Parton Showers, Matching & Merging

Bryan Webber

UNIVERSITY OF CAMBRIDGE

Outline

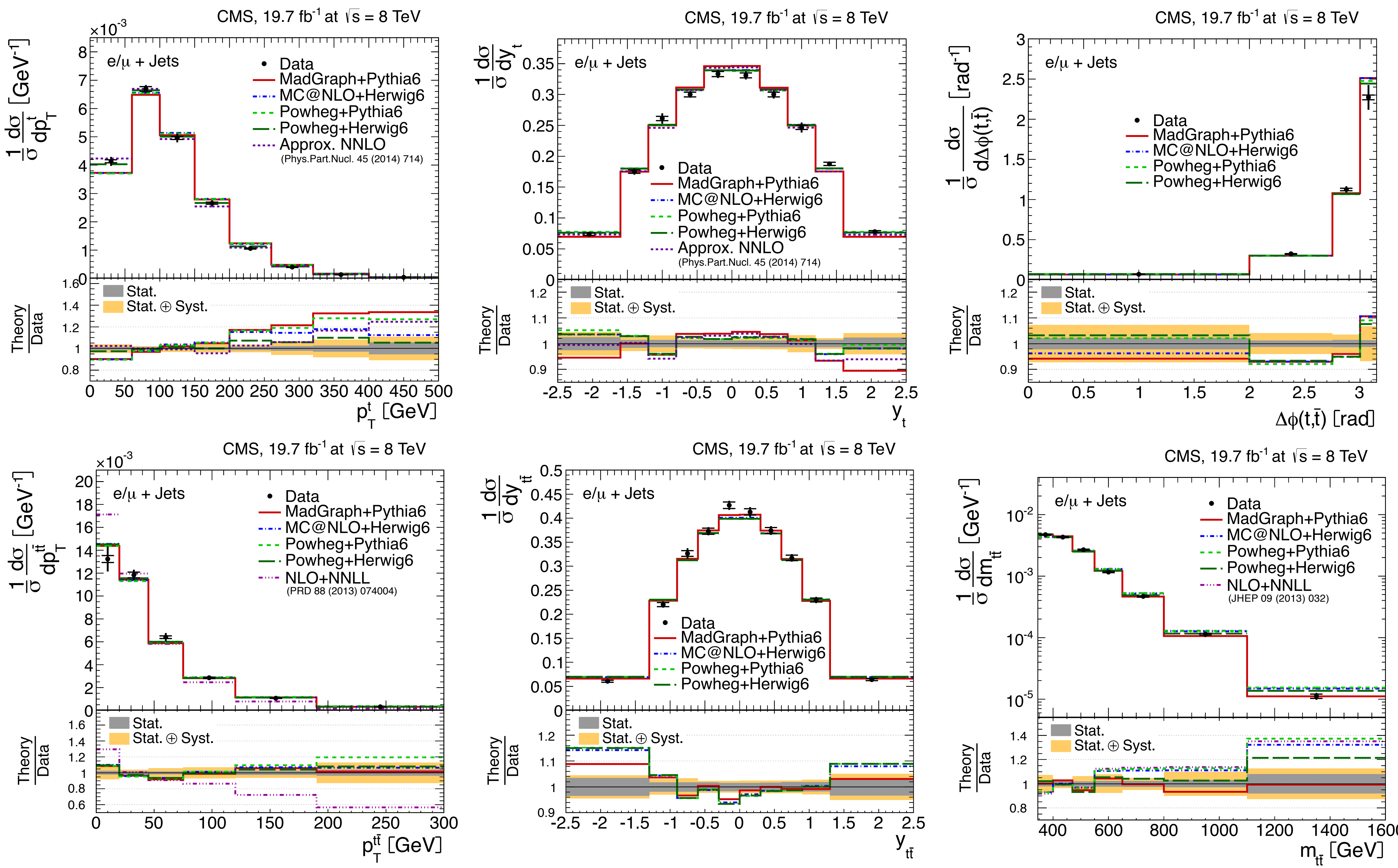
- Matching to a fixed order
  - NLO: MC@NLO, POWHEG, KrkNLO; automation
  - NNLO: NNLOPS, UN^{2}LOPS, Geneva
- Matching to multiple fixed orders
  - MEPS@NLO, FxFx, UNLOPS,…
- Shower variables and improvements
  - Coherence, colour, spin, …
- Conclusions

NLO matching

NLO matching

- Full inclusive NLO, extra jet LO
- Still mostly MC@NLO or POWHEG
- MC@NLO:
  - PS-specific; beyond NLO is PS only; some negative weights
- POWHEG:
  - Any PS; extra terms beyond NLO; positive weights
- New: KrkNLO

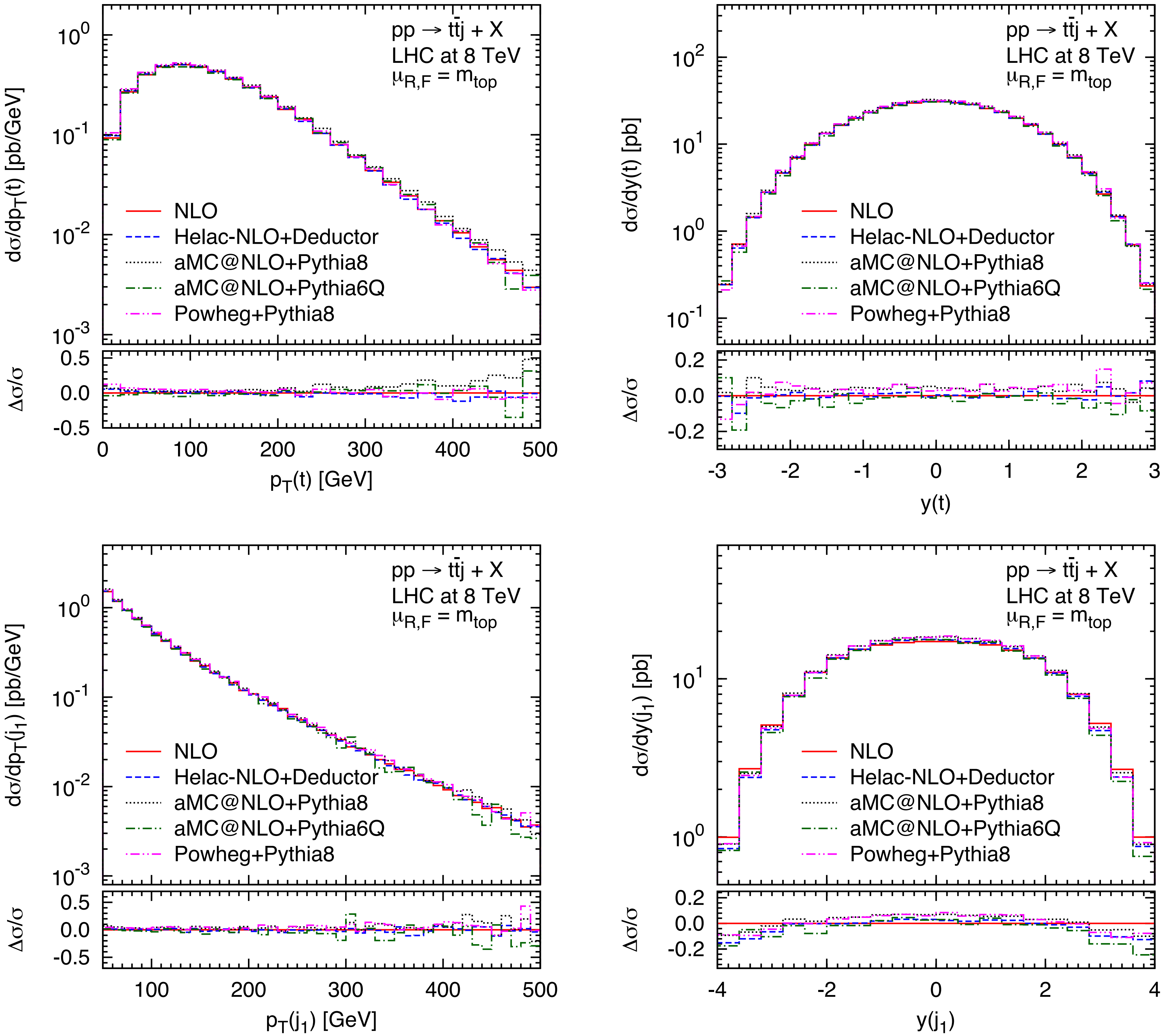
Top pairs at 8 TeV



- Differences are small!

CMS, 1505.04480

DEDUCTOR MC@NLO

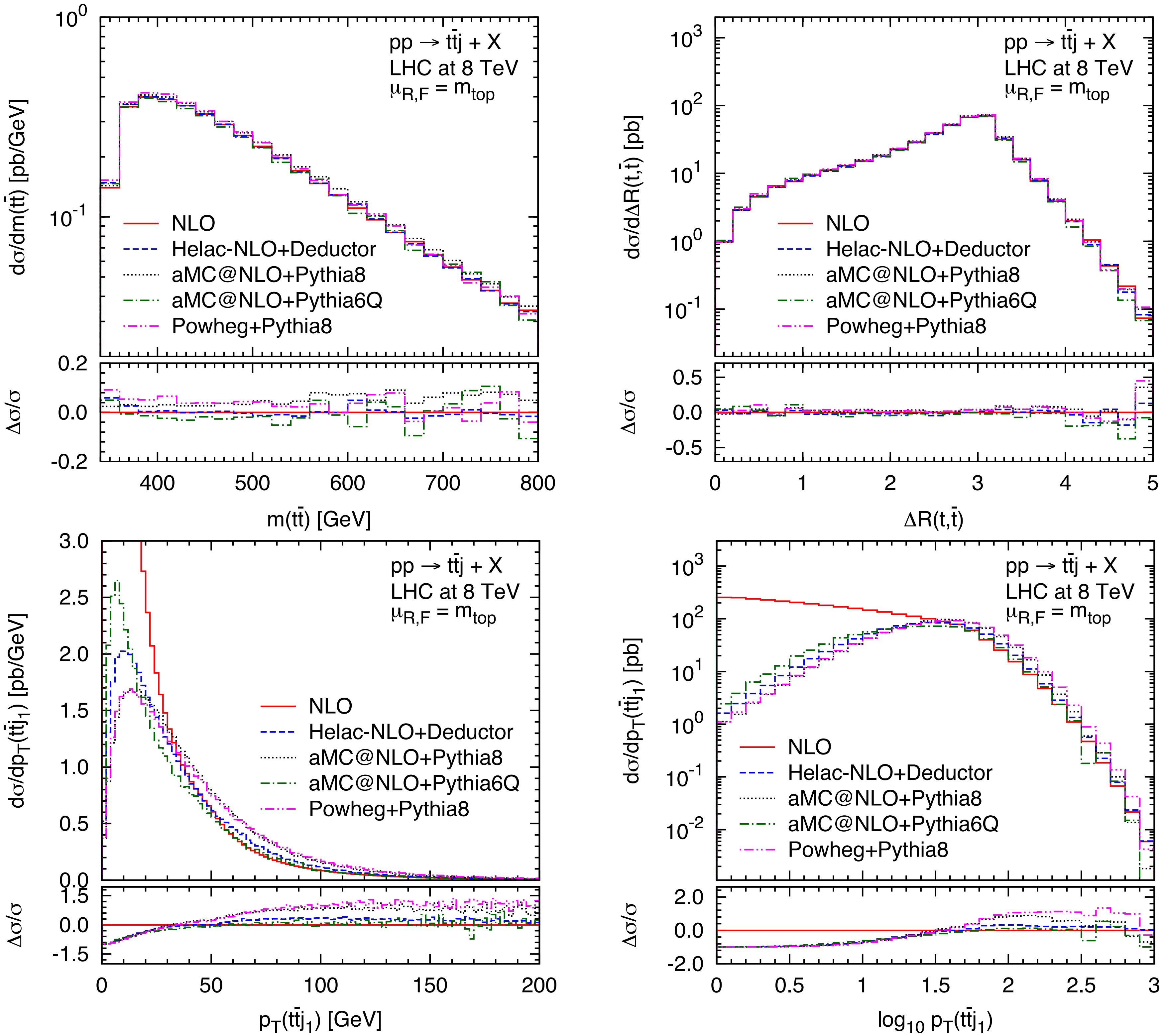


- $p\,p \rightarrow t\,\bar{t}\,j + X$

Nagy, Soper, 1401.6364

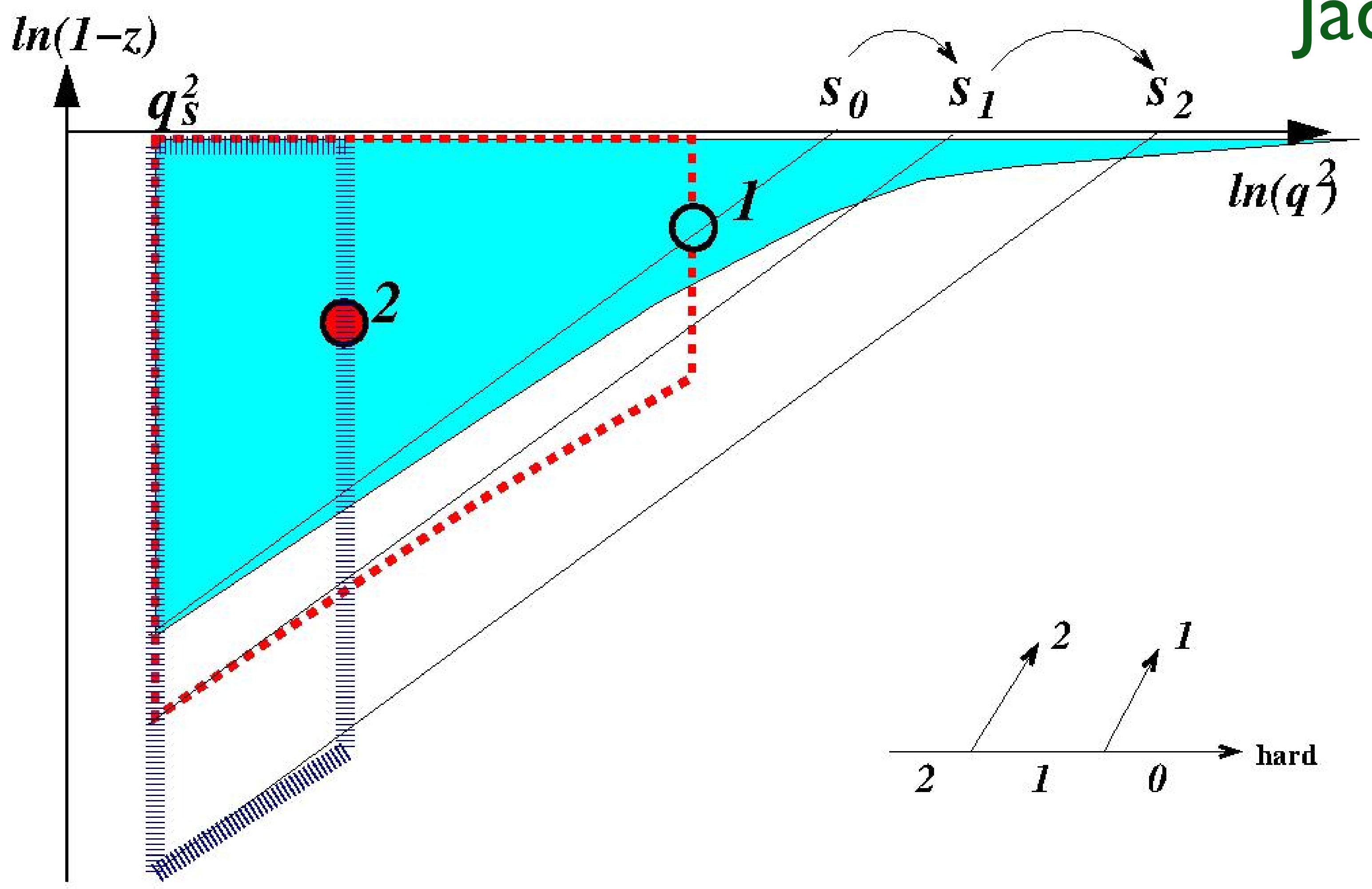
Czakon et al., 1502.00925

Deductor MC@NLO



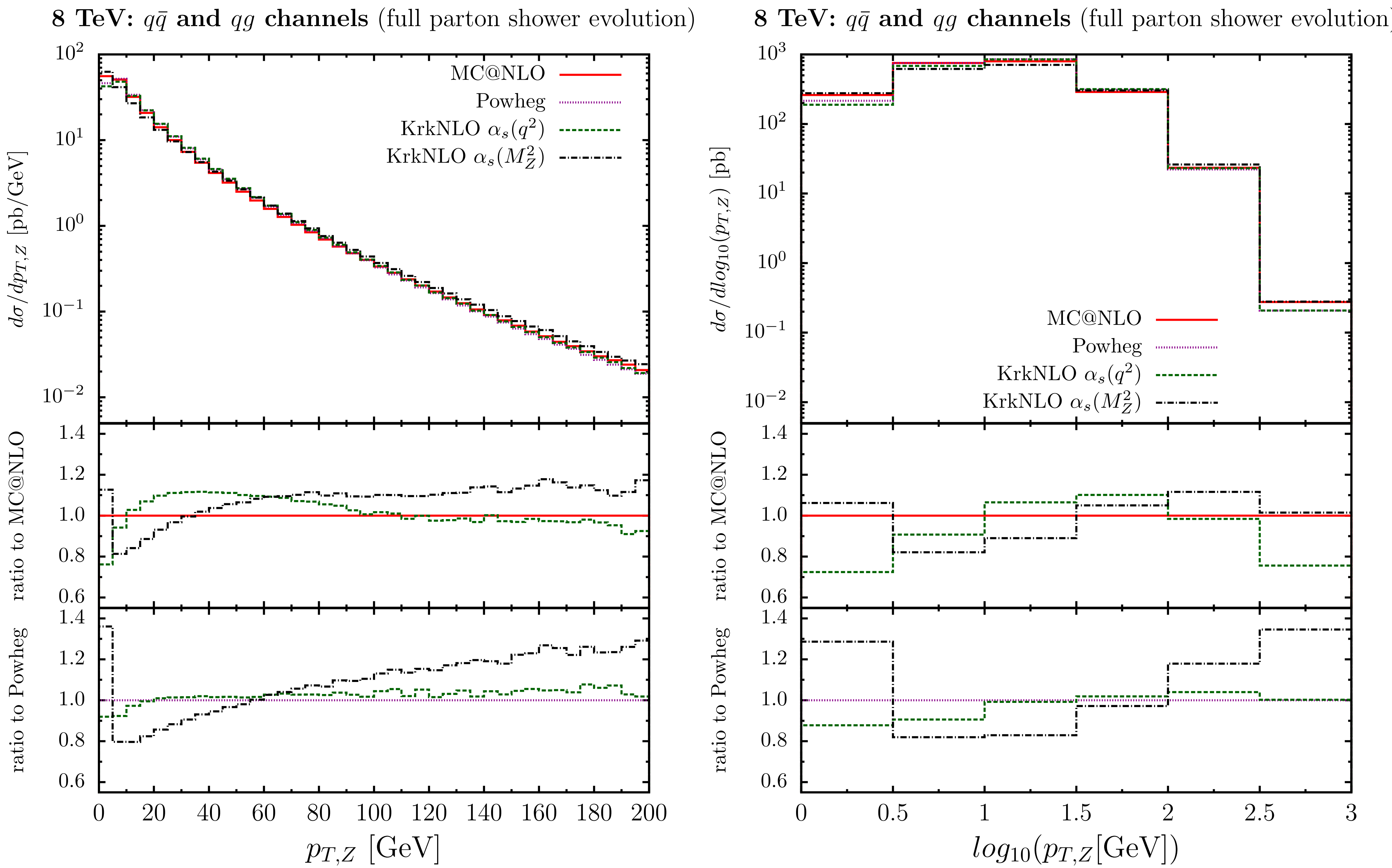
KrkNLO method

Jadach et al., 1503.06849



- Modified LO PS and PDF for full phase space
- NLO correction by multiplicative weight

KrkNLO results



- α_s choice is for first emission

Automatic NLO matching

- MC@NLO-type
  - MadGraph5_aMC@NLO (MadLoop5)

    Alwall et al., 1405.0301
  - Sherpa+OpenLoops

    Höche et al., 1111.1220; 1201.5882
  - Herwig++ Matchbox+OpenLoops/GoSam

    Plätzer, Gieseke, 1109.6256; Bellm et al., 1310.6877
- POWHEG-type
  - MadGraph4+POWHEG+MCFM/GoSam

    Campbell et al.,1202.5475; Luisoni et al., 1502.01213
  - Herwig++ Matchbox+OpenLoops/GoSam

