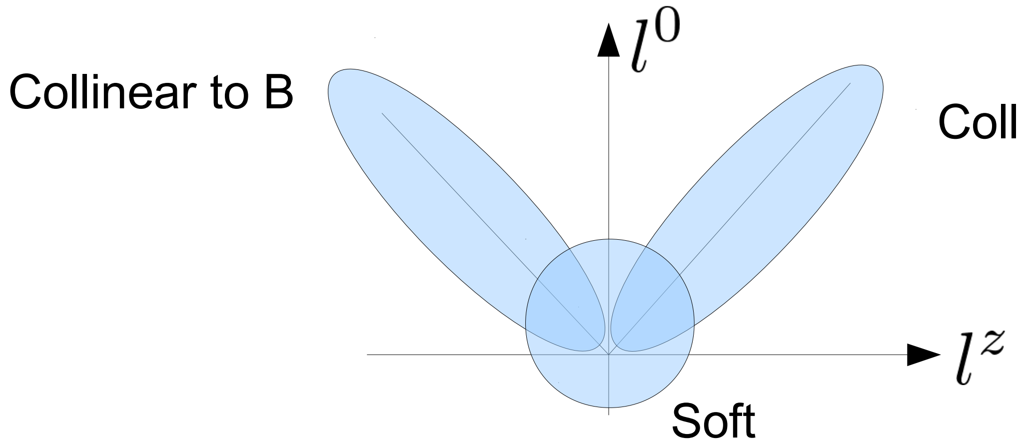


SCET_II

Soft-Collinear Overlaps

In the beam function calculation we perform integrations over emitted particles over all momentum space.

We need to remove soft region, since this is handled by soft function.



In SCET necessary machinery is called zero-bin subtractions.

Manohar, Stewart, Phys. Rev. D 76 (2007) 074002,

Nice feature of N-jettiness – all such overlap contributions automatically vanish at all orders in perturbation theory when using pure dimensional regularisation.

Other ingredients at NNLO

Are the other ingredients known at NNLO?

Jet functions: Yes

Becher, Neubert, Phys. Lett. B 637 (2006) 251{259, Becher, Bell, Phys. Lett. B 695 (2011) 252{258

Soft functions: Known analytically for 0-jettiness case.

Kelley, Schwartz, Schabinger, Zhu, Phys. Rev. D 84 (2011) 045022,
Monni, Gehrmann, Luisoni, JHEP 1108 (2011) 010
Hornig, Lee, Stewart, Walsh, Zuberi, JHEP 1108 (2011) 054,
Kang, Labun, Lee, arXiv:1504.04006.

Computed numerically for 1-jettiness case.

Boughezal, Liu, Petriello, arXiv:1504.02540.

For arbitrary N-jet processes, can in principle be obtained from known results for two-loop soft amplitudes.

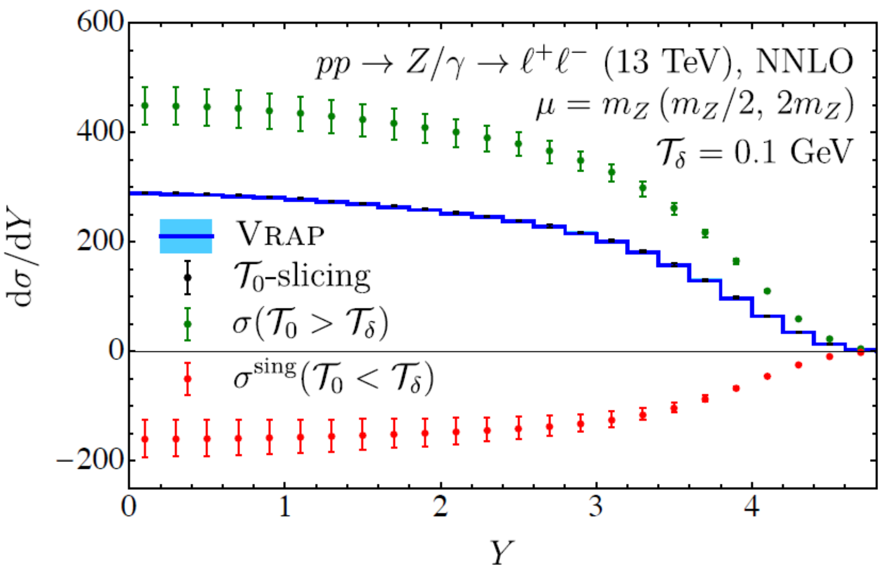
Boughezal, Liu, Petriello, arXiv:1504.02540.

