Chris White, University of Glasgow

Radiation beyond the soft approximation

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Overview

- Soft-collinear factorisation.
- Generalisation to next-to-leading power.
- Checks in Drell-Yan production.
- Further applications.

General structure of threshold corrections

- If ξ is a kinematic variable that is zero at threshold for some process:

$$\frac{d\sigma}{d\xi} = \sum_{n,m} \alpha^n \left[c_{nm}^{(0)} \left(\frac{\ln^m \xi}{\xi} \right)_+ + c_{nm}^{(1)} \ln^m \xi + \ldots \right].$$

- First set of terms correspond to (leading) threshold logs: pure soft and / or collinear.
- Second set of terms is next-to-leading power (NLP) threshold logs: next-to-soft and / or collinear.
- Can we classify / resum these terms?

Factorisation

- Many approaches exist for resumming threshold logs: diagrammatic (Sterman; Catani, Trentadue), Wilson lines (Korchemsky, Marchesini), RGEs (Forte, Ridolfi), soft-collinear effective theory (SCET) (Becher, Neubert; Schwartz; Stewart).
- Crucial to all of these approaches is the notion of *factorisation*:

  Soft and collinear physics is process-independent, so can be written in terms of universal functions.

- These functions are well-known at leading threshold level...

Soft-collinear factorisation

- A general n-point amplitude has the form:

$$\mathcal{A}_n = \mathcal{H}_n\left(\frac{Q^2}{\mu^2}, \{p_i\}, \{n_i\}, \alpha_s(\mu^2), \epsilon\right) \times \mathcal{S}\left(\{\beta_i\}, \alpha_s(\mu^2), \epsilon\right) \times \prod_{i=1}^{n}\left[\frac{J(p_i, n_i, \alpha_s(\mu^2), \epsilon)}{\mathcal{J}(\beta_i, n_i, \alpha_s(\mu^2), \epsilon)}\right]$$

- $\mathcal{H}_n$: hard function (IR finite).
- $\mathcal{S}$: soft function (all soft singularities).
- J: jet function (collinear singularities).
- $\mathcal{J}$: eikonal jet (soft-collinear, removes double-counting).

- The soft and jet functions are universal:

$$\mathcal{S}\left(\{\beta_i\}, \alpha_s(\mu^2), \epsilon\right) = \left\langle 0 \left| \prod_i \Phi(\beta_i) \right| 0 \right\rangle;$$

$$J\left(p, n, \alpha_s(\mu^2), \epsilon\right) u(p) = \langle 0 \left| \Phi(n) \Psi(0) \right| p \rangle,$$

defined in terms of Wilson lines:

$$\Phi(\beta) = \mathcal{P} \exp\left[i g_s \int_0^\infty d\lambda \beta \cdot A(\lambda\beta) \right]$$

- Hard function $\mathcal{H}_n$ defined by matching to the full amplitude, after computing $\mathcal{S}$, J, $\mathcal{J}$ to a given order.

Next-to-leading power logs

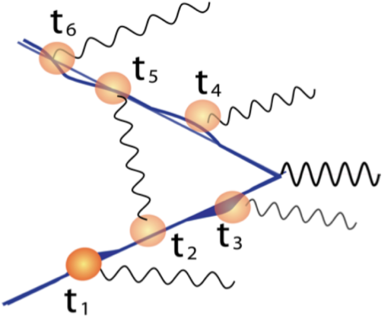
- To date, much less has been known about NLP effects.
- Known for a while to be numerically significant e.g. in Higgs production (Kramer Laenen, Spira; Harlander, Kilgore; Catani, de Florian, Grazzini, Nason).
- This has been confirmed by recent N^3LO Higgs results (Anastasiou, Duhr, Dulat, Herzog, Mistlberger).
- There are thus two good reasons to study NLP logs:
  1. Resummation of them will improve precision.
  2. Even without resummation, NLP logs may provide good approximate N^nLO cross-sections.
- The study of NLP effects has a surprisingly long history...

