Coulomb gluons and the ordering variable

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Outline

1.- Motivation

2.- Breaking of collinear factorisation

3.- A possible algorithm to sum soft corrections.

4.- An interesting application: Gaps between jets and the ordering problem

5.- Progress in the ordering problem

6.- Summary & conclusions

Motivation: Factorisation breaking

- Is it possible to factorise all collinear singularities into process independent & universal function?

"Only for sufficiently inclusive observables or for processes involving 0 (e+e-) or 1 (DIS) incoming parton". Collins, Soper & Sterman (Nucl. Phys. B 308(1988) 833)

-Recent progress in understanding how factorisation is broken
Forshaw, Seymour & Siodmok JHEP 11(2012)066
Catani, de Florian, Rodrigo JHEP 07(2012)026

"…in hadron-hadron collisions, the violations of strict collinear factorisation has implications on the non-abelian structure of logarithmic enhanced terms in perturbative calculations (starting from the N^2LO) and on various factorisation issues of mass singularities (starting from the N^3LO)".

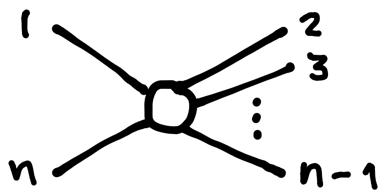
e.g.

- Non-global observables, Gaps between jets (Kyrieleis, Forshaw, Seymour JHEP 0809(2008)128)
- MPIs observables (Gaunt JHEP 07(2014)110)

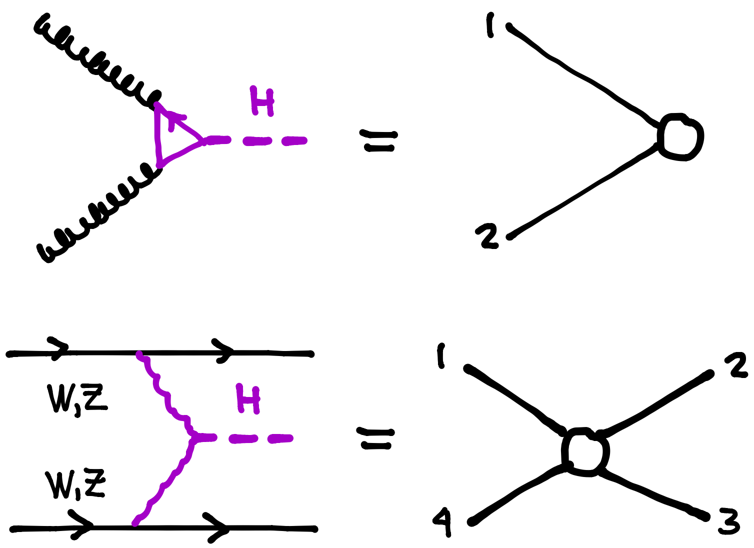
Although progress has been made it is not entirely clear how to include these effects.

Notation I: One soft emission correction to a hard process

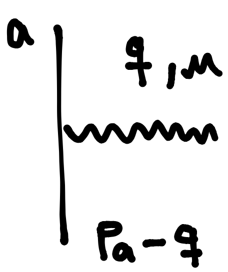
Hard process amplitude:
a vector in colour and spin spaces.

$|n\rangle \equiv$ 

e.g.



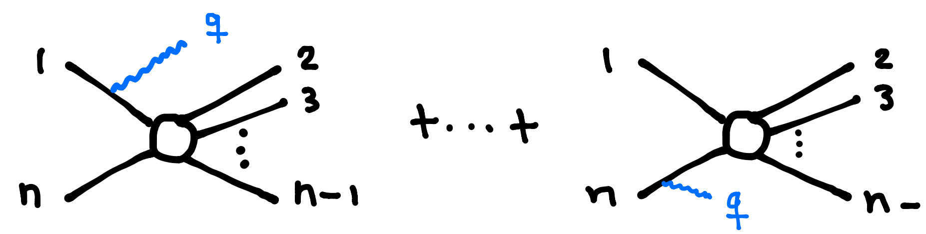
Soft gluon corrections (Eikonal rules)



$$\simeq g \, \frac{T_a \, p_a^\mu}{p_a \cdot q + \delta}$$

$$\delta = \begin{cases} 0 \text{ if } q \text{ Real} \\ i\epsilon \text{ if } q \text{ Virtual \& } a \text{ Outgoing} \\ -i\epsilon \text{ if } q \text{ Virtual \& } a \text{ Incoming} \end{cases}$$

Amplitude can be expressed as

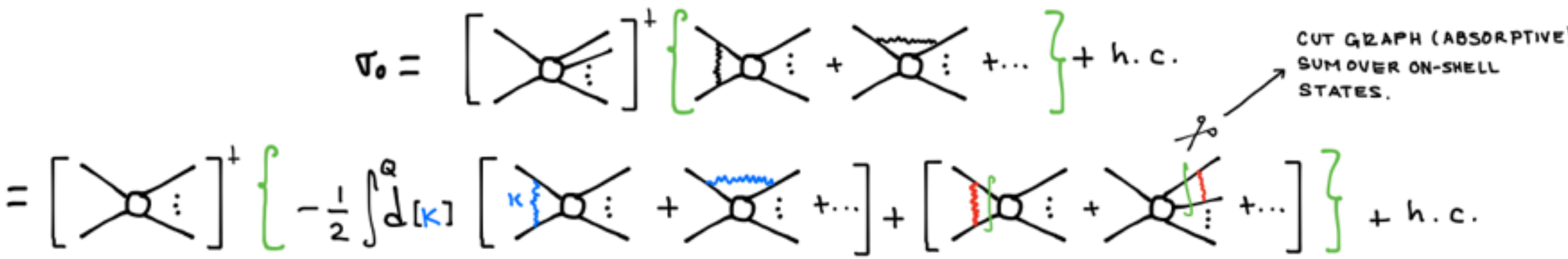


$$J(q)|n\rangle \equiv \left[g \frac{T_i \, P_i \cdot \varepsilon}{P_i \cdot q} + \cdots + g \frac{T_n \, P_n \cdot \varepsilon}{P_n \cdot q} \right] |n\rangle$$

$$\sigma_1 = \int^Q d[q] \langle n | \, J(q) \cdot J(q) \, | n \rangle \phi(q)$$

Eikonal gluons (real)

Notation II: One loop soft correction to a hard process.



$$= \frac{1}{2}\langle n| \left[-\int^Q d[k]\, J(k)\cdot J(k) + \sum_{a,b\,\in\,\text{in or out}} C^{ab}(0,Q) \right] |n\rangle + h.c.$$

Eikonal gluons (virtual)

Coulomb gluons
Only between pairs of incoming or outgoing partons

$$\left(J(k)\cdot J(k)\right)^\dagger = J(k)\cdot J(k)$$

$$C^{ab}(\alpha,\beta) = -i\pi\, g_s^2\, T_a\cdot T_b \int_{\alpha^2}^{\beta^2} \frac{dk_T^2}{k_T^2}$$