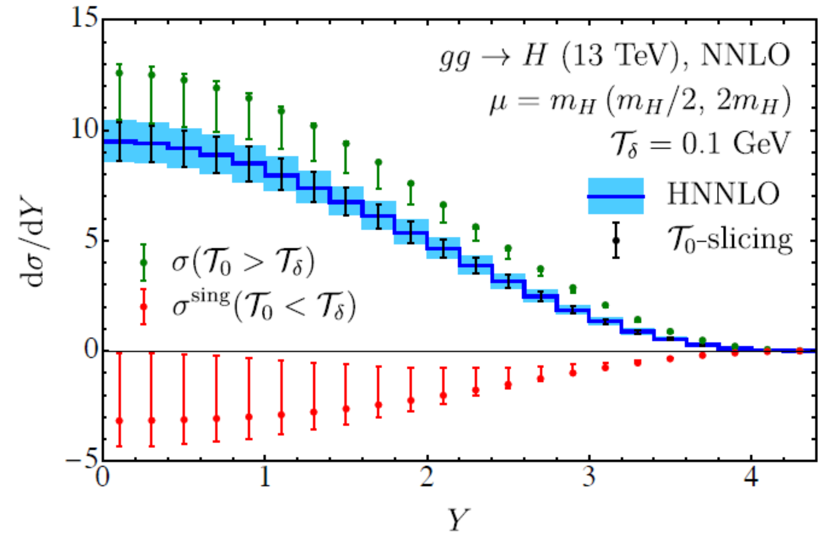
Hard functions: These are just the two-loop virtuals – some of them known, many not.

NNLO Z and H using 0-jettiness

As an example to show method works, we computed NNLO Z and H rapidity spectra at NNLO using the technique.

NLO Z/H + j computed using MCFM. Simplest phase space slicing method used.

