# Generalised Resummation for QCD Jet Observables

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Based on work in collaboration with:
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A. Banfi, H. McAslan (University of Sussex)
and ongoing work

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# Outline

- All-order perturbation theory in the Sudakov regime
- Practical relevance of resummation for QCD jet observables
- Different approaches to resummation and factorisation theorems
- Devising a general technique without any required factorisation theorem at NNLL
    - Observable's properties
    - Amplitudes
    - Relevant phase space regions
- Applications in e+e- -> 2 jets and p p -> V production
- Future work and perspectives

# Fixed order QCD and resummation

- High energy strong interaction can be very well described by perturbative QCD (PT) through a power series in the (small) coupling constant
- Each extra power of the coupling corresponds to an extra real emission and relative virtual corrections
- Each real emission has a different scale for its strong coupling which is of the order of its transverse momentum
- In a PT expansion the coupling is evaluated at some common scale of the order of the hard scale(s) of the process - this ensures the coupling to be small (high transverse momentum)
- The PT expansion can be taken and the hard event is usually very well described by the first few terms in the power series

# Fixed order QCD and resummation

- When the transverse momentum of the QCD radiation is constrained to be small, the strong coupling becomes large and an arbitrary amount of QCD emissions become equally important - Need for a all-orders description of the reaction

- The large coupling manifests itself in terms of large single logarithms in the perturbative series

$$\alpha_s(k_t^2) \sim \frac{\alpha_s(Q^2)}{1 - \alpha_s(Q^2)\beta_0 \ln \frac{Q^2}{k_t^2}}$$

- One way to account for the large coupling effects is to sum up all these logarithms to all orders in PT series

- Resummation recasts predictive power of the PT series in this regime and provides a reliable description for the reaction

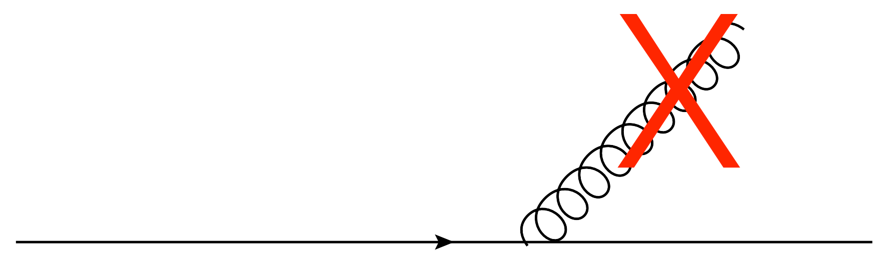
# Fixed order QCD and resummation

- Constraining the real radiation's kinematics leads to additional (double and single) logarithms

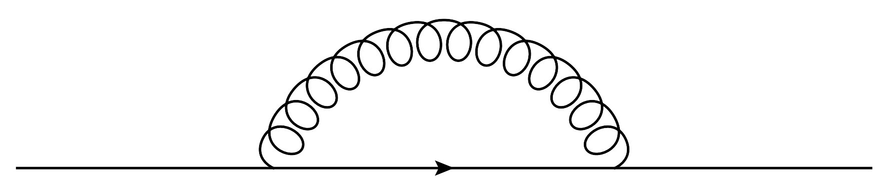
e.g. soft-collinear emission:

- real emission forced to be soft and/or collinear to the emitter
- virtual corrections are unaffected

$$\frac{dk_t}{k_t} d\eta \alpha_s(k_t^2)$$



$$-\frac{dk_t}{k_t} d\eta \alpha_s(k_t^2)$$



$$P(k_t < v) \sim 1 - \frac{\# \alpha_s C_F}{2\pi} \ln^2 v + \ldots$$

# Fixed order QCD and resummation

- In the perturbative regime these logarithms can become as large as (breakdown of the PT below this limit)

$$L \sim \frac{1}{\alpha_s}$$

- This makes "higher order" corrections as large as leading order ones, i.e. $(\alpha_s L)^n L \sim \alpha_s L^2$

- The PT series breaks down and the probability of the reaction diverges logarithmically in the large L limit instead of being suppressed

- The resummation of the large logarithms to all perturbative orders restores the correct physical (Sudakov) suppression and rescues the predictive power of perturbation theory

# Logarithmic accuracy

- Double logarithms due radiation kinematics commonly happen to exponentiate exactly (see later)
  - non-exponentiating observables are avoided because of issues with the simulation in event generators, e.g. JADE algorithm
- For such observables we can define a new perturbative order by expressing the cross section as an exponential function

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\rm Born}} \frac{d\sigma}{dv'} dv' \sim e^{\overbrace{\alpha_s^n L^{n+1}}^{\rm LL} + \overbrace{\alpha_s^n L^n}^{\rm NLL} + \overbrace{\alpha_s^n L^{n-1}}^{\rm NNLL} + \ldots}$$

- In the region where $L \sim 1/\alpha_s(Q)$, LL are enhanced w.r.t. the Born, NLL are as large as the Born cross section itself, NNLL count as NLO corrections, and so on

# Are these regimes of interest ?

- The large-logarithms (Sudakov) regime is probed in several collider reactions whenever tight phase space cuts are applied in the definition of the physical observables (e.g. event-shapes, jet rates, definition of fiducial/control regions for signal or background,…).

- Phenomenological interests:
  - fit of the strong coupling constant
  - precise simulation of background/signal
  - tuning/developing Monte Carlo event generators
  - design of better-behaved observables (e.g. substructure)

- Theoretical interests:
  - properties of the QCD radiation to all-orders
  - understanding of IRC singular structure (subtraction)
  - unveiling perturbative scalings in the deep IRC region
  - probing the boundary with the non-perturbative regime, and study of non-perturbative dynamics

# Different approaches

- Several NLL resummations exist for a number of observables at lepton-lepton (hadron) and hadron-hadron colliders (extensive literature)
- Different approaches to Sudakov resummation are available:
  - Use properties of QCD coherence (i.e. angular ordering, interference) and properties of the observable (e.g. rIRC safety) to set up a resummation program
    - Branching algorithm (many references)

      [Catani, Marchesini, Trentadue, Turnock, Webber et al.]
    - CAESAR/ARES method

      [Banfi, Salam, Zanderighi; Banfi, McAslan, Monni, Zanderighi]
  - Leading kinematical regions emerge from an observable-dependent (all-orders) factorisation theorem (many references)
    - Soft Collinear Effective Theory (see T. Becher's talk)

      [Bauer, Fleming, Pirjol, Stewart; Beneke, Chapovski, Diehl, Feldmann et al.]
    - Collins-Soper-Sterman approach

      [Collins, Soper, Sterman et al.]