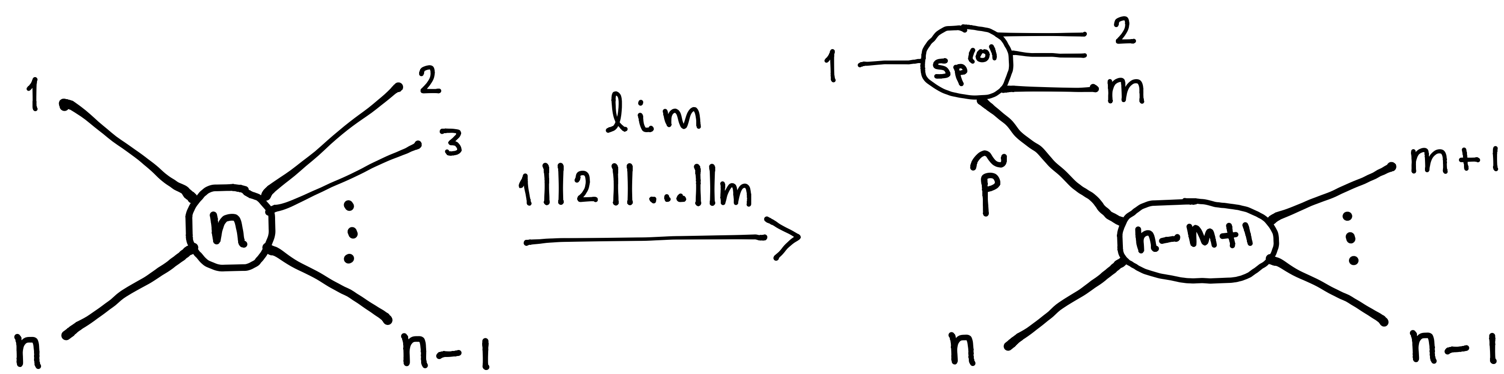
$$C = -C^\dagger$$

$$\Rightarrow \sigma_0 + \sigma_1 = \langle n| \int^Q d[q]\; J(q)\cdot J(q)\,[\phi(q) - 1]\,|n\rangle,$$

$$\phi \sim \alpha_s \ln^\gamma(u), \qquad \gamma = \{0,1,2\}.$$

Collinear factorisation at tree level

Catani, de Florian, Rodrigo JHEP 07(2012)026



$$|n\rangle \simeq Sp^{(0)} \, |n-m+1\rangle$$

($\tilde{p}$: SINGLE PARTON $(1+\ldots+m)$)

Depends only on collinear partons

$$Sp^{(0)} = Sp^{(0)}(\tilde{p}, 1, \ldots, m)$$

Depends only on non-collinear partons

$$|n-m+1(\tilde{p}, m+1, \ldots, n)\rangle$$

Hence, universal (process independent) contributions at cross section level

$$\langle n|n\rangle \simeq \langle n-m+1| \, Sp^{(0)\dagger} \, Sp^{(0)} \, |n-m+1\rangle ,$$

$$\equiv \langle n-m+1| \, P^{(0)} \, |n-m+1\rangle .$$

PDF's

Collinear factorisation (breaking) beyond tree level

Catani, de Florian, Rodrigo JHEP 07(2012)026
Forshaw, Seymour, Siodmok JHEP 11(2012)066

$$\mathrm{Sp}^{(\ell)}_{P} \; |n-m+1\rangle, \quad \ell\text{-loops}$$

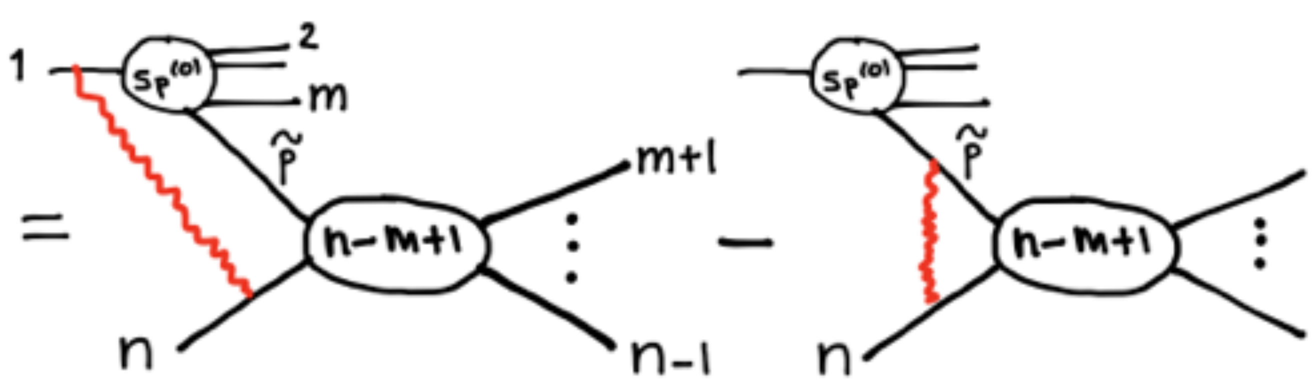
Exceptions:
- 0 incoming partons (e.g. e+e-).
- 1 incoming parton (e.g. DIS).
- Or sufficiently inclusive observables (e.g. inclusive Drell-Yan)

but, in general,

$$\mathrm{Sp}^{(\ell)}_{P} = \mathrm{Sp}^{(\ell)}_{P}(\tilde{p}, 1, \ldots, m, m+1, \ldots, n\rangle$$

The problem seeds at 1-loop order and is specific to the initial state

$$\mathrm{Sp}^{(1)}_{P_{NF}} \; |n-m+1\rangle =$$



$$\neq 0$$

$$= [C^{1n} - C^{\tilde{p}n}] \, \mathrm{Sp}^{(0)}_{P} \; |n-m+1\rangle.$$

Pole part $(k \to 0)$

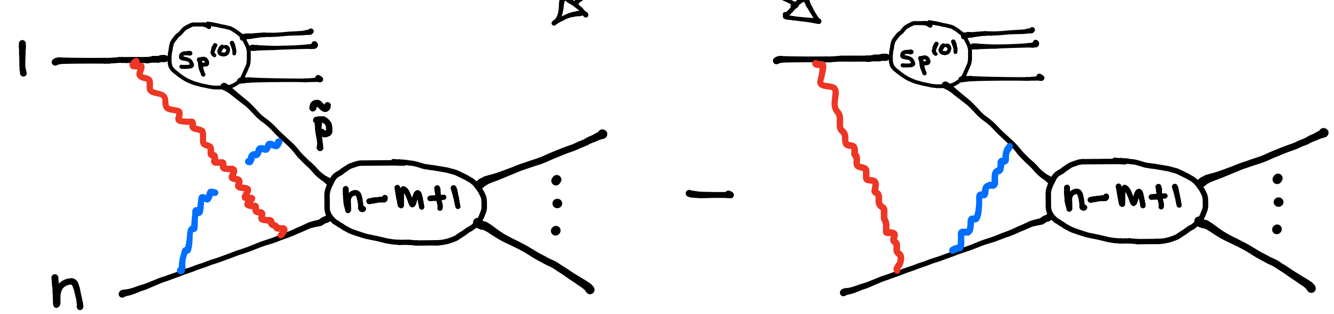
But they these cancel at cross section level

$$C^{1n} - C^{\tilde{p}n} + \text{h.c.} = 0. \quad \checkmark$$

2-loops

$$|Sp^{(2)}_{NF}|^2 \sim \left[\int d[q]\, J^2(q) \,,\, C^{1n} - C^{\tilde{p}n} \right] + \dots$$