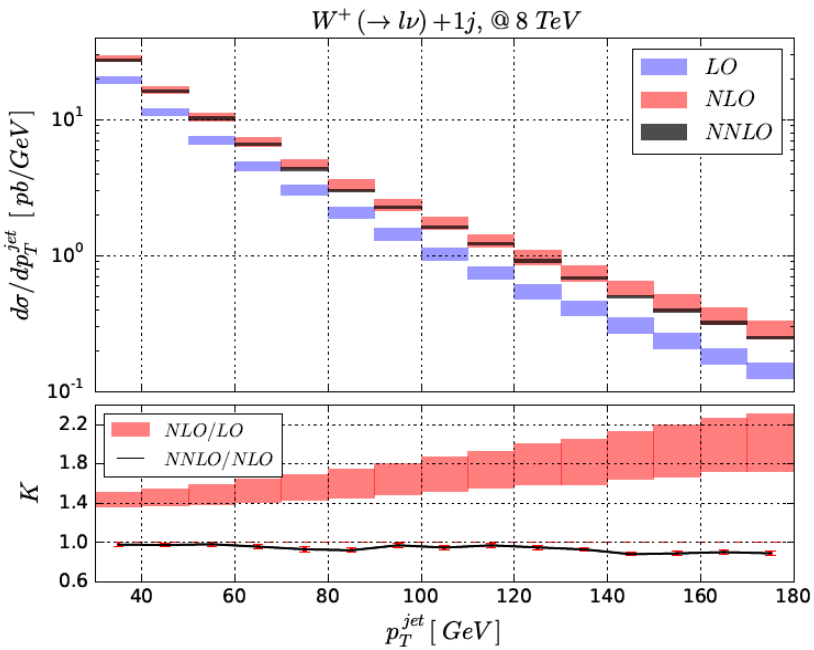




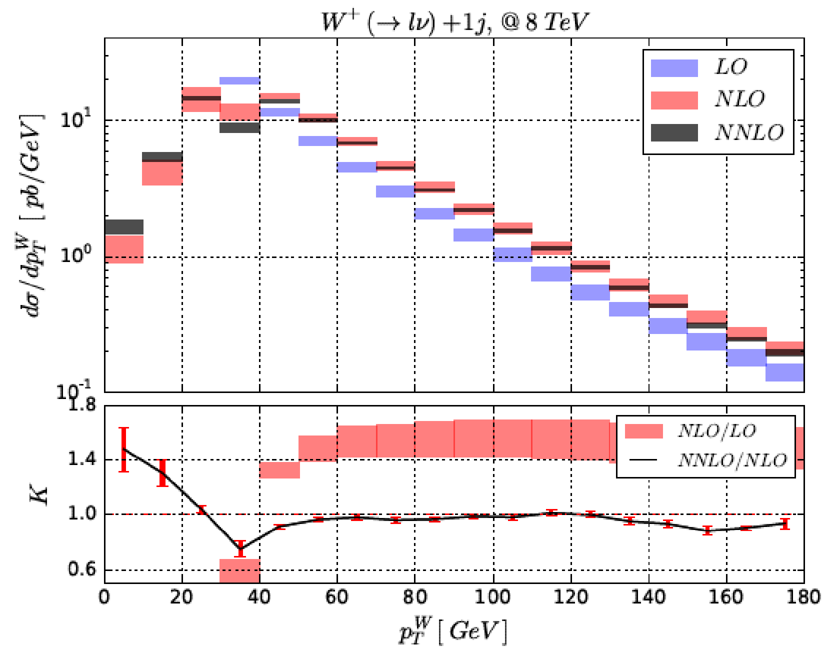
Good agreement found with existing codes VRAP and HNNLO.

NNLO W+j using 1-jettiness

W+j production at NNLO has also been computed very recently by Boughezal, Focke, Liu and Petriello using this technique. Again, a phase space slicing method was used. arXiv:1504.02131.



Leading jet p_t



W boson p_t

Last week – also H + j at NNLO using N-jettiness! Boughezal, Focke, Giele, Liu, Petriello, arXiv:1505.03893

Extensions: more-differential subtractions

Subtraction procedure defined thus far operates at a rather global level – all singular regions are projected onto one observable $\mathcal{T}_N$.

Conceivable that in order to improve numerical stability or convergence of the NNLO calculation one might want to use a more local subtraction.

In our approach, it is straightforward (at least conceptually) to progressively increase the locality of the subtractions. All one needs to do is:

a) Split $\mathcal{T}_N$ up into further IR-safe observables that cover the phase space and which are sensitive to observables in different regions

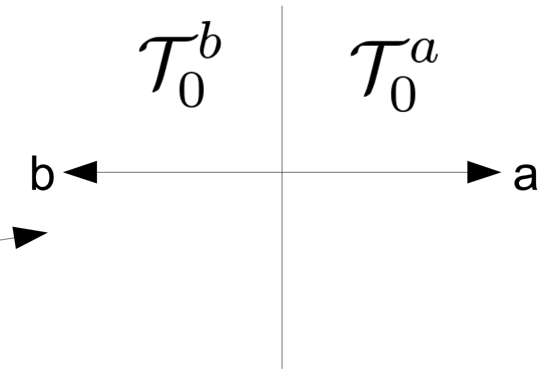
AND/OR

b) Introduce further observables that resolve the nature of emissions, e.g. allowing one to discriminate between double-real and single-real(+virtual) emissions in a given region.

Extensions: more-differential subtractions

Example of (a) for V + 0 jets:

Appropriate observable is 0-jettiness (aka beam thrust) – divides event into two hemispheres



Following the scheme discusssed previously we would use the total 0-jettiness $\mathcal{T}_0 = \mathcal{T}_0^a + \mathcal{T}_0^b$ to construct a subtraction

However, instead of taking the sum, we can also consider $\mathcal{T}_a \equiv \mathcal{T}_0^a$ and $\mathcal{T}_b \equiv \mathcal{T}_0^b$ separately, performing the subtraction differential in both of these observables.

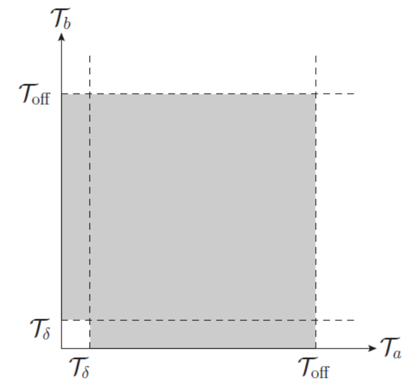
Each observable is only sensitive to a subset of the singular regions.

