NLL event-shape resummation within Sherpa

Towards soft-gluon resummation for multijet processes

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JHEP **1502** (2015) 106 [arXiv:1411.7325 [hep-ph]]

Outline

- **event-shape resummation** — the CAESAR approach
- **soft evolution of multi-parton amplitudes** — automated evaluation
- **patching up hard emissions** — a local matching scheme

Event-Shape Variables

classic probe of QCD dynamics

- continuous measure of final-state energy-momentum flows
- quantify departure from the Born limit
- sensitive to logarithmic & fixed-order corrections
- potential for QCD vs. New Physics separation

- consider observable $V(\{p\})$ with cumulant $\Sigma(v)$ for $V(\{p\}) < v$

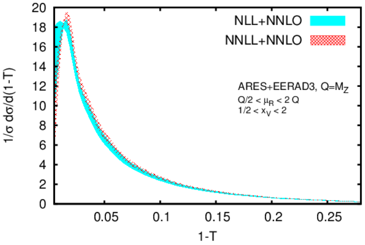
$$\Sigma(v) \propto e^{Lg_1(\alpha_s L)+g_2(\alpha_s L)+\ldots}$$

with $L = \ln(1/v)$

- thrust, broadenings, jet resolutions, ...
- e^+e^-, DIS, hadron collider
- analytic & Monte Carlo predictions

e^+e^- thrust at NNLL+NNLO

[Banfi et al. 1412.2126] [talk Monni]



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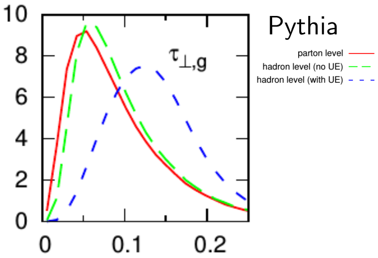
transverse thrust at LHC

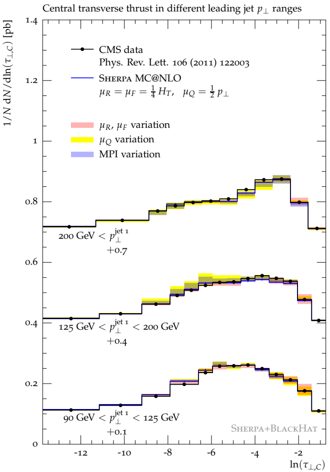
[Banfi et al. JHEP **1006** (2010) 038]

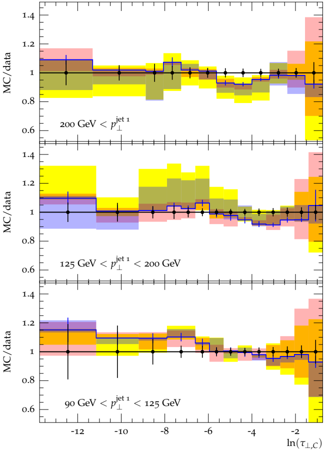


central transverse thrust at LHC

uncertainty study based on Sherpa MC@NLO [Höche, Schönherr Phys. Rev. D **86** (2012) 094042]

$\rightsquigarrow$ significant resummation scale dependence, sensitive to NP effects





The CAESAR Approach

automated NLL resummation of global event shapes [Banfi et al. JHEP **0503** (2005) 073]

- recursive IRC safe observable with Born limit $V(\{p\}) = 0$
  parametrization for single soft & collinear emission k

$$V(\{p\};k) = d_l \left(\frac{k_t^{(l)}}{Q}\right)^a e^{-b_l \eta^{(l)}} g_l(\phi^{(l)}) \quad k_t,\, \eta,\, \phi \text{ w.r.t. to leg } l$$