Pole part



$$d\sigma = \mathrm{Tr}\left(|\bar{n}\rangle\langle\bar{n}| \left[T_a \cdot T_b \,,\, T_e \cdot T_d \right] \right) = 0.$$

Cancel for pure QCD.
Seymour & Sjödahl
HEP/ 0810.5756

3-loops: No scape

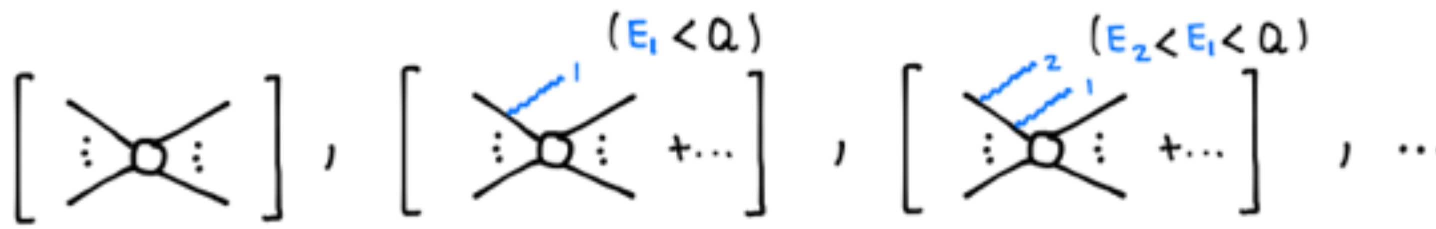
$$|Sp^{(3)}_{NF}|^2 \sim \left[\left[\int d[q]\, J^2(q) \,,\, C^{1n} - C^{\tilde{p}n} \right] , C^{1n} - C^{\tilde{p}n} \right] + \dots \neq 0$$

Pole part

The interplay between Eikonal and Coulomb gluons breaks factorisation!

In pursuit of a general algorithm

Soft gluon insertion technique (Catani, Ciafaloni Nucl.Phys. B249 (1985) 301): Leading infrared behaviour of soft corrections to a hard wide angle scattering. The real emission matrix elements yield



$$|n\rangle \quad , \quad J(1)|n\rangle \quad , \quad J(2)\,J(1)|n\rangle \quad , \quad \cdots$$

Similarly, within this approximation, virtual exchanges are assumed to be nearly on-shell (Eikonal) and, hence, they can be organised as follows

$$J(2)\,J(1)|n\rangle + J(2)\left[\tfrac{-1}{2}\int_{E_2}^{E_1}(J(k_1))^2\,dk_1\right]J(1)\,|n\rangle + J(2)\left[\left(\tfrac{-1}{2}\right)^2\int_{E_2}^{E_1}J^2(k_2)\,dk_2\int_{E_2}^{k_2^0}J^2(k_1)\,dk_1\right]J(1)\,|n\rangle + \ldots$$

$$= J(2)\,\tilde{\nabla}(2,1)\,J(1)\,|n\rangle \qquad \text{WITH} \qquad \tilde{\nabla}(2,1) \equiv \mathrm{EXP}\left(\left(\tfrac{-1}{2}\right)\int_{E_2}^{E_1}(J(k))^2\,dk\right)$$

Then, the m emissions amplitude to all orders yields (massive simplification!)

$$\tilde{\nabla}(0,m)\,J(m)\cdots J(2)\,\tilde{\nabla}(2,1)\,J(1)\,\tilde{\nabla}(1,Q)\,|n\rangle$$

But no Coulomb interactions!

Generalised Forshaw - Kyrielies - Seymour algorithm for the leading Eikonal and Coulomb gluons

The leading corrections due to 0,1,2,… real emissions at ALL orders

No emissions: $\sigma_0 = |V(0,Q)|n\rangle|^2$ $\longrightarrow$

Two loops
Catani JHEP07(2012)026
Eikonal exponentiation
Vladimirov, arxiv:1501.03316

1 real emission: $d\sigma_1 = |V(0,1)J(1)V(1,Q)|n\rangle|^2 \phi(q_1,q_2)\, d[q_1]$

2 real emissions: $d\sigma_2 = |V(0,2)J(2)\,V(2,1)J(1)V(1,Q)|n\rangle|^2 \theta(q_{2T}<q_{1T})\ \phi(q_1,q_2)\, d[q_1]\, d[q_2]$

$\vdots$

etc

But this time, V also inserts Coulomb interactions

$$V(a,b) = \exp\left\{ -\frac{1}{2}\int d[k]\ J(k)\cdot J(k)\ \theta(q_{aT}<k_T<q_{bT}) + \sum_{\substack{a\neq b \\ \text{in or out}}} C^{ab}(q_{aT},q_{bT}) \right\}$$