Extensions: more-differential subtractions

Two-variable subtraction formula looks as follows:

$$
\begin{aligned}
\sigma(X) = {} & \sigma^{\text{sing}}(X, \mathcal{T}_a < \mathcal{T}_{\text{off}}, \mathcal{T}_b < \mathcal{T}_{\text{off}}) \\
& \left.\begin{aligned}
& + \int_{\mathcal{T}_\delta}^{\mathcal{T}_{\text{off}}} \mathrm{d}\mathcal{T}_b \left[\frac{\mathrm{d}\sigma(X, \mathcal{T}_a < \mathcal{T}_\delta)}{\mathrm{d}\mathcal{T}_b} - \frac{\mathrm{d}\sigma^{\text{sing}}(X, \mathcal{T}_a < \mathcal{T}_\delta)}{\mathrm{d}\mathcal{T}_b} \right] \\
& + \int_{\mathcal{T}_\delta}^{\mathcal{T}_{\text{off}}} \mathrm{d}\mathcal{T}_a \left[\frac{\mathrm{d}\sigma(X, \mathcal{T}_b < \mathcal{T}_\delta)}{\mathrm{d}\mathcal{T}_a} - \frac{\mathrm{d}\sigma^{\text{sing}}(X, \mathcal{T}_b < \mathcal{T}_\delta)}{\mathrm{d}\mathcal{T}_a} \right]
\end{aligned}\right\} \\
& + \int_{\mathcal{T}_\delta}^{\mathcal{T}_{\text{off}}} \mathrm{d}\mathcal{T}_a \int_{\mathcal{T}_\delta}^{\mathcal{T}_{\text{off}}} \mathrm{d}\mathcal{T}_b \left[\frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_a\,\mathrm{d}\mathcal{T}_b} - \frac{\mathrm{d}\sigma^{\text{sing}}(X)}{\mathrm{d}\mathcal{T}_a\,\mathrm{d}\mathcal{T}_b} \right] \\
& + \int \mathrm{d}\mathcal{T}_a \int \mathrm{d}\mathcal{T}_b \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_a\,\mathrm{d}\mathcal{T}_b} \left[1 - \theta(\mathcal{T}_a < \mathcal{T}_{\text{off}})\,\theta(\mathcal{T}_b < \mathcal{T}_{\text{off}})\right] + \mathcal{O}(\delta_{\text{IR}})
\end{aligned}
$$

Integrated subtraction term



One variable nonzero, other integrated near origin → NLO calculation needed. These terms contain all real-virtual contributions, and singular X sec acts as real-virtual subtraction

Both $\mathcal{T}_a$ and $\mathcal{T}_b$ nonzero → two real emissions needed, so X sec requires only an LO calculation. Point-by-point subtraction in $\mathcal{T}_a$, $\mathcal{T}_b$ here.

Extensions: more-differential subtractions

Example of (b) for V + 0 jets:

Use both 0-jettiness $\mathcal{T}_0$ and the p_T of V. The relevant factorization formulae differential in both $\mathcal{T}_0$ and p_T have been written down in the SCET framework.

Jain, Procura, Waalewijn, JHEP 1204 (2012) 132,
Procura, Waalewijn, Zeune, JHEP 1502 (2015) 117.

The corresponding NNLO double differential beam functions have been derived for the quark case.

JG, Stahlhofen, JHEP 1412 (2014) 146

In the more general N-jet case, you can:

a) split $\mathcal{T}_N$ up into its components in different jet regions, performing subtractions differentially in these.

b) use the scalar sums of transverse momenta in each N-jettiness region with respect to the jet axis $E_{Ti} = \sum_{k \in i} |p_{Tk}|$

Extensions: more-differential subtractions

Note: to construct such more-differential subtractions, we ideally need

- the appropriate singular cross section differential in all of the chosen jet resolution variables...
- ...with these differential cross sections reproducing the correct singular behaviour in all of the relevant singular kinematic regimes.

Very active area of research – see e.g.

Jouttenus, Stewart, Tackmann, Waalewijn, Phys. Rev. D 83 (2011) 114030, Phys. Rev. D 88 (2013) 054031
Procura, Waalewijn, Zeune, JHEP 1502 (2015) 117
Larkoski, Moult, Neill, JHEP 1409 (2014) 046, arXiv:1501.0459.
Bauer, Tackmann, Walsh, Zuberi, Phys.Rev. D85 (2012) 074006,

Extensions: adding nonsingular terms to the subtraction

Recall the phase space slicing formula:

$$\sigma(X) = \sigma^{sing}(X, \tau_\delta) + \int_{\tau_\delta}^{1} d\tau \frac{d\sigma(X)}{d\tau} + \mathcal{O}(\tau_\delta)$$

Error here is order τ_δ because we only included the leading singular terms in σ^{sing}.