MG5_aMC@NLO

	Process	Syntax	Cross section (pb)	
Heavy quarks+vector bosons			LO 13 TeV	NLO 13 TeV
e.1	$pp \to W^{\pm}\, b\bar{b}$ (4f)	`p p > wpm b b~`	$3.074 \pm 0.002 \cdot 10^{2}$ $^{+42.3\%}_{-29.2\%}$ $^{+2.0\%}_{-1.6\%}$	$8.162 \pm 0.034 \cdot 10^{2}$ $^{+29.8\%}_{-23.6\%}$ $^{+1.5\%}_{-1.2\%}$
e.2	$pp \to Z\, b\bar{b}$ (4f)	`p p > z b b~`	$6.993 \pm 0.003 \cdot 10^{2}$ $^{+33.5\%}_{-24.4\%}$ $^{+1.0\%}_{-1.4\%}$	$1.235 \pm 0.004 \cdot 10^{3}$ $^{+19.9\%}_{-17.4\%}$ $^{+1.0\%}_{-1.4\%}$
e.3	$pp \to \gamma\, b\bar{b}$ (4f)	`p p > a b b~`	$1.731 \pm 0.001 \cdot 10^{3}$ $^{+51.9\%}_{-34.8\%}$ $^{+1.6\%}_{-2.1\%}$	$4.171 \pm 0.015 \cdot 10^{3}$ $^{+33.7\%}_{-27.1\%}$ $^{+1.4\%}_{-1.9\%}$
e.4*	$pp \to W^{\pm}\, b\bar{b}\, j$ (4f)	`p p > wpm b b~ j`	$1.861 \pm 0.003 \cdot 10^{2}$ $^{+42.5\%}_{-27.7\%}$ $^{+0.7\%}_{-0.7\%}$	$3.957 \pm 0.013 \cdot 10^{2}$ $^{+27.0\%}_{-21.0\%}$ $^{+0.7\%}_{-0.6\%}$
e.5*	$pp \to Z\, b\bar{b}\, j$ (4f)	`p p > z b b~ j`	$1.604 \pm 0.001 \cdot 10^{2}$ $^{+42.4\%}_{-27.6\%}$ $^{+0.9\%}_{-1.1\%}$	$2.805 \pm 0.009 \cdot 10^{2}$ $^{+21.0\%}_{-17.6\%}$ $^{+0.8\%}_{-1.0\%}$
e.6*	$pp \to \gamma\, b\bar{b}\, j$ (4f)	`p p > a b b~ j`	$7.812 \pm 0.017 \cdot 10^{2}$ $^{+51.2\%}_{-32.0\%}$ $^{+1.0\%}_{-1.5\%}$	$1.233 \pm 0.004 \cdot 10^{3}$ $^{+18.9\%}_{-19.9\%}$ $^{+1.0\%}_{-1.5\%}$
e.7	$pp \to t\bar{t}\, W^{\pm}$	`p p > t t~ wpm`	$3.777 \pm 0.003 \cdot 10^{-1}$ $^{+23.9\%}_{-18.0\%}$ $^{+2.1\%}_{-1.6\%}$	$5.662 \pm 0.021 \cdot 10^{-1}$ $^{+11.2\%}_{-10.6\%}$ $^{+1.7\%}_{-1.3\%}$
e.8	$pp \to t\bar{t}\, Z$	`p p > t t~ z`	$5.273 \pm 0.004 \cdot 10^{-1}$ $^{+30.5\%}_{-21.8\%}$ $^{+1.8\%}_{-2.1\%}$	$7.598 \pm 0.026 \cdot 10^{-1}$ $^{+9.7\%}_{-11.1\%}$ $^{+1.9\%}_{-2.2\%}$
e.9	$pp \to t\bar{t}\, \gamma$	`p p > t t~ a`	$1.204 \pm 0.001 \cdot 10^{0}$ $^{+29.6\%}_{-21.3\%}$ $^{+1.6\%}_{-1.8\%}$	$1.744 \pm 0.005 \cdot 10^{0}$ $^{+9.8\%}_{-11.0\%}$ $^{+1.7\%}_{-2.0\%}$
e.10*	$pp \to t\bar{t}\, W^{\pm} j$	`p p > t t~ wpm j`	$2.352 \pm 0.002 \cdot 10^{-1}$ $^{+40.9\%}_{-27.1\%}$ $^{+1.3\%}_{-1.0\%}$	$3.404 \pm 0.011 \cdot 10^{-1}$ $^{+11.2\%}_{-14.0\%}$ $^{+1.2\%}_{-0.9\%}$
e.11*	$pp \to t\bar{t}\, Z j$	`p p > t t~ z j`	$3.953 \pm 0.004 \cdot 10^{-1}$ $^{+46.2\%}_{-29.5\%}$ $^{+2.7\%}_{-3.0\%}$	$5.074 \pm 0.016 \cdot 10^{-1}$ $^{+7.0\%}_{-12.3\%}$ $^{+2.5\%}_{-2.9\%}$
e.12*	$pp \to t\bar{t}\, \gamma j$	`p p > t t~ a j`	$8.726 \pm 0.010 \cdot 10^{-1}$ $^{+45.4\%}_{-29.1\%}$ $^{+2.3\%}_{-2.6\%}$	$1.135 \pm 0.004 \cdot 10^{0}$ $^{+7.5\%}_{-12.2\%}$ $^{+2.2\%}_{-2.5\%}$
e.13*	$pp \to t\bar{t}\, W^{-}W^{+}$ (4f)	`p p > t t~ w+ w-`	$6.675 \pm 0.006 \cdot 10^{-3}$ $^{+30.9\%}_{-21.9\%}$ $^{+2.1\%}_{-2.0\%}$	$9.904 \pm 0.026 \cdot 10^{-3}$ $^{+10.9\%}_{-11.8\%}$ $^{+2.1\%}_{-2.1\%}$
e.14*	$pp \to t\bar{t}\, W^{\pm} Z$	`p p > t t~ wpm z`	$2.404 \pm 0.002 \cdot 10^{-3}$ $^{+26.6\%}_{-19.6\%}$ $^{+2.5\%}_{-1.8\%}$	$3.525 \pm 0.010 \cdot 10^{-3}$ $^{+10.6\%}_{-10.8\%}$ $^{+2.3\%}_{-1.6\%}$
e.15*	$pp \to t\bar{t}\, W^{\pm} \gamma$	`p p > t t~ wpm a`	$2.718 \pm 0.003 \cdot 10^{-3}$ $^{+25.4\%}_{-18.9\%}$ $^{+2.3\%}_{-1.8\%}$	$3.927 \pm 0.013 \cdot 10^{-3}$ $^{+10.3\%}_{-10.4\%}$ $^{+2.0\%}_{-1.5\%}$
e.16*	$pp \to t\bar{t}\, ZZ$	`p p > t t~ z z`	$1.349 \pm 0.014 \cdot 10^{-3}$ $^{+29.3\%}_{-21.1\%}$ $^{+1.7\%}_{-1.5\%}$	$1.840 \pm 0.007 \cdot 10^{-3}$ $^{+7.9\%}_{-9.9\%}$ $^{+1.7\%}_{-1.5\%}$
e.17*	$pp \to t\bar{t}\, Z\gamma$	`p p > t t~ z a`	$2.548 \pm 0.003 \cdot 10^{-3}$ $^{+30.1\%}_{-21.5\%}$ $^{+1.7\%}_{-1.6\%}$	$3.656 \pm 0.012 \cdot 10^{-3}$ $^{+9.7\%}_{-11.0\%}$ $^{+1.8\%}_{-1.9\%}$
e.18*	$pp \to t\bar{t}\, \gamma\gamma$	`p p > t t~ a a`	$3.272 \pm 0.006 \cdot 10^{-3}$ $^{+28.4\%}_{-20.6\%}$ $^{+1.3\%}_{-1.1\%}$	$4.402 \pm 0.015 \cdot 10^{-3}$ $^{+7.8\%}_{-9.7\%}$ $^{+1.4\%}_{-1.4\%}$

- Sampled from 172 processes
- Mostly new at NLO

Alwall et al., 1405.0301

MG5_aMC@NLO

Process		Syntax	Cross section (pb)			
Top quarks +bosons			LO 1 TeV		NLO 1 TeV	
j.1	$e^+e^- \to t\bar{t}H$	`e+ e- > t t~ h`	$2.018 \pm 0.003 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$1.911 \pm 0.006 \cdot 10^{-3}$	$^{+0.4\%}_{-0.5\%}$
j.2*	$e^+e^- \to t\bar{t}Hj$	`e+ e- > t t~ h j`	$2.533 \pm 0.003 \cdot 10^{-4}$	$^{+9.2\%}_{-7.8\%}$	$2.658 \pm 0.009 \cdot 10^{-4}$	$^{+0.5\%}_{-1.5\%}$
j.3*	$e^+e^- \to t\bar{t}Hjj$	`e+ e- > t t~ h j j`	$2.663 \pm 0.004 \cdot 10^{-5}$	$^{+19.3\%}_{-14.9\%}$	$3.278 \pm 0.017 \cdot 10^{-5}$	$^{+4.0\%}_{-5.7\%}$
j.4*	$e^+e^- \to t\bar{t}\gamma$	`e+ e- > t t~ a`	$1.270 \pm 0.002 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$1.335 \pm 0.004 \cdot 10^{-2}$	$^{+0.5\%}_{-0.4\%}$
j.5*	$e^+e^- \to t\bar{t}\gamma j$	`e+ e- > t t~ a j`	$2.355 \pm 0.002 \cdot 10^{-3}$	$^{+9.3\%}_{-7.9\%}$	$2.617 \pm 0.010 \cdot 10^{-3}$	$^{+1.6\%}_{-2.4\%}$
j.6*	$e^+e^- \to t\bar{t}\gamma jj$	`e+ e- > t t~ a j j`	$3.103 \pm 0.005 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$4.002 \pm 0.021 \cdot 10^{-4}$	$^{+5.4\%}_{-6.6\%}$
j.7*	$e^+e^- \to t\bar{t}Z$	`e+ e- > t t~ z`	$4.642 \pm 0.006 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$4.949 \pm 0.014 \cdot 10^{-3}$	$^{+0.6\%}_{-0.5\%}$
j.8*	$e^+e^- \to t\bar{t}Zj$	`e+ e- > t t~ z j`	$6.059 \pm 0.006 \cdot 10^{-4}$	$^{+9.3\%}_{-7.8\%}$	$6.940 \pm 0.028 \cdot 10^{-4}$	$^{+2.0\%}_{-2.6\%}$
j.9*	$e^+e^- \to t\bar{t}Zjj$	`e+ e- > t t~ z j j`	$6.351 \pm 0.028 \cdot 10^{-5}$	$^{+19.4\%}_{-15.0\%}$	$8.439 \pm 0.051 \cdot 10^{-5}$	$^{+5.8\%}_{-6.8\%}$
j.10*	$e^+e^- \to t\bar{t}W^{\pm}jj$	`e+ e- > t t~ wpm j j`	$2.400 \pm 0.004 \cdot 10^{-7}$	$^{+19.3\%}_{-14.9\%}$	$3.723 \pm 0.012 \cdot 10^{-7}$	$^{+9.6\%}_{-9.1\%}$
j.11*	$e^+e^- \to t\bar{t}HZ$	`e+ e- > t t~ h z`	$3.600 \pm 0.006 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$3.579 \pm 0.013 \cdot 10^{-5}$	$^{+0.1\%}_{-0.0\%}$
j.12*	$e^+e^- \to t\bar{t}\gamma Z$	`e+ e- > t t~ a z`	$2.212 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$2.364 \pm 0.006 \cdot 10^{-4}$	$^{+0.6\%}_{-0.5\%}$
j.13*	$e^+e^- \to t\bar{t}\gamma H$	`e+ e- > t t~ a h`	$9.756 \pm 0.016 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$9.423 \pm 0.032 \cdot 10^{-5}$	$^{+0.3\%}_{-0.4\%}$
j.14*	$e^+e^- \to t\bar{t}\gamma\gamma$	`e+ e- > t t~ a a`	$3.650 \pm 0.008 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$3.833 \pm 0.013 \cdot 10^{-4}$	$^{+0.4\%}_{-0.4\%}$
j.15*	$e^+e^- \to t\bar{t}ZZ$	`e+ e- > t t~ z z`	$3.788 \pm 0.004 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$4.007 \pm 0.013 \cdot 10^{-5}$	$^{+0.5\%}_{-0.5\%}$
j.16*	$e^+e^- \to t\bar{t}HH$	`e+ e- > t t~ h h`	$1.358 \pm 0.001 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$1.206 \pm 0.003 \cdot 10^{-5}$	$^{+0.9\%}_{-1.1\%}$
j.17*	$e^+e^- \to t\bar{t}W^+W^-$	`e+ e- > t t~ w+ w-`	$1.372 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$1.540 \pm 0.006 \cdot 10^{-4}$	$^{+1.0\%}_{-0.9\%}$

- All new at NLO except ttH

NNLO matching

NNLO matching

- Fully inclusive NNLO, one extra jet NLO
- So far, limited to DY, H
  - MiNLO-NNLOPS

    Hamilton et al., 1309.0017, 1407.3773
  - UN^{2}LOPS

    Höche, Li, Prestel, 1405.3607, 1407.3773
  - Geneva

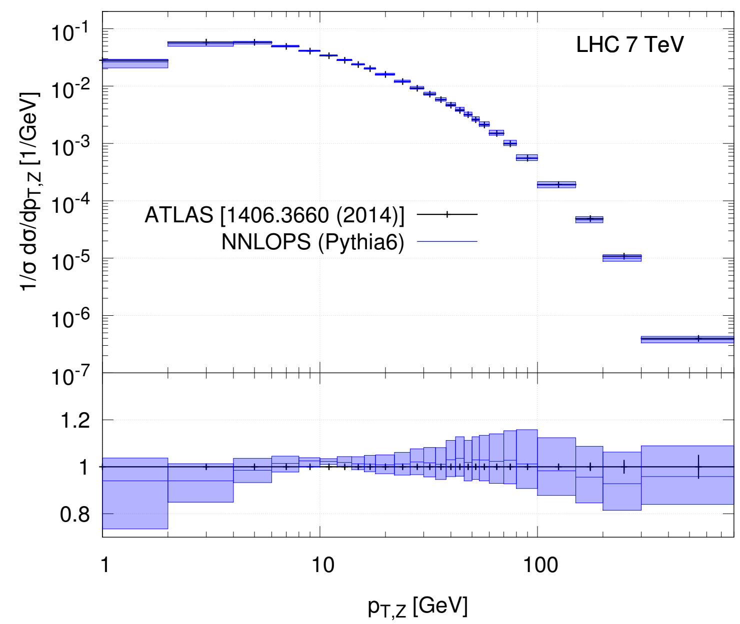
    Alioli et al., 1311.0286

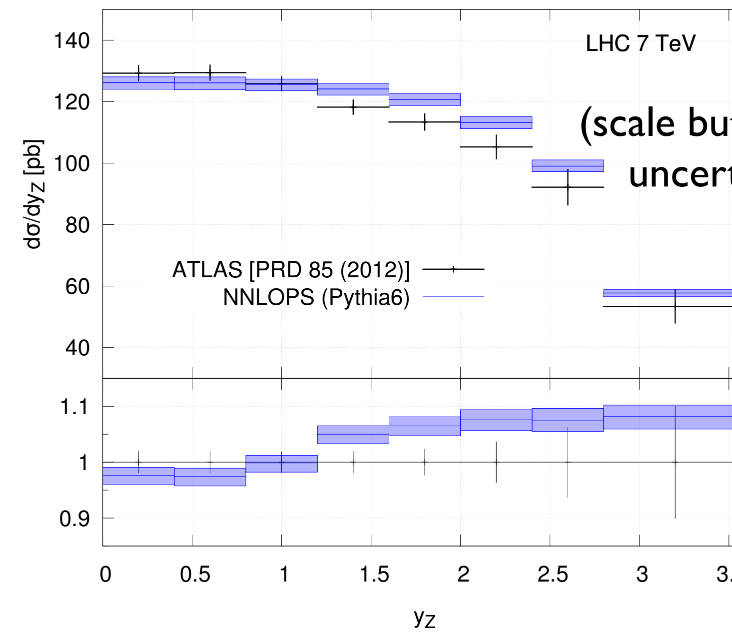
MiNLO-NNLOPS

- Modified DY/H+jet POWHEG with NNLL Sudakov (B_2)
- Reweighted to NNLO

Karlberg, Re, Zanderighi, 1407.2940

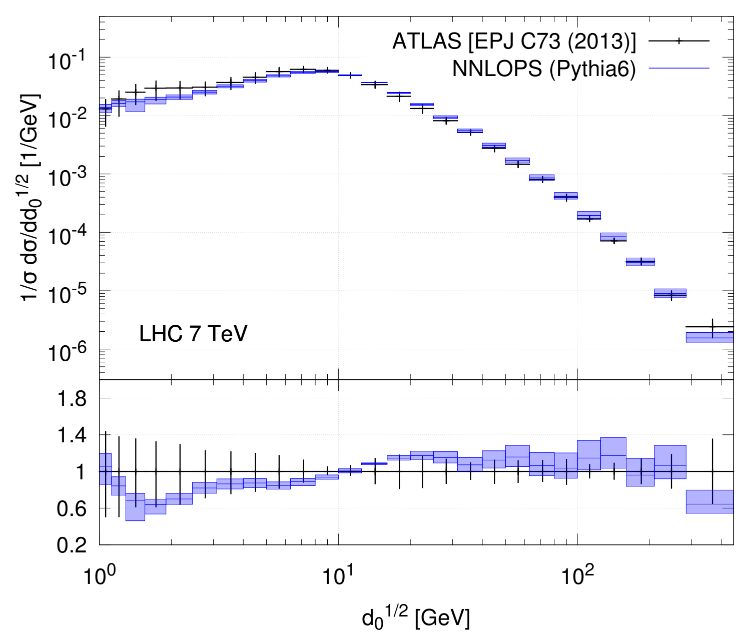
- Z p_T, rapidity

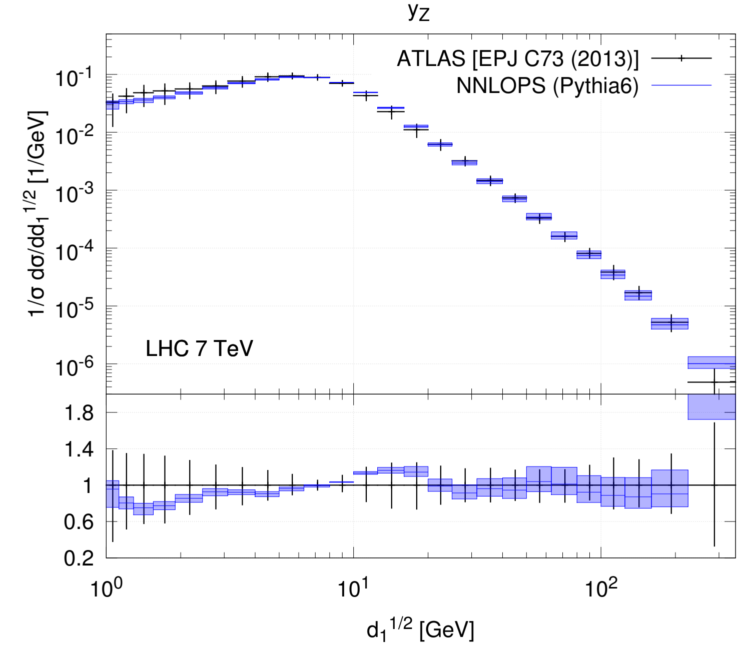




(scale but no PDF uncertainty)

- W $d_{0,1}$ (k_t alg.)





UN^{2}LOPS

Höche, Li, Prestel, 1405.3607

- Phase space slicing at $q_T \sim 1$ GeV (DY, H)
- q_T subtraction: NNLO in zero bin
- extra jet at NLO

$$
\begin{aligned}
\langle O\rangle^{(\mathrm{UN^2LOPS})} = & \int \mathrm{d}\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} \mathrm{d}\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} \mathrm{d}\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} \mathrm{d}\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} \mathrm{d}\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} \mathrm{d}\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} \mathrm{d}\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,O) \\
& + \int_{q_{T,\mathrm{cut}}} \mathrm{d}\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,O)
\end{aligned}
$$

UN²LOPS results (DY)

Höche, Li, Prestel, 1405.3607

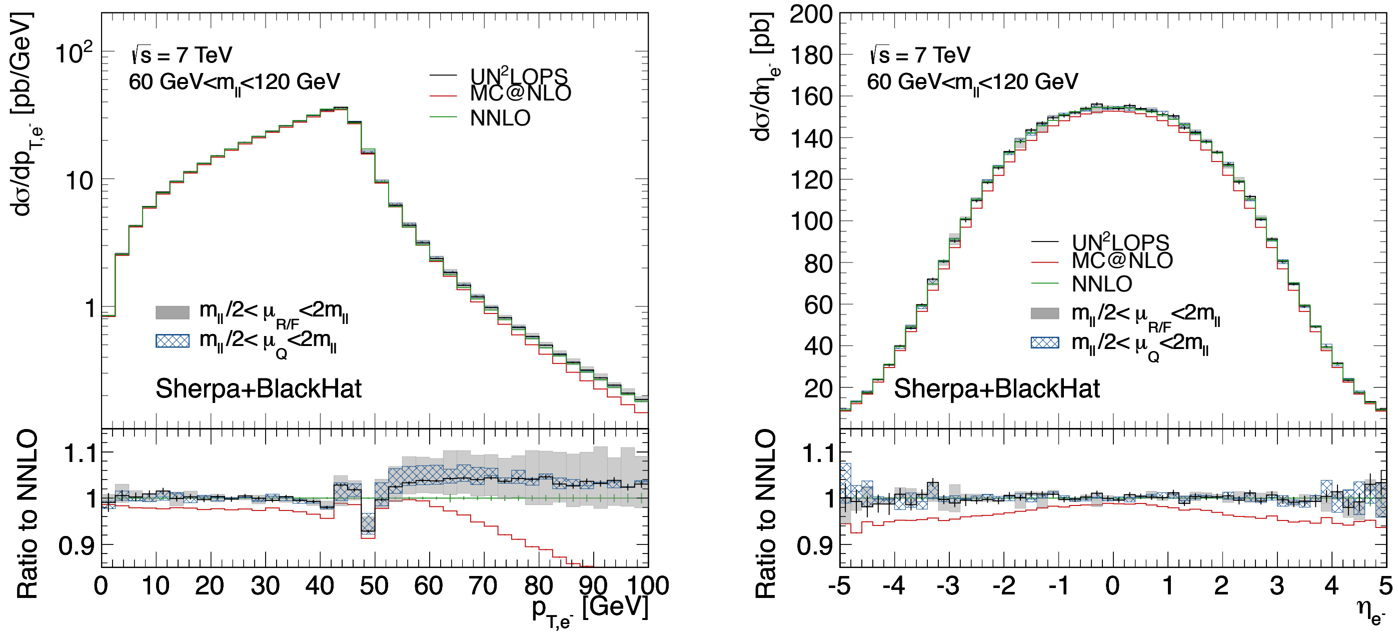


FIG. 2. Transverse momentum and rapidity spectrum of the electron. The gray solid (blue hatched) band shows scale uncertainties obtained by varying $\mu_{R/F}$ (μ_Q) in the range $m_{ll}/2 \leq \mu \leq 2\,m_{ll}$.

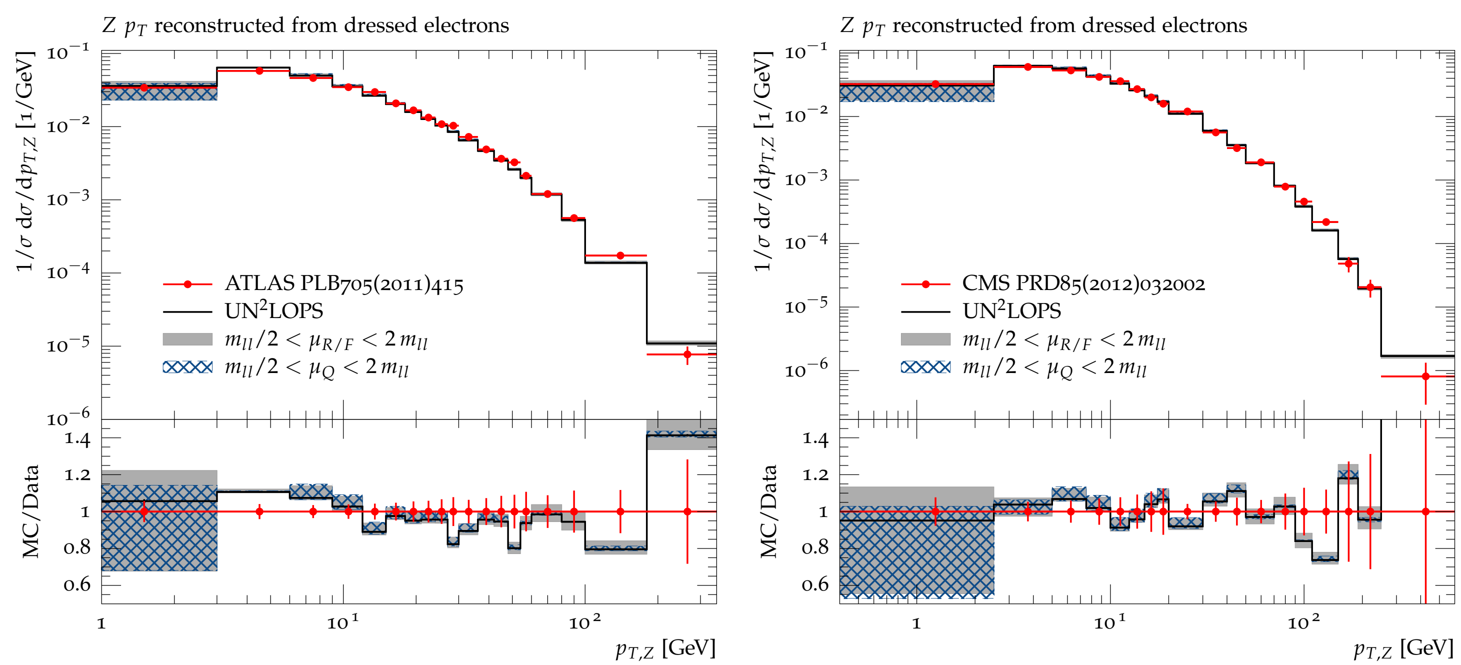


FIG. 3. UN²LOPS prediction for the transverse momentum spectrum of the Drell-Yan lepton pair in comparison to ATLAS data from [39] (left) and CMS data from [38] (right). The gray solid (blue hatched) band shows scale uncertainties obtained by varying $\mu_{R/F}$ (μ_Q) in the range $m_{ll}/2 \leq \mu \leq 2\,m_{ll}$.