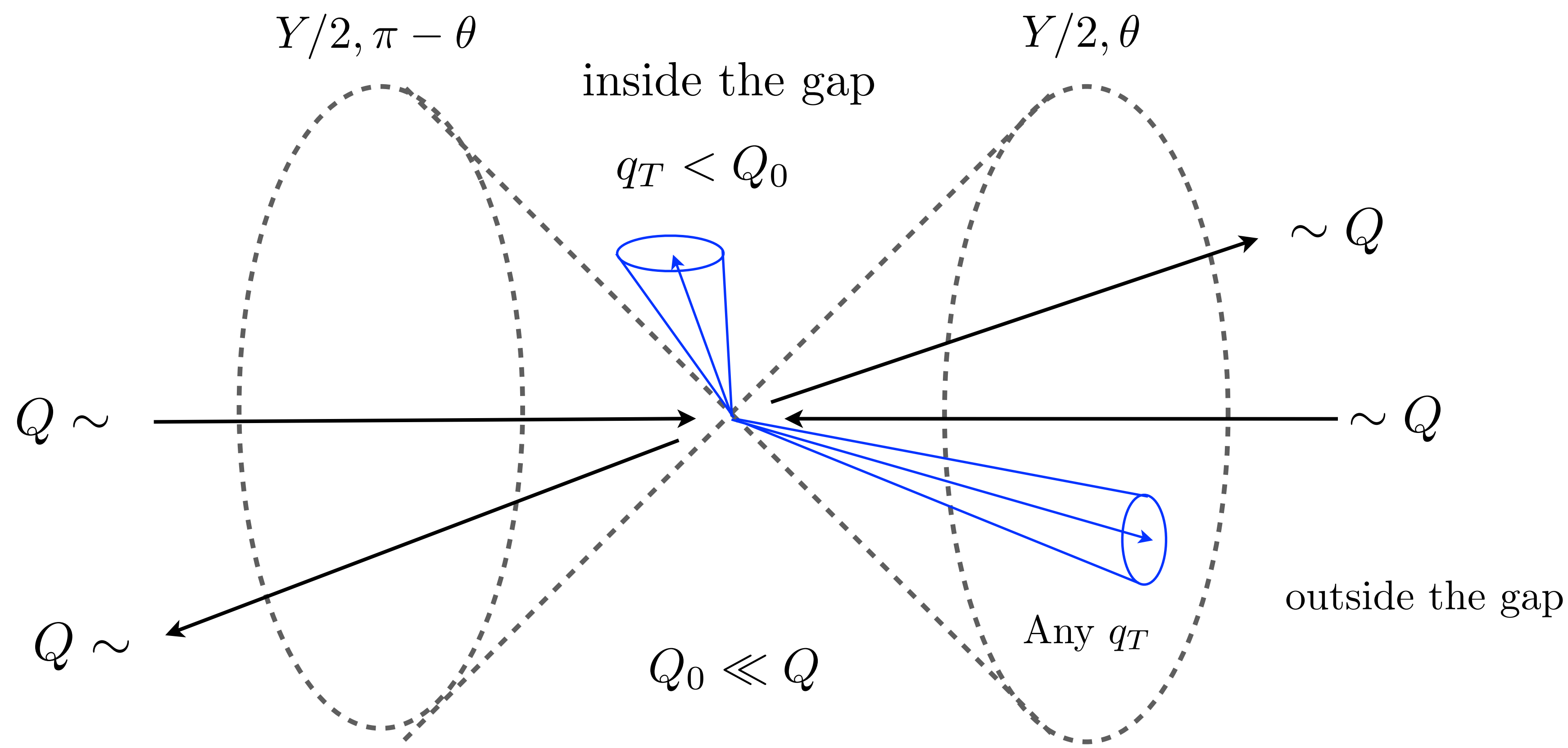
$$\boxed{C^{ab}(\alpha,\beta) = -i\pi\ g^2\ T_a\cdot T_b \int_{\alpha^2}^{\beta^2} \frac{dk_\perp^2}{k_\perp^2}}$$

Is this structure correct? and if it is, is it correct to order w.r.t. kT?

It could be applied to any observable!

An interesting application: Gaps between Jets

(Forshaw, Kyrieleis & Seymour hep /0604094 ; /0808.1269)



Soft corrections

$$d\sigma_m = |\mathcal{M}(q_1, ...q_m)|^2 \; \phi \; dPS$$

Leadings logs

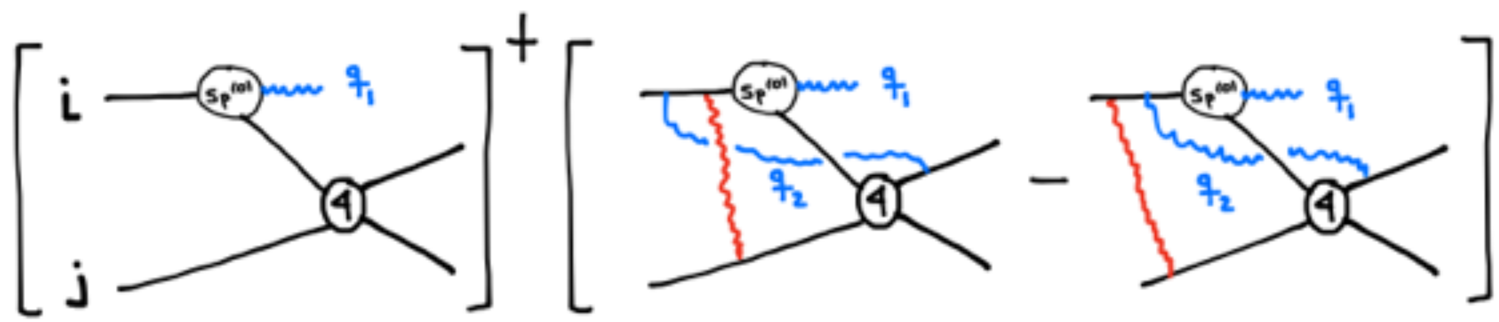
$$\sim \alpha_s^n \ln^n \left(\frac{Q^2}{Q_0^2}\right)$$

Super-leading logs
(due to Coulomb gluons)

$$\sim \alpha_s^3 \ln^4 \left(\frac{Q^2}{Q_0^2}\right), \alpha_s^4 \ln^5 \left(\frac{Q^2}{Q_0^2}\right)$$

Super-leading logs and the breaking of factorisation

$$\int^Q d\sigma_1 + \int^Q d\sigma_2 = \int^Q d[q_1]\, d[q_2]$$



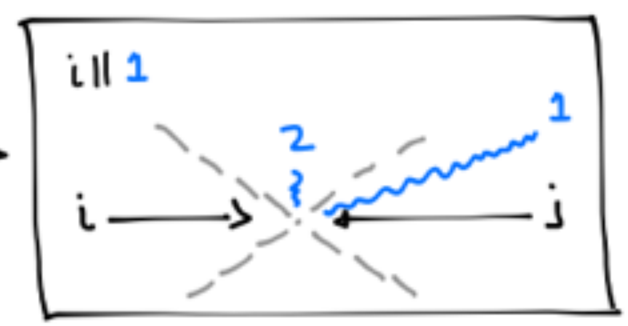
$$\times\ \Theta\,(q_{2T} - q_{1T})\,[\phi(q_2) - \phi(q_1, q_2)] \ + \dots$$

SAME AS
FACT. BREAKING
$i \parallel 1$

2-VIRTUAL (C & q_2)
1-REAL (q_1)

1-VIRTUAL (C)
2-REAL (q_2 & q_1)

$$= \alpha^3 \ln^4 \frac{Q}{Q_0} \langle 4 | \mathbf{T}_i^\dagger [\mathbf{T}_t^2, \mathbf{T}_i \cdot \mathbf{T}_j] \mathbf{T}_i | 4 \rangle + \mathcal{O}\left(\alpha_s^3 \ln^3 \frac{Q}{Q_0}\right).$$



The miscalculation of the terms that break factorisation results in super-leading logarithms!

Consistency cross-checks

- It yields gauge invariance amplitudes

$$|n+m\rangle\Big|_{\varepsilon(q_m)\to q_m} = 0$$

- Order by order corrections cancel if integrated inclusively (E and kT ordering)

$$\sigma_0 + \int_1 d\sigma_1 + \int_1 \int_2 d\sigma_2 + \cdots = \sigma_0$$

- Consistent with reported violations of collinear factorisation.