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# Different approaches

- NNLL corrections are generally sizeable (count as NLO) and important for precision physics - few NNLL results exist for 2 scale observables in e+e- collisions and even fewer for hadronic collisions
- Most important limitation being the “factorisability" requirement (see later) for the observable in some (smartly defined) conjugate space - resummation often leads to very tedious calculations (~ 14-16 yrs to go from NLL to the next order)
- GOAL: devise a (numerical) resummation approach that:
  - does not rely on factorisation properties of the observable
  - is NNLL accurate and extendable to higher orders
  - is fully general for a very broad category of observables (~all that can be possibly resummed to NNLL)
  - it is suitable for automation (only input: observable’s routine)

# Definition of the problem

- Describe the all-orders QCD radiation to a given logarithmic accuracy in a (as much as) general way
- To achieve that, we need to study the behaviour of both the squared amplitude and the observable in the presence of an arbitrary number of emissions
- We divide the problem (and its solution) in three parts
    - Observable's properties
    - Amplitudes in the logarithmic regime
    - Relevant phase space regions

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# Observable's properties to all orders

- We consider an Infrared and Collinear (IRC) safe observable normalised as $V = V(\{\tilde{p}\}, k_1, ..., k_n) \leq 1$, in the limit $V \to 0$

- In this limit the radiative corrections are described exclusively by virtual corrections, and collinear and/or soft real emissions - QCD amplitudes factorise in these regimes w.r.t. the Born up to regular (giving rise to non-logarithmic corrections) terms

$$|\mathcal{M}(\{\tilde{p}\}, k_1, ..., k_n)|^2 \simeq |M_{\text{Born}}(\{\tilde{p}\})|^2 |M(k_1, ..., k_n)|^2 + \dots$$

- All-order treatment is possible only if factorisation of the QCD amplitudes remains true at all orders (often assumed)

- Cases of collinear factorisation breaking due to miscancellation of Glauber modes (i.e. Coulomb phases) have been found at high orders in multijet squared amplitudes

[J. R. Forshaw, A. Kyrieleis, and M. Seymour 2006-2009]
[S. Catani, D. de Florian, and G. Rodrigo 2012]
[J. R. Forshaw, M. H. Seymour, and A. Siodmok 2012]

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- "Standard" approach to resummation: factorisation for the cross section is achievable if also the (constrained) phase space factorises (contains observable's definition). Resummation can be achieved once a factorisation theorem is set up (modulo complex computations)

# Observable's properties to all orders

- The observable does not trivially factorise in a product of terms arising from each kinematic mode contributing to the factorised amplitude - it often requires to transform into a conjugate space where the factorisation is explicit (e.g. Mellin - Laplace, Fourier)
  - OK for simple semi-inclusive cases: e.g. thrust in e+e-

$$1 - T \simeq \sum_{i=1}^{n} \frac{k_{ti}}{Q} e^{-\eta_i} \qquad \to \qquad \Theta(1 - T < \tau) = \int \frac{d\nu}{2\pi i\nu} e^{\nu\tau} \prod_{i=1}^{n} e^{-\nu \frac{k_{ti}}{Q} e^{-\eta_i}}$$

  - more difficult for involved observables: e.g. jet broadening in e+e- or inclusive vector-boson kt in hadron collisions
  - tough/impossible for observables which mix various kinematic modes or require iterative optimisations: e.g. jet rates, thrust major
- Main idea: factorisation is an unnecessary requirement for resummation. All one needs is some scaling properties of the observable

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# Requirements on the observable

- The standard requirement of IRC safety implies that the value of the observable does not change in the presence of one or more unresolved emissions (i.e. very soft and/or collinear)

- In addition, we require recursive IRC (rIRC) safety (for a more precise formulation see backup slides), i.e. [Banfi, Salam, Zanderighi 2004]

  - that in the presence of multiple emissions the observable scales in the same fashion as for a single emission (IRC divergences have an exponential form)

    - this property ensures the exponentiation of leading logarithms and a cancellation of divergences to all orders;

  - that for sufficiently small $\bar{v}$ there exists some $\epsilon$ that can be chosen *independently* of $\bar{v}$ such that we can neglect any emissions at scales $\sim \epsilon\bar{v}$

$$V(\{\tilde{p}\}, k_1, \ldots, k_n, \ldots, k_m) \simeq V(\{\tilde{p}\}, k_1, \ldots, k_n) + \epsilon^p v$$

    - this property allows one to associate a logarithmic order to each real n-emission probability, establishing a logarithmic hierarchy between correlated branchings

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We limit ourselves to *continuously* global observables*, i.e. constrain the radiation equally everywhere in the phase space
(it ensures the absence of non-global logarithms)

[Dasgupta, Salam 2001; Banfi, Marchesini, Smye 2002]

*Not a real limitation, however currently the full NNLL structure of non-global logarithms is still unknown. Important progresses recently (see Caron-Huot's talk) [Caron-Huot 2015]

  - this property allows one to associate a logarithmic order to each real n-emission probability, establishing a logarithmic hierarchy at NLL and beyond

# Definition of the problem

- Describe the all-orders QCD radiation to a given logarithmic accuracy in a (as much as) general way
- To achieve that, we need to study the behaviour of both the squared amplitude and the observable in the presence of an arbitrary number of emissions
- We divide the problem (and its solution) in three parts
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# Virtual and (unresolved) real emissions

- RGE evolution for virtual corrections leads to a complete exponentiation of IRC singularities. To perform the cancellation of IRC poles to all orders, some sort of factorisation of the real corrections is required!

- Solution: consider primary emissions off the Born legs *inclusive* in secondary (gluon) branchings. We introduce a resolution scale $\epsilon$ and define a subset of (*unresolved*) emissions, such that $V_{\mathrm{sc}}(\{\tilde{p}\}, k_i) < \epsilon v$

- The observable for unresolved emissions takes the simple (and factorising) form

$$\Theta\left(\epsilon v - \max\{V_{\mathrm{sc}}(\{\tilde{p}\}, k_1), \ldots, V_{\mathrm{sc}}(\{\tilde{p}\}, k_n)\}\right)$$

  - the soft-collinear approximation of the observable for unresolved emissions is enough to ensure the cancellation of the IRC poles arising from the virtual corrections (a different choice can be adopted - final result is scheme-invariant)

  - details of the actual observable's scaling for several emissions away from the soft-collinear region and correct treatment of the gluon branchings are introduced at a later stage (next slides)

  - rIRC ensures that these emissions do not contribute significantly to the observable, their contribution factorises at all-orders. Use virtual RGE to cancel IRC singularities

[Work in progress]

# Virtual and (unresolved) real emissions

- The combination of unresolved real and virtual corrections gives rise to an exponential factor that defines the no-emission probability at scales larger than $\epsilon v$

$$P(\mathrm{no-emissions}) \sim e^{-R(\epsilon v)}$$

- Since we're interested in vetoing emissions above $v$, this can be further expanded as

$$P(\mathrm{no-emissions}) \sim e^{-R(v)-R'(v)\ln\frac{1}{\epsilon}+\frac{1}{2!}R''(v)\ln^2\frac{1}{\epsilon}+\ldots}$$

