A study of microjets

Parton Showers, Event Generators and Resummation, 28 May 2015

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based on (arXiv:1411.5182) and work in preparation

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Outline

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   - Perturbative properties of jets

2. Method

3. Observables
   - Filtering
   - Microjet vetoes
   - Inclusive jet spectrum

4. Conclusion

INTRODUCTION

What is a jet algorithm ?

Jets are collimated bunches of particles produced by hadronization of a quark or gluon.

A jet algorithm maps final state particle momenta to jet momenta.

$$\underbrace{\{p_i\}}_{\text{particles}} \Longrightarrow \underbrace{\{j_k\}}_{\text{jets}}$$

This requires an external parameter, the jet radius R, specifying up to which point separate partons are recombined into a single jet.

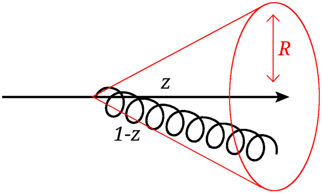


Figure – Gluon emission and emitting quark combined into a single jet.

Generalised k_t algorithm with incoming hadrons

Basic idea is to invert QCD branching process, clustering pairs which are closest in metric defined by the divergence structure of the theory.

Definition

1. For any pair of particles i, j find the minimum of

$$d_{ij} = \min\{k_{ti}^{2p}, k_{tj}^{2p}\} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = k_{ti}^{2p}, \quad d_{jB} = k_{tj}^{2p}$$

   where $\Delta R_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$.

2. If the minimum distance is d_{iB} or d_{jB}, then the corresponding particle is removed from the list and defined as a jet, otherwise i and j are merged.
3. Repeat until no particles are left.

The index p defines the specific algorithm, with $p = \pm 1, 0$.

Choice of jet radius

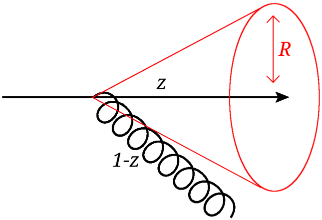
In recent years, jet radii have become ever smaller.

What are usual values for the jet radius R ?

- Most common choice is $R = 0.4 - 0.5$.
- In some environments (eg. heavy ions), values down to $R = 0.2$ are used to mitigate high pileup and underlying event contamination.
- Many modern jet tools (eg. trimming and filtering) resolve small subjets (typically with $R_{\text{sub}} = 0.2 - 0.3$) within moderate & large R jets.

Perturbative properties of jets

Jet properties will be affected by gluon radiation and $g \to q\bar{q}$ splitting.



Emissions outside of the jet reduce the jet energy.

We will try to investigate the effects of perturbative radiation on a jet analytically, in the small radius limit.

Perturbative properties: parton energy ≠ jet energy

Average energy difference between hardest final state jet and initial quark, considering emissions beyond the reach of the jet

$$\left\langle \frac{\text{quark } E - \text{jet } E}{\text{quark } E} \right\rangle = \int^{\mathcal{O}(1)} \frac{d\theta^2}{\theta^2} \int dz(\max[z, 1-z] - 1) \times \frac{\alpha_s}{2\pi} p_{qq}(z) \Theta(\theta - R)$$

$$= \frac{C_F}{\pi} \left(2 \ln 2 - \frac{3}{8}\right) \alpha_s \ln R + \ldots$$

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$\alpha_s \ln R$ implies large corrections for small R.

How relevant are small-R effects?

Energy loss has big effect on jet spectrum.

R	correction to jet spectrum
0.4	$\mathcal{O}(-25\%)$
0.2	$\mathcal{O}(-50\%)$

"In the small R limit, new clustering logarithms [of R ...] arise at each order and cannot currently be resummed."

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We aim to resum all leading logarithmic $(\alpha_s \ln R)^n$ terms in the limit of small R for a wide variety of observables.