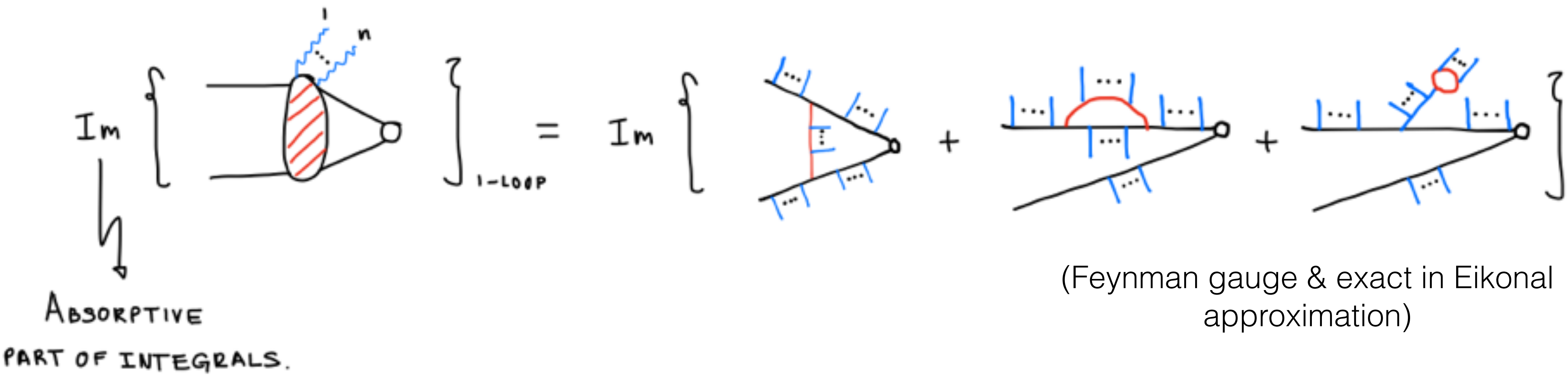
But, there is an ordering problem
(Banfi, Salam, Zanderighi JHEP06(2010)038)

The coefficient of super-leading log in GBJ varies for different ordering variables:

- Angular ordering: zero.
- Energy ordering: infinite.
- Transverse momentum ordering: finite.
- Virtuality ordering: 1/2.

Strategy for testing the ordering variable

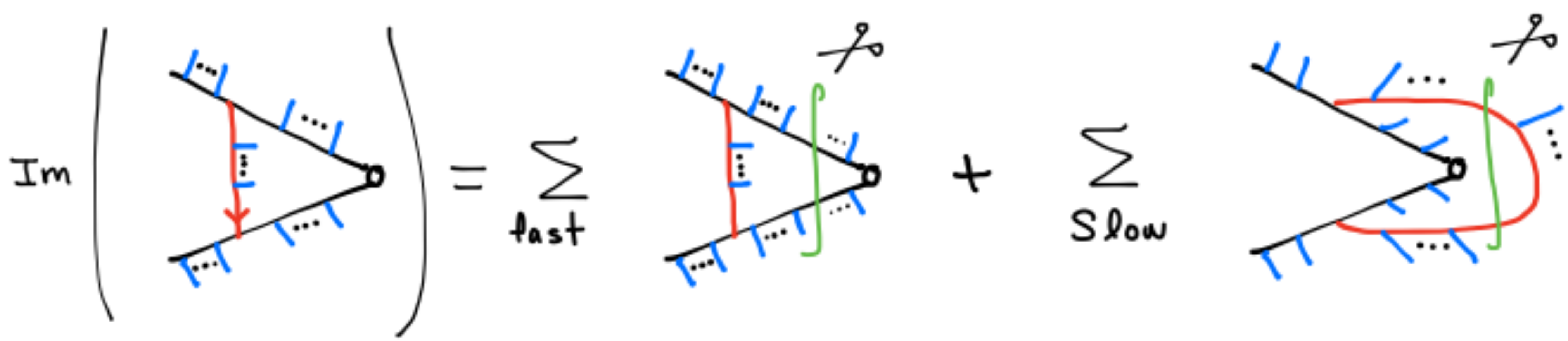
Compare with 1-loop fixed order calculation



(Feynman gauge & exact in Eikonal approximation)

Key step: Integration without assuming any ordering between virtual and real emissions!

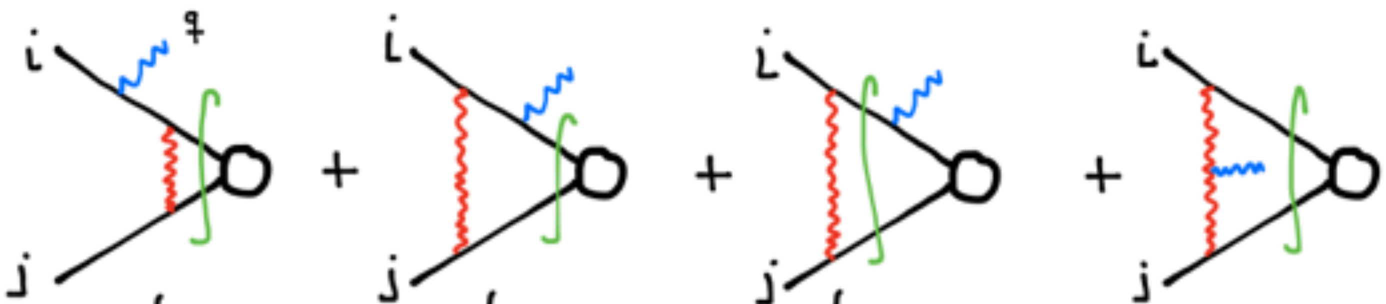
Step 1: Cutting rules



Bottom line: Sum over cuts over fast (Eikonal) or slow (soft gluon) lines.

One emission case

$(Q = 2p_i \cdot p_j)$



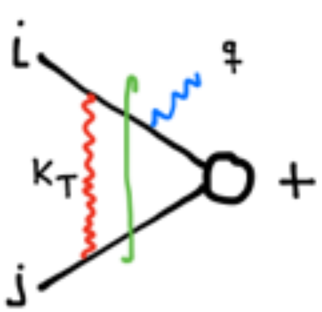
$$-i\pi \frac{p_i\cdot\epsilon}{p_i\cdot q}\int_0^{Q^2}\frac{dk_T^2}{k_T^2}\left\{ T_i(T_i\cdot T_j) - T_i\cdot T_j\, T_i + T_i\cdot T_j\, T_i \right\}|2\rangle$$

Non-abelian

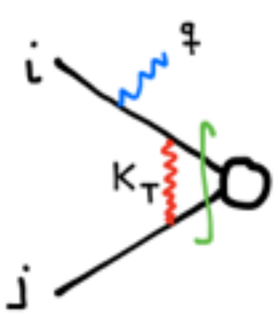
$$-i\pi \frac{p_i\cdot\epsilon}{p_i\cdot q}\underbrace{\int \frac{dk_T^2}{k_T^2}\,\frac{q_T^2}{k_T^2+q_T^2}}_{\simeq \int_0^{q_T^2}\frac{dk_T^2}{k_T^2}}\left[T_i\cdot T_j\, T_i - T_i\, T_i\cdot T_j\right]|2\rangle$$

$$+(i\leftrightarrow j)$$

$\overset{!}{\simeq} \int_0^{q_T^2}\frac{dk_T^2}{k_T^2}$



$+ \int_{q_T^2}^{Q^2}\frac{dk_T^2}{k_T^2}$



The triple gluon vertex graph triggers kT ordering!