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- resummed exponent based on observable specific parameters ($\lambda = \alpha_s\beta_0 L$)

$$g_1^{(\delta)}(\alpha_s L) = -\sum_{l=1}^{n} \frac{C_l}{2\pi\beta_0\lambda b_l}\left[(a-2\lambda)\ln\left(1-\frac{2\lambda}{a}\right) - (a+b_l-2\lambda)\ln\left(1-\frac{2\lambda}{a+b_l}\right)\right]$$

$$\begin{aligned} g_2^{(\delta,\mathcal{B})}(\alpha_s L) \;=\; & -\sum_{l=1}^{n} C_l\left[\frac{r_l^{(2)}}{b_l} + B_l\, T\left(\frac{L}{a+b_l}\right)\right] + \partial_L\left[L g_1^{(\delta)}(\alpha_s L)\right]\left(\ln \bar{d}_l - b_l \ln\frac{2E_l}{Q}\right) \\ & + \sum_{l=1}^{n_{\text{initial}}} \ln\frac{q^{(l)}(x_l, \mu_F^2 e^{-\frac{2L}{a+b_l}})}{q^{(l)}(x_l,\mu_F^2)} + \ln\mathcal{F}\left(\partial_L L g_1^{(\delta)}(\alpha_s L)\right) \\ & - T(L/a)\sum_{l=1}^{n} C_l \ln\frac{Q_{12}}{Q} + \ln\mathcal{S}\left(T(L/a)\right) \text{ with } T(L) = \frac{1}{\pi\beta_0}\ln\frac{1}{1-2\alpha_s\beta_0 L} \end{aligned}$$

correlated emissions

large-angle soft radiation

The CAESAR Approach cont'd

the soft function – $\mathbf{S(\xi)}$ [Kidonakis et al. Nucl. Phys. B **531** (1998) 365]

$$S(\xi) = \frac{\langle m_0 | e^{-\frac{\xi}{2}\boldsymbol{\Gamma}^\dagger} e^{-\frac{\xi}{2}\boldsymbol{\Gamma}} | m_0 \rangle}{\langle m_0 | m_0 \rangle}$$ captures process dependent soft radiation

- $|m_0\rangle$ Born amplitude in color space
- soft-anomalous dimension matrix $\boldsymbol{\Gamma}$

$$\boldsymbol{\Gamma} = -2 \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{Q_{ij}}{Q_{12}} + i\pi \sum_{i,j=II,FF} \mathbf{T}_i \cdot \mathbf{T}_j$$

with process/sub-channel (δ) specific color operators $\mathbf{T}_i$

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Example: $2 \to 2$ parton scattering

$$\mathbf{\Gamma} = -(\mathbf{T}_1 \cdot \mathbf{T}_3 + \mathbf{T}_2 \cdot \mathbf{T}_4)\, T - (\mathbf{T}_1 \cdot \mathbf{T}_4 + \mathbf{T}_2 \cdot \mathbf{T}_3)\, U$$

with $T \equiv \ln \frac{-t}{s} + i\pi$ and $U \equiv \ln \frac{-u}{s} + i\pi$

Soft Evolution of Multi-Parton Amplitudes

automating the evaluation of the soft-function

(i) definition of suitable color-basis and metric
(ii) computation of color operators $\mathbf{T}_i$
(iii) decomposition of the Born amplitude $|m_0\rangle$

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- color-decomposed $n-$parton amplitude [Mangano, Parke Phys. Rept. **200** (1991) 301]

$$\mathcal{M}_0(\{p\};\alpha) = \langle c_\alpha|m_0\rangle = \sum_i C^{(i)}(\alpha)\, m_0^{(i)}(\{p\})$$

$C^{(i)}$ color coefficients for assignments $\alpha = \{\alpha_1 .., \alpha_n\}$