METHOD

Evolution variable t

Use an evolution variable t corresponding to the integral over the angular emission probability weighted with α_s

$$t = \int_{R^2}^{1} \frac{d\theta^2}{\theta^2} \frac{\alpha_s(p_t\theta)}{2\pi} \sim \frac{\alpha_s}{2\pi} \ln \frac{1}{R^2}$$

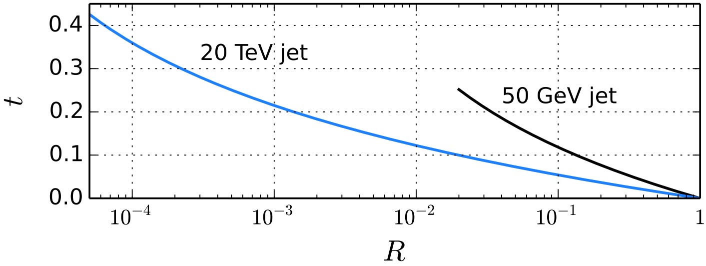


Figure – Plot of t as a function of R down to $Rp_t = 1\,\mathrm{GeV}$.

Generating functionals for microjets

Definition

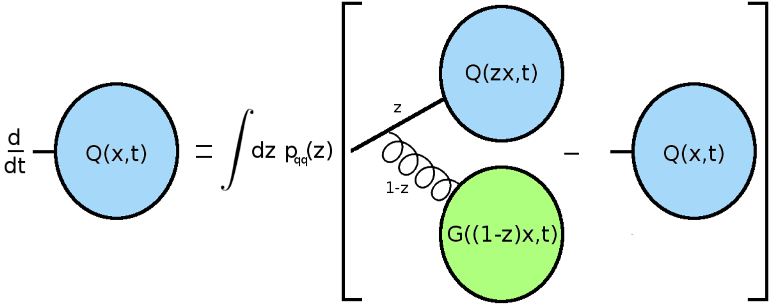
Quark generating functional $Q(x,t)$ encodes parton content observed when resolving an initial quark with momentum fraction x on an angular scale $t > 0$. (ie. $R < 1$).

We can formulate the evolution equation for the quark generating functional from

$$Q(x,t) = Q(x,t-\delta_t)\left(1-\delta t\int dz\,p_{qq}(z)\right) + \delta_t\int dz\,p_{qq}(z)\Big[Q(zx,t-\delta_t)G((1-z)x,t-\delta_t)\Big].$$

Gluon generating functional $G(x,t)$ defined the same way.

Evolution equation for the quark generating functional can be rewritten as a differential equation.

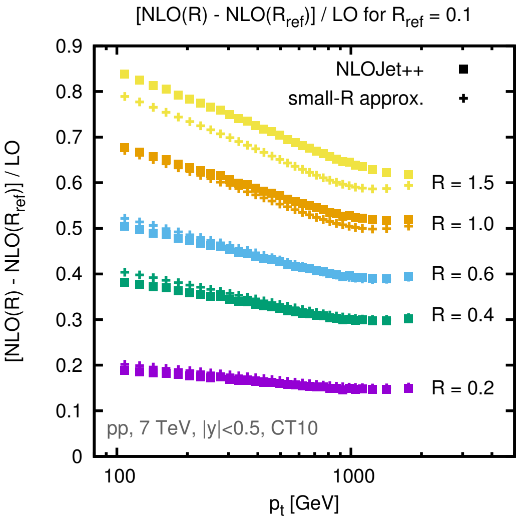


Blobs represent the generating functionals at a scale t.

A similar equation can be obtained for the gluon evolution.

These equations allow us to resum observables to all orders numerically. They effectively exploit angular ordering.

Small-R is a valid for $R \leq 1$.



Compare inclusive spectrum from NLOjet++ with small-R approximation

Agreement of squares and crosses indicates that the small-R approximation is good.

OBSERVABLES

Fragmentation functions

Two broad classes of observables:

- Inclusive microjet observables obtained from the inclusive microjet fragmentation function $f_{j/i}^{\text{incl}}(z,t)$.
  In this case, momentum conservation translates to

$$\sum_j \int dz\, z\, f_{j/i}^{\text{incl}}(z,t) = 1.$$

- Hardest microjet observables obtained from the momentum distribution of the hardest microjet $f^{\text{hardest}}(z,t)$.
  Here we have a probability sum rule

$$\int dz\, f^{\text{hardest}}(z,t) = 1.$$

Observables

We computed small-R effects for a number of observables

- Filtering (= keep n_{filt} hardest subjets).
- Jet vetoes in H & Z production.
- Inclusive jet spectrum (resummation is DGLAP-like).
- Jet flavour (eg. hardest microjet flavour-change probability).
- Trimming (= keep subjets with $p_t^{\text{subjet}} > f_{\text{cut}}\, p_t^{\text{jet}}$)

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Filtering

Definition

Reclustering of a jet on a smaller angular scale $R_{\text{sub}} < R$, discarding all but the n_{filt} hardest subjets.

