Towards NLL threshold resummation for $2 \to 3$ type processes

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Resummation for more final state particles

- General formulation available for $2 \to n$ *[Bonciani et al. , '03][Mert-Aybat et al. , '06]*
- Recent advances in automatization of resummation in $2 \to 5$ (massless) *[Gerwick et al. , '14]*
- Relation to event shapes and N-jettiness in SCET
- Practical applications of threshold resummation (in traditional, conjugate space formulation) to production of more than two particles in the final state?

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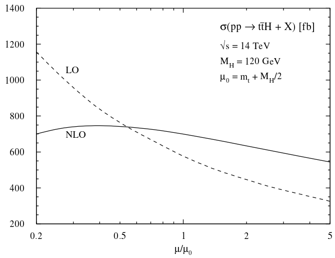
Importance of $pp \to t\bar{t}H$

- A Higgs boson found with a mass of 125 GeV
- Precision study needed to determine if it is SM Higgs
- Direct way to access Yukawa coupling
- QCD Corrections up to NLO *[Beenakker et al. , '02] [Dawson et al. , '02]*
- Matched to parton showers by: aMC@NLO *[Frederix et al. , '11]*, PowHel *[Garzelli et al. , '11]*, Sherpa *[Hoeche et al., '12]*, POWHEG-BOX *[Hartanto et al. , '14]*
- Electroweak correction *[Frixione et al. , '14][Zhang, '14]*



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Currently used results

$\sqrt{S}$	m_H [GeV]	σ [pb]	Scale%	(PDF+α_s) %
8.0 TeV	125.0	0.1293	$^{+3.8}_{-9.3}$	$^{+8.1}_{-8.1}$
13.0 TeV	125.0	0.5085	$^{+5.7}_{-9.3}$	$^{+8.8}_{-8.8}$
13.5 TeV	125.0	0.5608	$^{+5.8}_{-9.3}$	$^{+8.9}_{-8.9}$
14.0 TeV	125.0	0.6113	$^{+5.9}_{-9.3}$	$^{+8.9}_{-8.9}$

[LHCHXSWG, '15]

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Why resummation for $t\bar{t}H$?

Gains

- NNLO corrections out of reach
- Resummation can help reduce scale uncertainty
- Good process to start:
  - Simple color structure
  - Massive particles $\rightarrow$ no final state collinear divergences

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Pitfalls

- $2 \to 3$ phase space supressed near threshold ($\sigma \propto \beta^4$)
- Resummation not an approximation for NNLO

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Definition of Threshold

Threshold variable $\hat{\tau} = \frac{M^2}{\hat{s}}$ (Other definitions also possible)

$M = m_H + 2m_t$

$\sqrt{\hat{s}}$: the partonic center of mass energy

$$\beta^2 = 1 - \hat{\tau} = 1 - \frac{M^2}{\hat{s}} \sim \frac{\text{maximum energy of the emitted gluons}}{\text{total available energy}}$$

At NLO: *[Beenakker et al. , '02]*

$$\sigma_{\mathrm{LO}} \times \frac{2\alpha_s}{\pi} \left\{ (T_1^2 + T_2^2) \left[2\log^2\beta - 3\log\beta - 2\log\beta\log\left(\frac{\mu_F}{2m_t + m_H}\right)\right] - (T_3 + T_4)^2 \log\beta \right\}$$

Coulomb enhanced terms not treated in the resummation

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Mellin Transform

Mellin transform is used with respect to τ (needed for factorization of phase space):

$$\begin{aligned}\tilde{\sigma}_{pp\to t\bar{t}H}(N) &\equiv \int_0^1 d\tau\, \tau^{N-1}\sigma_{pp\to t\bar{t}H}(\tau,\mu_R,\mu_F) \\ &= \sum_{i,j}\tilde{f}_{i/p}(N+1,\mu_F)\tilde{f}_{j/p}(N+1,\mu_F)\tilde{\hat{\sigma}}_{ij\to t\bar{t}H}(N,\mu_R,\mu_F)\end{aligned}$$

- $\tilde{f}_{i/p}(N+1,\mu_F)$: Mellin transform with respect to x
- $\tilde{\hat{\sigma}}_{ij\to t\bar{t}H}(N,\mu_R,\mu_F)$: Mellin transform with respect to $\hat{\tau}$

$\log^n(1-\hat{\tau}) \Rightarrow \log^n N$ and threshold $\hat{\tau}\to 1 \sim N\to\infty$

First application for $2 \to 3$ in Mellin space

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Orders of Resummation

Large logarithms $\log N \equiv L$ for $N \to \infty$

Perturbation needs to be reordered in α_s and L:

$$\tilde{\sigma} \sim \tilde{\sigma}_{LO} \times \mathcal{C}(\alpha_\mathrm{s}) \exp\left[L g_1(\alpha_\mathrm{s} L) + g_2(\alpha_\mathrm{s} L) + \alpha_\mathrm{s} g_3(\alpha_\mathrm{s} L) + \cdots \right]$$