UN²LOPS results (Higgs)

Höche, Li, Prestel, 1407.3773

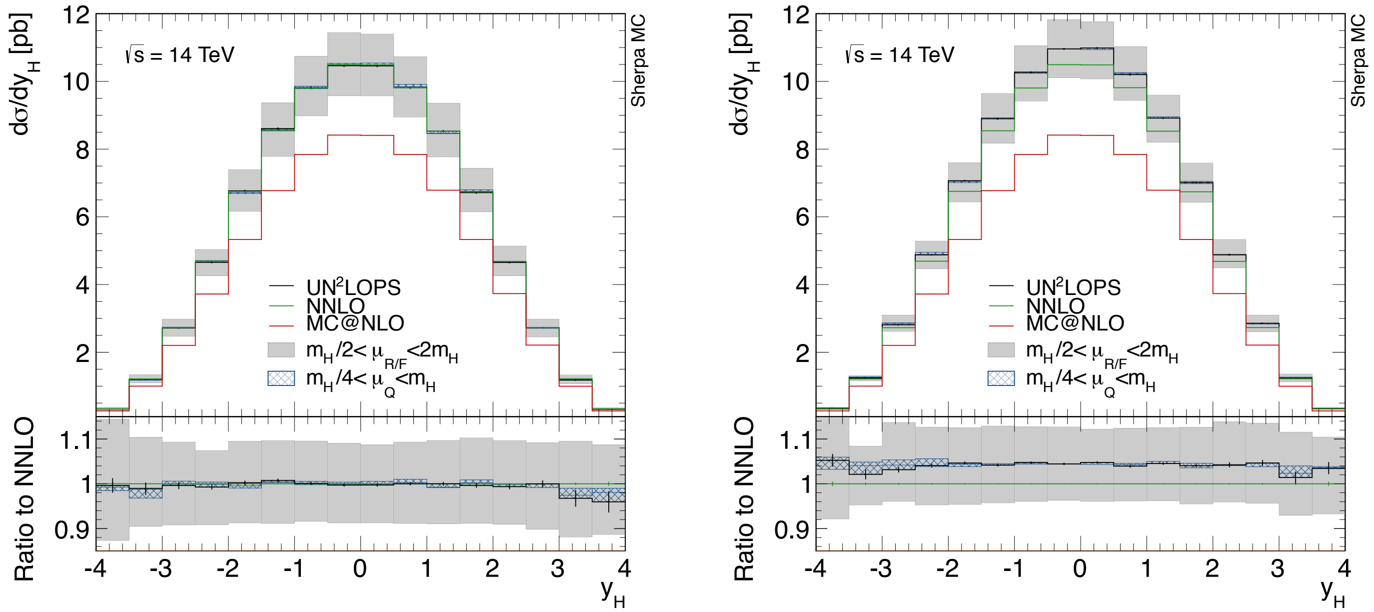


FIG. 2. Rapidity spectrum of the Higgs boson in individual matching (left) and factorized matching (right). See Sec. IV for details.

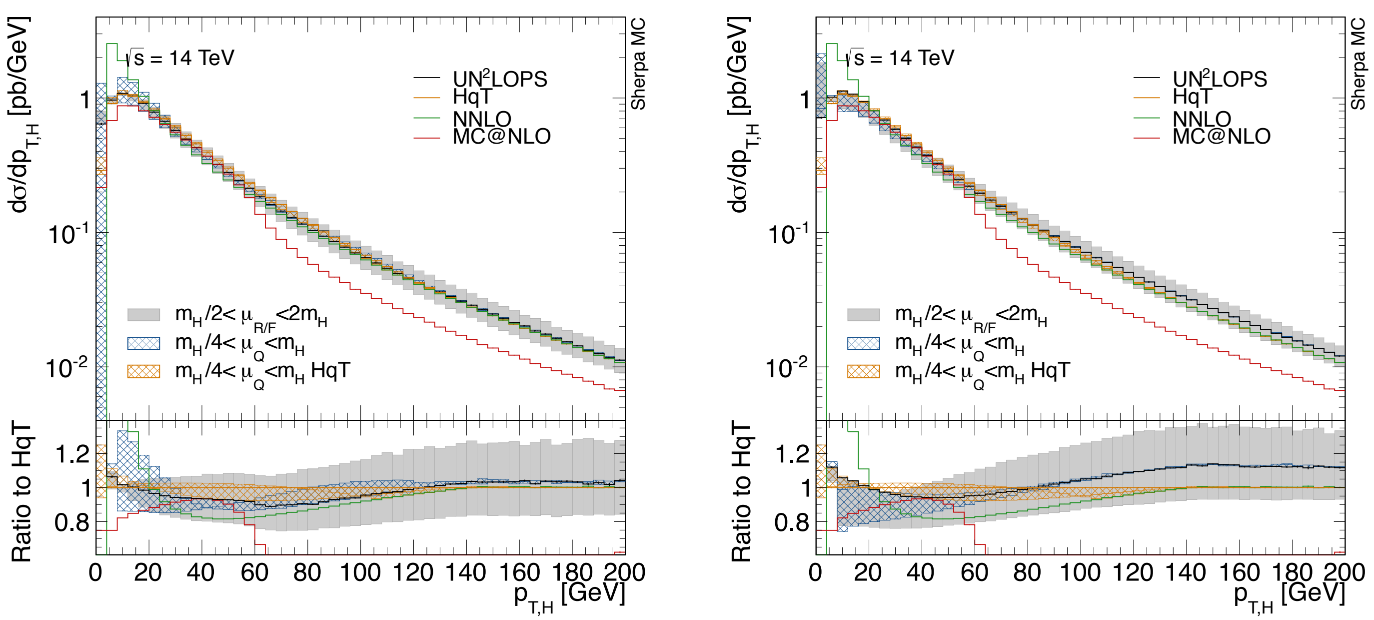


FIG. 3. Transverse momentum spectrum of the Higgs boson in individual matching (left) and factorized matching (right). See Sec. IV for details.

Matching to multiple fixed orders

Merging at NLO (?)

- Separate samples by jet resolution, e.g. d_{cut}
- Make NLO for $d_{i+1} < d_{cut} < d_i$
- Avoid double counting
- Reduce d_{cut} dependence
  - MEPS@NLO: Höche et al., 1207.5030
  - FxFx: Frederix, Frixione, 1209.6215
  - Geneva: Alioli et al., 1211.7049
  - UNLOPS: Lönnblad, Prestel, 1211.7278, Plätzer, 1211.5467

MEPS@NLO

- W+0,1,2 jets at NLO
- W+3,4 jets at LO

Höche et al., 1207.5030

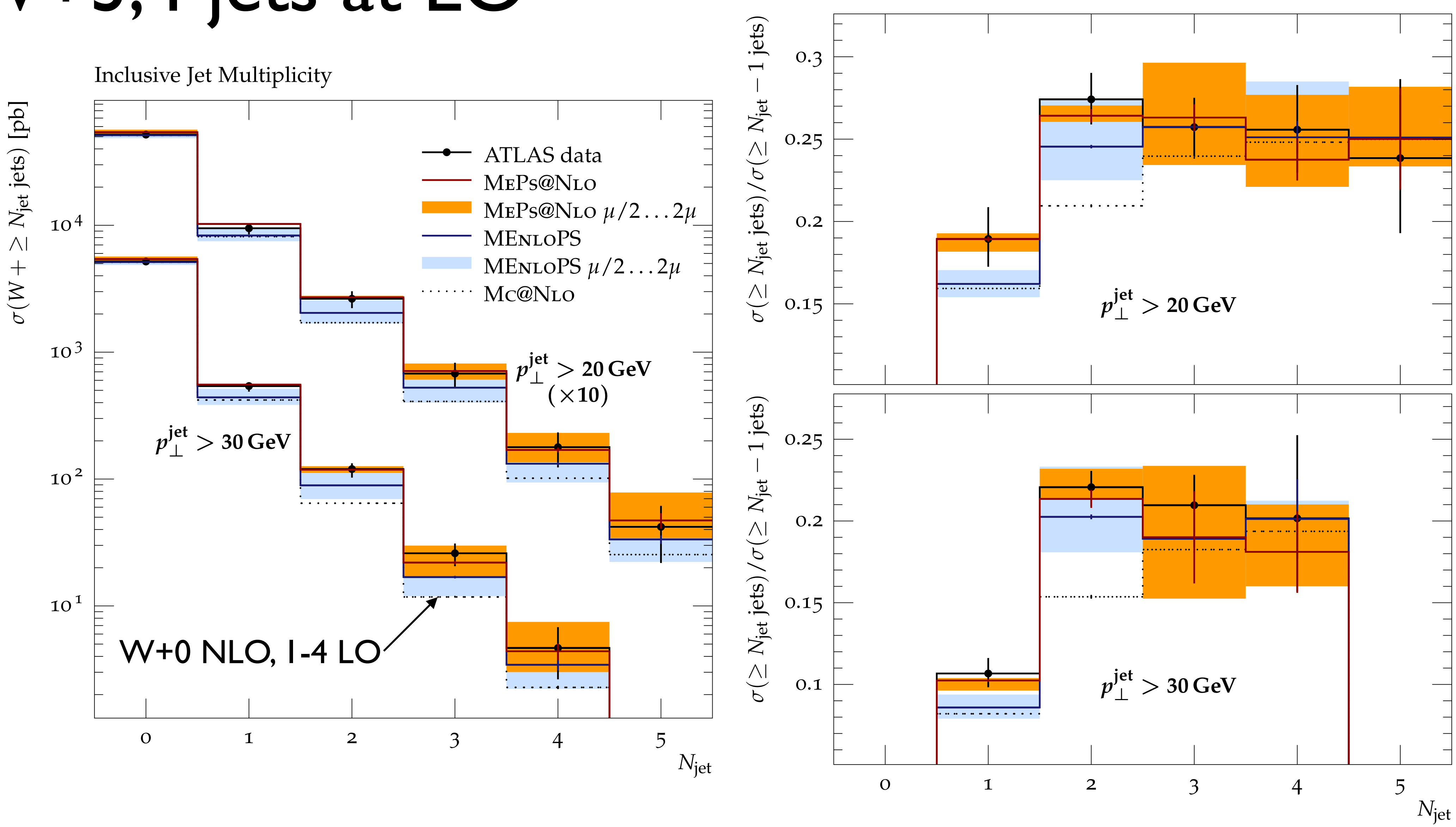


Figure 1: Cross section as a function of the inclusive jet multiplicity (left) and their ratios (right) in W+jets events measured by ATLAS [50].

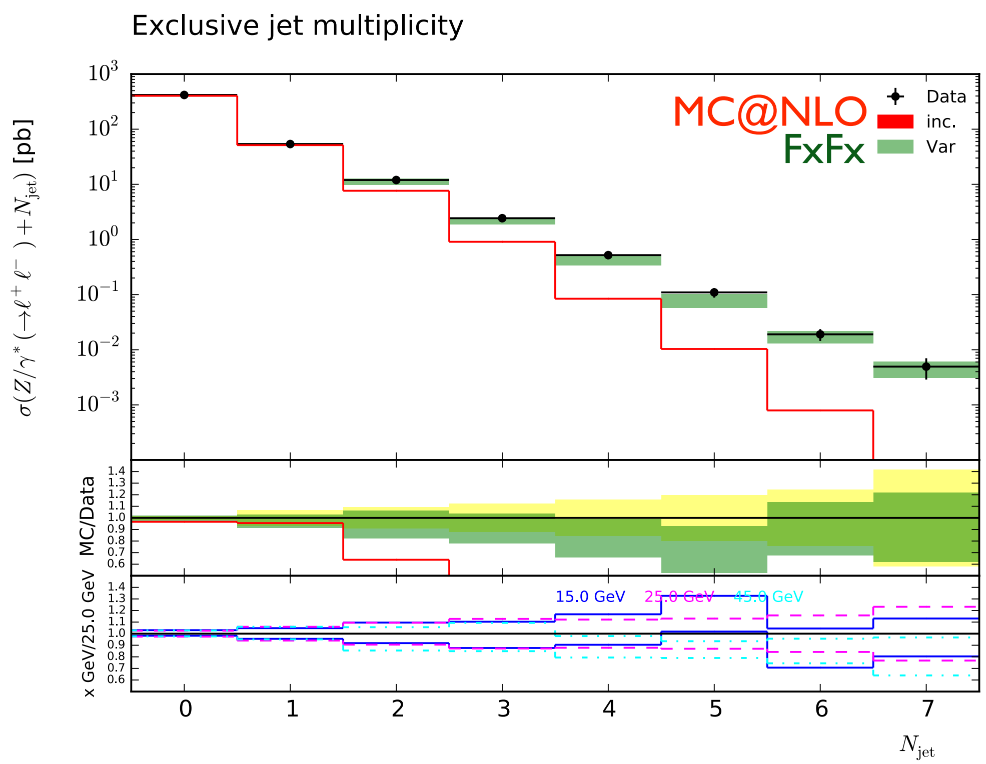
FxFx merging

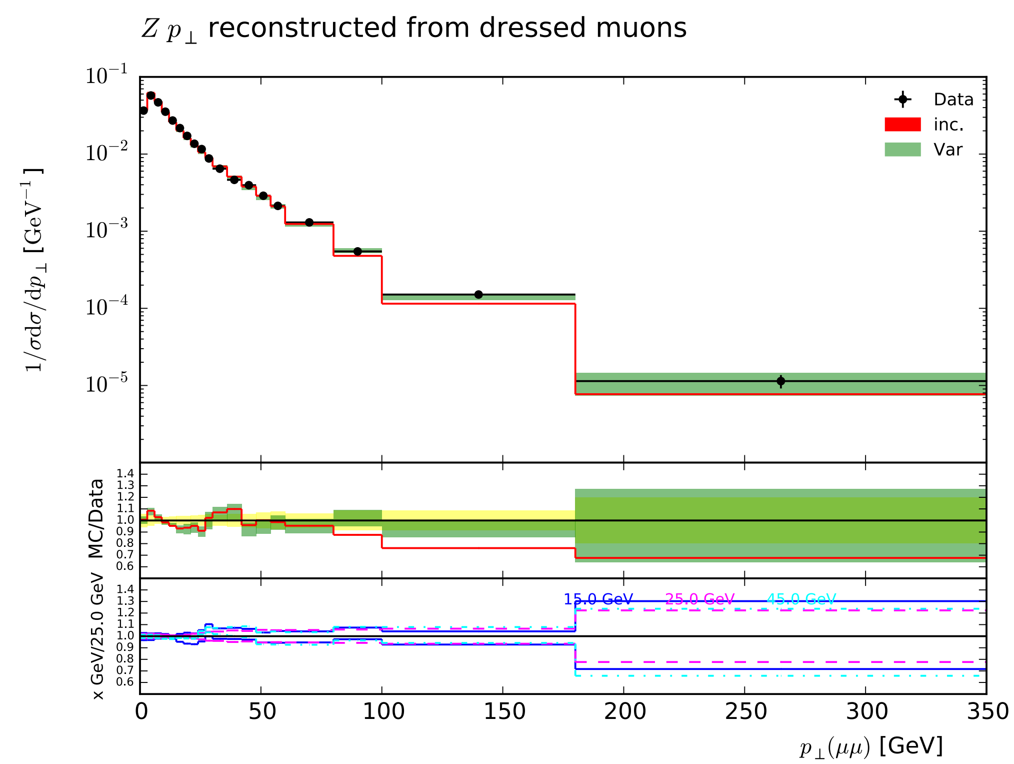
Frederix, Frixione, 1209.6215

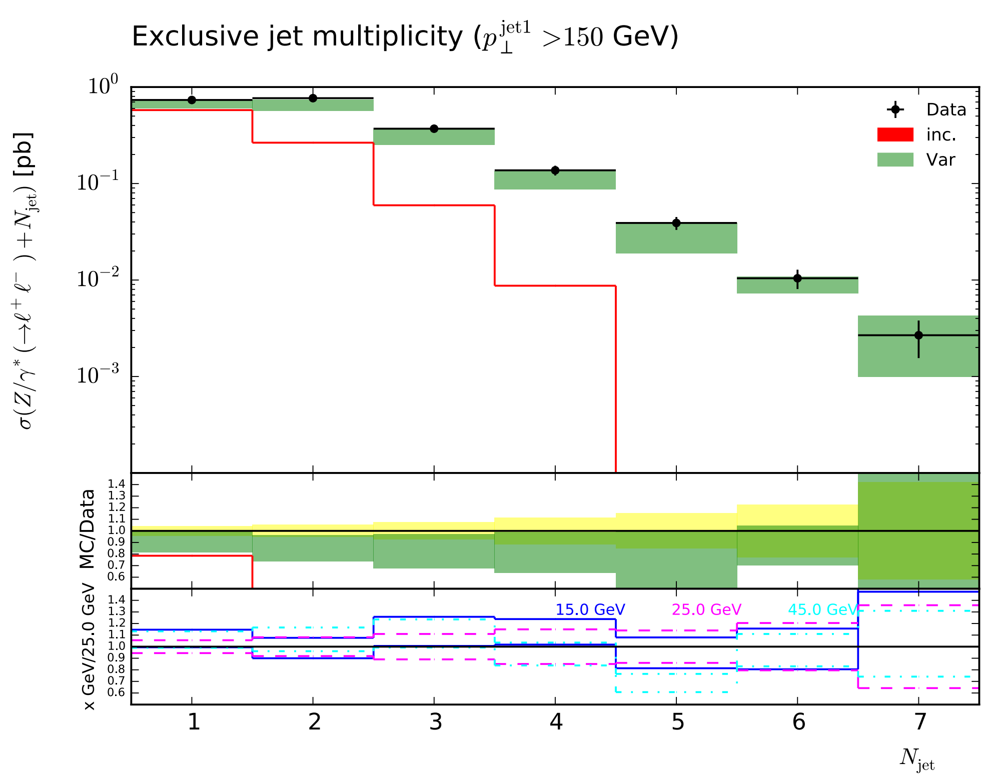
- $\mathbb{S}$ and $\mathbb{H}$ event samples for each multiplicity ($D(d_i) \approx \Theta(\mu_2 - d_i)$)

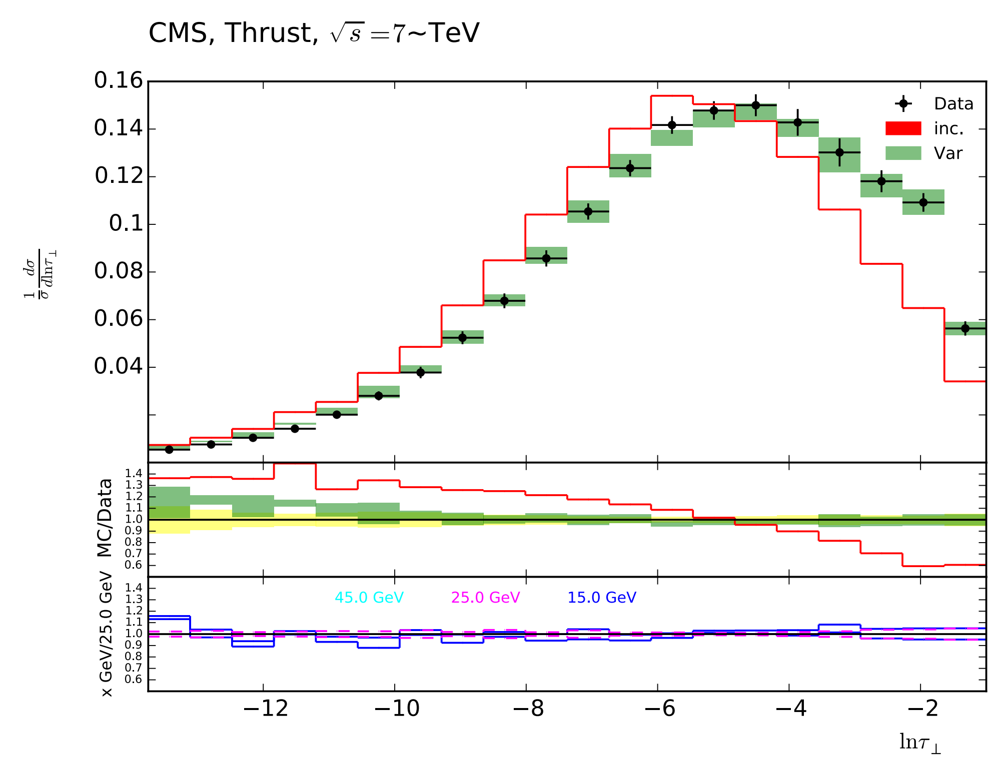
$$
\begin{aligned}
d\bar{\sigma}_{\mathbb{S},0} &= T_0 + V_0 - T_0\mathcal{K} + T_0\mathcal{K}_{\rm MC}D(d_1(\Xi_{\mathbb{H},0}))\,, \\
d\bar{\sigma}_{\mathbb{H},0} &= \Big[T_1 - T_0\mathcal{K}_{\rm MC}\Big]\,D(d_1(\Xi_{\mathbb{H},0}))\,, \\
d\bar{\sigma}_{\mathbb{S},i} &= \Big[T_i + V_i - T_i\mathcal{K} + T_i\mathcal{K}_{\rm MC}D(d_{i+1}(\Xi_{\mathbb{H},i}))\Big] \\
&\quad \times \left(1 - D(d_i(\Xi_{\mathbb{S},i}))\right)\,\Theta\left(d_{i-1}(\Xi_{\mathbb{S},i}) - \mu_2\right)\,, \\
d\bar{\sigma}_{\mathbb{H},i} &= \Big[T_{i+1}\left(1 - D(d_i(\Xi_{\mathbb{H},i}))\right)\Theta\left(d_{i-1}(\Xi_{\mathbb{H},i}) - \mu_2\right) \\
&\quad - T_i\mathcal{K}_{\rm MC}\left(1 - D(d_i(\Xi_{\mathbb{S},i}))\right)\Theta\left(d_{i-1}(\Xi_{\mathbb{S},i}) - \mu_2\right)\Big]\,D(d_{i+1}(\Xi_{\mathbb{H},i}))\,, \\
d\bar{\sigma}_{\mathbb{S},N} &= \Big[T_N + V_N - T_N\mathcal{K} + T_N\mathcal{K}_{\rm MC}\Big] \\
&\quad \times \left(1 - D(d_N(\Xi_{\mathbb{S},N}))\right)\Theta\left(d_{N-1}(\Xi_{\mathbb{S},N}) - \mu_2\right)\,, \\
d\bar{\sigma}_{\mathbb{H},N} &= T_{N+1}\left(1 - D(d_N(\Xi_{\mathbb{H},N}))\right)\Theta\left(d_{N-1}(\Xi_{\mathbb{H},N}) - \mu_2\right) \\
&\quad - T_N\mathcal{K}_{\rm MC}\left(1 - D(d_N(\Xi_{\mathbb{S},N}))\right)\Theta\left(d_{N-1}(\Xi_{\mathbb{S},N}) - \mu_2\right)\,.
\end{aligned}
$$