$m_0^{(i)}$ color-ordered partial amplitudes from planar diagrams

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- defines hard-matrix element ($H^{\alpha\beta}$) and color metric ($c_{\alpha\beta} = \langle c_\alpha | c_\beta \rangle$)

$$\mathrm{Tr}(cH) = c_{\alpha\beta} H^{\alpha\beta} = c_{\alpha\beta} \langle m_0 | c_\alpha \rangle \langle c_\beta | m_0 \rangle \;\rightsquigarrow\; S(\xi) = \frac{c_{\alpha\beta} H^{\gamma\sigma} \mathcal{G}^\dagger_{\gamma\rho} c^{\rho\beta} c^{\alpha\delta} \mathcal{G}_{\delta\sigma}}{c_{\alpha\beta} H^{\alpha\beta}}$$

$\rightsquigarrow$ employ Sherpa's built-in ME generator COMIX [Gleisberg,Höche JHEP **0812** (2008) 039]

Soft Evolution of Multi-Parton Amplitudes

use non-orthogonal color bases: $\mathbf{c}_{\alpha\beta} \neq \delta_{\alpha\beta}$ [talk Sjödahl]

- processes involving quarks: trace basis [Mangano, Parke Phys. Rept. **200** (1991) 301]

$$\mathcal{M}_0(\{p\}; i_1, a_2, .., j_n) = \sum_{\sigma \in P(n-2)} (T^{a_{\sigma_2}} .. T^{a_{\sigma_{n-1}}})_{i_1}^{j_n} \, m_0(p_1, p_{\sigma_2}, .., p_{\sigma_{n-1}}, p_n)$$

- purely gluonic processes: adjoint basis [Del Duca et al. Nucl. Phys. **B568** (2000) 211]

$$\mathcal{M}_0(\{p\}; a_1, .., a_n) = \sum_{\sigma \in P(n-2)} (F^{a_{\sigma_2}} .. F^{a_{\sigma_{n-1}}})_{a_1 a_n} \, m_0(p_1, p_{\sigma_2}, .., p_{\sigma_{n-1}}, p_n)$$

$\rightsquigarrow$ bases (often) over-complete, featuring zero-eigenvalues at $N_c = 3$

$\rightsquigarrow$ inverse metric $(c_{\alpha\beta})^{-1} = c^{\alpha\beta}$ singular at $N_c = 3$

$\rightsquigarrow$ resort to numerical matrix inversion in $N_c = 3 + \epsilon$

$\rightsquigarrow$ $S(\xi)_{N_c = 3+\epsilon} = S(\xi)_{N_c = 3} + \mathcal{O}(\epsilon)$

Soft Evolution of Multi-Parton Amplitudes

soft evolution of multi-parton amplitudes

basis properties for 4−, and 5−parton processes [e.g. Kidonakis et al. '98, Forshaw et al. '06]

sub-process	$gggg$	$q\bar{q}gg$	$q\bar{q}q\bar{q}$	$ggggg$	$q\bar{q}ggg$	$q\bar{q}q\bar{q}g$
dim. basis	5	3	2	16	10	4
dim. Born	2	2	2	6	6	4
zero EV	0	0	0	0	0	0

basis properties for 6− and 7−parton processes

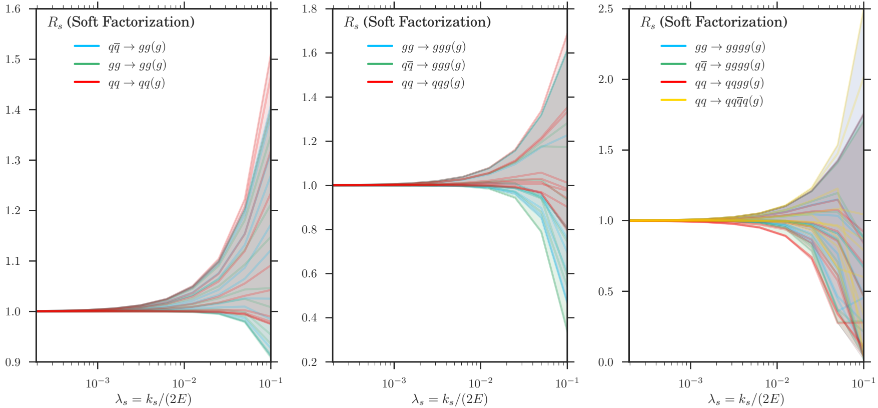
sub-process	$6g$	$q\bar{q}4g$	$q\bar{q}q\bar{q}gg$	$qqq\bar{q}\bar{q}\bar{q}$	$7g$	$q\bar{q}5g$	$q\bar{q}q\bar{q}3g$	$q\bar{q}q\bar{q}q\bar{q}g$
dim. basis	79	46	14	6	421	252	62	18
dim. Born	24	24	12	6	120	120	48	18
zero EV	5	6	1	0	70	75	12	1