If we could also include terms proportional to τ_δ, then accuracy of slicing method (also subtraction method) would be improved.

These terms correspond to subleading power corrections in the resummation language. Such power corrections can in principle be computed – e.g. in SCET, where they may be computed using subleading Lagrangians and operators.

See e.g. Freedman, arXiv:1303.1558, Freedman, Goerke, Phys. Rev. D 90 (2014) 11 114010, for work in this direction for thrust in e^+e^-

Extensions: heavy quarks

So far we've restricted ourselves to massless partons.

Construction of analogous $\mathcal{T}_N$ subtractions for processes involving massive quarks is possible using same techniques.

Can distinguish two cases:

$$m_q \ll Q$$

Leibovich, Ligeti, Wise, Phys. Lett. B 564 (2003) 231{234,
Fleming, Hoang, Mantry, Stewart, Phys. Rev. D 77 (2008) 074010, Phys. Rev. D 77 (2008) 114003
Jain, Scimemi, Stewart, Phys. Rev. D 77 (2008) 094008,

Consider a massive quark jet with its own N-jettiness axis making use of the tools in SCET developed for treatment of massive collinear quarks

$$m_q \sim Q$$

e.g. $t\bar{t}$ production and similar processes

Treat the heavy quarks as part of the hard interaction (without its own N -jettiness axis) together with a more complicated soft function to account for soft gluon emissions from the heavy quarks – as has been suggested already in the context of qT subtraction method.

Zhu, Li, Li, Shao, Yang, Phys. Rev. Lett. 110 (2013), 8 082001
Catani, Grazzini, Torres, Nucl.Phys. B890 (2014) 518{538,

Summary

- I explained how a global subtraction method for NNLO calculations can be constructed using the N-jettiness variable. This method can in principle be used for processes with arbitrary numbers of jets and for pp, ep or ee.
- Subtraction term here is the appropriate fixed order expansion of the N-jettiness singular cross section/factorization formula. This can be computed efficiently using SCET. In this context SCET breaks up the calculation of the subtraction term into pieces that are easier to compute, with beam and jet functions being reusable in many processes.
- Showed an example of the method working in practice – Z and H rapidity spectra at NNLO.
- Gave suggested extensions that could improve numerical covergence/stability:
  - More-differential subtractions
  - Adding leading nonsingular contributions to subtractions
- Brief discussion of how to treat processes involving massive quarks.

Extra plots for NNLO Z and H

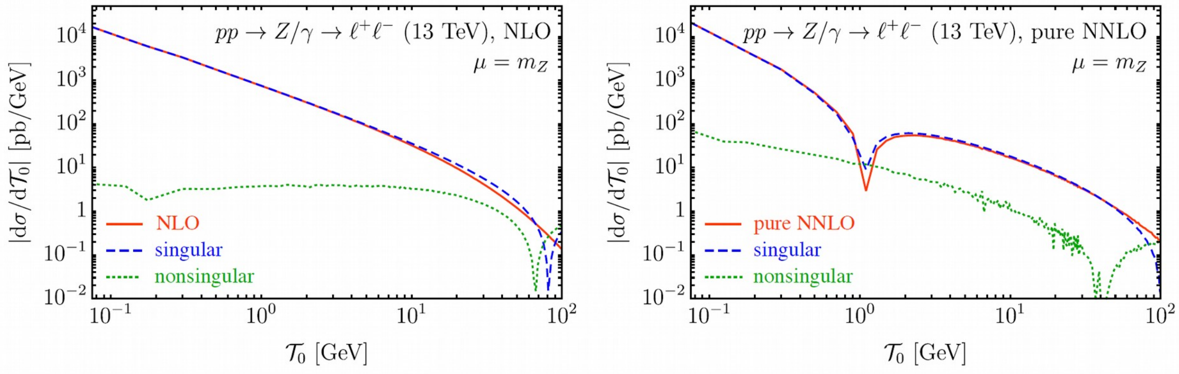


Figure 3. The full, singular, and nonsingular contributions to the $\mathcal{T}_0$ spectrum for Drell-Yan production. The NLO $\mathcal{O}(\alpha_s)$ corrections are shown on the left, and the pure NNLO $\mathcal{O}(\alpha_s^2)$ corrections are on the right.