- The factor $R(v)$ is called the *radiator*, and defines the physical region where no radiation is allowed

- The logarithmic dependence on the resolution parameter cancels when all-order resolved real emissions are taken into account (rIRC safety)

- Owing to the above definition of unresolved emissions, the radiator is universal for all observables with the same soft-collinear scaling in the presence of a single emission

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# Virtual and (unresolved) real emissions

- The resulting cumulative resummed cross section takes the form

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\rm Born}} \frac{d\sigma}{dv'} dv' \sim e^{-R(v)} \mathcal{F}(v)$$

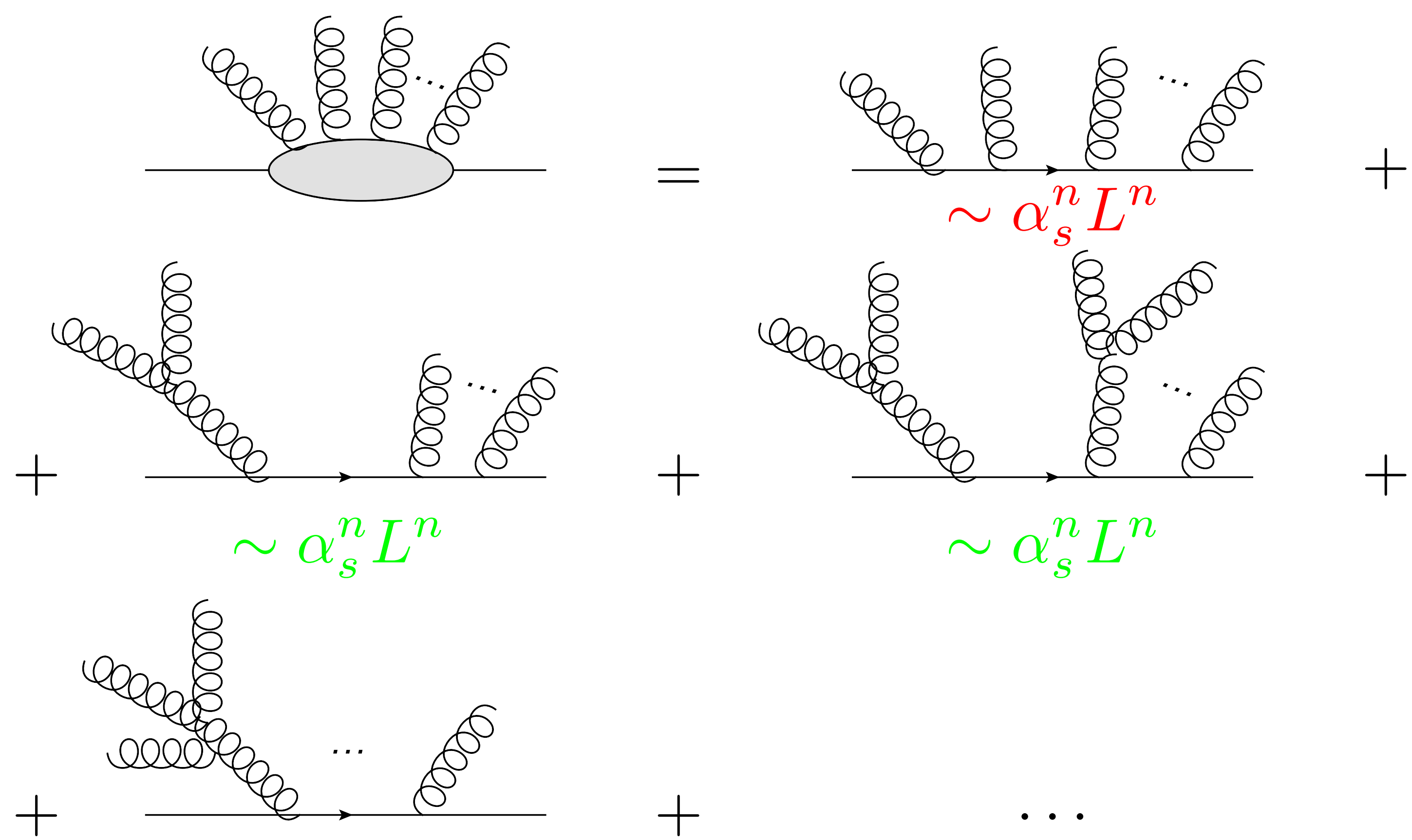
Multiple emissions function

- rIRC safety ensures that all double logarithms are contained in the radiator, and that the multiple emissions function is non trivial (at most) at NLL level

- Because of the lower cutoff defined by the resolution parameter, the remaining (resolved) real emissions are bounded both from above and from below (i.e. each emission "loses" one logarithm power)

# Soft (resolved) matrix elements

- It is useful to decompose the matrix element for n soft emissions (w.r.t. the Born) as a sum of terms with an increasing number of colour-correlated emissions (i.e. non-abelian contributions)