	$\Downarrow$	$\Downarrow$	$\Downarrow$
With orders of precision:	LL	NLL	NNLL
	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$\alpha_\mathrm{s}^n \log^{n+1}(N)$	$\alpha_\mathrm{s}^n \log^{n}(N)$	$\alpha_\mathrm{s}^{n+1} \log^{n}(N)$

Exponential functions are universal for initial state emission

[Kodaira, Trentadue, '82][Sterman, '87][Catani, d'Emilio, Trentadue, '88][Catani, Trentadue, '89]

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Soft anomalous dimension

Color space

- Using s-channel color basis
- Same as for example $pp \to t\bar{t}$

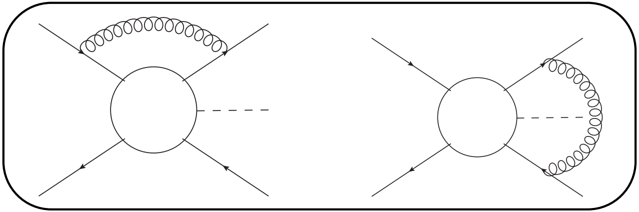
Soft anomalous dimension

Calculated at the hand of UV divergences of eikonal integrals

$$\Gamma^{(1)}_{ij\to klB,IJ} = -C^{ab}_{IJ}\mathrm{Res}\left\{\omega^{ab}\right\} - \Gamma^{(1)}_{ij\to C}\,\delta_{IJ}$$

- I and J: color indices
- a and b: colored particle indices
- C^{ab}_{IJ}: color factor of the exchange
- ω^{ab}: UV divergent terms of eikonal integrals

Eikonal integrals



Using feynman rules approximated for soft gluon emission (eikonal)

Soft anomalous dimension matrix

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,11} = -\frac{\alpha_s}{2\pi} 2C_F \left(L_{\beta_{34}} + 1\right)$$

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,12} = \frac{\alpha_s}{2\pi} \frac{2C_F}{N_C} \Omega_3 = \frac{C_F}{2N_C} \Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,21}$$

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,22} = \frac{\alpha_s}{2\pi} \left[(N_C - 2C_F)\left(L_{\beta_{34}} + 1\right) + N_C \Lambda_3 + (8C_F - 3N_C)\,\Omega_3\right]$$

$$\Omega_3 = \left(T_{13} + T_{24} - T_{14} - T_{23}\right)/2$$

$$\Lambda_3 = \left(T_{13} + T_{24} + T_{14} + T_{23}\right)/2$$

$$T_{ij} = \log\left(\frac{m_j^2 - t_{ij}}{m_j\sqrt{s}}\right) + \frac{i\pi - 1}{2}$$

with $t_{ij} = (p_i - p_j)^2$ and $\beta_{34}^2 = 1 - (m_3 + m_4)^2/s_{34}$

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Agrees with $q\bar{q} \to Q\bar{Q}$

for $p_5 \to 0$ and $m_5 \to 0$

$$\Omega_3 = \left(T_{13} + T_{24} - T_{14} - T_{23}\right)/2$$

$$\Lambda_3 = \left(T_{13} + T_{24} + T_{14} + T_{23}\right)/2$$

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with $t_{ij} = (p_i - p_j)^2$ and $\beta_{34}^2 = 1 - (m_3 + m_4)^2/s_{34}$

$\Downarrow$ $\Downarrow$

For $2 \to 2$: $t_{13} = t_{24}$, $t_{23} = t_{14}$; $s_{34} = \hat{s}$, $\beta_{34} = \beta$

Soft wide-angle emission

Threshold limit for soft anomalous dimension:

$$2\mathcal{R}e\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}B,IJ,\mathrm{thr.}} = \frac{\alpha_{\mathrm{s}}}{\pi}\mathrm{diag}\left(0,-N_C\right)$$

$$2\mathcal{R}e\Gamma^{(1)}_{gg\to Q\bar{Q}B,IJ,\mathrm{thr.}} = \frac{\alpha_{\mathrm{s}}}{\pi}\mathrm{diag}\left(0,-N_C,-N_C\right)$$

results in soft wide-angle contribution:

$$\Delta_I^{\mathrm{NLL}} = \exp\left[h_2(\alpha_{\mathrm{s}}L,-C_I)\right]$$

With C_I the quadratic Casimir invariant

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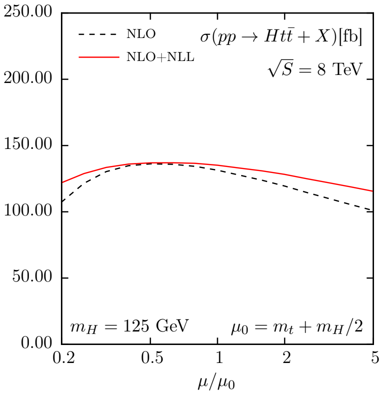
Results