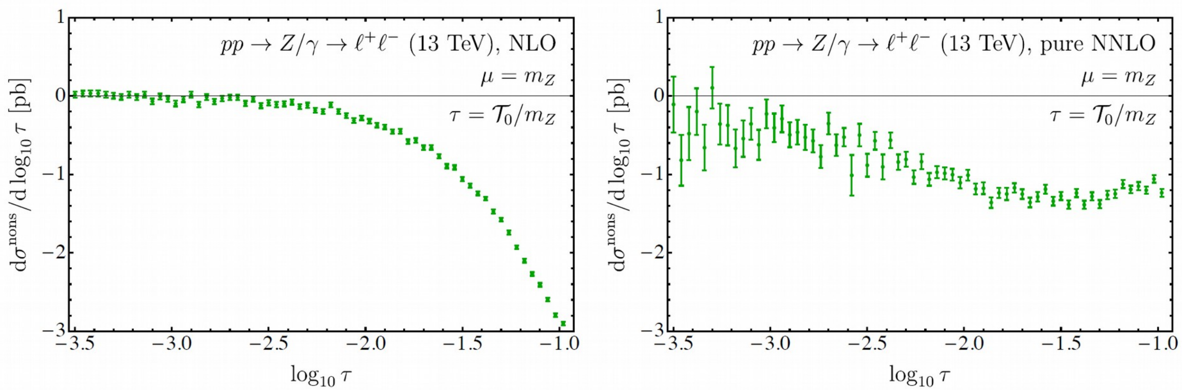


Figure 4. The nonsingular $\mathcal{T}_0$ spectrum for Drell-Yan as a function of $\tau = \mathcal{T}_0/m_Z$. The NLO $\mathcal{O}(\alpha_s)$ corrections are shown on the left, and the pure NNLO $\mathcal{O}(\alpha_s^2)$ corrections are on the right.

Extra plots for NNLO Z and H

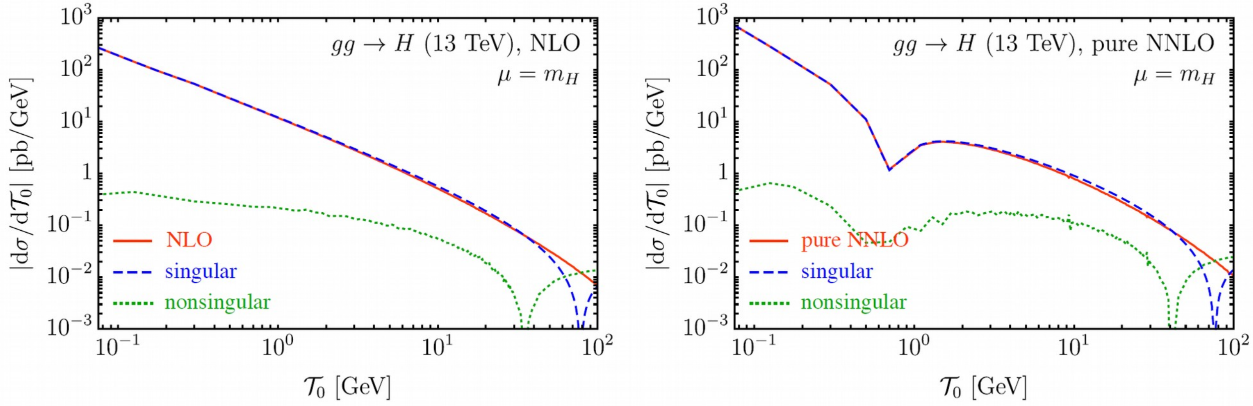


Figure 5. The full, singular, and nonsingular contributions to the $\mathcal{T}_0$ spectrum for gluon-fusion Higgs production. The NLO $\mathcal{O}(\alpha_s)$ corrections are shown on the left, and the pure NNLO $\mathcal{O}(\alpha_s^2)$ corrections are on the right.