FxFx Z results (1)



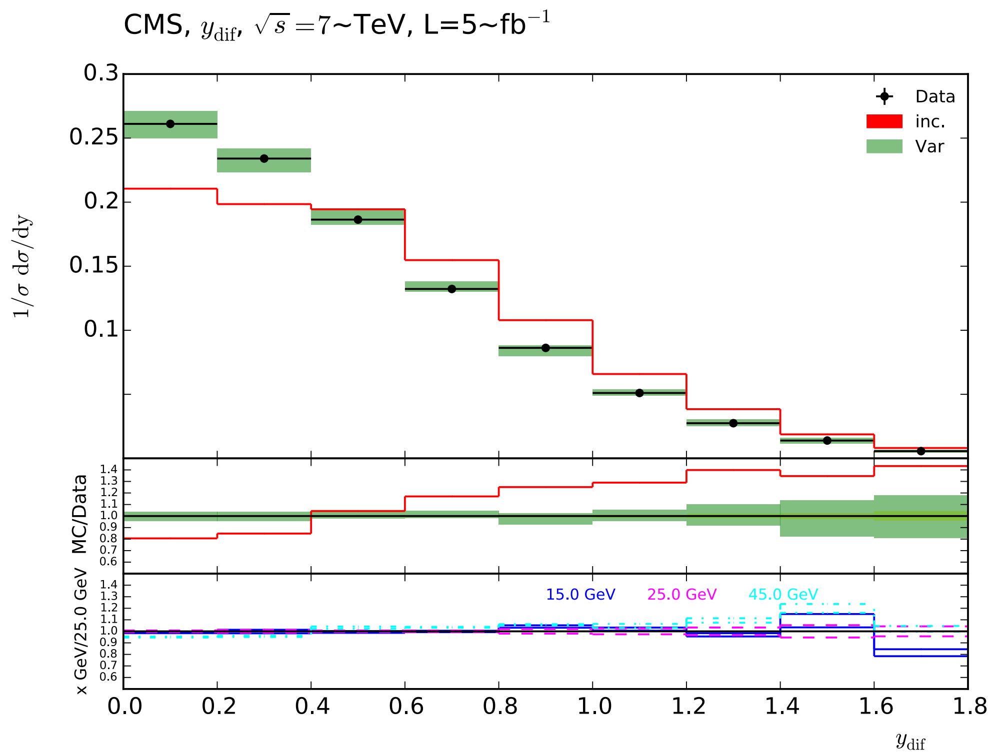


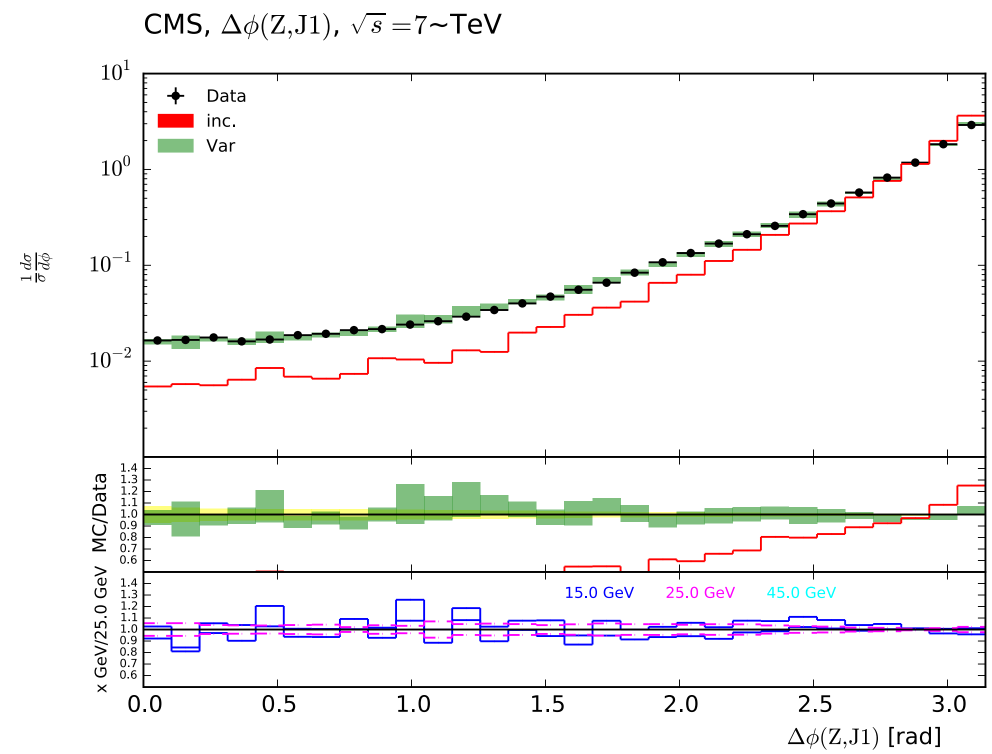


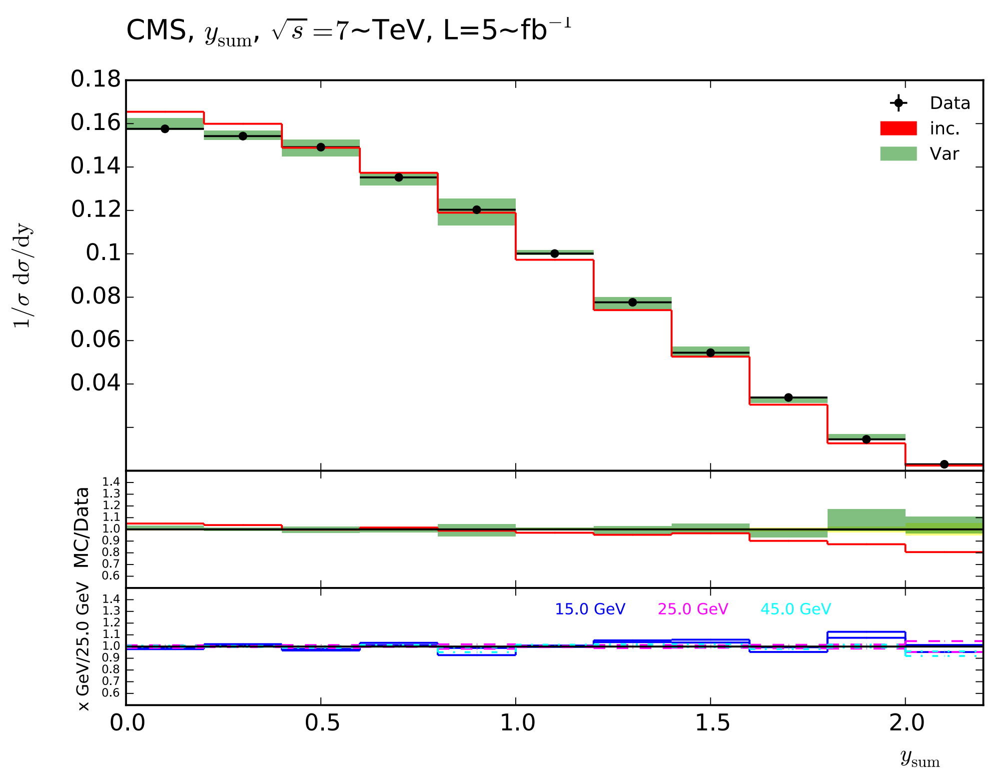


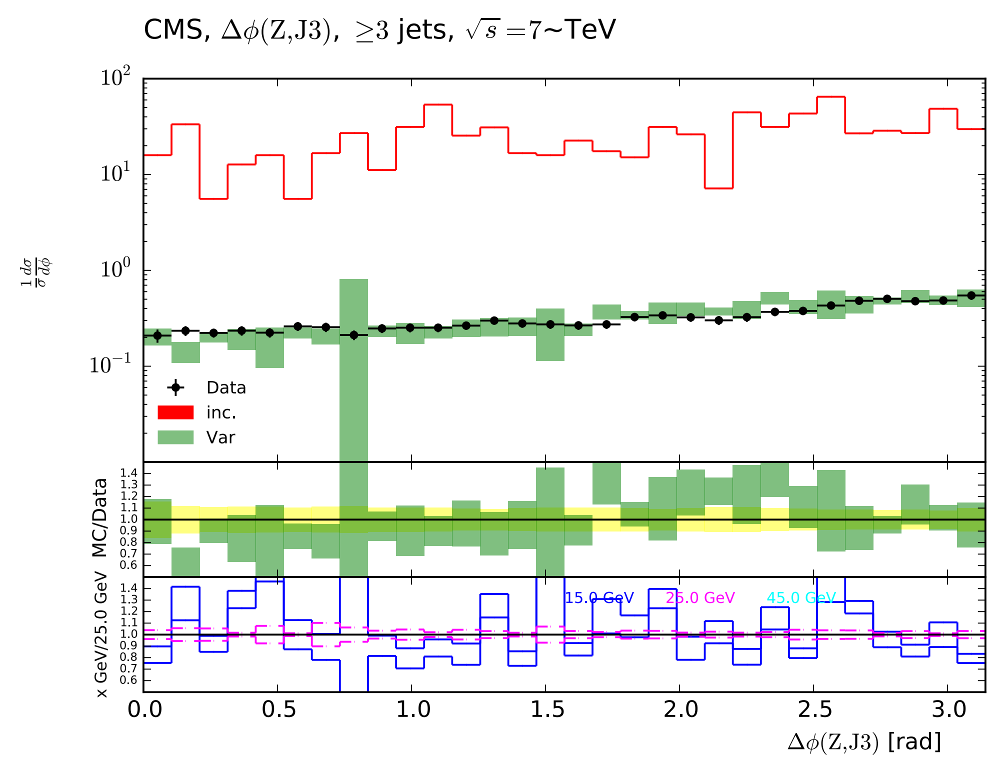
Frederix, Frixione, Papaefstathiou, Prestel, Torrielli, in prep.

FxFx Z results (2)

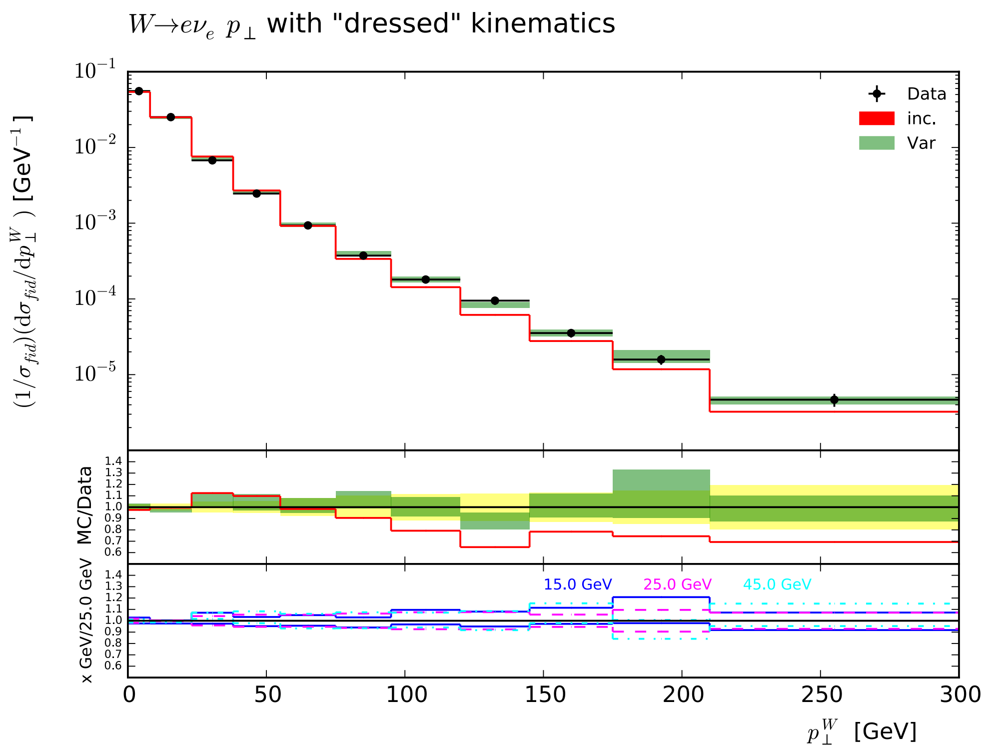


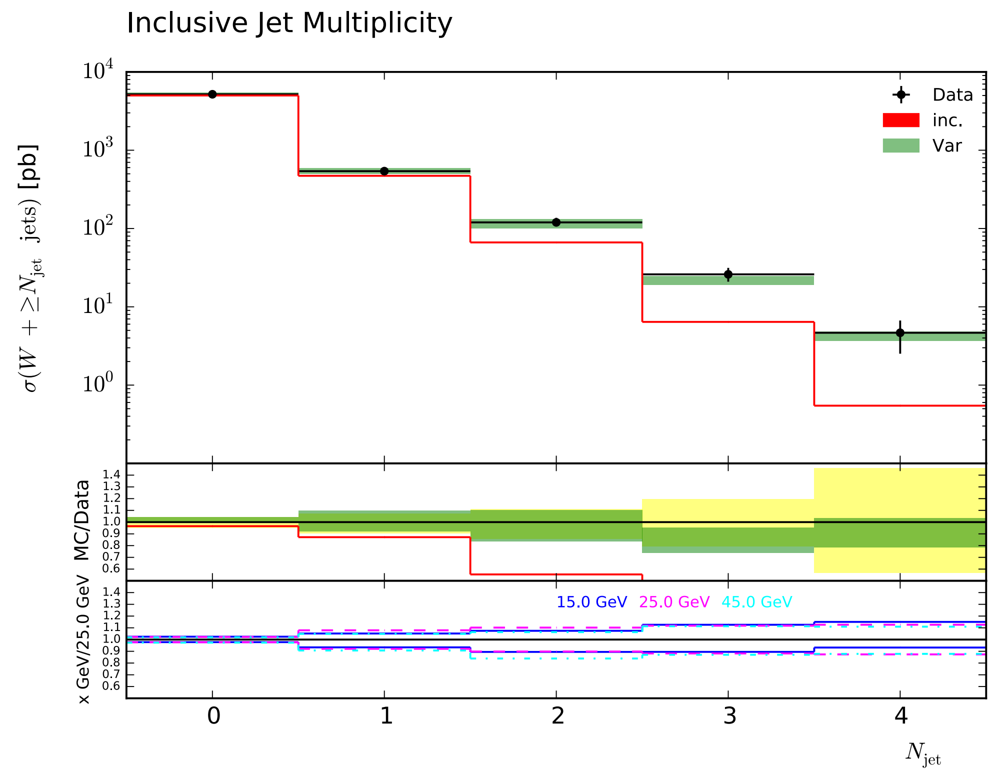


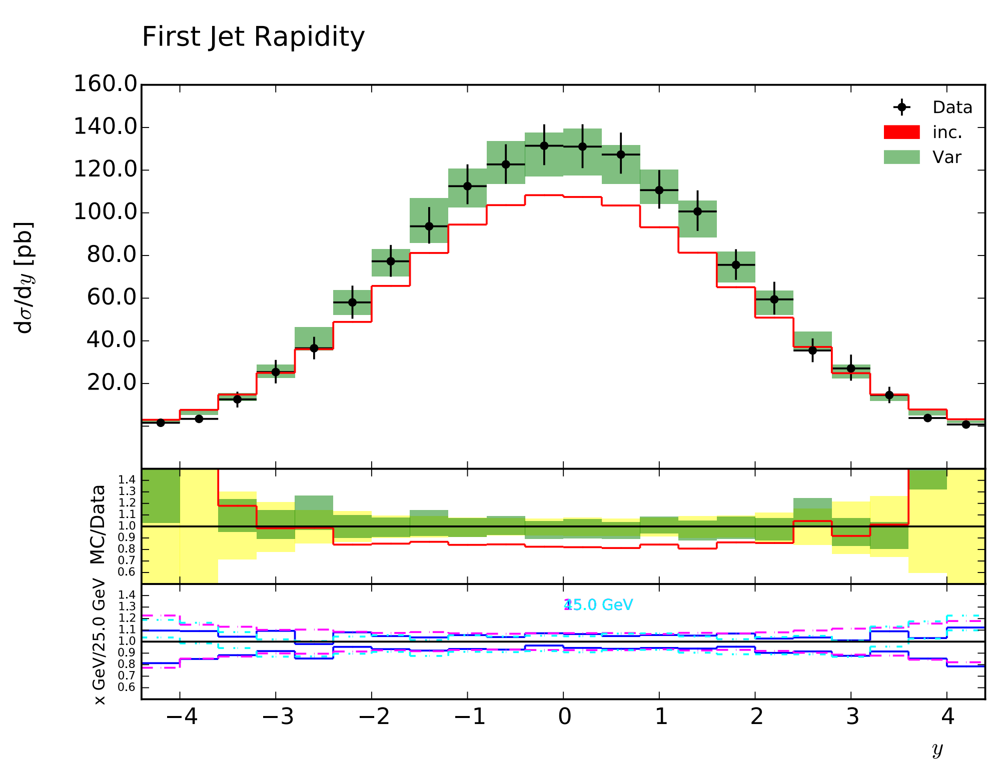


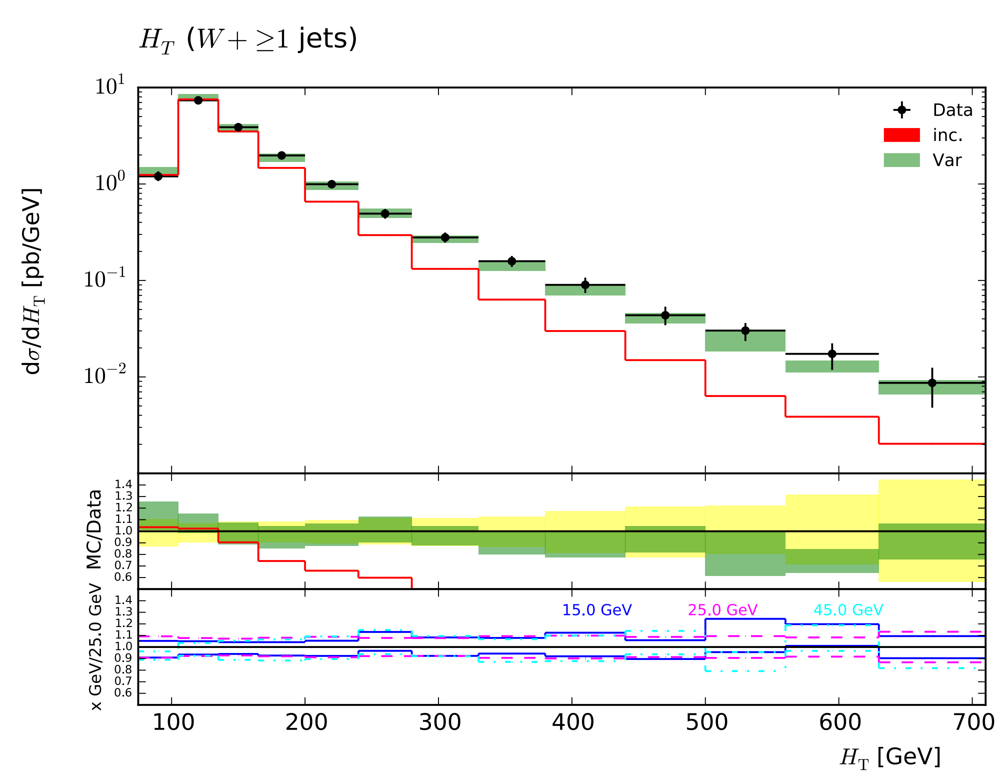


FxFx W results (1)