check for emission of single soft gluon with momentum $\mathbf{p_s}$, $|\mathbf{p_s}| = \mathbf{k_s}$

$$R_s = \frac{\alpha_s}{\pi}\mathrm{Tr}\left[H_n \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \frac{p_i \cdot p_j}{p_i \cdot p_s\, p_j \cdot p_s}\right] \frac{1}{\mathrm{Tr}\,\left(c\, H_{n+1}\right)} \quad \text{for} \quad \lambda_s = k_s/(2E) \to 0$$

Soft Evolution of Multi-Parton Amplitudes

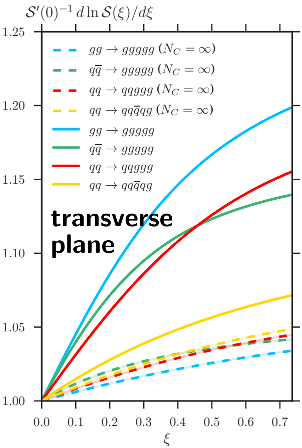
- in the limit of soft-gluon emission $\lim_{\lambda_s \to 0} R_s = 1$
- sample over non-collinear kinematics in the transverse plane ($\eta = 0$)
- sanity check for non-vanishing n and $n+1$ Born elements

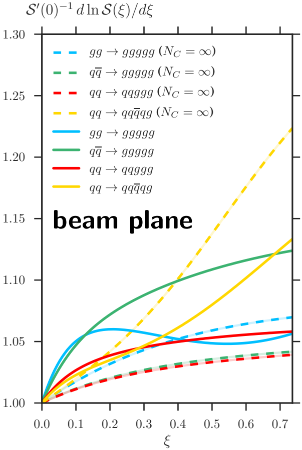


Soft Evolution of Multi-Parton Amplitudes

the full soft function – finite N_c effects

- fixed final-state momenta on circle in transverse/beam plane
- compare large-N_c limit (dashed) to full $N_c = 3$ result (solid)





Local Matching to Real Emission ME

patching up hard emissions

$$\frac{d\sigma^{\rm matched}}{dv} = \frac{d\sigma^{\rm resummed}}{dv} + \underbrace{\left(\frac{d\sigma^{\rm fixed-order}}{dv} - \frac{d\sigma^{\rm expanded}}{dv}\right)}_{\text{subtracted matrix-element calculation}}$$

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aiming for tree-level accuracy here – subtract α_s expansion

$$\frac{d}{dL}\frac{d\Sigma^{(\delta)}}{d\mathcal{B}} = \frac{2\alpha_s}{\pi}\frac{d}{dL}\left[\frac{G_{12}}{2}L^2 + G_{11}L\right] + \mathcal{O}\left(\alpha_s^2\right),$$