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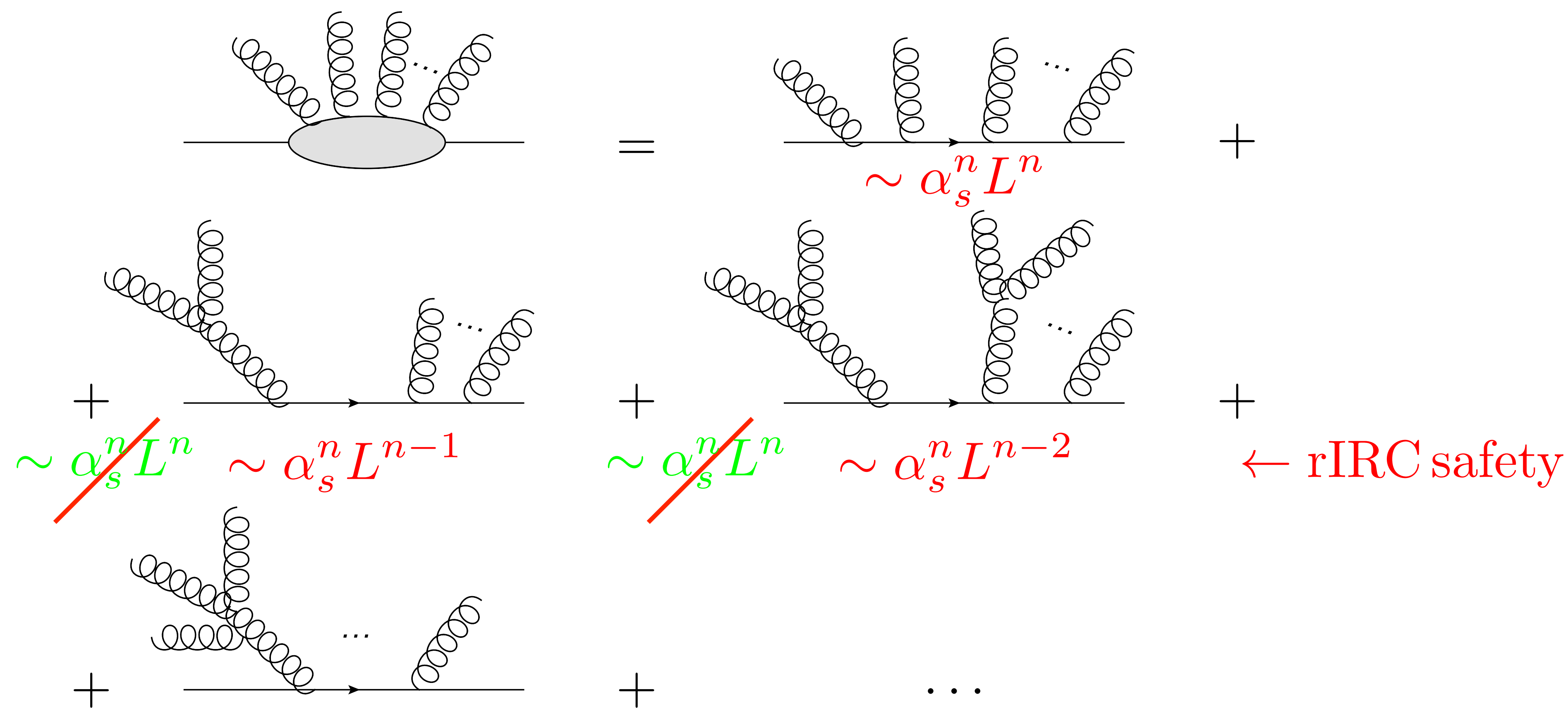


- Which diagrams do we need to achieve NNLL, i.e. neglect terms of order $\sim \alpha_s^n L^{n-2}$ ?

# Collinear (resolved) matrix elements

- The above counting allows us to keep (at NNLL) only configurations with an arbitrary number of independent soft-collinear emissions, and a single soft gluon branching either in a quarks or gluons pair

- This neglects contributions from the real matrix element which would give rise to subleading contributions

- Analogously, if we repeat the same analysis in the hard collinear limit, we find that only a single hard emission can be emitted collinearly to the Born hard leg at this logarithmic order (accompanied by the usual ensemble of soft-collinear emissions)

- Terms beyond this accuracy can be systematically included for extensions to higher orders (if ever necessary)

# Definition of the problem

- Describe the all-orders QCD radiation to a given logarithmic accuracy in a (as much as) general way
- To achieve that, we need to study the behaviour of both the squared amplitude and the observable in the presence of an arbitrary number of emissions
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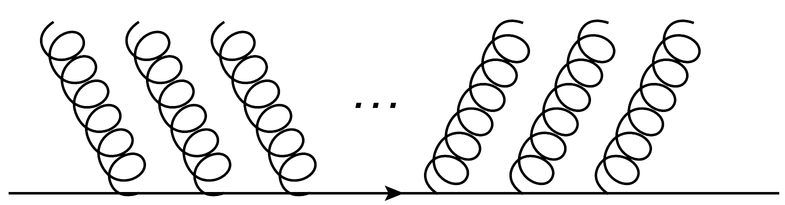
# Phase space at NLL with n=2 legs

- At NLL the multiple emission function is given by an ensemble of soft and collinear independent (abelian) emissions widely separated in rapidity (coherence)

- Non-abelian effects are completely accounted for in the radiator (i.e. inclusive gluon branching treatment -> CMW scheme)

Measure defined by the soft-collinear ensemble



$$= \int d\mathcal{Z}[\{R'_{\mathrm{NLL},\ell_i}, k_i\}]\,\Theta\left(1 - \lim_{v\to 0}\frac{V_{\mathrm{sc}}(\{\tilde p\},\{k_i\})}{v}\right)$$

$$\mathcal{F}_{\mathrm{NLL}}(v) = \langle \Theta(1 - \lim_{v\to 0}\frac{V_{\mathrm{sc}}(\{\tilde p\},\{k_i\})}{v})\rangle$$

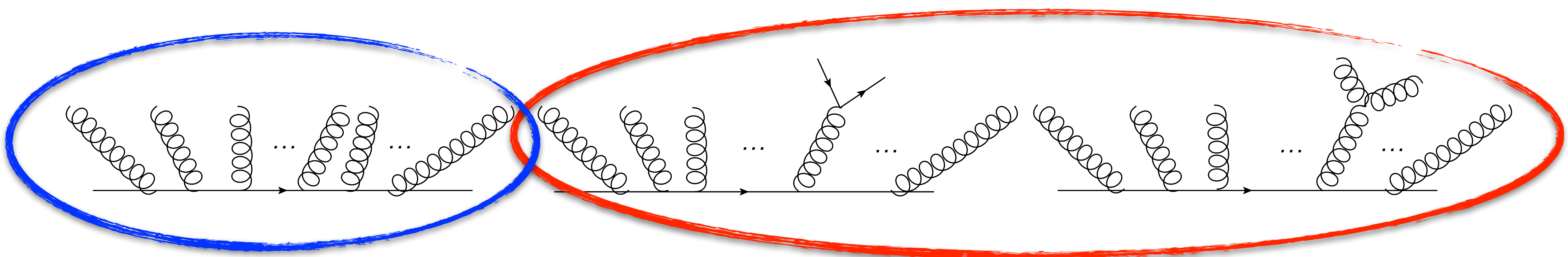
[Banfi, Salam, Zanderighi]

- Differences in rapidity bounds of different emissions contribute at NNLL (can be neglected at this order)

# Phase space at NNLL with n=2 legs

[Banfi, McAslan, Monni, Zanderighi]

- Extension to NNLL involves additional kinematic configurations:
  - (at most) two soft-collinear emissions get close in rapidity



Clustering correction
(jet algorithms only)

Correlated corrections

$$\delta\mathcal{F}_{\rm correl}(\lambda) = \int_0^\infty \frac{d\zeta_a}{\zeta_a}\int_0^{2\pi}\frac{d\phi_a}{2\pi}\sum_{\ell_a=1,2}\left(\frac{2C_{\ell_a}\lambda}{\beta_0}\frac{R''_{\ell_a}(v)}{\alpha_s(Q)}\right)\int_0^\infty\frac{d\kappa}{\kappa}\int_{-\infty}^{\infty}d\eta\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{1}{2!}C_{ab}(\kappa,\eta,\phi)\times$$