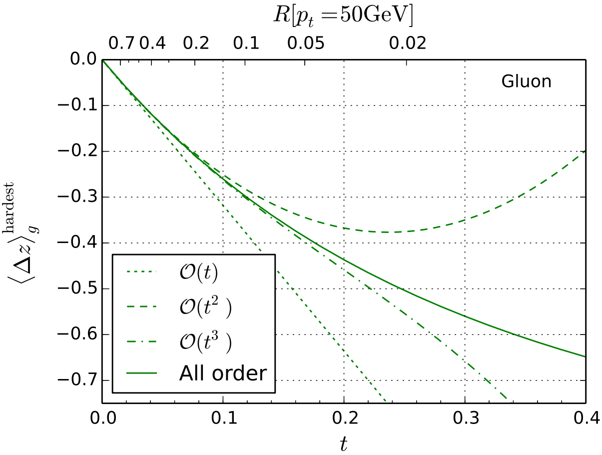
Define $f^{k\text{-hardest}}(z)$ the probability that the k-th hardest subjet carries a momentum fraction z of the initial parton.

We can express the energy loss between the filtered jet and the initial parton as

$$\langle \Delta z \rangle^{\text{filt},n} = \left[\sum_{k=1}^{n} \int dz\, z\, f^{k\text{-hardest}}(z) \right] - 1.$$

Filtering

We can observe that the filtered jet retains more of the original parton momentum compared to the momentum of the hardest microjet.



Average jet energy loss Δz after filtering with $n_{\text{filt}} = 1$.

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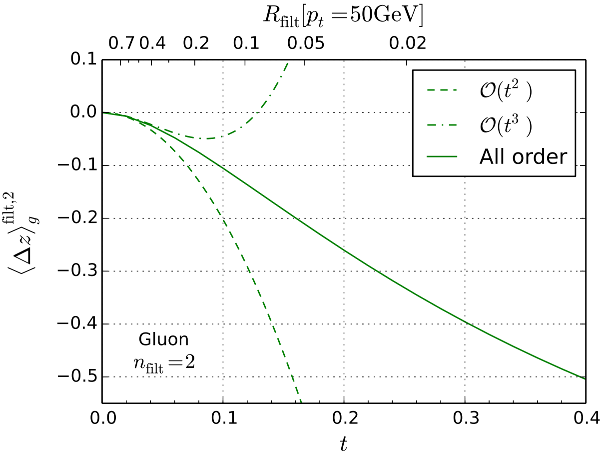
Convergence of the power series is extremely slow.

Resummation essential to get a stable answer.



Average jet energy loss Δz after filtering with $n_{\rm filt} = 2$.

Microjet vetoes

Jet veto resummation for Higgs production has terms

$$\alpha_s^m \ln^{2m} \frac{Q}{p_t} + \text{subleading}$$

Among the subleading terms there are small-R enhanced terms

$$\alpha_s^{m+n} \ln^m \frac{Q}{p_t} \ln^n \frac{1}{R^2} + \dots$$

Suspected of having important impact, and calculated by several groups

NNLL jet vetoes	$n = 1$	[Banfi, Monni, Salam, Zanderighi PRL 109 (2012) 202001]
		+ [Becher, Neubert, Rothen JHEP 1301 (2013) 125]
		+ [Stewart, Tackmann, Walsh, Zuberi PRD 89 (2014) 054001]
Alioli & Walsh	$n = 2$ (numerically)	[JHEP 1403 (2014) 119, corr. in arXiv-v3]
Our work	$n = 2$ (analytically)	[JHEP 1504 (2015) 039]
	+ $n \to \infty$ (numerically)	

Microjet vetoes

Writing the probability of no gluon emissions above a scale p_t as

$$P(\text{no primary-parton veto}) = \exp\left[-\int_{p_t}^{Q} \frac{dk_t}{k_t}\bar{\alpha}_s(k_t) 2\ln\frac{Q}{k_t}\right],$$

one can show that including small-R corrections and applying the veto on the hardest microjet, we have

$$\begin{aligned}\mathcal{U} &\equiv P(\text{no microjet veto})/P(\text{no primary-parton veto})\\ &= \exp\left[-2\bar{\alpha}_s(p_t)\ln\frac{Q}{p_t}\int_0^1 dz\, f^{\text{hardest}}(z, t(R,p_t))\ln z\right].\end{aligned}$$