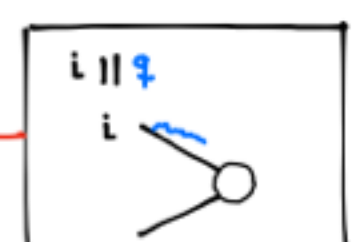
Putting all terms together

ABSORPTIVE PART

$$= \left[C^{ij}(0, q_T)\, J(q) + J(q)\, C^{ij}(q_T, Q) \right] |2\rangle$$

$$\left(C^{ij}(\alpha, \beta) = -i\pi\, g^2\, T_i \cdot T_j \int_{\alpha^2}^{\beta^2} \frac{dk_\perp^2}{k_\perp^2} \right)$$

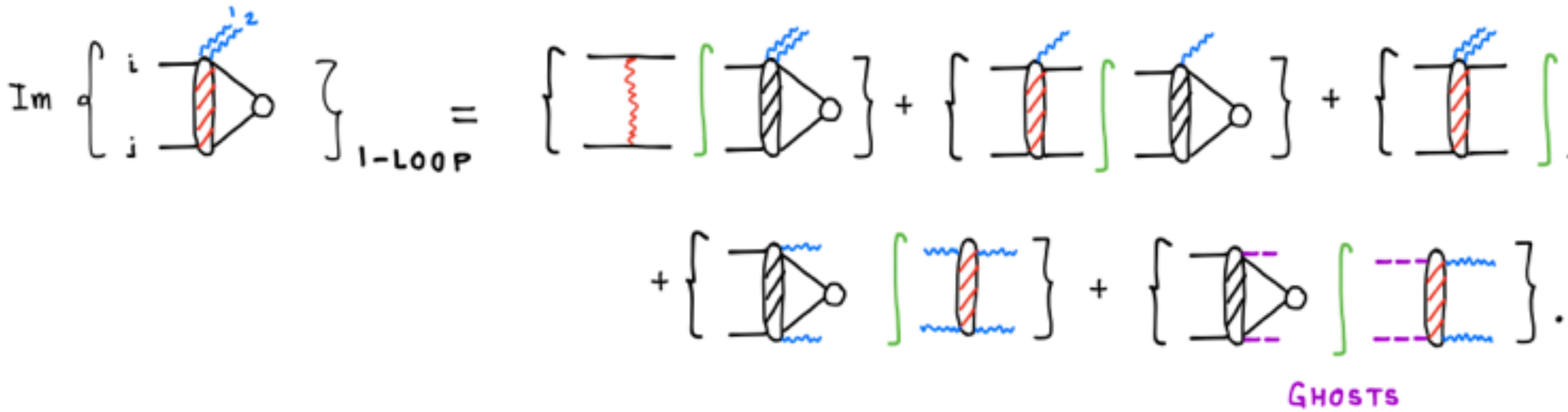
$$\simeq \left[C^{ij}(0, q_T)\, J_i(q) + J_i(q)\, C^{ij}(q_T, Q) \right] |2\rangle$$



$$\left(J_i(q) \equiv g\, T_i \frac{p_i \cdot \varepsilon}{p_i \cdot q} \right)$$

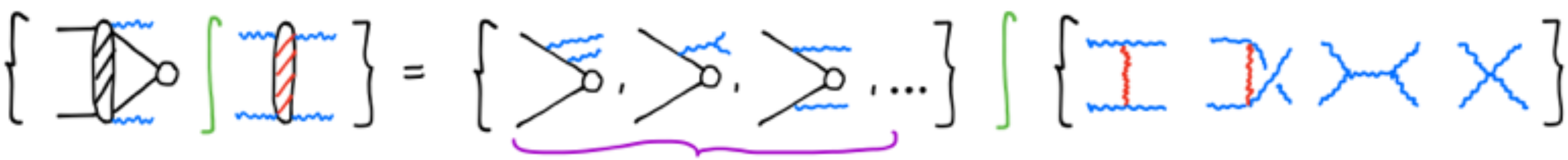
$$\checkmark\ = \mathrm{Im} \left\{ V_{01}\, J(1)\, V_{1Q}\, |2\rangle \right\}_{1\text{-LOOP}} .$$

Two emissions case



(132 cuts)

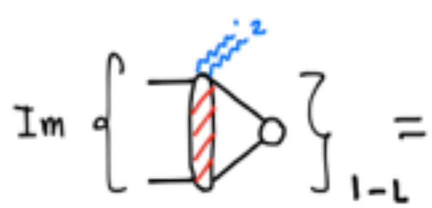
e.g.



$$\equiv \quad K(1,2)\,|2\rangle \xrightarrow{1\gg 2} J(2)\,J(1)\,|2\rangle$$

A complicated operator (for general kinematics) in spin-colour space but the UV and IR divergences have a simple structure

$\ln Q^2$-terms



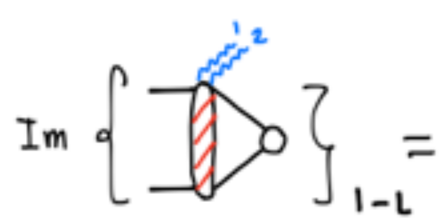
$$= K(1,2)\; C^{ij}(Q)\,|2\rangle\,, \qquad C^{ij}(Q) = -i\pi\, \mathbf{T}_i\cdot\mathbf{T}_j \int^{Q^2} \frac{dk_T^2}{k_T^2}\,,$$

Soft poles

\+

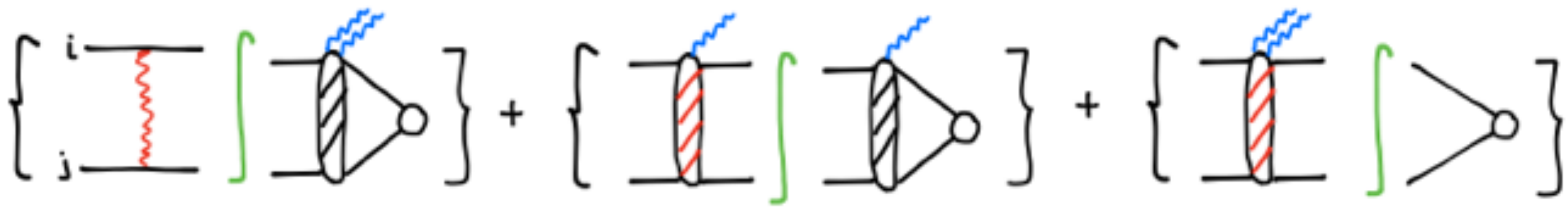
~~Collinear poles~~

Cancel



$$= \left[C^{ij}(0) + C^{12}(0)\right] K(1,2)\,|2\rangle\,, \qquad C^{ab}(0) = -i\pi\, g_s\, \mathbf{T}_a\cdot\mathbf{T}_b \int_0 \frac{dk_T^2}{k_T^2}\,.$$