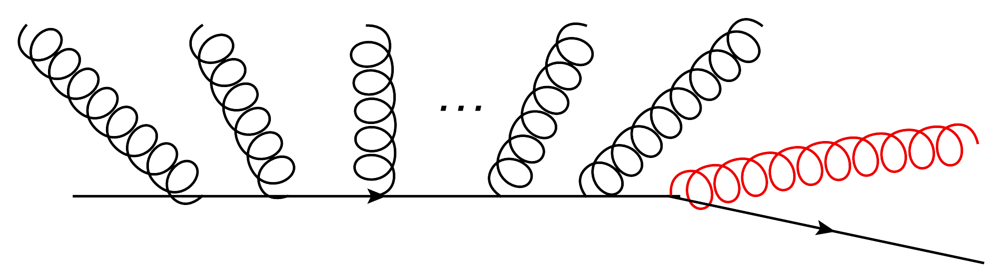
$$\times\int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i},k_i\}]\left[\Theta\left(v - V_{\rm sc}(\{\tilde{p}\},k_a,k_b,\{k_i\})\right) - \Theta\left(v - V_{\rm sc}(\{\tilde{p}\},k_a+k_b,\{k_i\})\right)\right]$$

$$C_{ab}(\kappa,\eta,\phi) = \frac{\tilde{M}^2(k_a,k_b)}{M^2_{\rm sc}(k_a)M^2_{\rm sc}(k_b)}$$

All corrections in terms of four-dimensional integrals

# Phase space at NNLL with n = 2 legs

- Extension to NNLL involves additional kinematic configurations:
  - (at most) one collinear emission can carry a significant fraction of the energy of the hard emitter (which recoils against it)



  - Corrections affect both matrix element (hc) and observable (rec)

$$\delta\mathcal{F}_{\rm hc}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b_\ell)}Q)}{\alpha_s(Q)(a+b_\ell)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int d\mathcal{Z}[\{R'_{\rm NLL,\ell_i}, k_i\}] \times$$
$$\times \int_0^1 \frac{dz}{z}\,(z p_\ell(z) - 2C_\ell)\left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, \{k_i\})}{v}\right)\Theta(1-\zeta)\right]$$

$$\delta\mathcal{F}_{\rm rec}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b_\ell)}Q)}{\alpha_s(Q)(a+b_\ell)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int d\mathcal{Z}[\{R'_{\rm NLL,\ell_i}, k_i\}] \times$$
$$\times \int_0^1 dz\, p_\ell(z) \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm hc}^{(k')}(\{\tilde{p}\}, k', \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right)\right]$$

# Phase space at NNLL with n = 2 legs**

- Extension to NNLL involves additional kinematic configurations:
  - (at most) one soft-collinear emission has the correct rapidity bounds (approximated in the NLL ensemble)

$$\delta\mathcal{F}_{\rm sc}(\lambda) = \frac{\pi}{\alpha_s(Q)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \sum_{\ell=1,2} \left(\delta R'_{{\rm NNLL},\ell} + R''_{\ell_i} \ln \frac{d_\ell g_\ell(\phi)}{\zeta}\right) \int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i}, k_i\}] \times$$
$$\times \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right) - \Theta(1-\zeta)\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, \{k_i\})}{v}\right)\right],$$

  - (at most) one soft emission can have very small rapidity (wide angle)

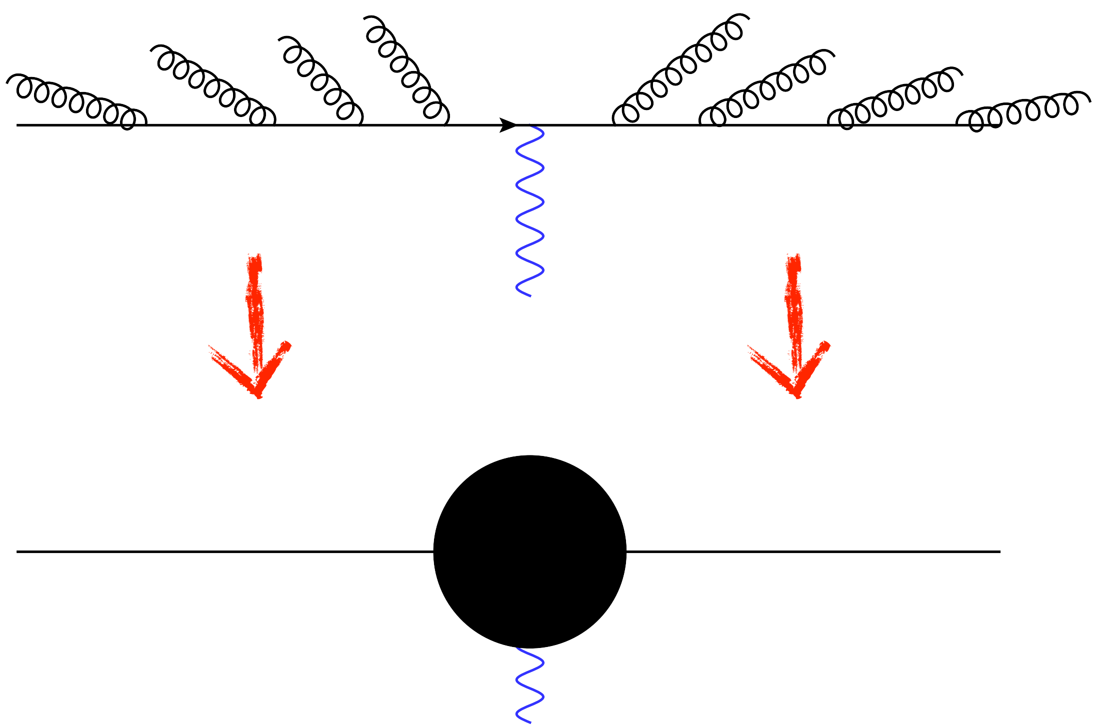
$$\delta\mathcal{F}_{\rm wa}(\lambda) = \frac{2C_F}{a} \frac{\alpha_s(v^{1/a}Q)}{\alpha_s(Q)} \int_0^\infty \frac{d\zeta}{\zeta} \int_{-\infty}^\infty d\eta \int_0^{2\pi} \frac{d\phi}{2\pi} \int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i}, k_i\}]$$
$$\times \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm wa}^{(k)}(\{\tilde{p}\}, k, \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right)\right]$$