The R-dependent correction generates a series of terms

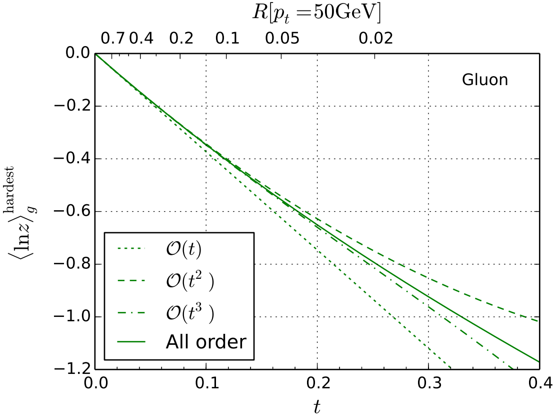
$$\alpha_s^{m+n}(Q)\ln^m(Q/p_t)\ln^n R\,.$$

Logarithmic moment $\langle \ln z \rangle$

The logarithmic moment of f^{hardest} is

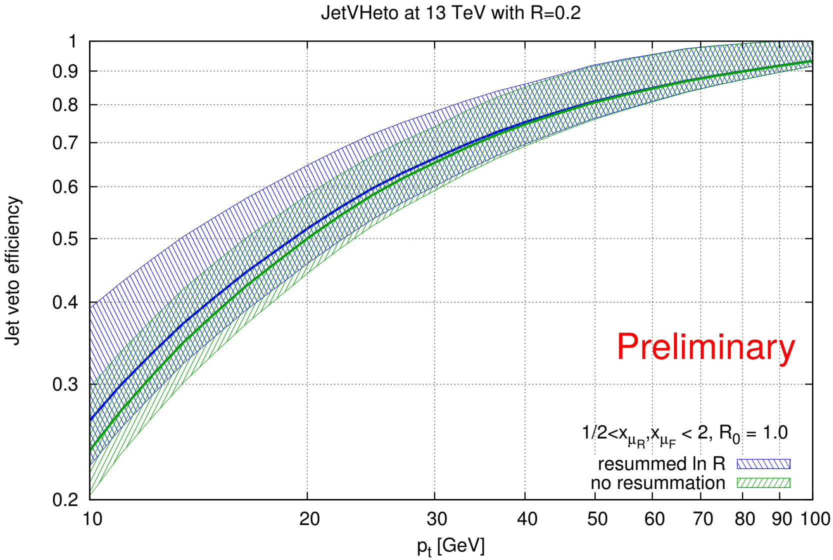
$$\langle \ln z \rangle^{\text{hardest}} \equiv \int_0^1 dz\, f^{\text{hardest}}(z) \ln z\,.$$

This seems to have a particularly stable perturbative expansion.



Small-R effects in jet veto efficiency

Inclusion of small-R terms leads to better handle on uncertainties.



Inclusive jet spectrum

The jet spectrum can be obtained from the convolution of the inclusive microjet fragmentation function with the inclusive partonic spectrum from hard $2 \to 2$ scattering

$$\frac{d\sigma_{\text{jet}}}{dp_t} = \sum_i \int_{p_t} \frac{dp'_t}{p'_t} \frac{d\sigma_i}{dp'_t} f^{\text{incl}}_{\text{jet}/i}(p_t/p'_t, t),$$

Small-R effects enhanced by $\ln n$ factors

$$\sim \alpha_s \ln \frac{1}{R^2} \ln n$$

To estimate corrections, assume that the partonic spectrum is dominated by a single flavour i and that its p_t dependence is $d\sigma_i/dp_t \sim p_t^{-n}$ then

$$\frac{d\sigma_{\text{jet}}}{dp_t} \simeq \frac{d\sigma_i}{dp_t} \int_0^1 dz\, z^{n-1} f^{\text{incl}}_{\text{jet}/i}(z, t) \equiv \frac{d\sigma_i}{dp_t} \langle z^{n-1} \rangle^{\text{incl}}_i .$$