Leading behaviour: Eikonal cuts



Strongly ordered energies

$$\simeq \left[C^{ij}(0, q_{2T})\, J(2)\, J(1) + J(2)\, C^{ij}(q_{2T}, q_{1T})\, J(1) + J(2)\, J(1)\, C^{ij}(q_{1T}, Q_{ij}) \right] |2\rangle .$$

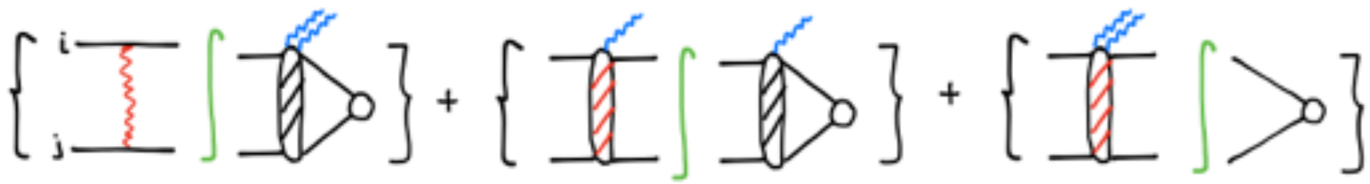
$$\boxed{q_1^0 \gg q_2^0}$$

Single collinear limits

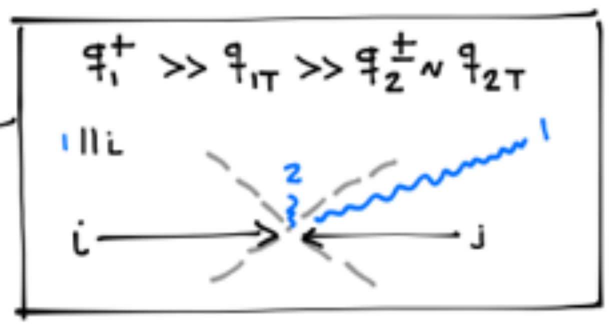
$$\simeq \left[C^{ij}(0, q_{2T})\, J_i(2)\, J(1) + J_i(2)\, C^{ij}(q_{2T}, q_{1T})\, J(1) + J_i(2)\, J(1)\, C^{ij}(q_{1T}, Q_{ij}) \right] |2\rangle .$$

$q_1^{\pm} \sim q_{1T}\,,\; q_2^{+} \gg q_{2T}$

$$\left(J_i(q) \equiv g\, T_i \frac{p_i \cdot \varepsilon}{p_i \cdot q} \right)$$



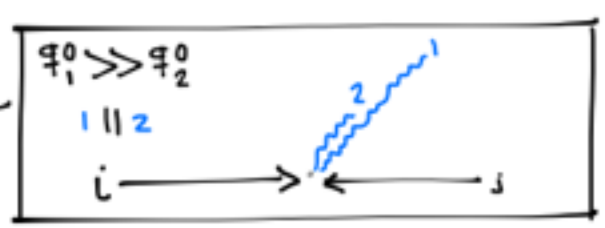
$$\simeq \left[C^{ij}(0, q_{2T})\, J(2)\, J_i(1) + J(2)\, C^{ij}(q_{2T}, q_{1T})\, J_i(1) + J(2)\, J_i(1)\, C^{ij}(q_{1T}, Q_{ij}) \right] |2\rangle .$$



$$\left(J_i(q) \equiv g\, T_i \frac{p_i \cdot \varepsilon}{p_i \cdot q} \right)$$

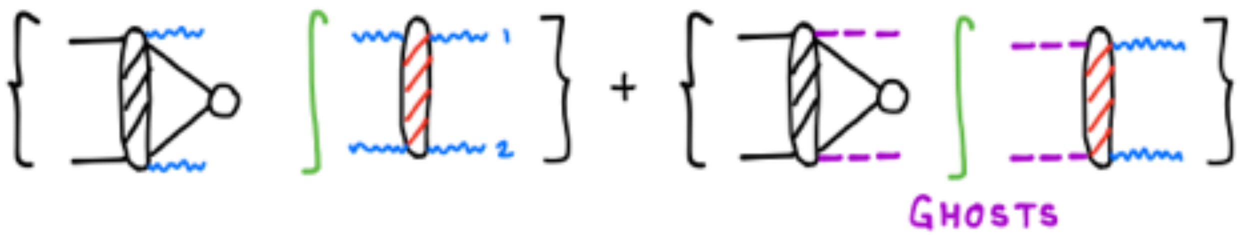
In exact agreement with the calculation of the super-logs!

$$\simeq \left[C^{ij}(0, q_{1T})\, J_1(2)\, J(1) + J_1(2)\, J(1)\, C^{ij}(q_{1T}, Q_{ij}) \right] |2\rangle .$$



$$\left(J_1(q_2) \equiv g\, T_{q_1} \frac{q_1 \cdot \varepsilon_2}{q_1 \cdot q_2} \right)$$

Leading behaviour: soft gluon cuts



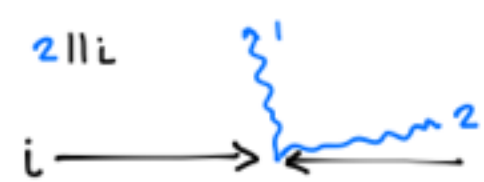
$$\simeq \left[C^{12}(0,q_{2T}) \left(J_i(2) + J_j(2) \right) + C^{12}(0,q'_{2T})\, J_1(2) \right] J(1)\,|2\rangle .$$