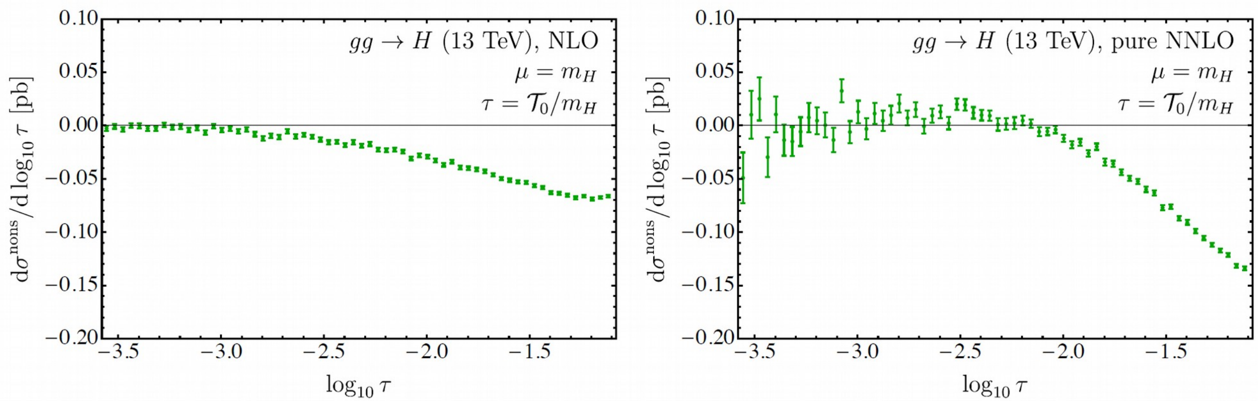


Figure 6. The nonsingular $\mathcal{T}_0$ spectrum for gluon-fusion Higgs production as a function of $\tau = \mathcal{T}_0/m_H$. The NLO $\mathcal{O}(\alpha_s)$ corrections are shown on the left, and the pure NNLO $\mathcal{O}(\alpha_s^2)$ corrections are on the right.