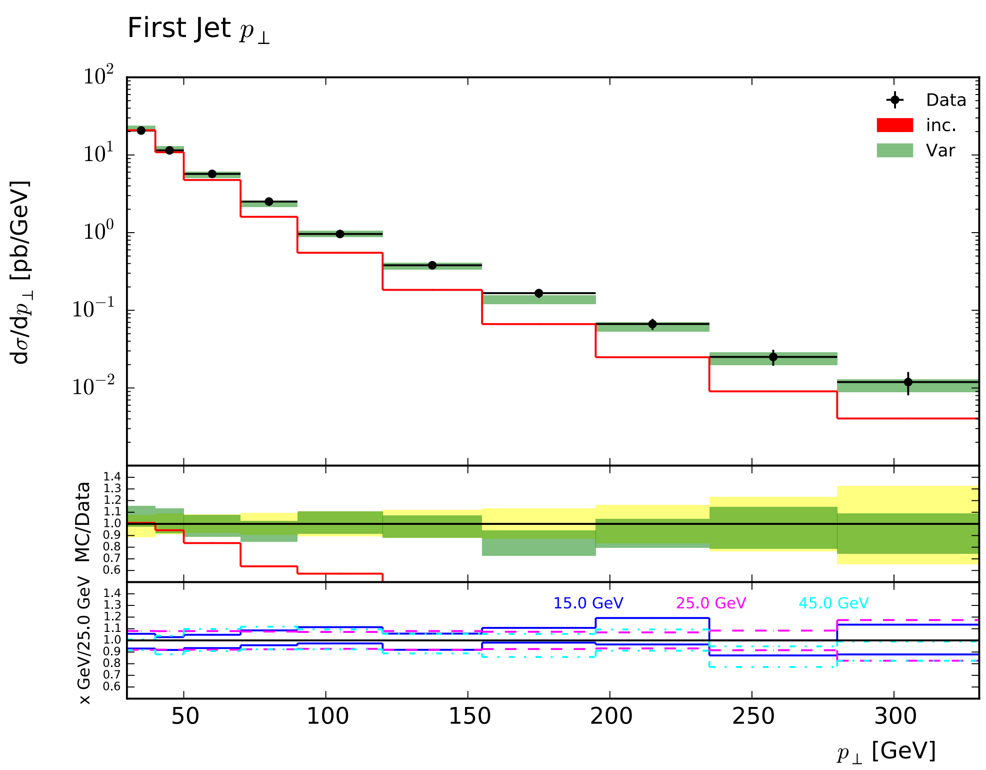


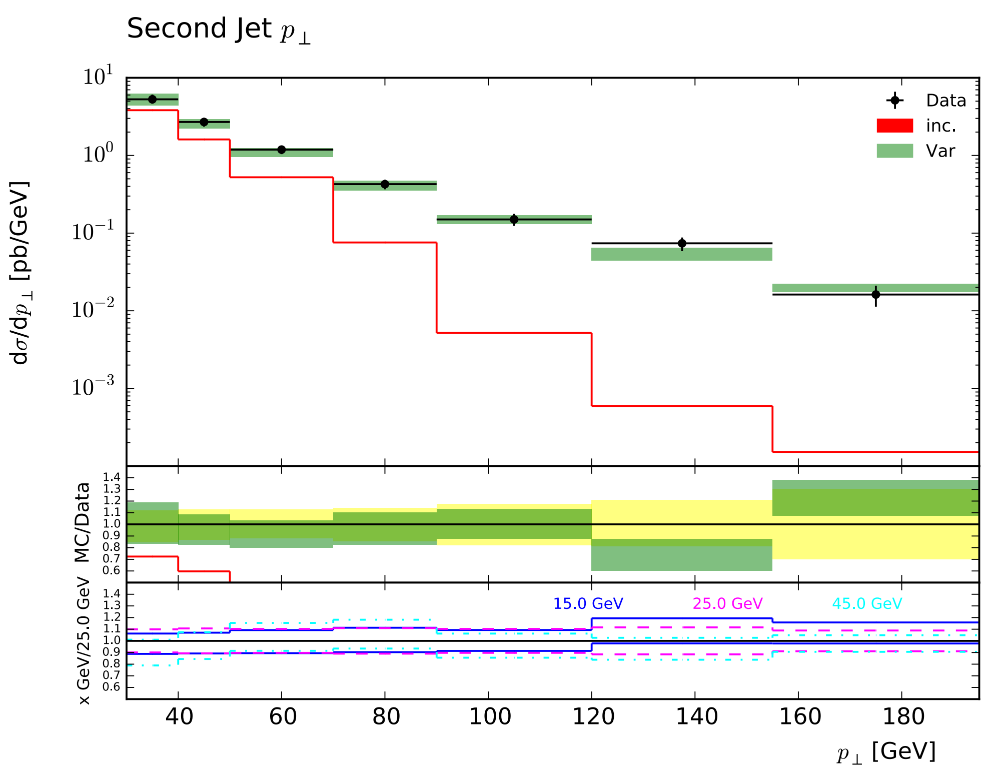


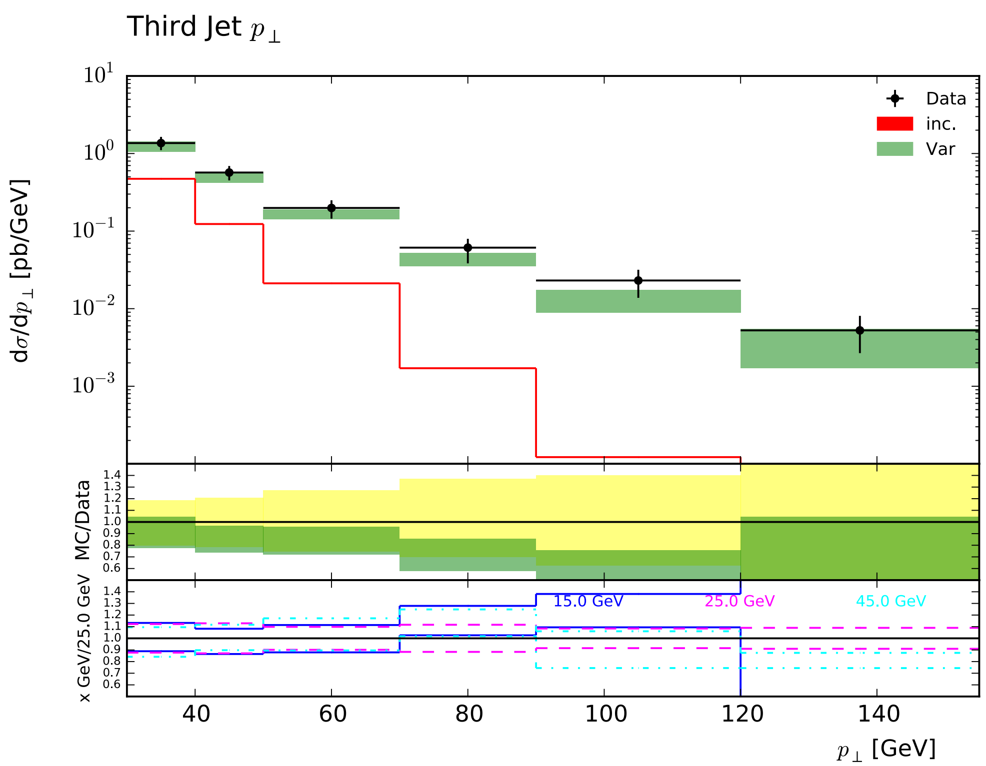


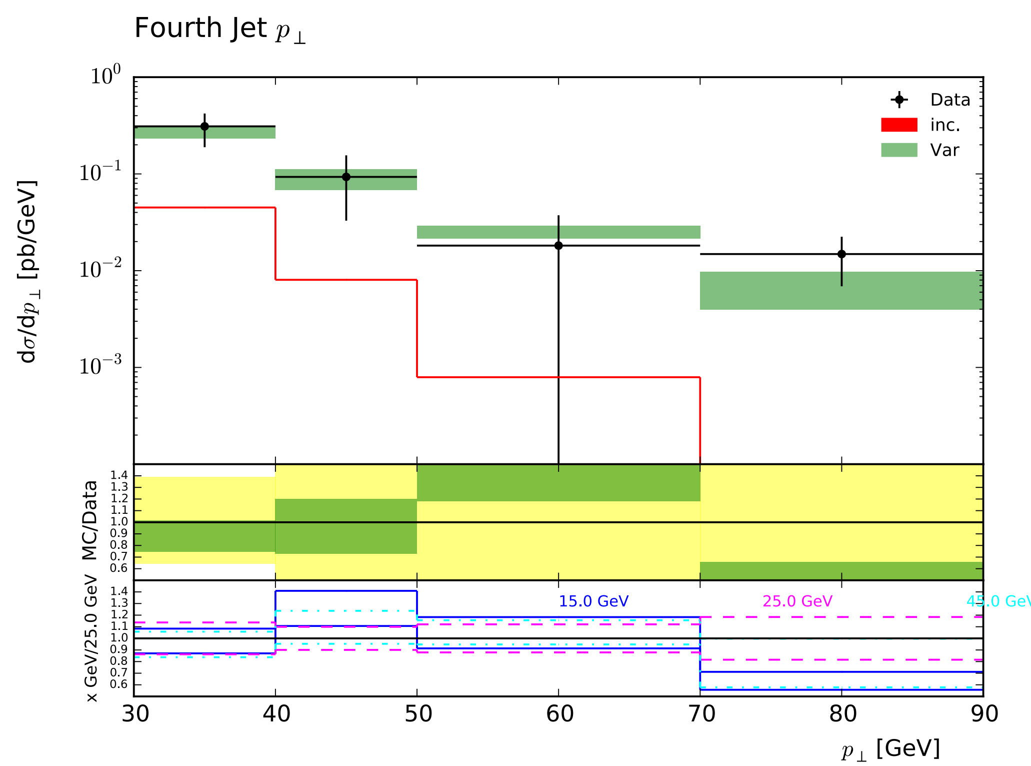


FxFx W results (2)

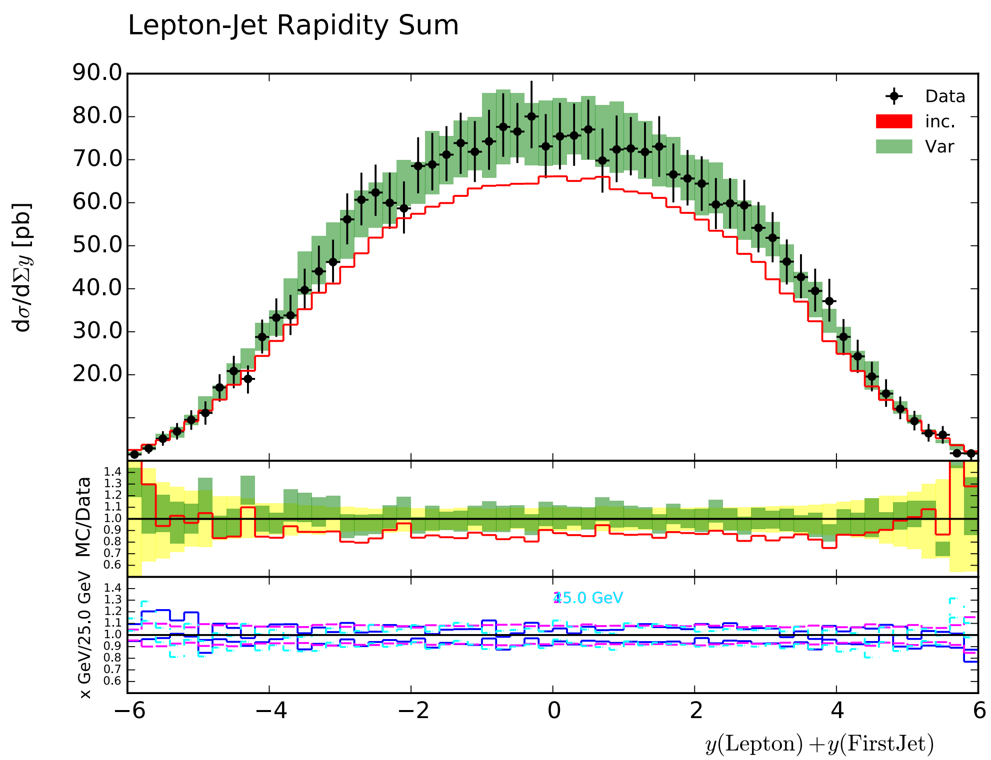


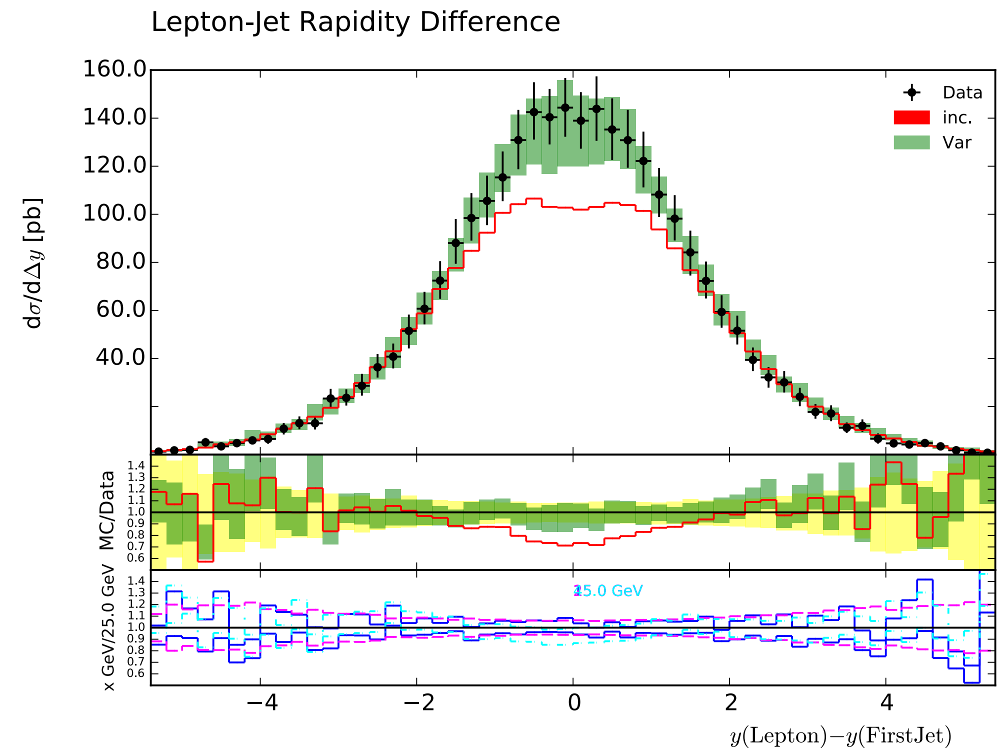


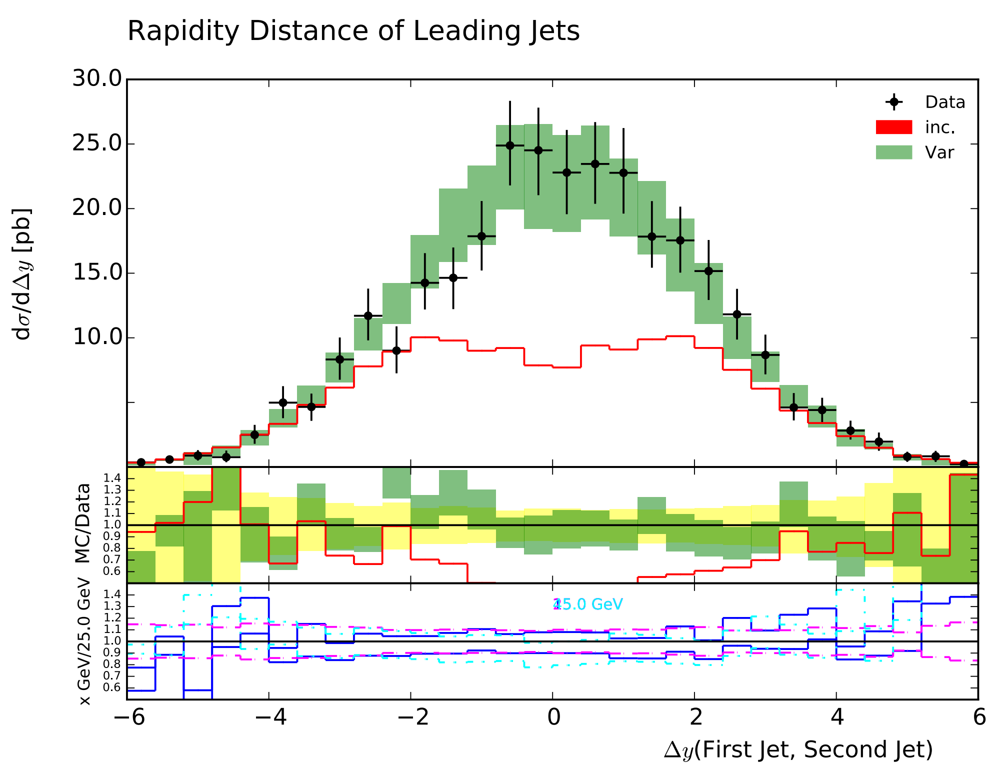


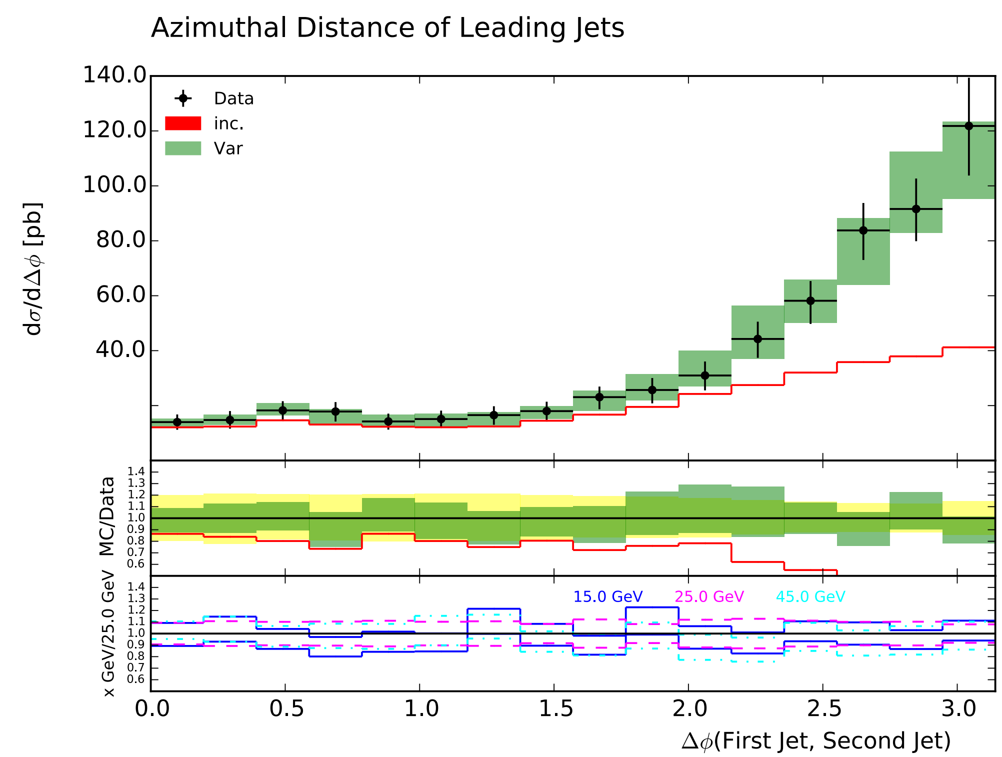


FxFx W results (3)



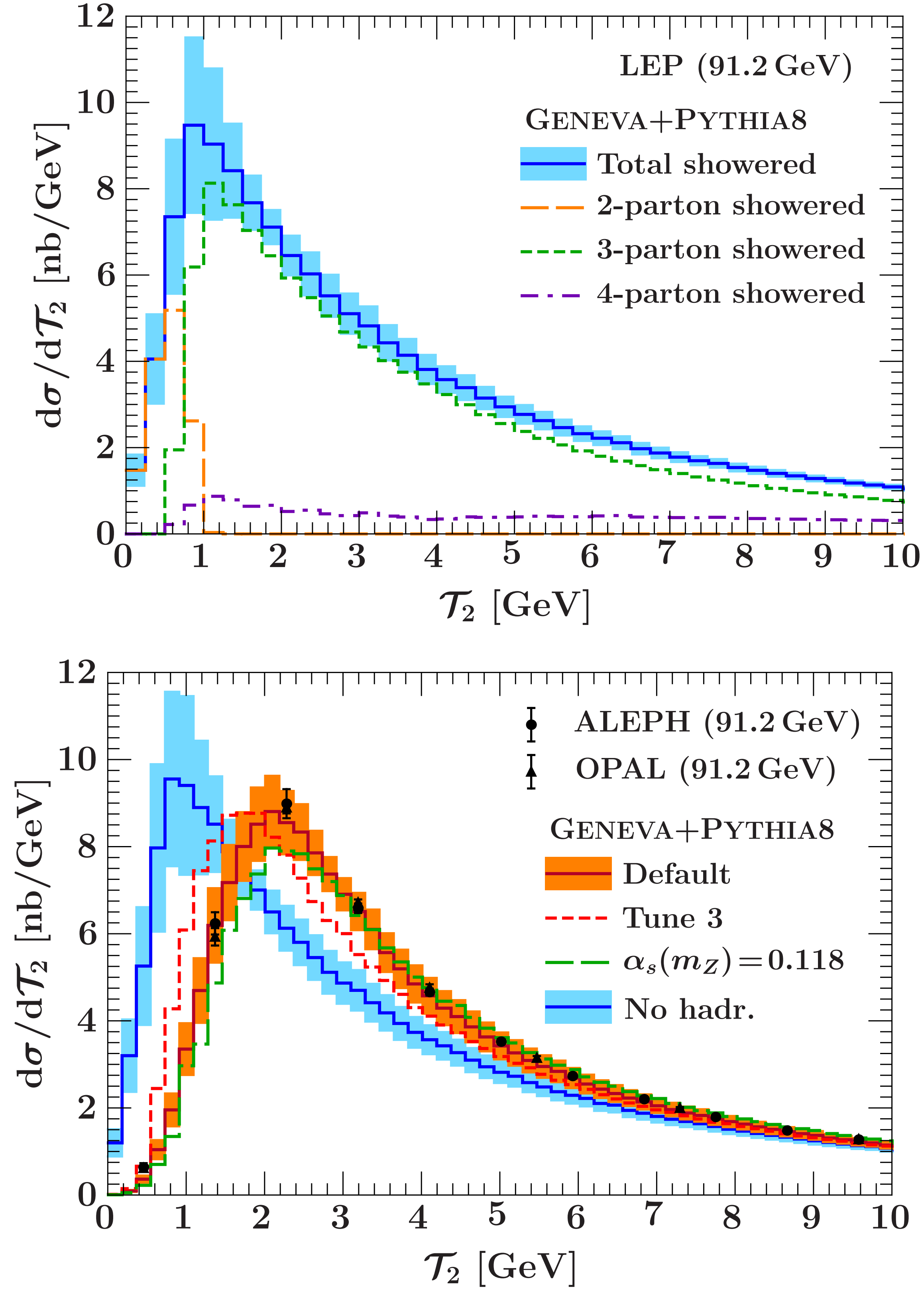






Geneva merging (e^+e^-)