within CAESAR formalism

$$G_{12} = -\sum_{l=1}^{n}\frac{C_l}{a(a+b_l)}$$

$$G_{11} = -\left[\sum_{l=1}^{n} C_l\left(\frac{B_l}{a+b_l} + \frac{1}{a(a+b_l)}\left(\ln\bar{d}_l - b_l\ln\frac{2E_l}{Q}\right) + \frac{1}{a}\ln\frac{Q_{12}}{Q}\right)\right.$$
$$\left. + \frac{1}{a}\frac{\mathrm{Re}[\Gamma_{\alpha\beta}]\,H^{\alpha\beta}}{c_{\alpha\beta}H^{\alpha\beta}} + \sum_{l=1}^{n_{\text{initial}}}\frac{\int_{x_l}^{1}\frac{dz}{z}P_{lk}^{(0)}\left(\frac{x_l}{z}\right)q^{(k)}(z,\mu_F^2)}{2(a+b_l)q^{(l)}(x_l,\mu_F^2)}\right].$$

Local Matching to Real Emission ME

generate α_s expansion from local subtraction terms

- variant of Sherpa's Catani–Seymour subtraction [Gleisberg, Krauss Eur. Phys. J. C **53** (2008) 501]

$$\mathcal{D}_{ij,k}(1,..,n) = -\frac{1}{2p_i p_j} \times \langle m_0(1,..,ij,..,k,..,n)| \frac{\mathbf{T}_i \cdot \mathbf{T}_k}{\mathbf{T}_{ij}^2} \hat{V}_{ij,k}(z,k_T) |m_0(1,..,ij,..,k,..,n)\rangle$$

- integrate suitable splitting operators over real-emission phase space
- use identical color insertion operators $\mathbf{T}$ as in calculation of $\mathbf{\Gamma}$

$\rightsquigarrow$ obtain $Re[\Gamma_{\alpha\beta}]H^{\alpha\beta}$ for $\hat{V}_{ij,k} \to 1/a \ln(Q_{(ij)k}/Q_{12})$

$\rightsquigarrow$ G_{12} and B_l term in G_{11} from $\hat{V}_{ij,k} \to P_{ij,i}$

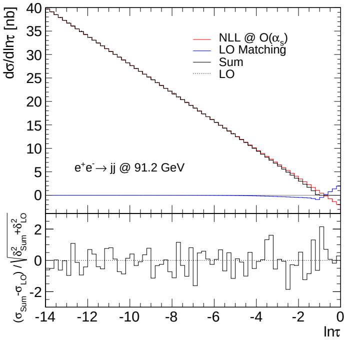
$\rightsquigarrow$ collinear counterterm generated separately (quasi-local)

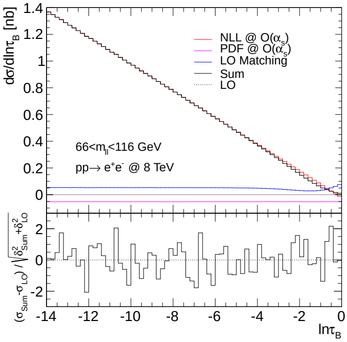
Local Matching to Real Emission ME

test for local cancellation – the case of thrust

- pure FF (thrust) and II (beam thrust) dipole contributions
- explicit NLL expansion (red) plus local matching term (blue,magenta)
- compared to independent LO matrix-element calculation (dashed)