Low-Burnett-Kroll theorem

- Next-to-soft effects were first studied in gauge theory (QED) by Low (1958).
- He considered external scalars; generalised to fermions by Burnett and Kroll (1968).
- Both groups only considered massive particles: all threshold effects soft.
- Del Duca (1990) generalised the Low-Burnett-Kroll result to include collinear effects.
- He gave a generalised factorisation formula at NLP level.

Path integral approach

- Can describe next-to-soft effects by writing external propagators as spacetime path integrals (Laenen, Stavenga, White).



- Leading term is classical trajectory (soft limit).
- First-order wobbles give next-to-soft behaviour.
- Reproduces NNLO real emission in Drell-Yan.
- Also works for gravity (White).

- Collinear singularities not accounted for.

Physical Evolution Kernel approach

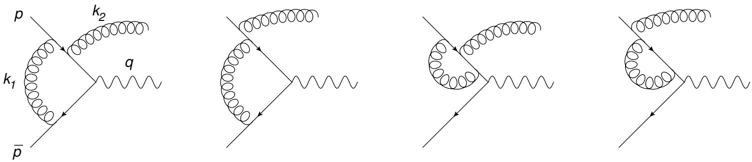
- In DIS, can set up evolution equations for physical structure functions directly:

$$\frac{dF_i(x, Q^2)}{d \ln Q^2} = \sum_j K_{ij} \otimes F_i.$$

- The *physical evolution kernels* K_{ij} are known to NNLO in some cases.
- Fixed order information can be used to motivate a partial resummation prescription for NLP effects (Almasy, Moch, Presti, Soar, Vermaseren, Vogt).
- Can be generalised to DY, Higgs production, e^+e^- etc.
- NLP results found to give excellent approximation of full fixed-order result in some cases.

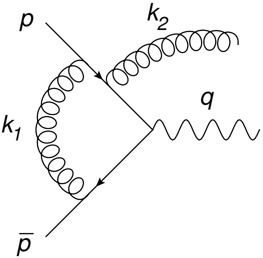
Method of regions approach

- For specific processes, can classify all (next-to) soft / collinear effects using the *method of regions* (Beneke, Smirnov, Pak, Jantzen).
- Example: Drell-Yan production with 1-real and 1-virtual gluon (Bonocore, Laenen, Magnea, Vernazza, White).



- The virtual gluon momentum k_1 can be soft, hard or collinear.
- Each of these is a "region", and they do not overlap.

Method of regions



- Introduce n_+, n_- via

$$p^\nu = \frac{\sqrt{\hat{s}}}{2} n_+^\mu, \quad \bar{p}^\mu = \frac{\sqrt{\hat{s}}}{2} n_-^\mu.$$

- Then write (Sudakov)

$$k_1^\mu = \frac{(n_- \cdot k_1)}{2} n_+^\mu + \frac{(n_+ \cdot k_1)}{2} n_-^\mu + k_{1\perp}^\mu.$$

- Split k_1 into different regions:

$$\text{Hard}: \quad k_1 \sim \sqrt{\hat{s}}\,(1,1,1)\ ; \qquad \text{Soft}: \quad k_1 \sim \sqrt{\hat{s}}\left(\lambda^2,\lambda^2,\lambda^2\right)\ ;$$
$$\text{Collinear}: \quad k_1 \sim \sqrt{\hat{s}}\left(1,\lambda,\lambda^2\right)\ ; \quad \text{Anticollinear}: \quad k_1 \sim \sqrt{\hat{s}}\left(\lambda^2,\lambda,1\right)\ .$$

- The method of regions reproduces the known 1-real, 1-virtual DY K-factor at NNLO.
- Thus, it works also at next-to-soft level.
- All possible collinear and (next-to) soft logs classified (Bonocore, Laenen, Magnea, Vernazza, White).
- Valuable data for constraining / testing NLP factorisation.
- Method of regions has also been applied recently to Higgs production at N^3LO (Anastasiou, Duhr, Dulat, Herzog, Mistlberger).
- No formal proof that it works at NLP level, but it clearly seems to!

A quick summary

- There are many ways of thinking about NLP effects, spanning 60 years!
- None of these previous approaches, though, offers a complete understanding.
- They provide strong hints, though, of an underlying structure.
- What we want is to be able to predict NLP logs in an arbitrary process.
- Can they be written in terms of universal functions (like LP effects)?
- Encouraging recent progress on two fronts...