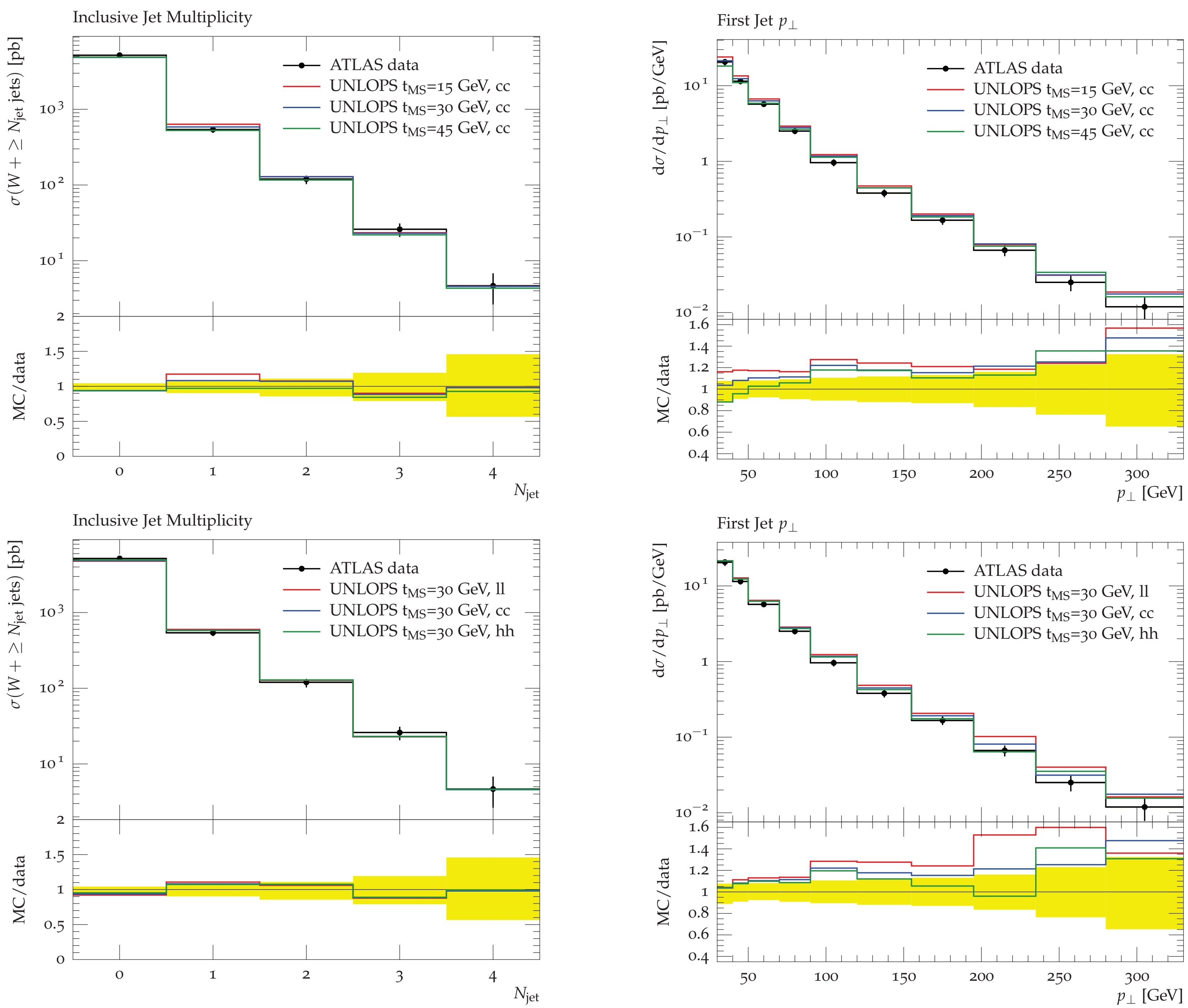
Alioli et al., 1211.7049



UNLOPS merging

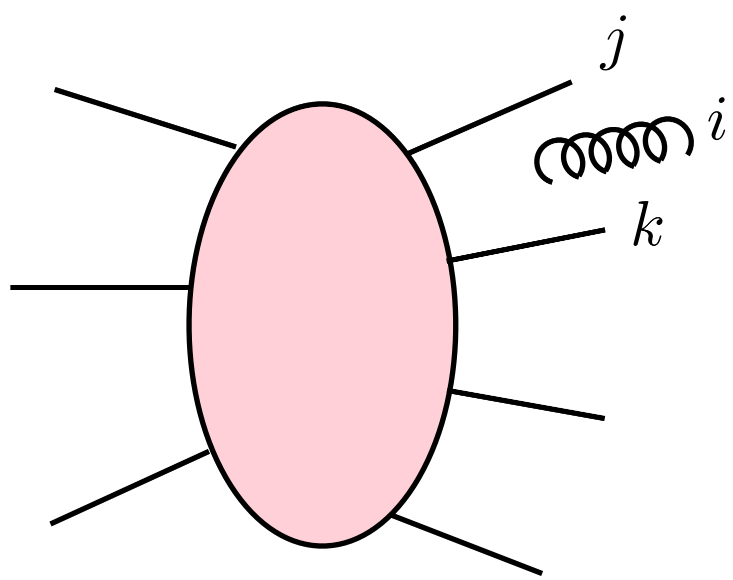
Lönnblad, Prestel, 1211.7278

- Merging scale dependence
- Ren/fac scale dependence



Parton Showers

Antenna (Lund) shower



In the rest frame of dipole (jk), emission i has transverse momentum p_t and rapidity y where

$$p_t^2 = 2\frac{p_i \cdot p_j p_i \cdot p_k}{p_j \cdot p_k}, \quad (1)$$

$$y = \frac{1}{2}\ln\frac{p_i \cdot p_k}{p_i \cdot p_j} \quad (2)$$

and the leading contribution of this emission is

$$d\sigma_{n+1}^{(jk)} = d\sigma_n \frac{\alpha_s}{2\pi}(-2\mathbf{T}_j \cdot \mathbf{T}_k)\frac{dp_t^2}{p_t^2}dy\,. \quad (3)$$

The phase space for emission is $p_i \cdot p_j\,,\ p_i \cdot p_k < p_j \cdot p_k = Q^2/2$ where Q is the dipole mass. Hence

$$-\ln\frac{Q}{p_t} < y < \ln\frac{Q}{p_t}. \quad (4)$$

- **Three jets in e^+e^- annihilation**

The p_t resolution is Q_0 where $Q_0^2/Q^2 = y_{\rm cut}$. Hence the 3-jet rate is

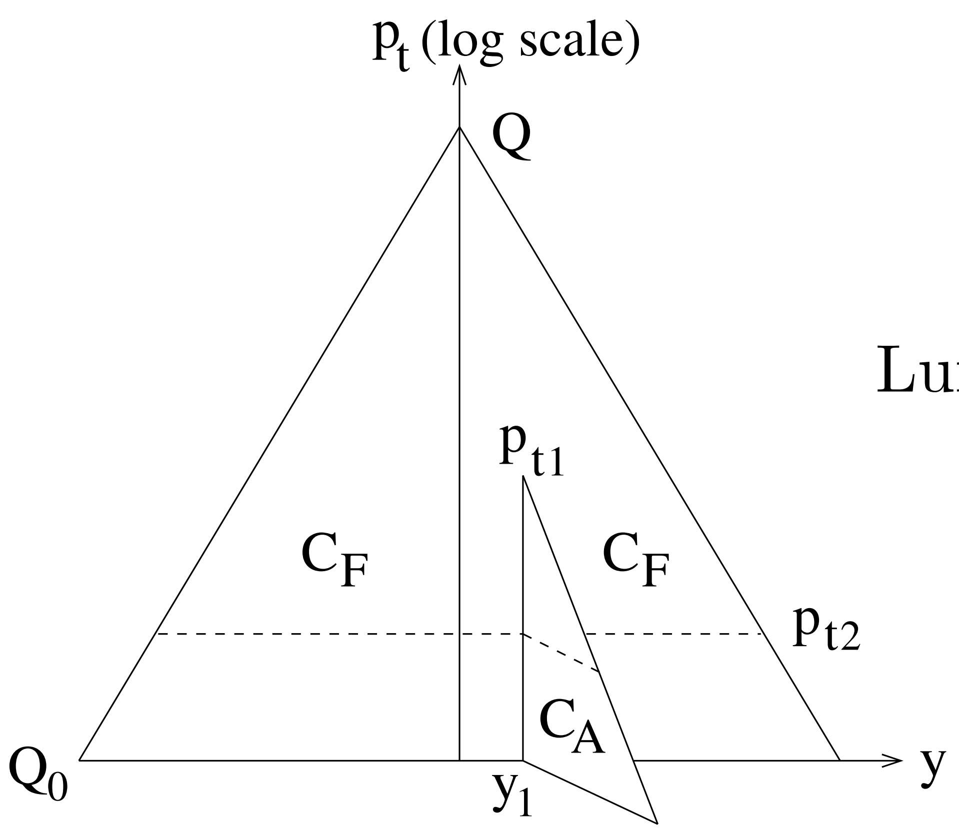
$$R_3^d = \frac{\alpha_s}{2\pi}2C_F\int_{Q_0}^{Q} 2\frac{dp_t}{p_t}\int_{-\ln Q/p_t}^{\ln Q/p_t} dy = \frac{1}{2}C_F a L^2 \quad (5)$$

where $a = \alpha_s/\pi$ and $L = \ln y_{\rm cut}$.

Antenna shower

- **Four jets in e^+e^- annihilation**

$$
\begin{aligned}
R_4^d &= \left(\frac{\alpha_s}{2\pi}\right)^2 2C_F \int_{Q_0}^{Q} 2\frac{dp_{t1}}{p_{t1}} 2\ln\frac{Q}{p_{t1}} \left[2C_F \int_{Q_0}^{p_{t1}} 2\frac{dp_{t2}}{p_{t2}} 2\ln\frac{Q}{p_{t2}} + C_A \int_{Q_0}^{p_{t1}} 2\frac{dp_{t2}}{p_{t2}} 2\ln\frac{p_{t1}}{p_{t2}}\right] \\
&= \left(\frac{\alpha_s}{2\pi}\right)^2 16C_F \int_{Q_0}^{Q} \frac{dp_{t1}}{p_{t1}} \ln\frac{Q}{p_{t1}} \left[2C_F\left(\ln^2\frac{Q}{Q_0} - \ln^2\frac{Q}{p_{t1}}\right) + C_A \ln^2\frac{p_{t1}}{Q_0}\right] \\
&= \left(\frac{1}{8}C_F^2 + \frac{1}{48}C_F C_A\right) a^2 L^4\,,
\end{aligned}
$$



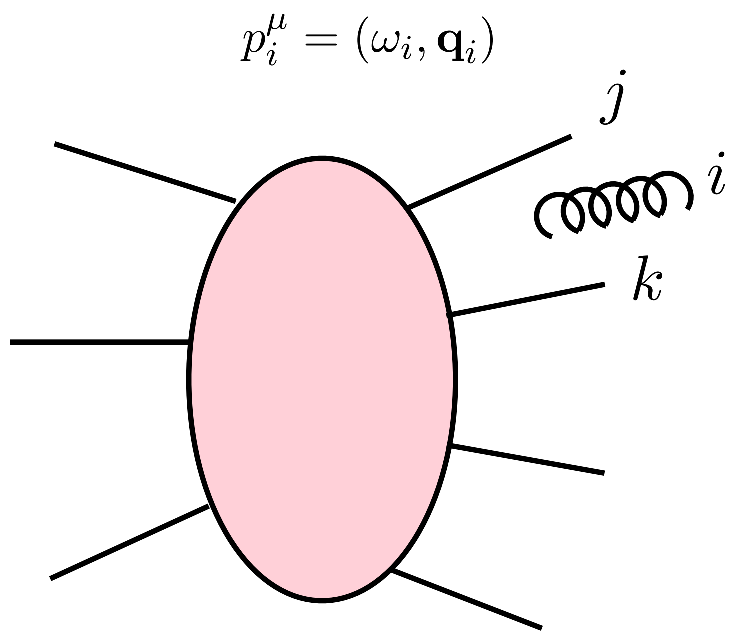
Lund “origami” diagram for $e^+e^- \to 4$ jets.

Dipole partition

$$D_{jk} = \frac{p_j \cdot p_k}{p_j \cdot q \, p_k \cdot q} = D_{jk}^{(j)} + D_{jk}^{(k)}$$

- Catani-Seymour $$D_{jk}^{(j)} = D_{jk} \frac{p_k \cdot q}{(p_j + p_k) \cdot q}$$
- Nagy-Soper $$D_{jk}^{(j)} = D_{jk} \frac{p_k \cdot q \, p_j \cdot Q}{p_j \cdot q \, p_k \cdot Q + p_k \cdot q \, p_j \cdot Q}$$
- Angular-ordered $$D_{jk}^{(j)} = \frac{1}{2} D_{jk} + \frac{1}{2\, q \cdot Q} \left(\frac{p_j \cdot Q}{p_j \cdot q} - \frac{p_k \cdot Q}{p_k \cdot q} \right)$$

AO parton shower



- Coherent emission from (jk)

$$
\begin{aligned}
d\sigma_{n+1}^{(jk)} &= g_s^2 d\sigma_n \frac{d^3\mathbf{q}_i}{(2\pi)^3 2\omega_i}(-\mathbf{T}_j\cdot\mathbf{T}_k)\frac{p_j\cdot p_k}{p_j\cdot p_i\,,p_k\cdot p_i} \\
&= \frac{\alpha_s}{2\pi}d\sigma_n \frac{d\omega_i}{\omega_i}\frac{d\Omega_i}{2\pi}(-\mathbf{T}_j\cdot\mathbf{T}_k)\frac{\xi_{jk}}{\xi_{ij}\,\xi_{ik}}
\end{aligned}
$$

where $\xi_{jk} = 1 - \cos\theta_{jk}$. Now

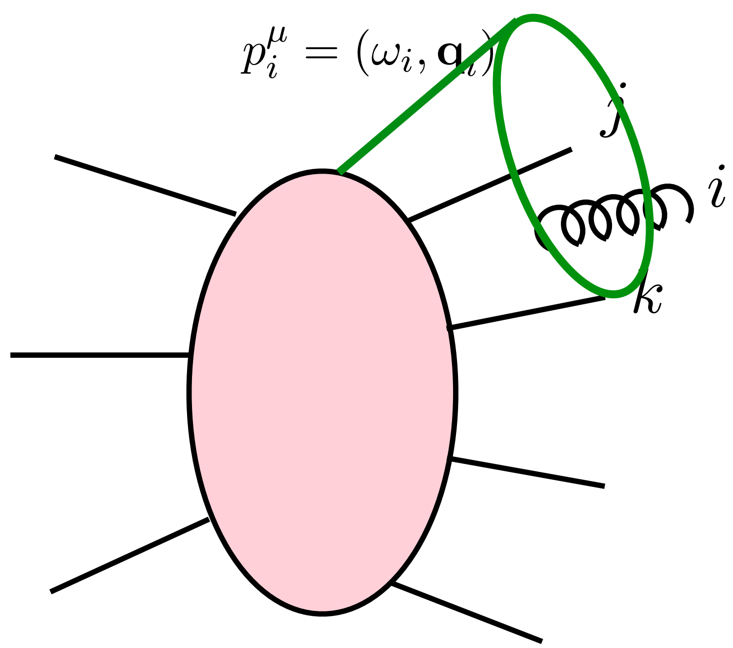
$$
\frac{d\Omega_i}{2\pi}\frac{\xi_{jk}}{\xi_{ij}\,\xi_{ik}} = \frac{d\xi_{ij}}{\xi_{ij}}\frac{d\phi_{ij}}{2\pi}\frac{1}{2}\left(\frac{\xi_{jk}-\xi_{ij}}{\xi_{ik}}+1\right) + (j \leftrightarrow k)
$$

- After azimuthal integration, this is exactly

$$
\frac{d\xi_{ij}}{\xi_{ij}}\Theta(\xi_{jk}-\xi_{ij}) + \frac{d\xi_{ik}}{\xi_{ik}}\Theta(\xi_{jk}-\xi_{ik})
$$

- Each parton j,k radiates into cone θ_{ij}, $\theta_{ik} < \theta_{jk}$

AO parton shower



- Coherent emission from (jk)

$$d\sigma_{n+1}^{(jk)} = g_s^2 d\sigma_n \frac{d^3\mathbf{q}_i}{(2\pi)^3 2\omega_i}(-\mathbf{T}_j\cdot\mathbf{T}_k)\frac{p_j\cdot p_k}{p_j\cdot p_i\,,p_k\cdot p_i}$$

$$= \frac{\alpha_s}{2\pi} d\sigma_n \frac{d\omega_i}{\omega_i}\frac{d\Omega_i}{2\pi}(-\mathbf{T}_j\cdot\mathbf{T}_k)\frac{\xi_{jk}}{\xi_{ij}\,\xi_{ik}}$$

where $\xi_{jk} = 1-\cos\theta_{jk}$. Now

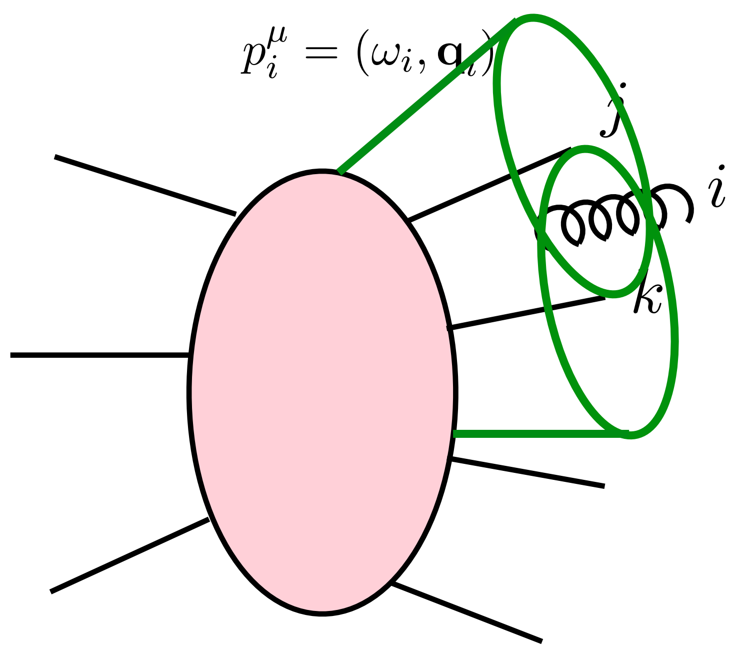
$$\frac{d\Omega_i}{2\pi}\frac{\xi_{jk}}{\xi_{ij}\,\xi_{ik}} = \frac{d\xi_{ij}}{\xi_{ij}}\frac{d\phi_{ij}}{2\pi}\frac{1}{2}\left(\frac{\xi_{jk}-\xi_{ij}}{\xi_{ik}}+1\right)+(j\leftrightarrow k)$$

- After azimuthal integration, this is exactly

$$\frac{d\xi_{ij}}{\xi_{ij}}\Theta(\xi_{jk}-\xi_{ij})+\frac{d\xi_{ik}}{\xi_{ik}}\Theta(\xi_{jk}-\xi_{ik})$$

- Each parton j,k radiates into cone θ_{ij} , $\theta_{ik} < \theta_{jk}$

AO parton shower



- Coherent emission from (jk)

$$
\begin{aligned}
d\sigma_{n+1}^{(jk)} &= g_s^2 d\sigma_n \frac{d^3\mathbf{q}_i}{(2\pi)^3 2\omega_i} (-\mathbf{T}_j \cdot \mathbf{T}_k) \frac{p_j \cdot p_k}{p_j \cdot p_i \, , p_k \cdot p_i} \\
&= \frac{\alpha_s}{2\pi} d\sigma_n \frac{d\omega_i}{\omega_i} \frac{d\Omega_i}{2\pi} (-\mathbf{T}_j \cdot \mathbf{T}_k) \frac{\xi_{jk}}{\xi_{ij}\, \xi_{ik}}
\end{aligned}
$$

where $\xi_{jk} = 1 - \cos\theta_{jk}$. Now

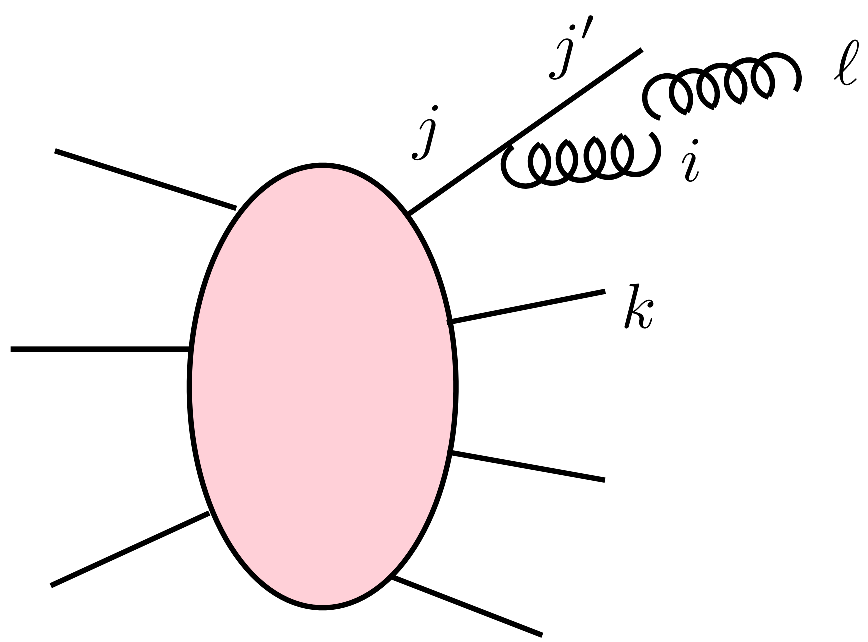
$$
\frac{d\Omega_i}{2\pi} \frac{\xi_{jk}}{\xi_{ij}\, \xi_{ik}} = \frac{d\xi_{ij}}{\xi_{ij}} \frac{d\phi_{ij}}{2\pi} \frac{1}{2} \left(\frac{\xi_{jk} - \xi_{ij}}{\xi_{ik}} + 1 \right) + (j \leftrightarrow k)
$$

- After azimuthal integration, this is exactly

$$
\frac{d\xi_{ij}}{\xi_{ij}} \Theta(\xi_{jk} - \xi_{ij}) + \frac{d\xi_{ik}}{\xi_{ik}} \Theta(\xi_{jk} - \xi_{ik})
$$

- Each parton j radiates into cone $\theta_{ij} < \theta_{jk}$

AO parton shower



- Two gluon emission

$$d\sigma_{n+2}^{(ij)} = \frac{\alpha_s}{\pi} d\sigma_{n+1}^{(j)} \frac{d\omega_\ell}{\omega_\ell} \Bigg\{ (-\mathbf{T}_i \cdot \mathbf{T}'_j) \int^{\xi_{ij}} \left(\frac{d\xi_{\ell i}}{\xi_{\ell i}} + \frac{d\xi_{\ell j}}{\xi_{\ell j}} \right) + \sum_{k\neq i,j} \left[(-\mathbf{T}_i \cdot \mathbf{T}_k) \int^{\xi_{ik}} \frac{d\xi_{\ell i}}{\xi_{\ell i}} + (-\mathbf{T}'_j \cdot \mathbf{T}_k) \int^{\xi_{jk}} \frac{d\xi_{\ell j}}{\xi_{\ell j}} \right] \Bigg\}$$

where $\mathbf{T}'_j = \mathbf{T}_j - \mathbf{T}_i$ and $\mathbf{T}_i + \mathbf{T}'_j + \sum_{k\neq i,j} \mathbf{T}_k = 0$