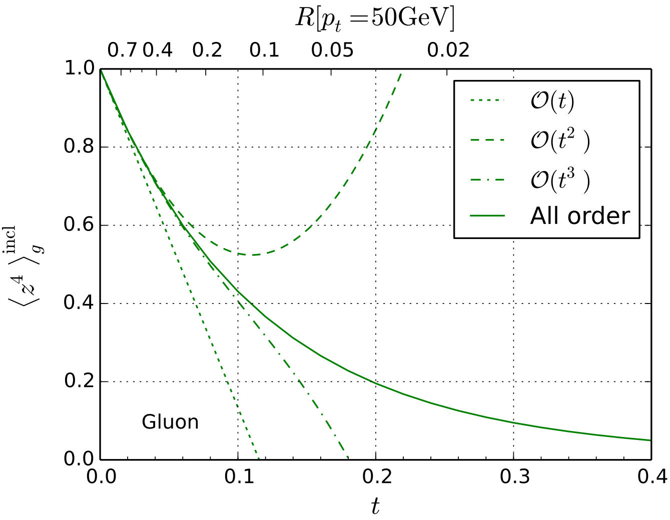
At the LHC typical n values for the partonic spectrum range from about 4 at low p_t to 7 or even higher at high p_t.

Moment of inclusive microjet spectrum $\langle z^{n-1} \rangle$

Convergence is slow, and small-R terms are important, amounting to a 30 – 50% effect on gluonic inclusive spectrum.

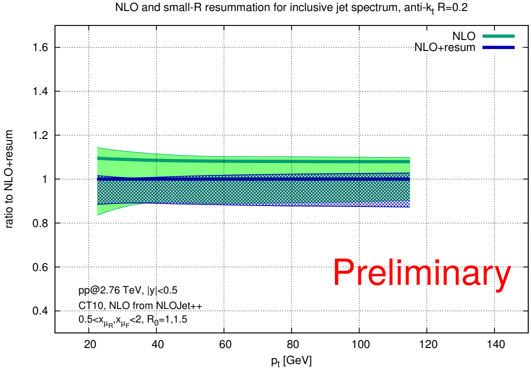
Corresponds to correction to $1/p_t^5$ spectrum.

NNLO corrections (order t^2) deviate noticeably from all-orders result below $R = 0.3$.



Different ways of estimating scale-dependence of incl. jet spect.

Small-R resummation changes the scale dependence.



There are different ways of estimating uncertainties due to the matching scheme.

$$\max_{\mu_F,\mu_R}\left(\text{resum}\times\frac{\text{NLO}+\text{LO}-\text{NLO}_{\text{small-}R}}{\text{LO}}\right)$$

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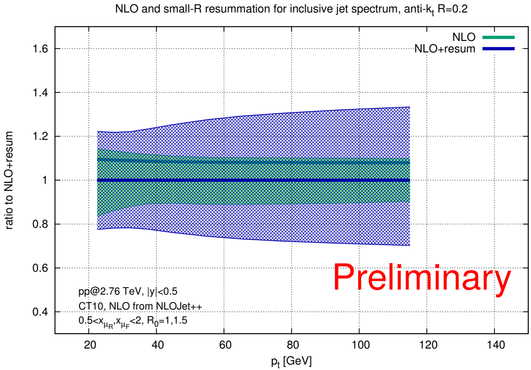

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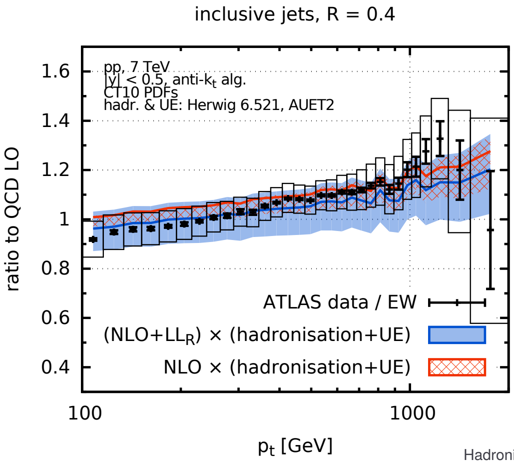
$$\max_{\mu_F,\mu_R}\left(\frac{\text{resum}}{\text{LO}}\right)\otimes\max_{\mu_F,\mu_R}\left(\text{NLO}+\text{LO}-\text{NLO}_{\text{small-}R}\right)$$