**With n>2 there are additional (NLL) contributions due to the quantum interference between hard legs

# Physical interpretation

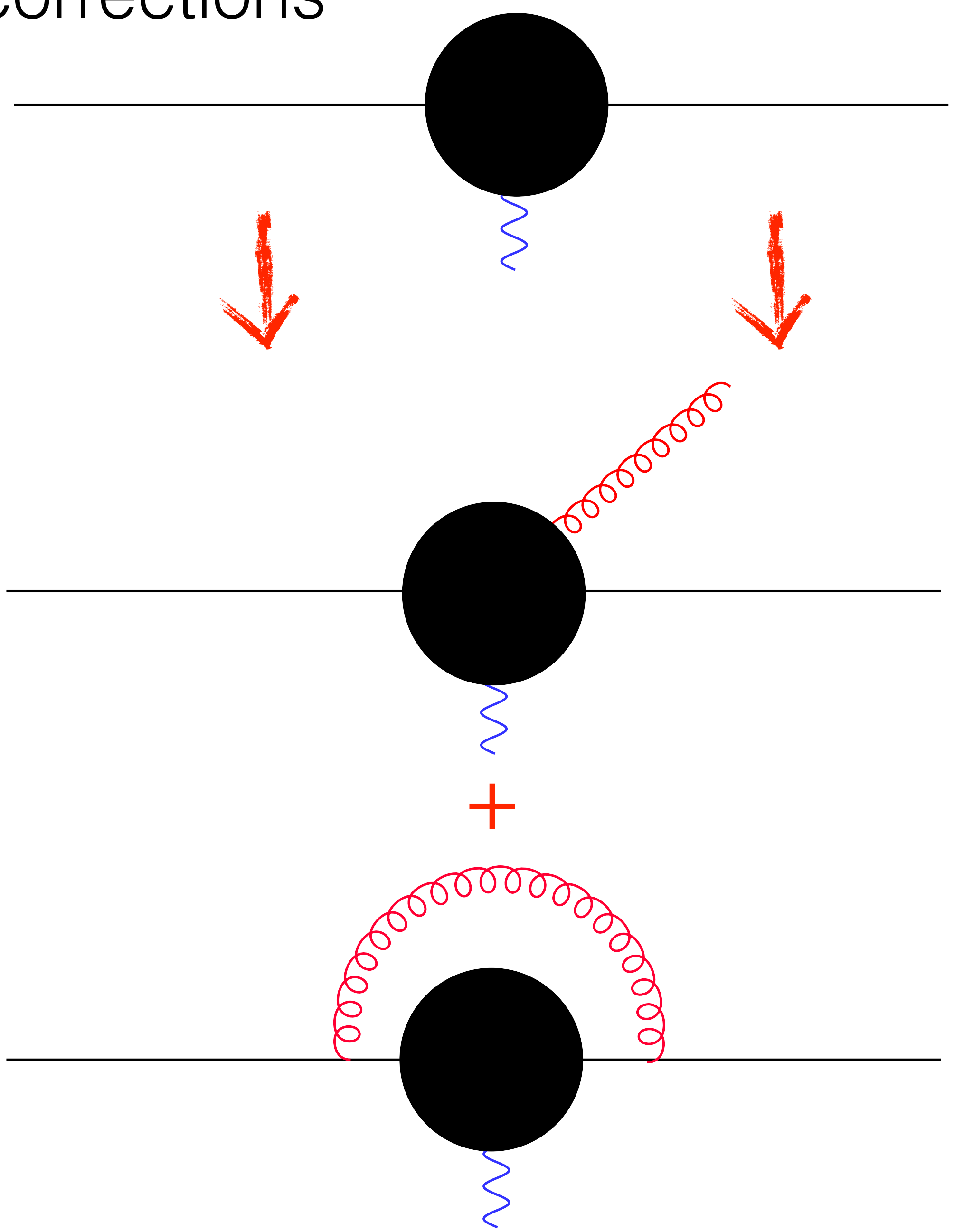
- The NLL multiple emission function is given by soft-collinear gluons strongly ordered in angle (i.e. far in rapidity), each inclusive in the secondary branching (i.e. running coupling scheme)

- This ensemble defines the Born level answer (i.e. NLL order)



# Physical interpretation

- The NNLL answer corresponds precisely to computing the NLO correction to this configuration, i.e. one exact soft and/or collinear real emission (everywhere in the phase space) and the corresponding (full) one-loop virtual corrections




- In addition, add corrections to the running coupling scheme (i.e. exact treatment of the secondary gluon splitting)

- Definition of NNLL real corrections can be seen as a subtraction scheme

# Application to processes with n=2 legs

[Banfi, McAslan, Monni, Zanderighi]

- Observables with very different logarithmic structures can be modelled with the same method

| correction type | $p_{t,\mathrm{veto}}$ | $1-T$ | $B_T$ | $B_W$ | $C$ | $\rho_H$ | $T_M$ | $O$ | $y_3^{\mathrm{Dur.}}$ | $y_3^{\mathrm{Cam.}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{F}_{\mathrm{NLL}}$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\delta\mathcal{F}_{\mathrm{rap}}$ | x | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | x |
| $\delta\mathcal{F}_{\mathrm{wa}}$ | x | x | x | x | ✓ | x | x | x | ✓ | ✓ |
| $\delta\mathcal{F}_{\mathrm{hc}}$ | x | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | x |
| $\delta\mathcal{F}_{\mathrm{rec}}$ | x | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | x |
| $\delta\mathcal{F}_{\mathrm{clust}}$ | ✓ | x | x | x | x | x | x | x | ✓ | ✓ |
| $\delta\mathcal{F}_{\mathrm{correl}}$ | ✓ | x | ✓ | ✓ | x | x | ✓ | ✓ | ✓ | ✓ |

- Reproduce existing results in the literature up to NNLL:

jet veto, [Banfi, Monni, Salam, Zanderighi] [Becher, Neubert, Rothen] [Stewart, Tackmann, Walsh, Zuberi]

thrust, [Becher, Schwartz] [Gehrmann, Luisoni, Monni]

jet broadenings, [Becher, Bell]