- Collecting terms in $\xi_{\ell i}$ and $\xi_{\ell j}$, we find

$$d\sigma_{n+2}^{(ij)} = \frac{\alpha_s}{\pi} d\sigma_{n+1}^{(j)} \frac{d\omega_\ell}{\omega_\ell} \Bigg\{ \mathbf{T}_i \cdot \mathbf{T}_i \int^{\xi_{ij}} \frac{d\xi_{\ell i}}{\xi_{\ell i}} + \mathbf{T}'_j \cdot \mathbf{T}'_j \int^{\xi_{ij}} \frac{d\xi_{\ell j}}{\xi_{\ell j}} - \mathbf{T}_j \cdot \sum_{k\neq i,j} \mathbf{T}_k \int_{\xi_{ij}}^{\xi_{jk}} \frac{d\xi_{\ell j}}{\xi_{\ell j}} - \mathbf{T}_i \cdot \sum_{k\neq i,j} \mathbf{T}_k \left(\int_{\xi_{ij}}^{\xi_{ik}} \frac{d\xi_{\ell i}}{\xi_{\ell i}} - \int_{\xi_{ij}}^{\xi_{jk}} \frac{d\xi_{\ell j}}{\xi_{\ell j}} \right) \Bigg\}.$$

each parton emits into its cone

coherent emission outside cones, done in previous step

non-singular, 2 logs down, neglected

$e^+e^- \to q\bar{q}gg$

- AO parton shower:

$$d\sigma^{(ij)}_{n+2} = \frac{\alpha_S}{\pi} d\sigma^{(j)}_{n+1} \frac{d\omega_\ell}{\omega_\ell} \left\{ \mathbf{T}_i \cdot \mathbf{T}_i \int^{\xi_{ij}} \frac{d\xi_{\ell i}}{\xi_{\ell i}} + \mathbf{T}'_j \cdot \mathbf{T}'_j \int^{\xi_{ij}} \frac{d\xi_{\ell j}}{\xi_{\ell j}} - \mathbf{T}_j \cdot \sum_{k \neq i,j} \mathbf{T}_k \int_{\xi_{ij}}^{\xi_{jk}} \frac{d\xi_{\ell j}}{\xi_{\ell j}} \right\}$$

$$d\sigma^{(iq)}_4 = \frac{\alpha_S}{\pi} d\sigma^{(q)}_3 \frac{d\omega_\ell}{\omega_\ell} \left\{ C_A \int^{\xi_{iq}} \frac{d\xi_{\ell i}}{\xi_{\ell i}} + C_F \int^{\xi_{q\bar{q}}} \frac{d\xi_{\ell q}}{\xi_{\ell q}} \right\}$$ where $d\sigma^{(q)}_3 = \frac{\alpha_S}{\pi} \sigma_2 \, C_F \frac{d\omega_i}{\omega_i} \int^{\xi_{q\bar{q}}} \frac{d\xi_{iq}}{\xi_{iq}}$

- 4-jet rate (k_t-algorithm):

$$y_{\ell q} = 2\omega_\ell^2 \xi_{\ell q}/Q^2 > y_c \,, \quad y_{\ell i} = 2\omega_m^2 \xi_{\ell i}/Q^2 > y_c$$ where $\omega_m = \min\{\omega_\ell, \omega_i\}$

$$\sigma^{(iq)}_4 = \left(\frac{\alpha_S}{\pi}\right)^2 \sigma_2 \, C_F \int_{Q\sqrt{y_c}/2}^{Q/2} \frac{d\omega_i}{\omega_i} \int_{Q\sqrt{y_c}/2}^{Q/2} \frac{d\omega_\ell}{\omega_\ell} \int_{Q^2 y_c/2\omega_i^2}^{2} \frac{d\xi_{iq}}{\xi_{iq}} \left\{ C_A \ln\left(\frac{2\omega_m^2 \xi_{iq}}{Q^2 y_c}\right) + C_F \ln\left(\frac{4\omega_\ell^2}{Q^2 y_c}\right) \right\}$$

$$R_4 = (\sigma^{(iq)}_4 + \sigma^{(i\bar{q})}_4)/\sigma_2 = \left(\frac{\alpha_s}{\pi}\right)^2 C_F \left(\frac{1}{8} C_F + \frac{1}{48} C_A\right) L^4$$ where $L = \log(1/y_c)$

$e^+e^- \to q\bar{q}gg$

- 4-jet rate (k_t-algorithm) vs $L = \log(1/y_{\rm cut})$

$$\frac{1}{\sigma_{\rm B}}\frac{d\sigma_4}{dL} = \left[\frac{\alpha_{\rm S}(s)}{\pi}\right]^2 (4AL^3 + 3BL^2 + 2CL + \ldots)$$

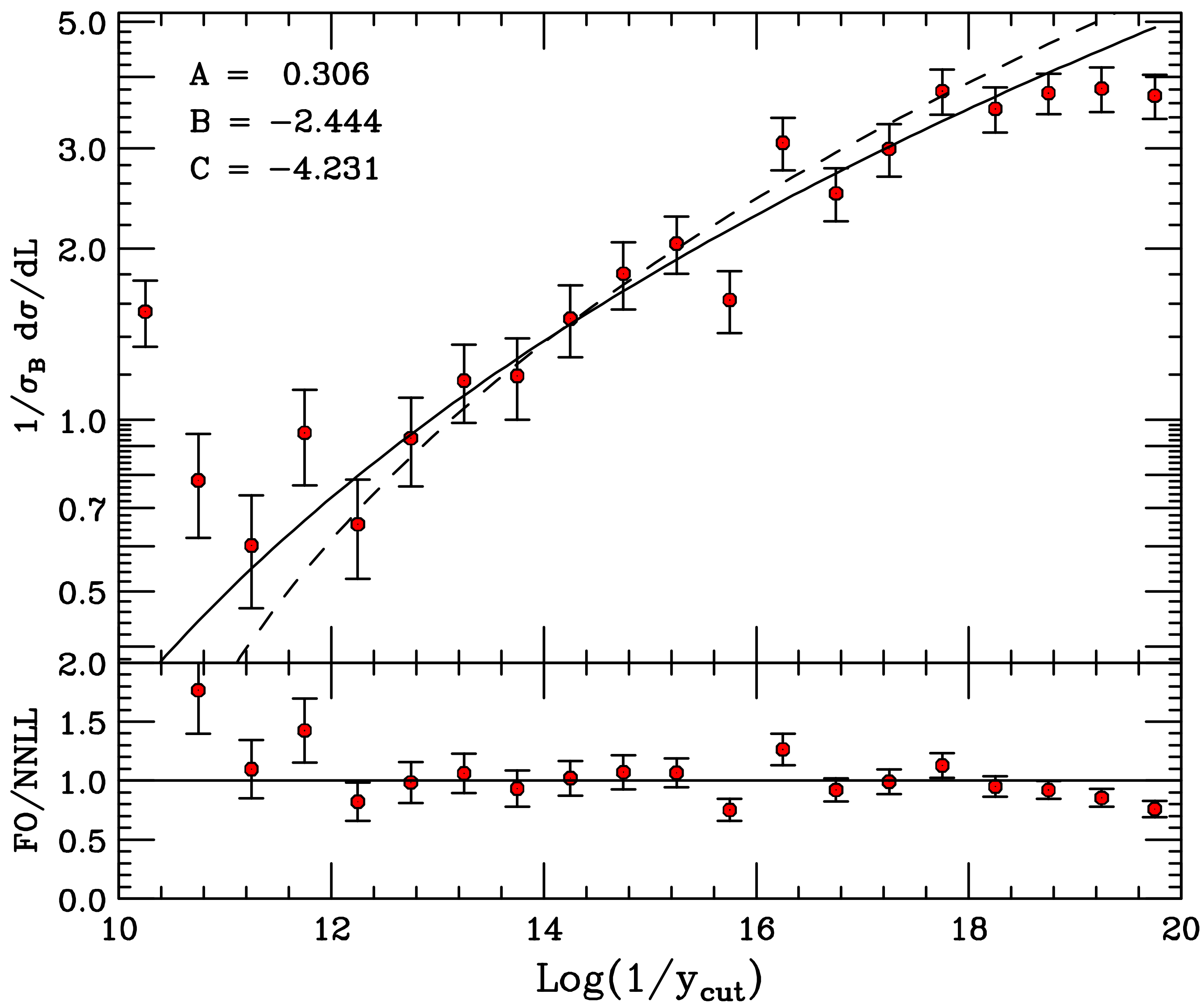
$$A = C_F^2/8 + C_F C_A/48$$

$$B = -3C_F^2/4 - 5C_F C_A/18$$

$C = ??$ (fitted)

- Compare with MadGraph5 at 1 TeV
  - $M_{ij} > 100$ MeV → $L < 18.4$

$e^+e^- \to q\bar{q}gg$



- Dashed is LCA (refitting C)

$e^+e^- \to q\bar{q}ggg$

- 5-jet rate (k_t-algorithm) vs $L = \log(1/y_{\rm cut})$

$$\frac{1}{\sigma_{\rm B}}\frac{d\sigma_5}{dL} = \left[\frac{\alpha_{\rm S}(s)}{\pi}\right]^3 (6AL^5 + 5BL^4 + 4CL^3 + \ldots)$$

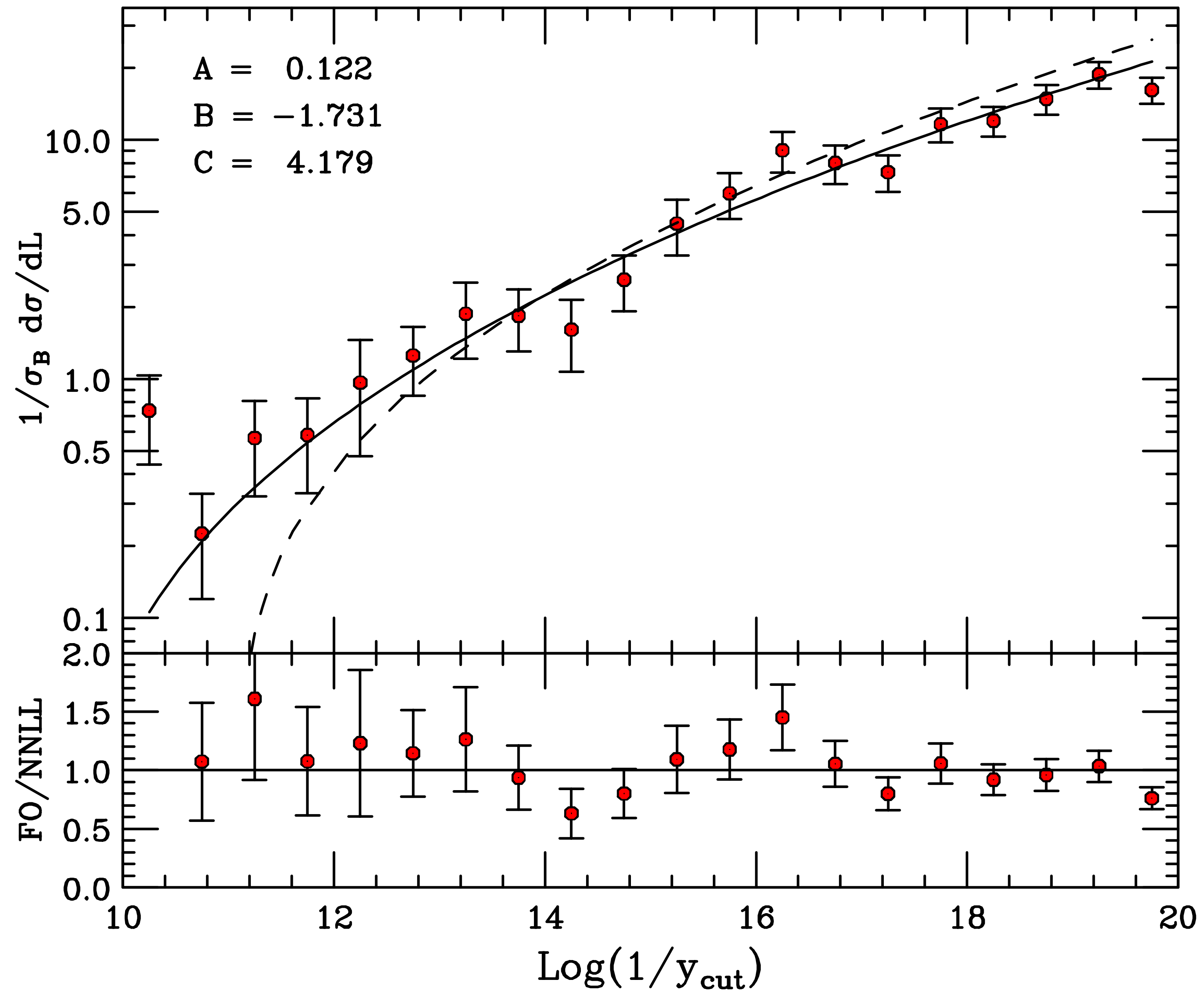
$$A = C_F^3/48 + C_F^2 C_A/96 + C_F C_A^2/720$$

$$B = -3C_F^3/16 - 49C_F^2 C_A/288 - 91C_F C_A^2/2880$$

$C = ??$ (fitted)

- Compare with MadGraph5 at 1 TeV again

$e^+e^- \to q\bar{q}ggg$



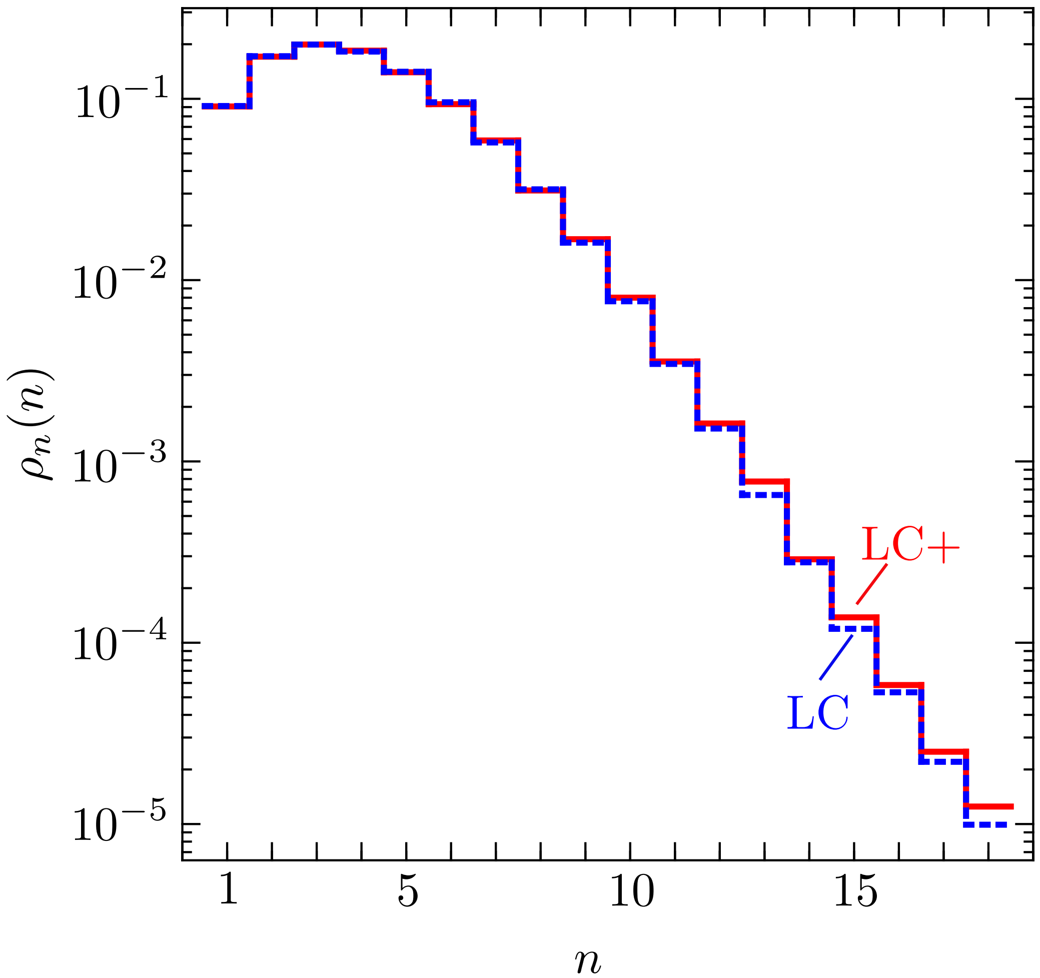
- Dashed is LCA (refitting C)

Subleading colour

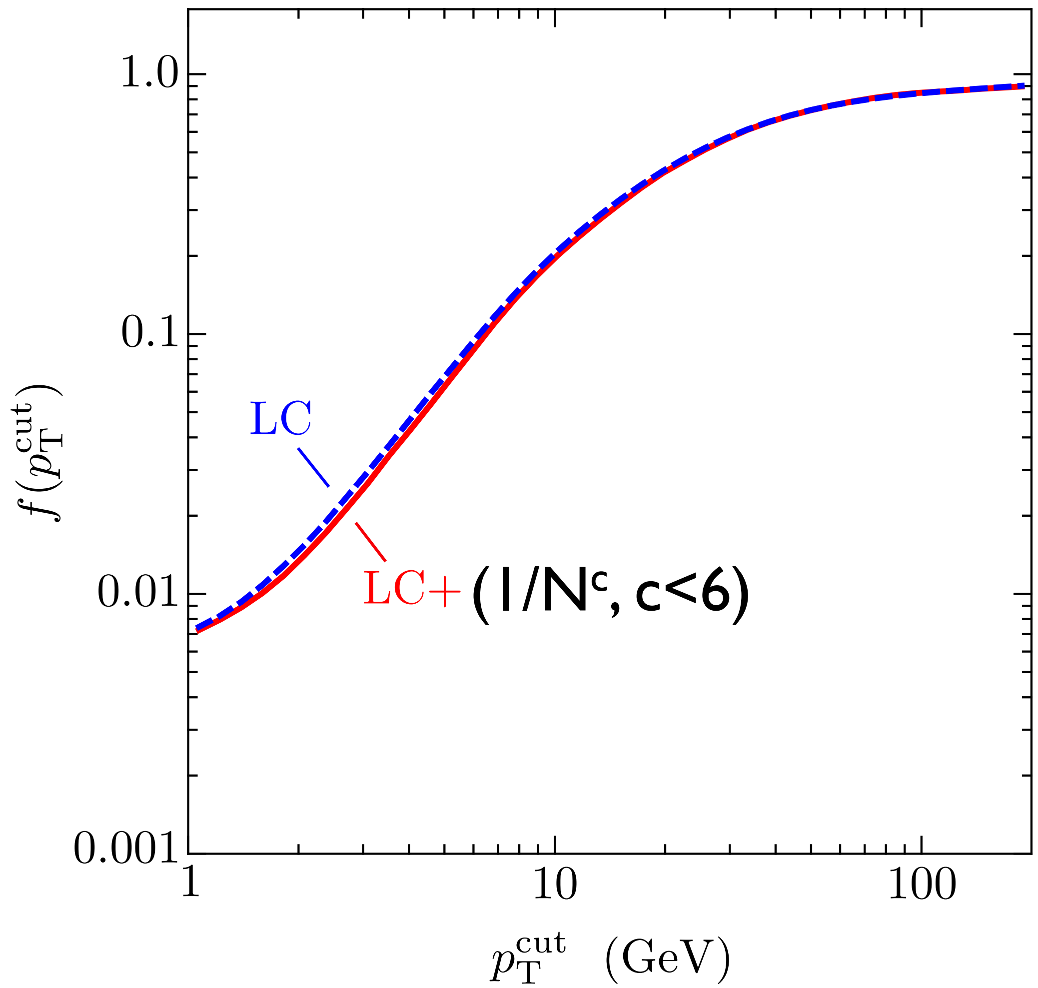
- DEDUCTOR pp at 14 TeV

Nagy, Soper, 1202.4496,1501.00778

- Number of partons (p_{Tjet}>200 GeV, k_{tmin}=1GeV)



- Gap fraction (|y|<2)



Subleading colour

- q q→q q evolution matrix elements Plätzer, 1312.2448

Diagonal

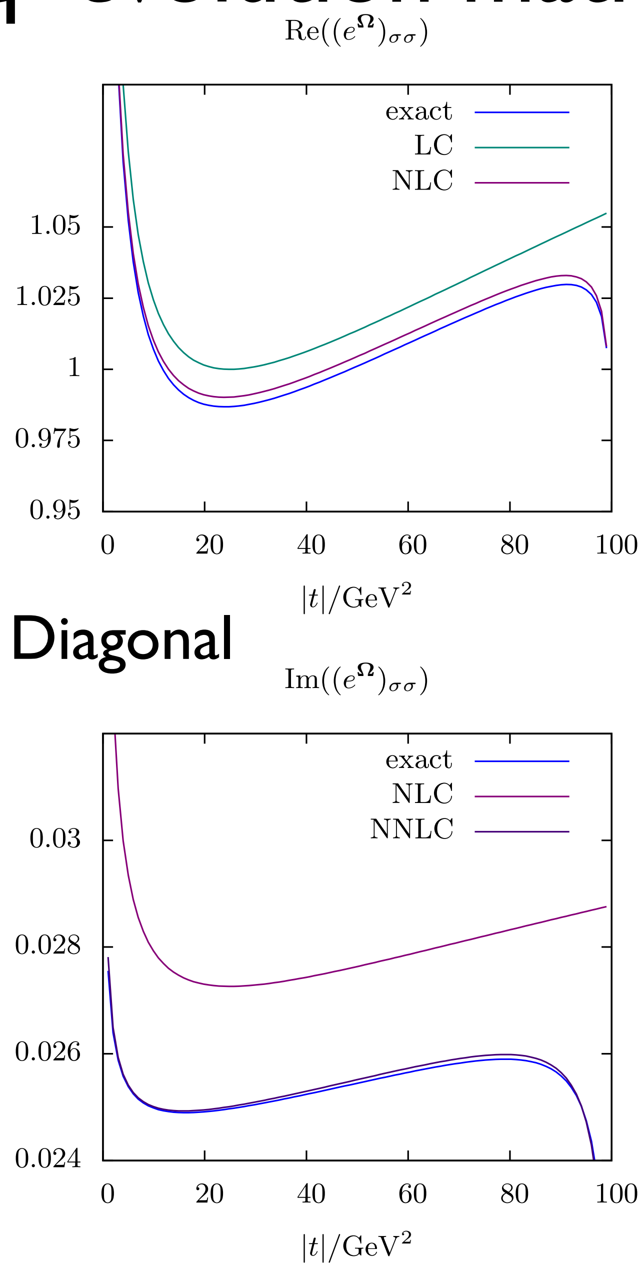


Fig. 3. Real and imaginary parts of a diagonal evolution matrix element for quark-quark scattering at $s = 100\ \mathrm{GeV}^2$, $\mu^2 = 25\ \mathrm{GeV}^2$ as a function of the momentum transfer $|t|$, comparing the exact results to various approximations. This matrix elements describes the amplitude to keep a t-channel colour flow σ.