Extra plots for NNLO Z and H

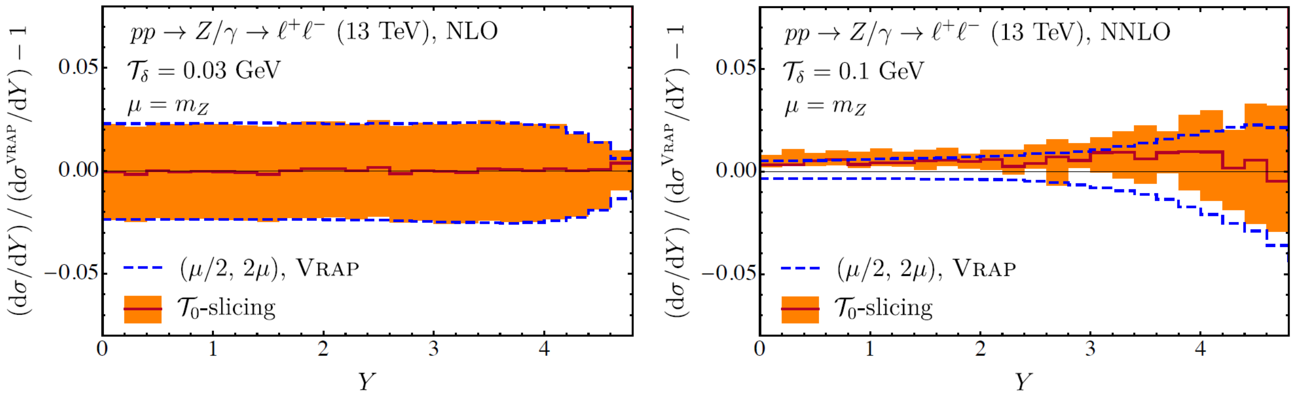


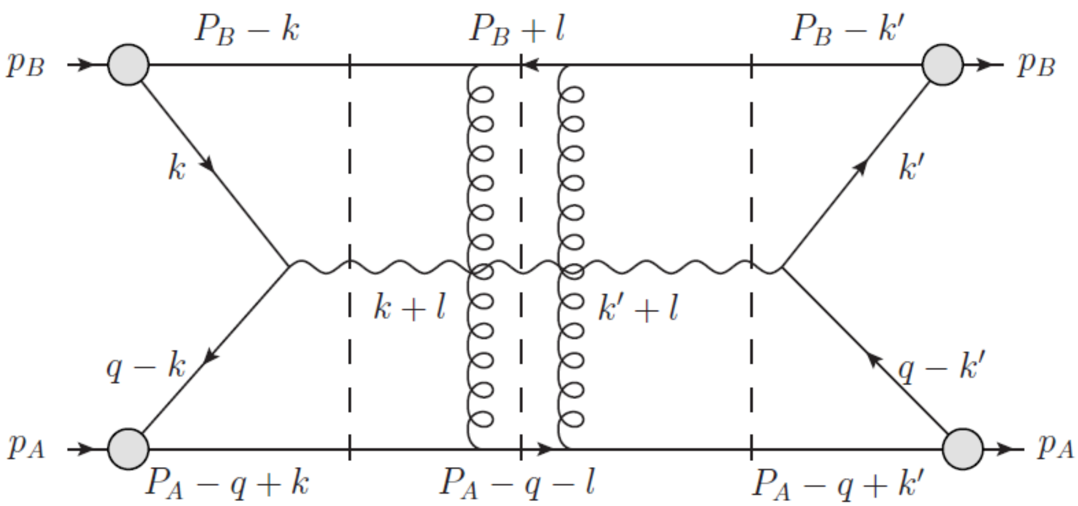
Figure 8. The scale certainty band in the Drell-Yan rapidity distribution for both VRAP and $\mathcal{T}_0$-slicing, relative to the central scale from VRAP at NLO (right) and NNLO (left).

Glauber modes and N-jettiness

Small caveat: this factorization formula misses so-called Glauber modes, which are an IR mode that is known not to cancel for N-jettiness → this factorization formula is not complete.

JG, JHEP 1407 (2014) 110,

However non-cancelling Glauber modes are associated with diagrams of following structure:



This corresponds to N^{4}LO – much higher order than we are concerned with here.

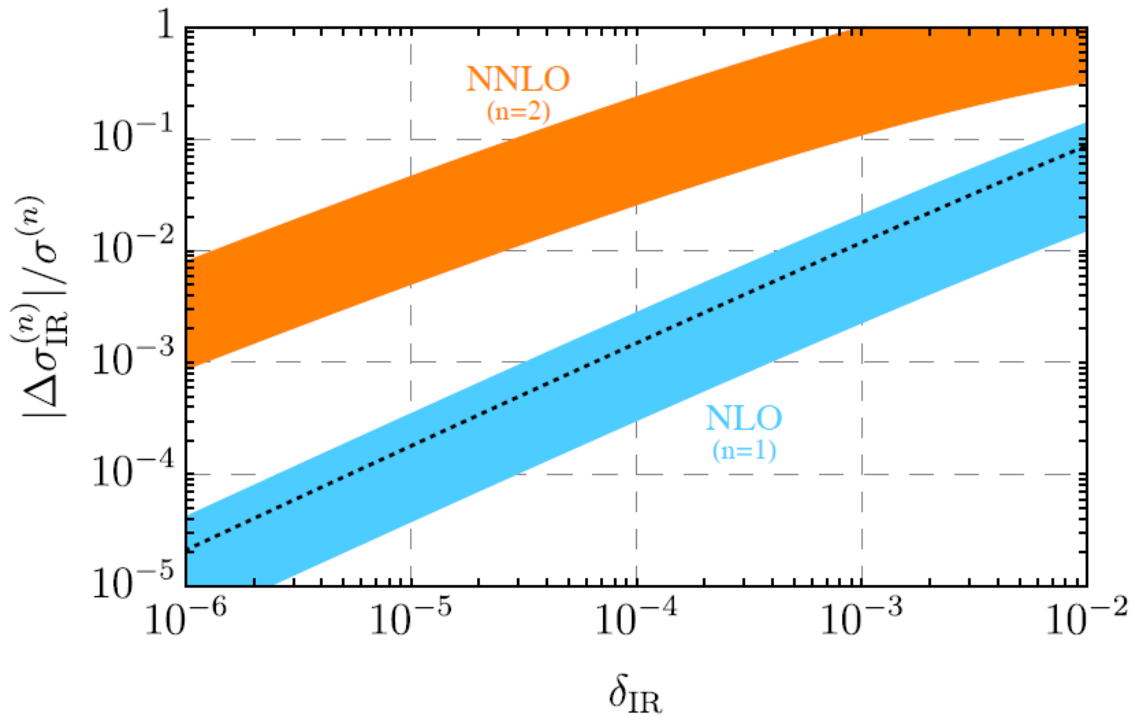


Figure 1. Estimated size of the missing nonsingular terms below $\tau = \delta_{\rm IR}$ as a fraction of the full correction at NLO (blue band) and NNLO (orange band), see eq. (2.25). The dashed line shows the known exact result for thrust.