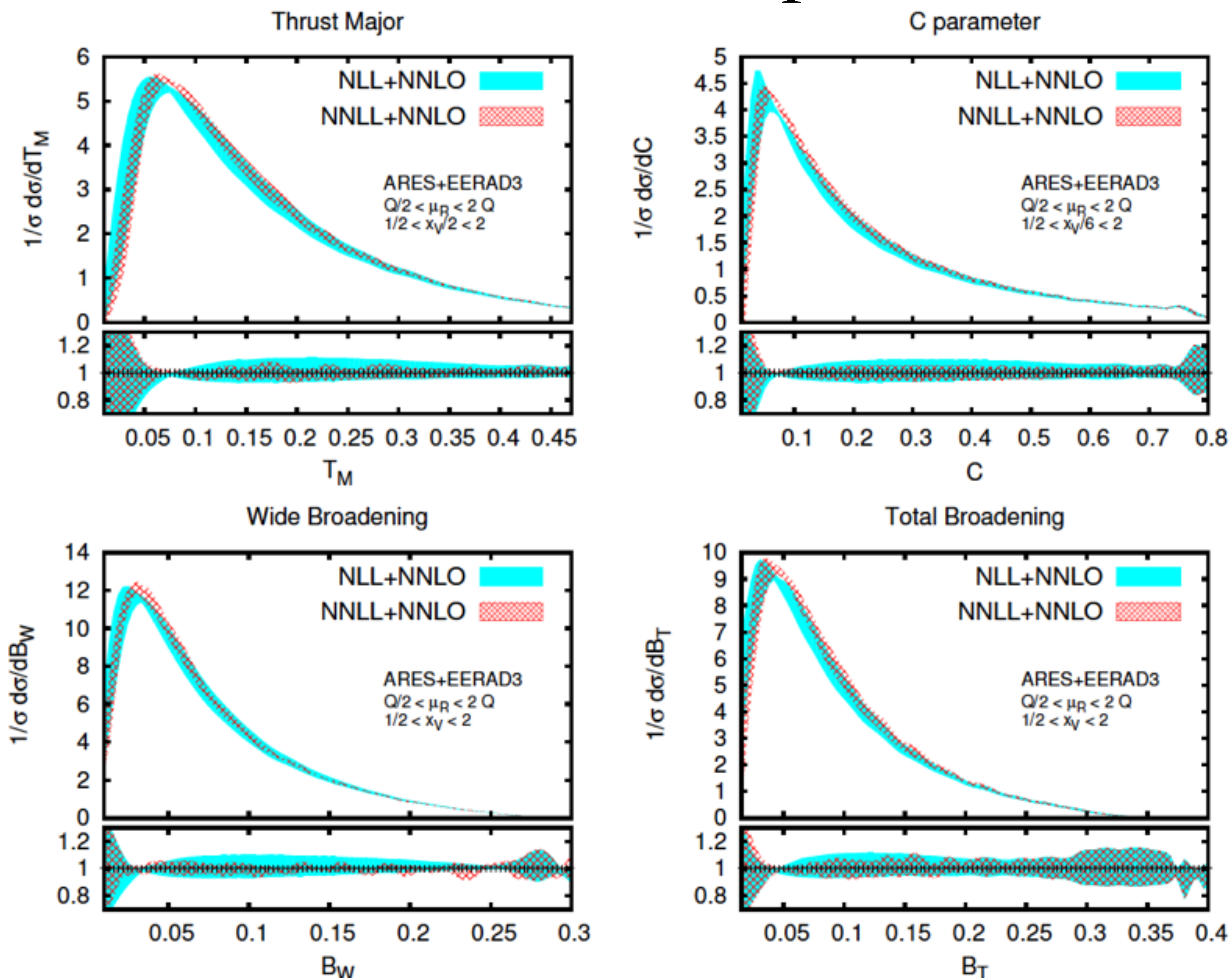
heavy-jet mass, [Chien, Schwartz]

C-parameter [Hoang et al.]

- Obtain new results for involved observables (no factorisation theorem)

e.g. Thrust major: $T_M \equiv \max_{\vec{n}\cdot\vec{n}_T=0} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$

# e+e- event shapes*




*NNLO fixed order from EERAD3



[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich]

# New results: the two-jet rate in e+e-

- The three-jet resolution parameter defines the maximum value of $y_{\rm cut}$ that leads to two QCD jets in the final state

- Clustering is performed according to a jet algorithm defined by a ordering variable $v_{ij}$ and a test variable $y_{ij}$

  - Durham kt clustering:

$$y_{ij}^{(D)} = v_{ij}^{(D)} = 2\frac{\min\{E_i, E_j\}^2}{Q^2}(1-\cos\theta_{ij})$$

  - Cambridge kt clustering:

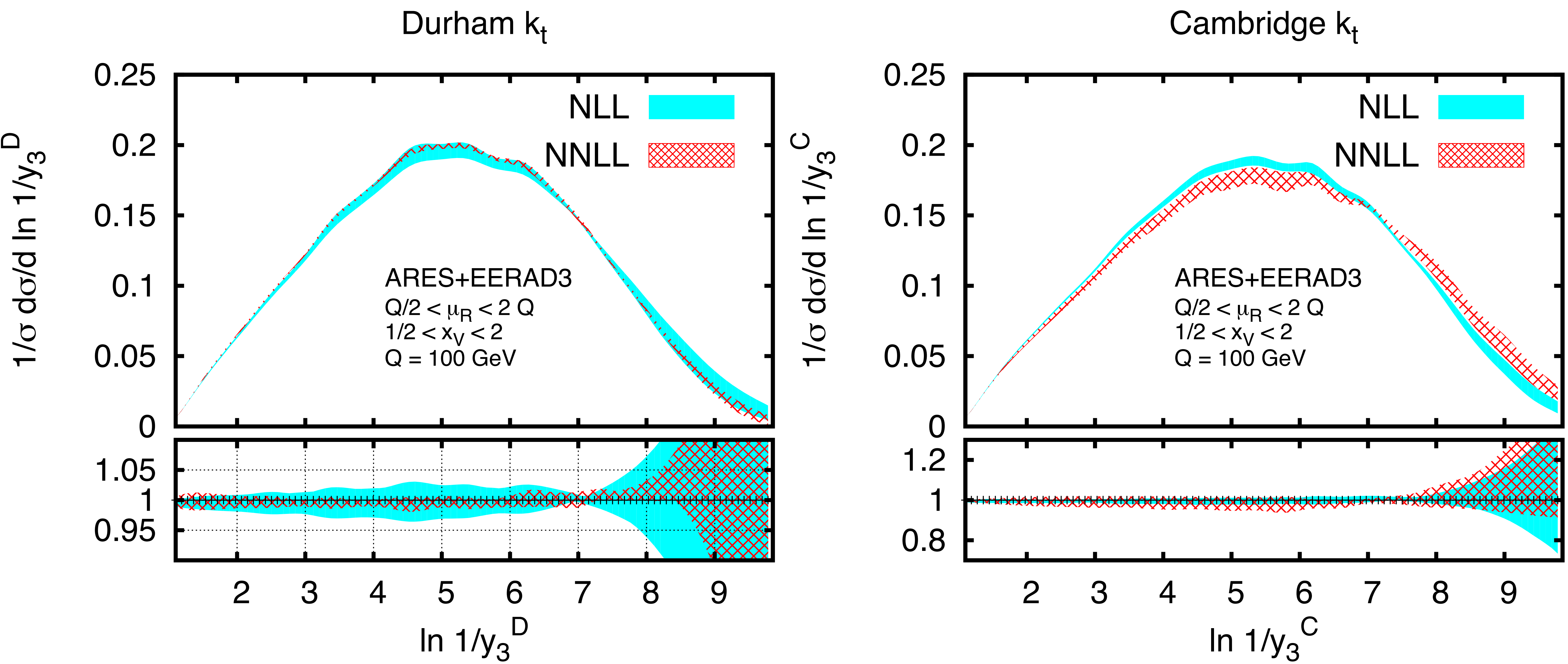
$$y_{ij}^{(C)} = y_{ij}^{(D)}\,, \qquad v_{ij}^{(C)} = 2\,(1-\cos\theta_{ij})\,, \quad + \text{ soft freezing}$$