Off-diagonal

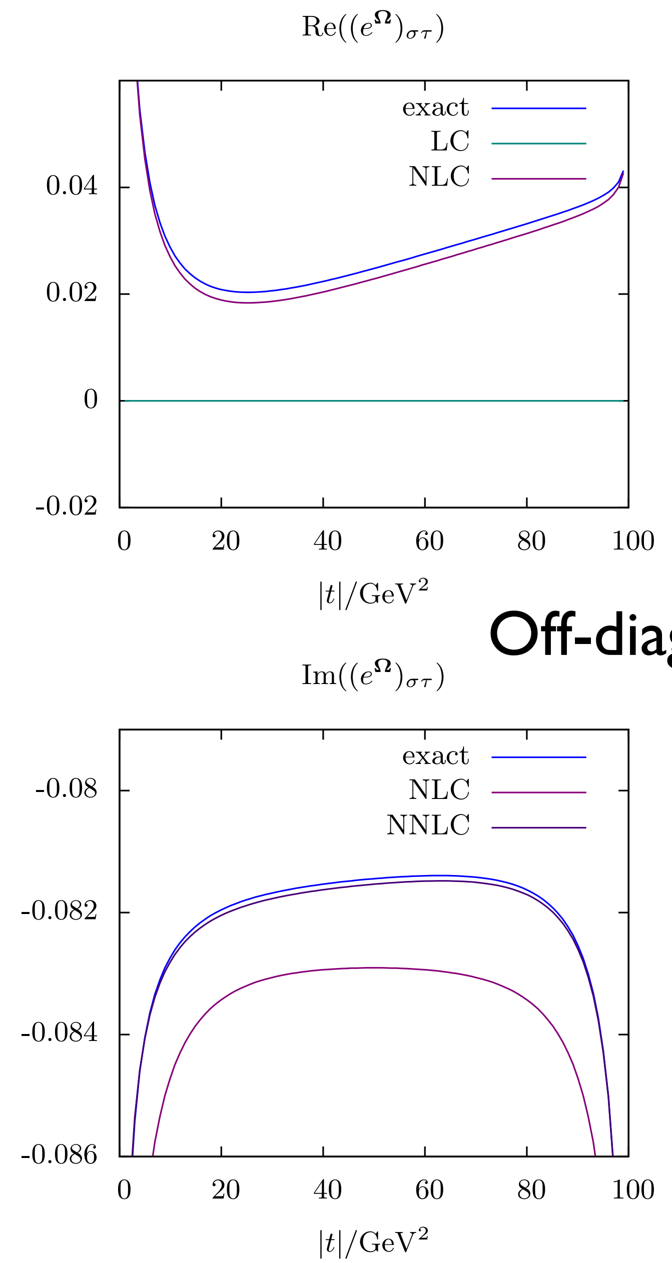
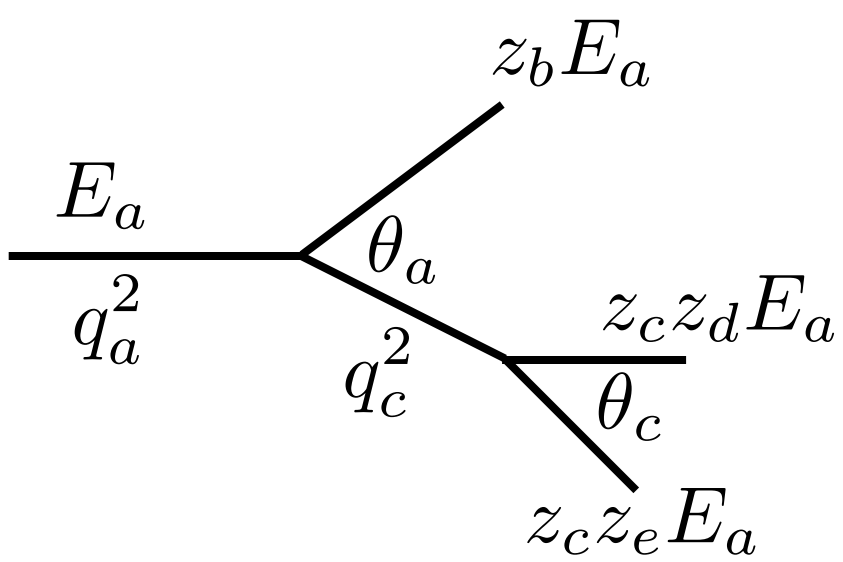


Fig. 4. Same as figure 3 for an off-diagonal matrix element. The matrix element considered describes the transition from a u-channel colour flow τ to a t-channel one, σ.

Shower ordering

- Virtuality-ordered shower

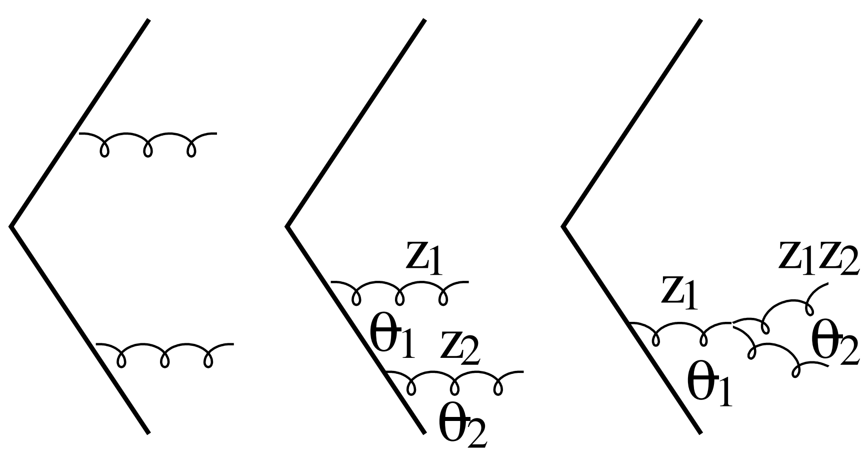
$$q_a^2 = \frac{q_b^2}{z_b} + \frac{q_c^2}{z_c} + \frac{q_T^2}{z_b z_c}$$



$$q_T \simeq z_b z_c E_a \theta_a \qquad q_a^2 \simeq z_b z_c (E_a \theta_a)^2$$

$$q_c^2 \simeq z_c^2 z_d z_e (E_a \theta_c)^2 < z_c q_a^2 \simeq z_c^2 z_b (E_a \theta_a)^2$$

$$\boxed{z_d z_e \theta_c^2 < z_b \theta_a^2}$$



(a) (b) (c)

(b) $z_2\theta_2^2 < z_1\theta_1^2$ → same as AO

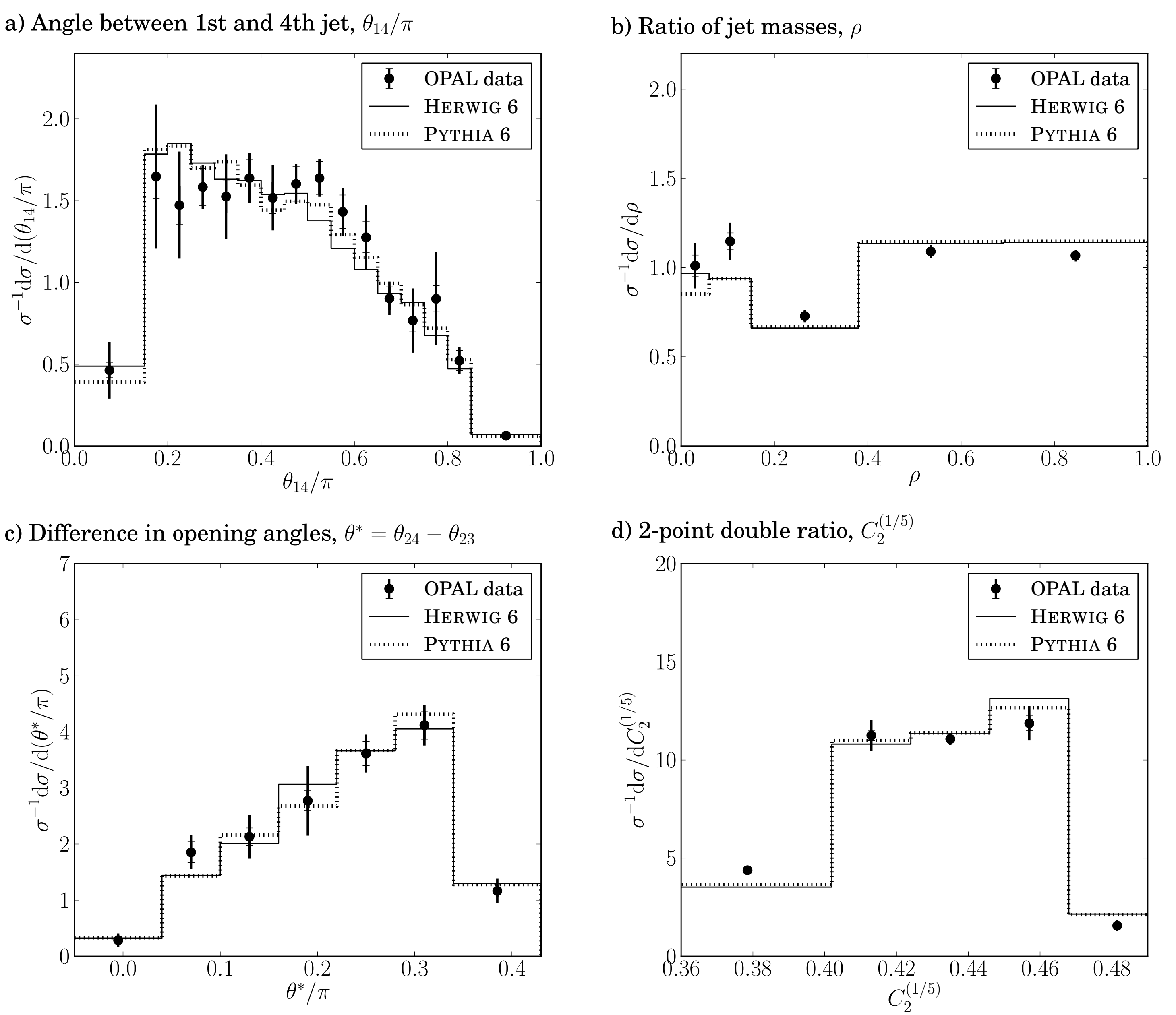
(c) $z_2\theta_2^2 < \theta_1^2$ → larger than AO

$$\rightarrow R_4^{q^2} = \left(\frac{\alpha_S}{\pi}\right)^2 \frac{C_F}{8}\left(C_F + \frac{1}{4}C_A\right) L^4 \quad \leftarrow \frac{25}{12}$$

$$\text{vs } R_4^{AO} = \left(\frac{\alpha_S}{\pi}\right)^2 \frac{C_F}{8}\left(C_F + \frac{1}{6}C_A\right) L^4 \quad \leftarrow \frac{22}{12}$$

Coherence tests

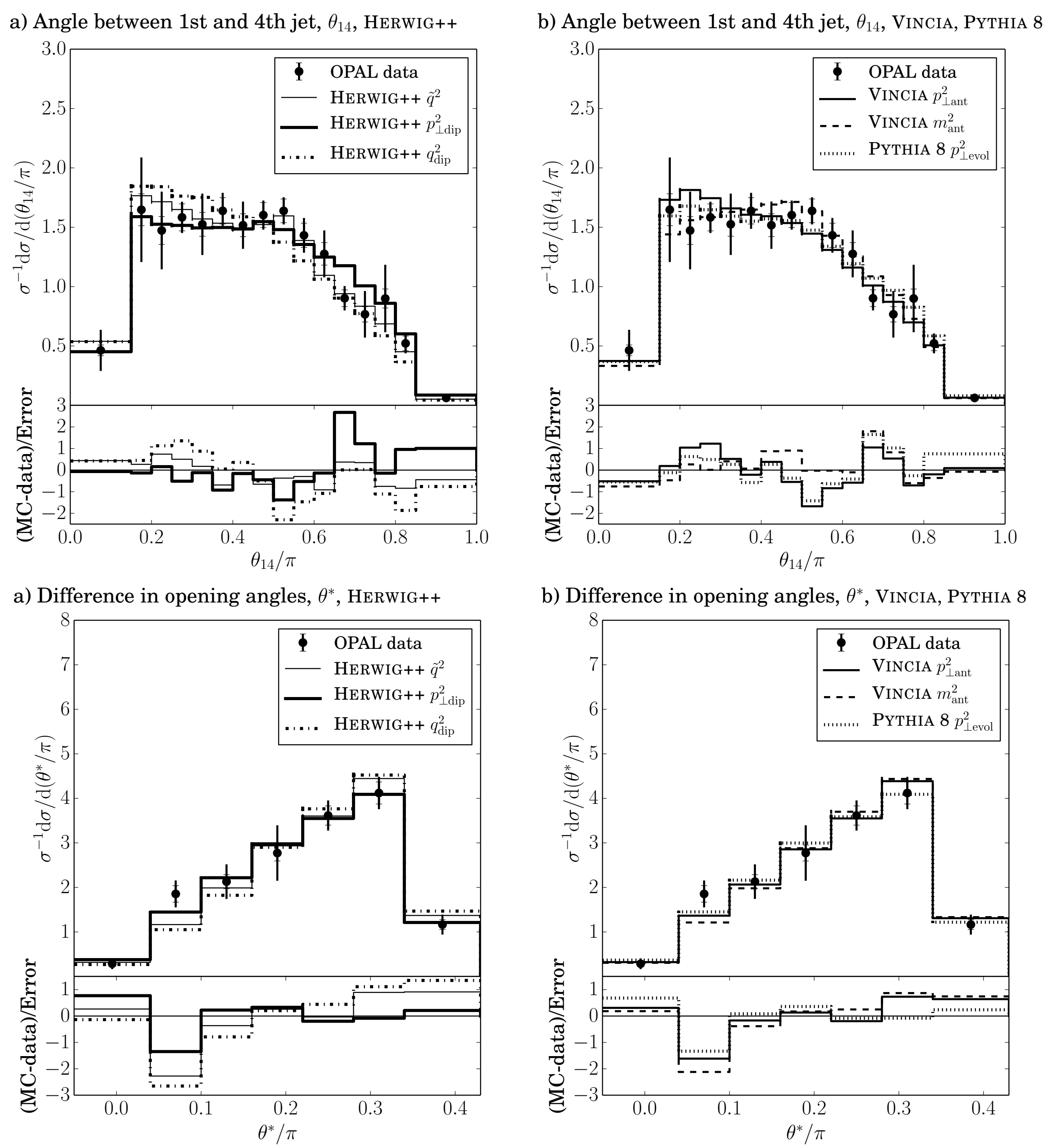
- $Z^0 \to$ 4 jets (LEP OPAL data) Fischer et al., 1505.01636



- Herwig 6, Pythia 6 comparisons

Coherence tests

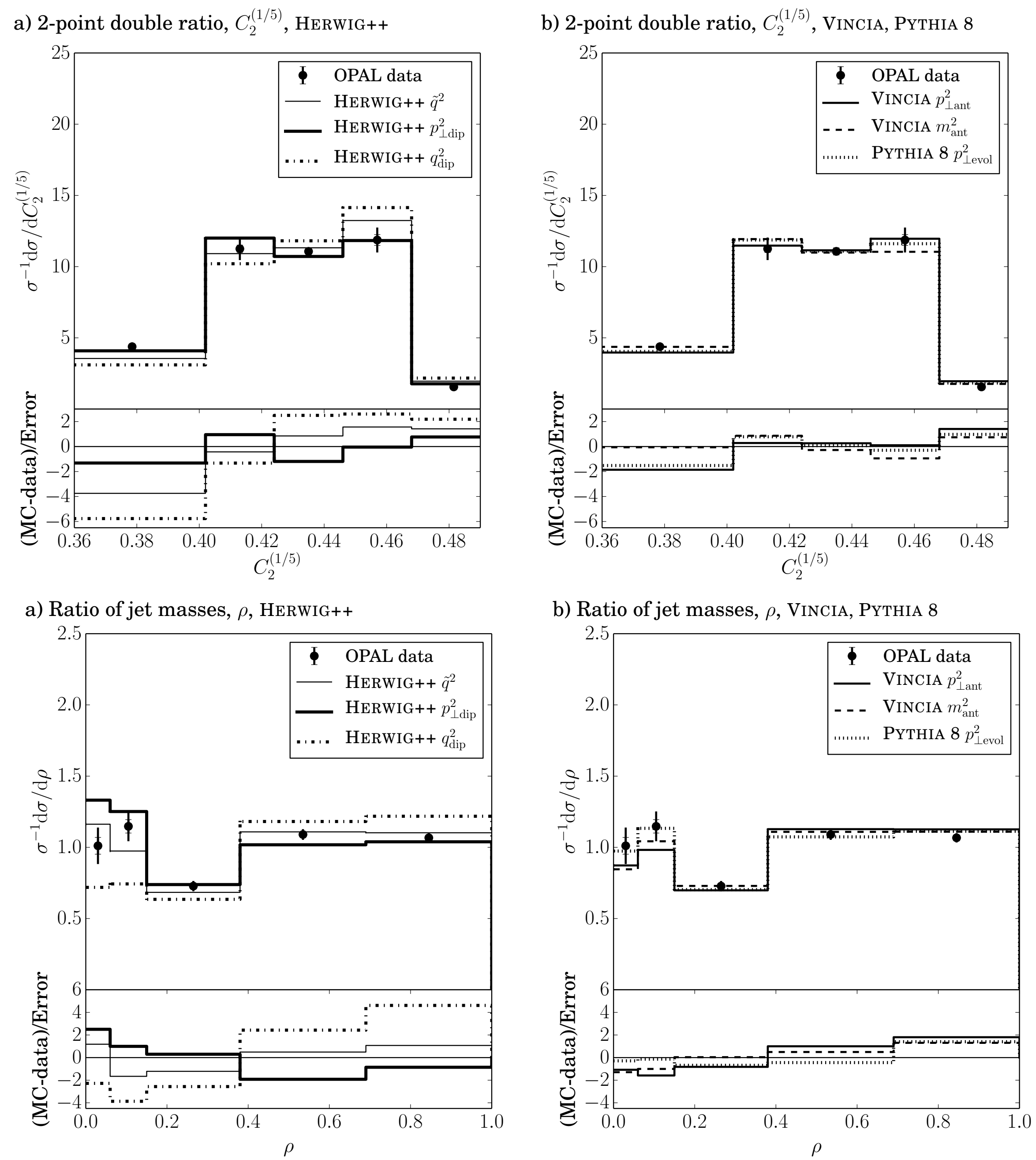
- $Z^0 \to$ 4 jets (LEP OPAL data) Fischer et al., 1505.01636



- Herwig++, Pythia 8 comparisons

Coherence tests

- $Z^0 \rightarrow$ 4 jets (LEP OPAL data) Fischer et al., 1505.01636



- Herwig++ virtuality ordering does worst

Spin in showers

$$< s|\hat{P}_{qq}(z,k_\perp;\epsilon)|s' >= \delta_{ss'}\, C_F\, \left[\frac{1+z^2}{1-z} - \epsilon(1-z)\right] \quad ,$$

$$< s|\hat{P}_{qg}(z,k_\perp;\epsilon)|s' >= \delta_{ss'}\, C_F\, \left[\frac{1+(1-z)^2}{z} - \epsilon z\right] \quad ,$$

$$< \mu|\hat{P}_{gq}(z,k_\perp;\epsilon)|\nu >= T_R \left[-g^{\mu\nu} + 4z(1-z)\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right] \quad ,$$

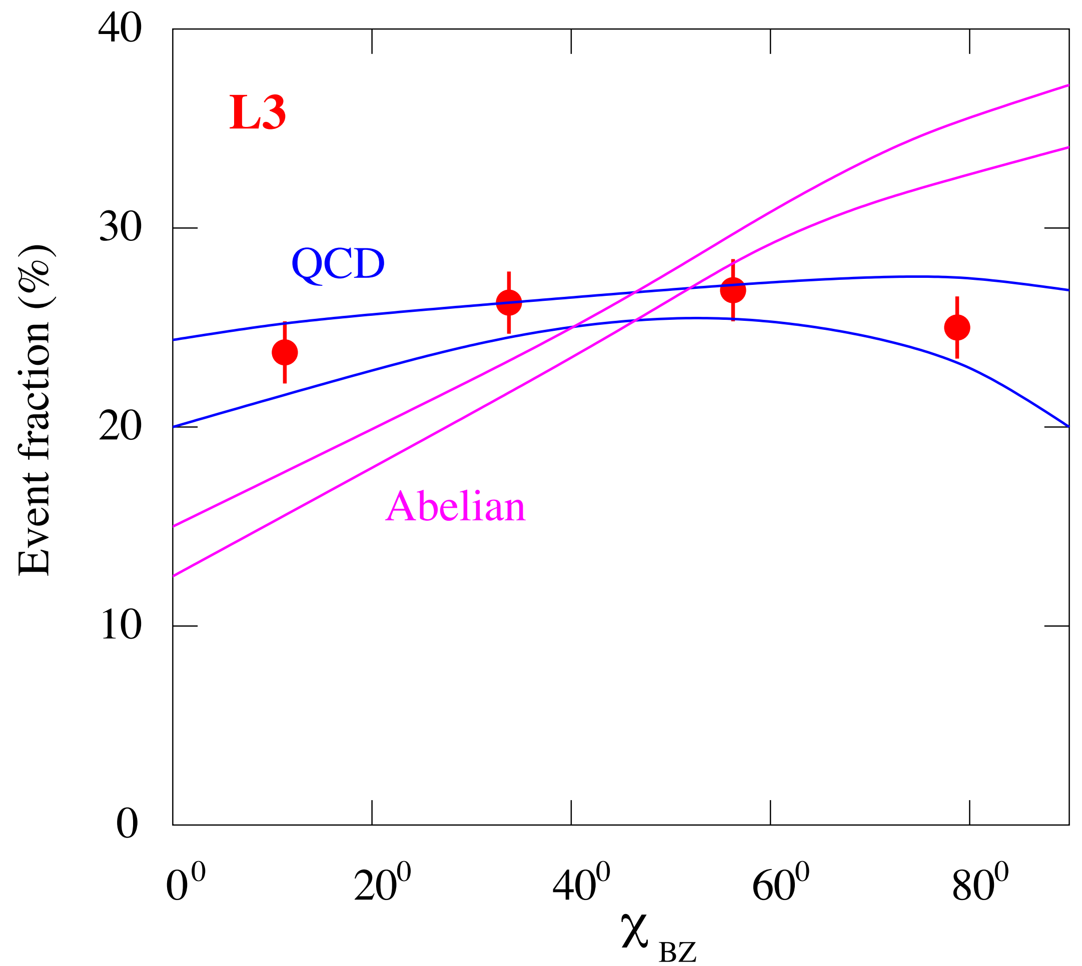
$$< \mu|\hat{P}_{gg}(z,k_\perp;\epsilon)|\nu >= 2C_A \left[-g^{\mu\nu}\left(\frac{z}{1-z} + \frac{1-z}{z}\right) - 2(1-\epsilon)z(1-z)\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right]$$

- No effect in q→qg (helicity conservation)
- Opposite in g→gg and g→q$\bar{q}$
  - Cancel when $N_f = N_c$

Spin in showers

- Bengtsson-Zerwas angle in $e^+e^- \to 4$ jets

$$\cos\chi_{BZ} = \frac{(\boldsymbol{p}_1 \times \boldsymbol{p}_2)\cdot(\boldsymbol{p}_3 \times \boldsymbol{p}_4)}{|\boldsymbol{p}_1 \times \boldsymbol{p}_2|\,|\boldsymbol{p}_3 \times \boldsymbol{p}_4|}\ .$$



- Abelian theory has only $g \to q\bar{q}$

Conclusions

- Matching at NLO
  - SM processes automated, EW and BSM soon
- Matching at NNLO
  - So far only DY & H, others much harder
- Merging at NLO
  - Still in a state of flux; FxFx automated
- Parton showers
  - Coherence effects visible
  - Spin and subleading colour effects small
  - Hadronization important, but little effort/progress

Thanks for listening!