$$\boxed{q_1^0 \gg q_2^0}$$

$$\simeq \; C^{12}(0,q_{2T})\; J_i(2)\; J(1)\,|2\rangle$$

$q_1^\pm \sim q_{1T}\,,\; q_2^+ \gg q_{2T}$

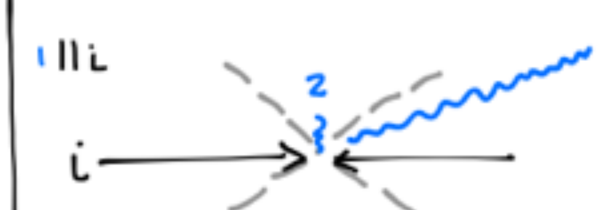
$2 \parallel i$



$$\simeq \; C^{12}(0,q_{2T})\; J(2)\; J_i(1)\,|2\rangle$$

$q_1^+ \gg q_{1T} \gg q_2^\pm \sim q_{2T}$

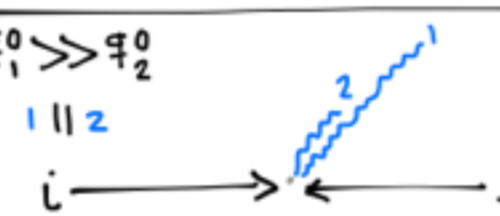
$1 \parallel i$



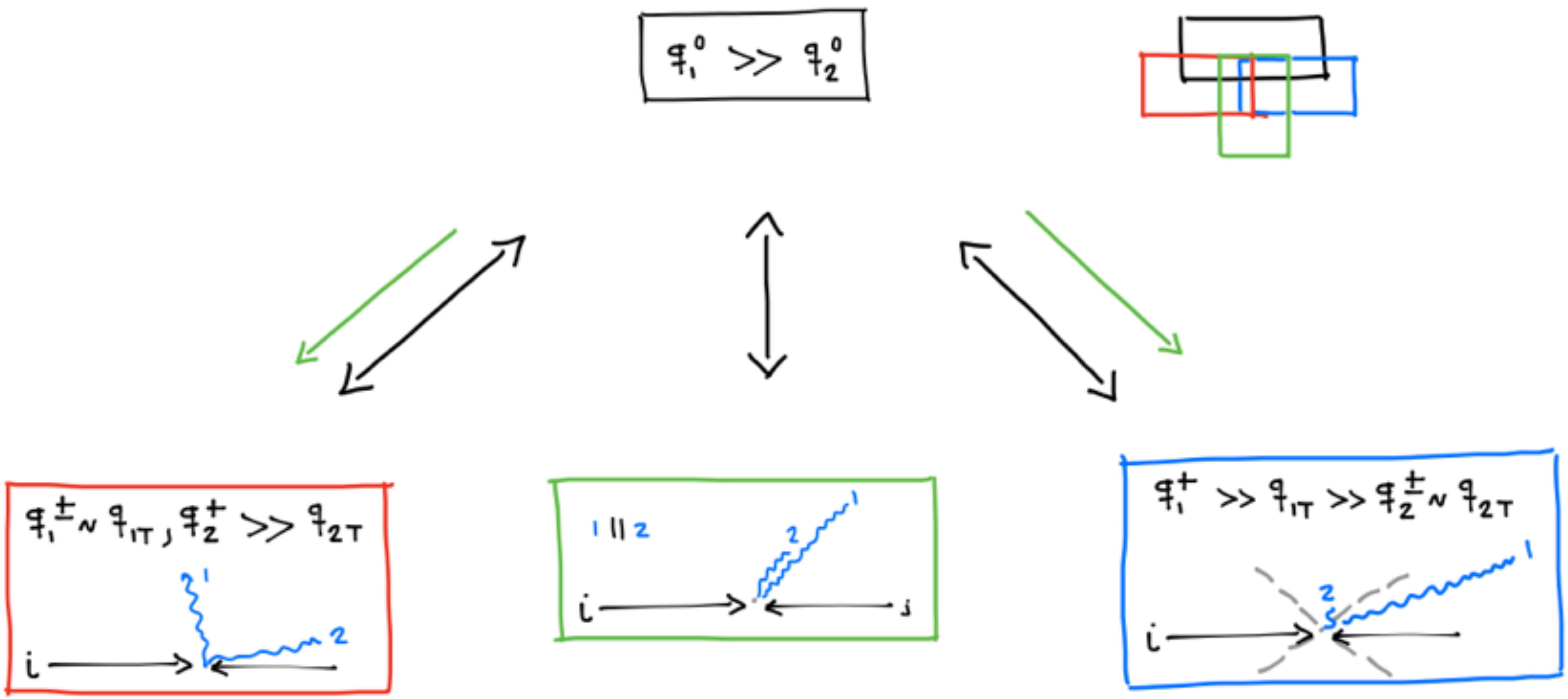
$$\simeq \; C^{12}(0,q'_{2T})\; J_1(2)\; J(1)\,|2\rangle$$

$q_1^0 \gg q_2^0$

$1 \parallel 2$



An interesting property



Limits commute

Can be derived from

Summary

The leading soft behaviour of these amplitudes yields

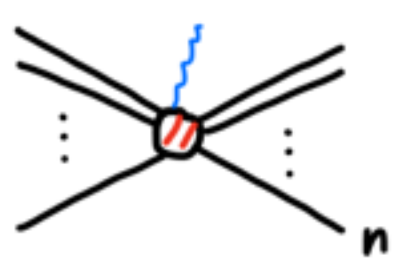
$$\mathrm{Im}\left\{ V_{01}\, J(1)\, V_{1Q}\, |2\rangle \right\} = C^{ij}(0,q_{1T})\, J(1) + J(1)\, C^{ij}(q_{1T},Q) + \mathcal{O}(2\text{-loops})$$

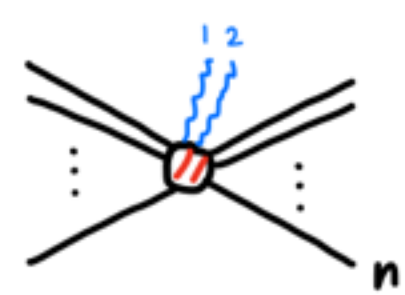
$$\mathrm{Im}\left\{ V_{02}\, J(2)\, V_{21}\, J(1)\, V_{1Q}\, |2\rangle \right\} =$$

$$\left[C^{ij}(0,q_{2T})\, J(2)\, J(1) + J(2)\, C^{ij}(q_{2T},q_{1T})\, J(1) + J(2)\, J(1)\, C^{ij}(q_{1T},Q) \right] |2\rangle \; +$$

$$\left[C^{12}(0,q_{2T}) \big(J_i(2) + J_j(2) \big) + C^{12}(0,q^{1}_{2T})\, J_1(2) \right] J(1)\, |2\rangle + \mathcal{O}(2\text{-loops}) .$$

Paper in preparation with details. One more with generalisations

$$\begin{matrix}\mathrm{Im} \\ \mathrm{Re}\end{matrix} \Bigg\{$$ 

$$\Bigg\} ,\quad \mathrm{Im} \Bigg\{$$ 

$$\Bigg\}$$

Conclusions

- We studied the soft corrections to a hard wide-angle scattering (2->0) due to one virtual gluon and, one or two emissions. We focused the on the absorptive part of loop integrals and derived cutting rules to select it.

- The leading behaviour of these amplitudes obeys an simple pattern (analogous to the soft gluon insertion technique). Precisely, these can be written as a product of Eikonal emissions and Coulomb operators whose limits are ordered w.r.t. the kT of the real emissions.

- The generalisation of this pattern to higher orders could be use to dress a hard wide-angle scatterings with all its leading soft corrections, including Coulomb interactions.

- Our currents results already show non-trivial evidence of the existence of super-leading logarithms found in the gaps between jets observable.

- The exact UV and IR poles can be expressed in term of Coulomb operators acting on the external legs.