Factorisation approach

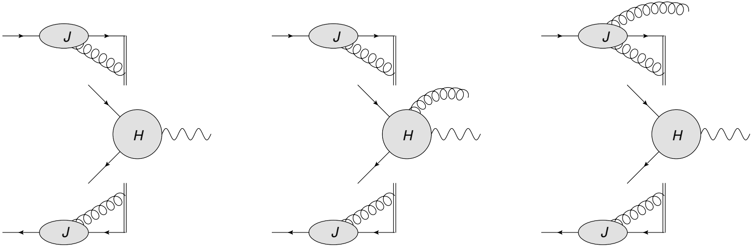
- Here will focus on diagrammatic approach - see also SCET (Larkoski, Neill, Stewart).
- Start with an amplitude with n hard particles (momenta $\{p_i\}$), and add an extra emission (momentum k).
- Can then write a formula for this valid at NLP level (Bonocore, Laenen, Magnea, Melville, Vernazza, White).
- This approach builds on the previous work of Del Duca.
- So far a formula has been given for all QED-type logs...

Towards a NLP factorisation formula

- An n-point amplitude can be written

$$\begin{aligned}\mathcal{A}_n &= \mathcal{H}_n \times \mathcal{S} \times \prod_i \left[\frac{J_i}{\mathcal{J}_i}\right] \\ &= \underbrace{\mathcal{H}_n \times \bar{\mathcal{S}}}_{H} \times \prod_i J_i.\end{aligned}$$

- Now consider emission of a soft gluon k from inside H or jets.



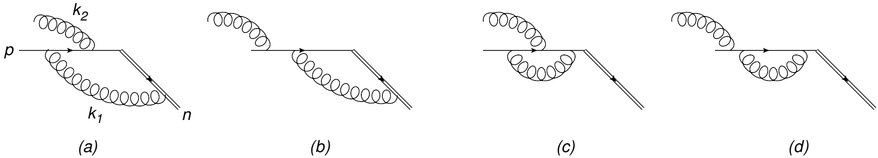
(a) (b) (c)

The jet emission function

- Emissions from the jets can be written in terms of

$$J_\mu(p, n, \alpha_s(\mu^2), \epsilon; k)u(p) = \left\langle 0 \left| \int d^d y \, e^{-i(p+k)\cdot y} \Phi_n(y, \infty)\Psi(y) j_\mu(0) \right| p \right\rangle$$

- Universal quantity in field theory. Introduced by Del Duca, but not previously calculated.



- Emissions from H related to this via Ward identities.

Factorisation formula for NLP effects

- Putting everything together, one finds

$$\mathcal{A}^{\mu}_{n+1}(\{p_i\};k) = \sum_i \Bigg[q_i \left(\frac{(2p_i - k)^\mu}{2p_i\cdot k - k^2} - G_i^{\nu\mu}\frac{\partial}{\partial p_i^\nu}\right)\mathcal{A}_n$$
$$+\mathcal{H}_n(p_i)\bar{\mathcal{S}}(\beta_i)G_i^{\nu\mu}\left(J_\nu(p_i,k) - q_i\frac{\partial}{\partial p_i^\nu}J(p_i,k)\right)\prod_{j\neq i}J(p_j)\Bigg],$$

where the "G-polarisation tensor" is a simple function of p_i and k.

- This is the generalisation of the soft-collinear factorisation formula to NLP level.
- Looks complicated, but each term can be traced to a clear physical interpretation.

Comments

- The first term of the formula is the leading soft-collinear factorisation result.
- All NLP corrections expressed in terms of J_μ, plus quantities already appearing at leading threshold level.
- Once J_μ has been calculated for a given spin, it can be used in any process.
- This result differs from that of Del Duca, in that it explicitly includes the (reduced) soft function.
- Can check its validity in Drell-Yan production.