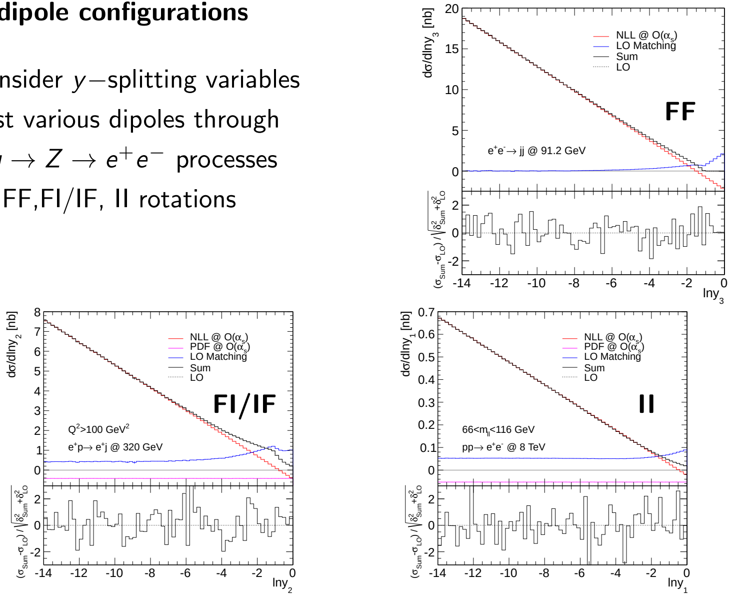
⇝ perfect agreement within statistics





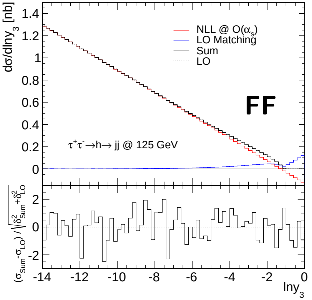
quark dipole configurations

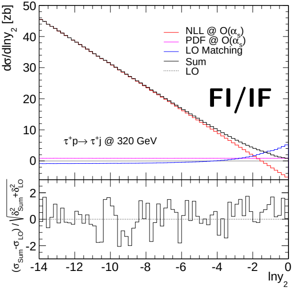
- consider $y-$splitting variables
- test various dipoles through $q\bar{q} \to Z \to e^+e^-$ processes in FF,FI/IF, II rotations

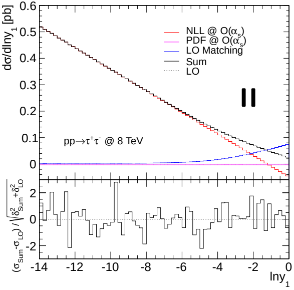


gluon dipole configurations

- consider $y-$splitting variables
- test various dipoles through $gg \to h \to \tau\tau$ processes in FF,FI/IF, II rotations







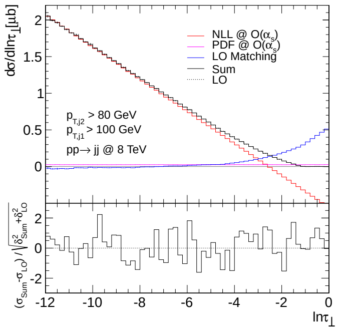
Proof of Concept: Transverse Thrust

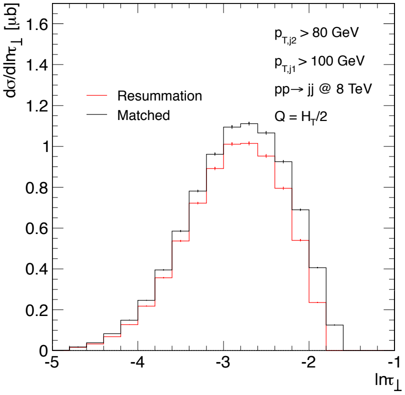
transverse thrust at 8 TeV LHC

- inclusive dijet production at LO
- NLL resummed transverse thrust

$$\tau_\perp \quad = \quad 1 - T_\perp$$

- NLL matched to three-jet ME





Summary: NLL event-shape resummation within Sherpa

automated evaluation of soft function

- automatic color basis calculations
- COMIX generator for partial amplitudes/hard matrix

$\rightsquigarrow$ high-multi processes, fast and efficient evaluation

novel matching scheme

- quasi-local matching to exact real-emission matrix elements
- based on (modified) Catani–Seymour subtraction terms

$\rightsquigarrow$ quick and efficient evaluation of matching contribution

framework for NLL resummation and shower comparison

- fully integrated into Sherpa framework, invoked as “shower” plugin
- needs observable parameters as input
- extendable in various directions: NLO, NNLL, ...