- No analytic expression for the observable beyond leading-log order - observable defined in a algorithmic way

- Clustering of emissions far in rapidity (e.g. soft and hard-collinear) is allowed in the Durham algorithm

# New results: the two-jet rate in e+e-**

[Banfi, McAslan, Monni, Zanderighi]




- Good convergence for the Durham algorithm, reduction of uncertainties at NNLL
- The Cambridge algorithm gets sensitive to multiple emissions at NNLL: sizeable corrections (uncertainties underestimated at NLL)



**Preliminary results

# Conclusions

- Novel general method for the resummation of any rIRC safe, global two-scales observable at NNLL order
  - rIRC safety as only applicability condition
  - Contribution of resolved real radiation formulated in terms of four dimensional integrals, which is suitable for efficient numerical implementation, currently automated for final-state radiation
  - NNLL corrections systematically derived, each correction has clear physical interpretation. Method extendable to higher orders
- Formulation in a "pseudo" parton-shower framework - important hints on what's needed for a NNLL parton shower
- Modulo technical work, NNLL resummation for this (broad) class of observables in processes with 2 coloured Born legs is a theoretically solved problem

# Conclusions

- Novel general method for the resummation of any rIRC safe, global two-scales observable at NNLL order
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# Outlook

- A number of problems need to be attacked in order to treat the most general resummable observable
- Extension to more than two hard legs requires a parametrisation of QCD interference between different hard emitters (trivial in the 2-legs case)
  - simple at NLL [Botts, Sterman (1989)] [Sterman et al. (1996 - 2000)] [Banfi, Marchesini, et al. (2000 - 2002)] [Bonciani, Catani, Mangano, Nason (2003)]
  - slightly more technical at NNLL, but not overly complex (singular structure for multi-jet amplitudes known)

    [Catani; Dixon, Magnea, Sterman; Gardi, Magnea; Becher, Neubert]
- Many jet observables (e.g. > 0 jet rates) at the LHC defined in a non-global way
  - solution for NNLL non-global logarithms (see Caron-Huot's talk)

    recent progress in [Caron-Huot (2015)]. See also [Larkoski, Moult, Neill (2015)]
- Some observables contain more sources of large logarithms at once (e.g. H +1 jet rate; small-R effects in H+0 jet rate)

  see e.g. [Liu, Petriello (2012)][Dasgupta, Dreyer, Salam, Soyez (2014)]
  - not clear how to resum them simultaneously at NNLL in a general way

# Thank you for your attention

# Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i \, ; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu) \, , \; \mu^2 = (\kappa_{ia} + \kappa_{ib})^2 / \kappa_{ti}^2$$

- The standard requirement of IRC safety implies that

$$\lim_{\zeta_{m+1} \to 0} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_m(\bar{v}\zeta_m), \kappa_{m+1}(\bar{v}\zeta_{m+1}))$$
$$= V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_m(\bar{v}\zeta_m))$$
$$\lim_{\mu \to 0} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \dots \kappa_m(\bar{v}\zeta_m))$$
$$= \quad V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \dots, \kappa_i(\bar{v}\zeta_i), \dots \kappa_m(\bar{v}\zeta_m))$$

- We limit ourselves to *continuously* global observables*, i.e. the transverse momentum dependence is the same everywhere (it ensures the absence of non-global logarithms)

*Not a real limitation, although currently NNLL structure of non-global logarithms unknown

# Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu)\,,\; \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ti}^2$$

- Impose the following conditions, known as recursive IRC (rIRC) safety [Banfi, Salam, Zanderighi]

$$\lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \qquad (1)$$

- The above limit must be well defined and non-zero (except possibly in a phase space region of zero measure)
- Condition (1) simply requires the observable to scale in the same fashion for multiple emissions as for a single emission (IRC divergences have an exponential form)
- It is enough to ensure the exponentiation of double logarithms to all orders

# Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\},\kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia},\kappa_{ib}\}(\zeta,\mu)\,,\ \mu^2 = (\kappa_{ia}+\kappa_{ib})^2/\kappa_{ti}^2$$

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[Banfi, Salam, Zanderighi]

$$\lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\kappa_m(\bar{v}\zeta_m)) \qquad (1)$$

$$\lim_{\zeta_{m+1}\to 0}\lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\kappa_m(\bar{v}\zeta_m),\kappa_{m+1}(\bar{v}\zeta_{m+1}))$$

$$= \lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{a})$$

$$\lim_{\mu\to 0}\lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\{\kappa_{ia},\kappa_{ib}\}(\bar{v}\zeta_i,\mu),\ldots\kappa_m(\bar{v}\zeta_m))$$

$$= \lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\kappa_i(\bar{v}\zeta_i),\ldots\kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{b})$$

# Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu)\,,\; \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ti}^2$$

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$$= \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{a})$$

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$$= \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_i(\bar{v}\zeta_i), \ldots \kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{b})$$

# Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i \,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu)\,,\; \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ti}^2$$

- Impose the following conditions, known as recursive IRC (rIRC) safety [Banfi, Salam, Zanderighi]

- Conditions (2.a) and (2.b), in addition to plain IRC safety, require that for sufficiently small $\bar{v}$ there exists some $\epsilon$ that can be chosen *independently* of $\bar{v}$ such that we can neglect any emissions at scales $\sim \epsilon\bar{v}$

- The order with which one takes the limit is different in fixed-order and resummed calculations, and the final result must not change

$$\lim_{\mu\to 0}\lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \ldots \kappa_m(\bar{v}\zeta_m))$$
$$= \lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_i(\bar{v}\zeta_i), \ldots \kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{b})$$

# Event shapes definition

- thrust and heavy jet mass

$$T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{Q} \qquad \rho_H \equiv \max_{i=1,2} \frac{M_i^2}{Q^2}, \qquad M_i^2 \equiv \left( \sum_{j \in \mathcal{H}^{(i)}} p_j \right)^2$$

- total and wide jet broadening

$$B_L \equiv \sum_{i \in \mathcal{H}^{(1)}} \frac{|\vec{p_i} \times \vec{n_T}|}{2Q}, \quad B_R \equiv \sum_{i \in \mathcal{H}^{(2)}} \frac{|\vec{p_i} \times \vec{n_T}|}{2Q} \qquad B_W \equiv \max\{B_L, B_R\} \qquad B_T \equiv B_L + B_R$$

- C parameter

$$C \equiv 3 \left( 1 - \frac{1}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)(p_j \cdot Q)} \right)$$

- thrust major and oblateness

$$T_M \equiv \max_{\vec{n} \cdot \vec{n_T} = 0} \frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{Q} \qquad T_{\mathrm{m}} \equiv \frac{\sum_i |p_{i,x}|}{Q} \qquad O \equiv T_M - T_{\mathrm{m}}$$