Drell-Yan check

- The simplest example that checks *all* aspects of the NLP factorisation formula is the 1-real, 1-virtual contribution to Drell-Yan production.
- In that case, the factorisation formula leads to three separate contributions:
    1. The non-radiative (NLO) result, dressed by an extra emission.
    2. Derivative of the non-radiative amplitude w.r.t. external momenta.
    3. A "G-emission" from each jet.
- These are straightforwardly calculated, and integrated over phase space (Bonocore, Laenen, Magnea, Melville, Vernazza, White).

Drell-Yan check

- The answer reproduces exactly the known NLP contribution to the NNLO Drell-Yan K factor (Hamberg, Matsuura, van Neerven; Harlander, Kilgore).

$$
K^{(2)}_{\mathrm{1r,1v}} = \left(\frac{\alpha_s C_F}{4\pi}\right)^2 \Bigg\{ \frac{32\mathcal{D}_0 - 32}{\epsilon^3} + \frac{-64\mathcal{D}_1 + 48\mathcal{D}_0 + 64L_1 - 96}{\epsilon^2}
+ \frac{64\mathcal{D}_2 - 96\mathcal{D}_1 + 128\mathcal{D}_0 - 196 - 64L_1^2 + 208L_1}{\epsilon}
- \frac{128}{3}\mathcal{D}_3 + 96\mathcal{D}_2 - 256\mathcal{D}_1 + 256\mathcal{D}_0 + \frac{128}{3}L_1^3 - 232L_1^2 + 412L_1 - 408 \Bigg\},
$$

where

$$
\mathcal{D}_i = \left(\frac{\log^i(1-z)}{1-z}\right)_+, \quad L_1 = \log(1-z).
$$

- Even constant (non-log) terms are reproduced!

What next?

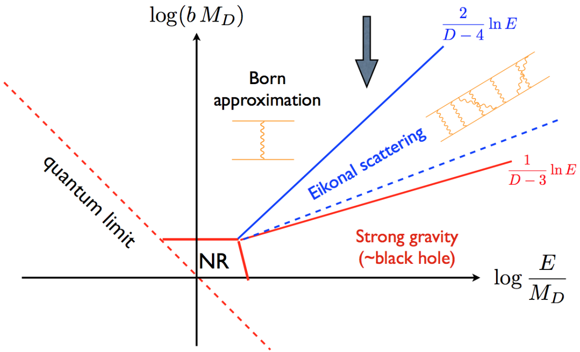
- The factorisation formula can be generalised to include all non-abelian contributions (in progress).
- Can also think of a formula at cross-section, rather than amplitude level (Laenen, Magnea, Stavenga, White).
- This would then pave the way for resummation of NLP effects.
- It would be interesting to compare the SCET and diagrammatic approaches (Larkoski, Neill, Stewart; Beneke, Campanario, Mannel, Pecjak)...
- ...and apply either of them to collider processes!

QCD versus gravity

- Remarkably, the structure of next-to-soft corrections appears to be similar in QCD and gravity.
- This may be related to the *BCJ double copy*, a recently conjectured relationship between the two theories (Bern, Carrasco, Johannson).
- QCD next-to-soft effects are important for collider physics...
- ...but there may also be phenomenological consequences of next-to-soft graviton emission!

Next-to-soft gravity

- Soft gravitons dominate in the high-energy (Regge) limit.



- Different regions in energy / impact parameter (Giddings, Schmidt-Sommerfeld, Andersen).
- Next-to-soft corrections probe unknown parts of this diagram.

Next-to-soft gravity

- A first study of next-to-soft corrections performed by Akhoury, Saotome, Sterman.
- Analysed a subset of terms when a light particle scatters on a black hole.
- Next-to-soft corrections found to exponentiate, modifying the leading behaviour by an amount proportional to the Schwarzschild radius.
- Further applications e.g. to black hole production?
- See also Amati, Ciafaloni, Veneziano, Colferai, Falcioni; 't Hooft; Verlinde^2; Jackiw, Kabat, Ortiz.

Conclusions

- Soft-collinear factorisation and resummation are widely applicable in collider physics.
- It is possible to extend this factorisation to include threshold corrections at next-to-leading power (NLP).
- Can be used to increase the precision of collider predictions.
- Links between QCD and gravity, including interesting conjectures in (non-)supersymmetric theories.
- Possible applications in quantum gravity phenomenology.