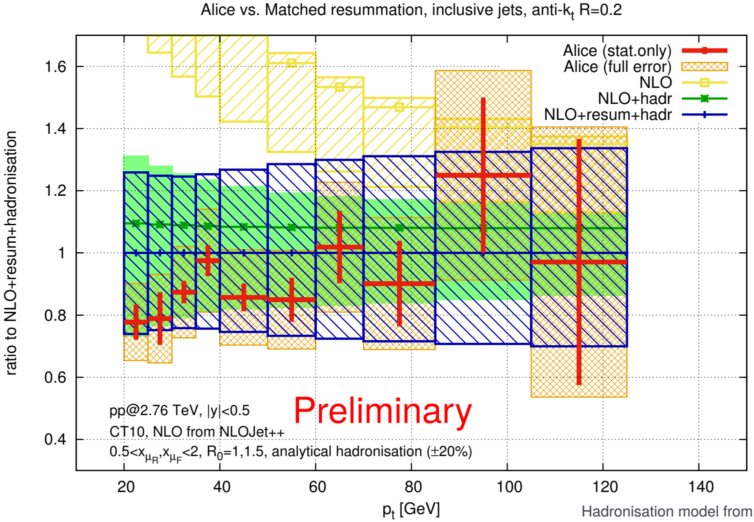
Small-R resummation somewhat improves agreement with Atlas $R = 0.4$ data, and changes the scale dependence of the NLO prediction.



Preliminary

Hadronisation model from
[Dasgupta, Magnea, Salam JHEP 0802 (2008) 055]

Small-R resummation somewhat improves agreement with Alice $R = 0.2$ data, and changes the scale dependence of the NLO prediction.



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CONCLUSION

Conclusion

- Small-R effects can be substantial, for example reducing the inclusive jet spectrum by $30 - 50\%$ for gluon jets for $R = 0.4 - 0.2$.
- Using generating-functional approach, carried out numerical leading logarithmic resummation of $\ln R$ enhanced-terms in small-R jets.
- Preliminary: small-R terms for jet veto efficiency in gluon-fusion Higgs production appear to suggest slight increase in uncertainties, little change to central values.
- Preliminary: small-R terms for inclusive jet spectrum have 10% effect beyond NLO and change scale uncertainties.

We intend to make the code public.

BACKUP SLIDES

Jet radius value

Jet radius values for different experiments, excluding substructure R choices

	ATLAS	CMS	ALICE	LHCb
R	$0.2^{*}, 0.4-0.6$	$0.3^{*}, 0.5, 0.7$	$0.2-0.4$	$0.5, 0.7$

* for PbPb only

Evolution equations

We can write the complete evolution equations as differential equations, for the quark the previous graph corresponds to

Quark

$$\frac{dQ(x,t)}{dt} = \int dz\, p_{qq}(z)\, [Q(zx,t)\, G((1-z)x,t) - Q(x,t)]\,.$$

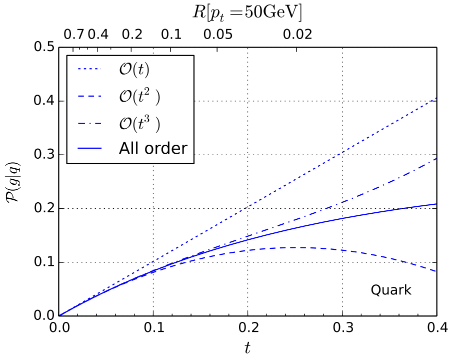
In the gluon case we find,

Gluon

$$\frac{dG(x,t)}{dt} = \int dz\, p_{gg}(z)\, [G(zx,t)G((1-z)x,t) - G(x,t)] + \int dz\, n_f\, p_{qg}(z)\, [Q(zx,t)Q((1-z)x,t) - G(x,t)]\,.$$

Jet flavour

Given a parton flavour, we look at the probability that the hardest resulting microjet has changed flavour.



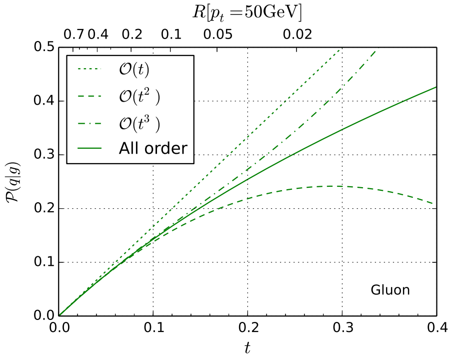


Figure – Flavour change probability.

Comparison with Pythia 8

Small-R resummation yields spectrum similar to parton shower.