[Kulesza, Motyka, Stebel, VT, in preparation]

PDFs used: MMHT2014NLO



NLO+NLL is NLL matched to NLO

preliminary

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PDFs used: MSTW2008NLO

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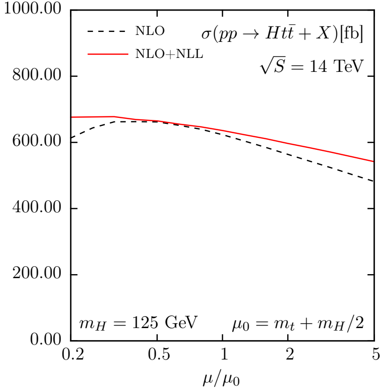

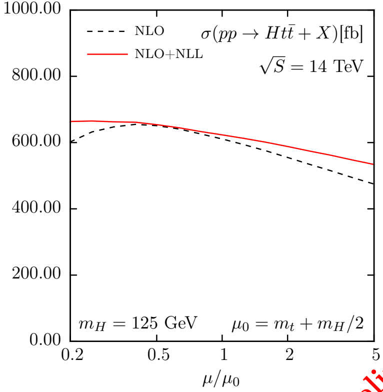

preliminary

NLO+NLL is NLL matched to NLO

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Results

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PDFs used: MMHT2014NLO

$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL [fb]	Pdf uncertainty	K-factor
8	$131.4^{+3.6\%}_{-9.0\%}$	$135.1^{+1.3\%}_{-5.0\%}$	$^{+3.0\%}_{-2.7\%}$	1.028
13	$506.3^{+5.7\%}_{-9.3\%}$	$517.0^{+4.0\%}_{-5.9\%}$	$^{+2.3\%}_{-2.3\%}$	1.021
14	$610.9^{+6.5\%}_{-9.1\%}$	$623.4^{+4.9\%}_{-5.6\%}$	$^{+2.3\%}_{-2.2\%}$	1.020

preliminary

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Resummation for differential observables

$pp \to Q\bar{Q}B + X \qquad \Rightarrow Q^2$

$$z_5 = \frac{\hat{s} - s_{345}}{\hat{s}}$$

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Resummation for differential observables

$pp \to Q\bar{Q}B + X \qquad \Rightarrow Q^2$

$$z_5 = \frac{\hat{s} - s_{345}}{\hat{s}}$$

$pp \to B + X[Q\bar{Q}] \qquad \Rightarrow P_{T,5}, y_5$

$$v_5 = \frac{(p_1 + p_2 - p_5)^2 - s_{34}}{\hat{s}} = \frac{\hat{s} + \tilde{t}_{15} + \tilde{t}_{25} + m_5^2 - s_{34}}{\hat{s}}$$

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$pp \to Q\bar{Q} + X[B] \qquad \Rightarrow P_{T,34}, y_{34}, Q_{34}^2$

$$s_5 = \frac{(p_1 + p_2 - p_3 - p_4)^2 - m_5^2}{\hat{s}} = \frac{\hat{s} + \tilde{t}_{13} + \tilde{t}_{14} + \tilde{t}_{23} + \tilde{t}_{24} + s_{34} - m_5^2}{\hat{s}}$$

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Threshold resummation in differential notation

$$
\begin{aligned}
W &= (1 - x_1)\, c_1 + (1 - x_2)\, c_2 + \omega \\
&= \omega_1 c_1 + \omega_2 c_2 + \omega_s
\end{aligned}
$$

$$
\int dW e^{-NW} \frac{d\sigma_{ij\to c+X}}{d\hat{\Pi}_3}
$$

$$
= \int dW e^{-NW} H_{ij}\left(\hat{\Pi}_3\right) \int d\omega_1 d\omega_2 d\omega_s \delta\left(W - \omega_1 c_1 - \omega_2 c_2 - \omega_s\right)
$$

$$
\psi_{i/i}\left(\omega_1, p_i, n\right) \psi_{j/j}\left(\omega_2, p_i, n\right) S\left(\omega_s Q^2/\mu^2, v_i, n\right)
$$

$$
= H_{ij}\left(\hat{\Pi}_3\right) \tilde{\psi}_{i/i}\left(N c_1, p_i, n\right) \tilde{\psi}_{j/j}\left(N c_2, p_i, n\right) \tilde{S}\left(\frac{Q^2}{\mu^2 N}, v_i, n\right)
$$

- $\omega_{1/2}$ Collinear weights
- ω_s Soft weight

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Summary

Conclusions

- Improvement in scale uncertainty (factor 2)
- Small corrections (2%)

Outlook

- Different threshold definitions
- Differential distributions

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Thank you for your attention

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