QCD corrections at higher orders
- status and perspective -

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Workshop on *Parton Showers, Event Generators and Resummation 2015*, Cracow, May 26, 2015

Highest energies at colliders until 202x

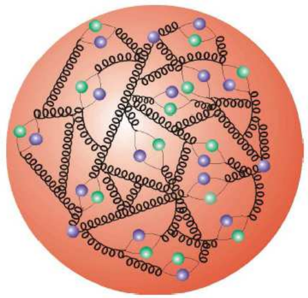
Energy frontier

- Study of Higgs boson properties, searches for new massive particles

$$E = m\,c^2$$

Hadron colliders

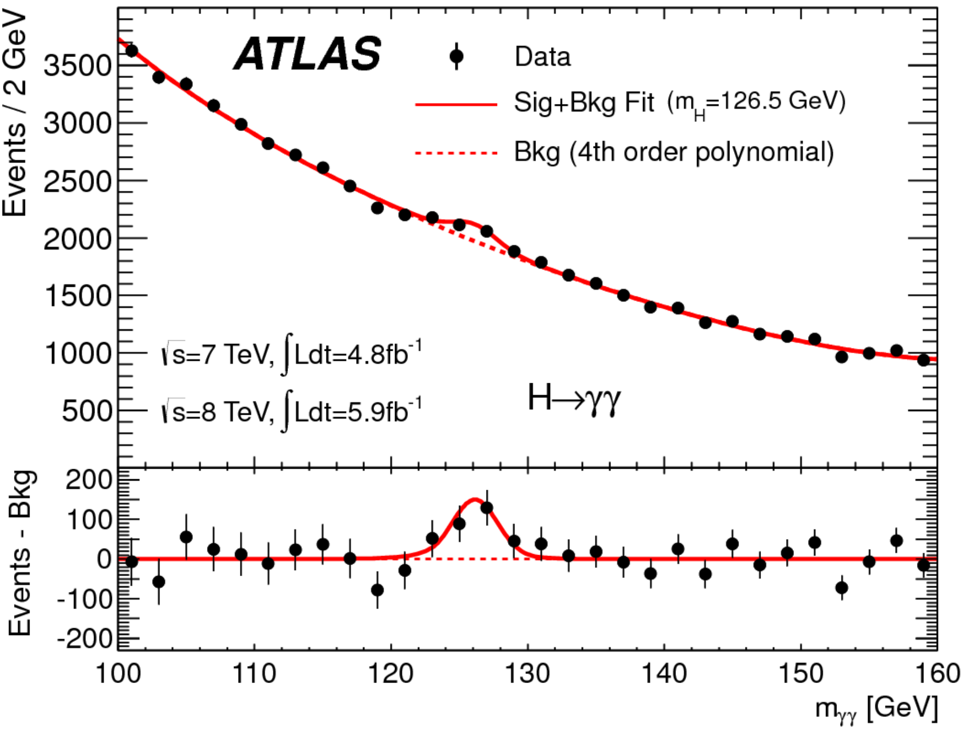
- Proton–proton collisions reach TeV-scale
  - LHC run I $\sqrt{S} = 7$ and 8TeV, run II $\sqrt{S} = 13$TeV

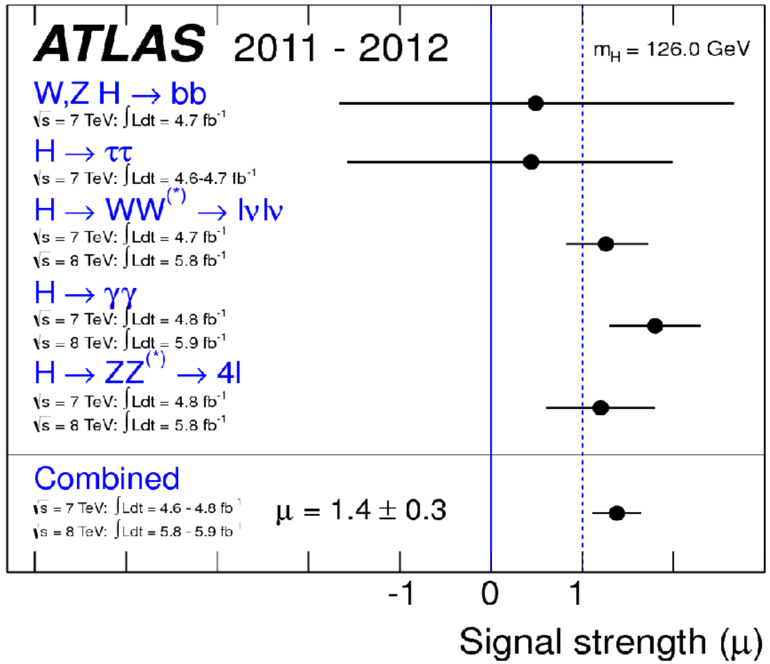


- Proton: composite multi-particle bound state
  - collider: "wide-band beams" of quarks and gluons
  - understand SM background (LHC is a QCD machine)
- Theory has to match or exceed accuracy of LHC data
  - perturbative QCD is essential and established part of toolkit
  - electroweak corrections important for precision predictions

Higgs discovery

Atlas coll. July 2012





- Measured $H \to \gamma\gamma$ decay mode (left)
- Signal strength of all analyzed decay modes normalized to SM expectation (right)
- Agreement with SM for $H \to ZZ$; excess of $H \to \gamma\gamma$ (new physics ?)

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Nobel Prizes and Laureates

Physics Prizes ‹ 2013 ›

▼ About the Nobel Prize in Physics 2013

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▸ François Englert

▸ Peter W. Higgs

All Nobel Prizes in Physics

All Nobel Prizes in 2013

The Nobel Prize in Physics 2013
François Englert, Peter W. Higgs

The Nobel Prize in Physics 2013

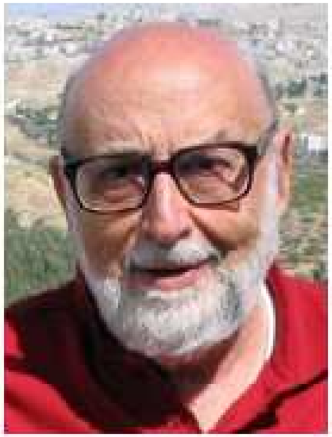


Photo: Pnicolet via Wikimedia Commons

François Englert

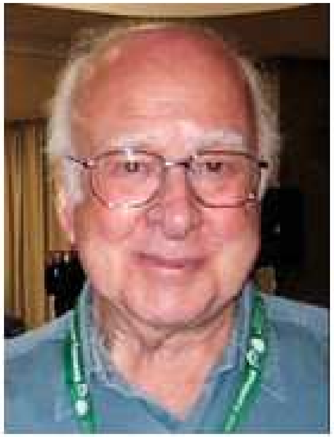


Photo: G-M Greuel via Wikimedia Commons

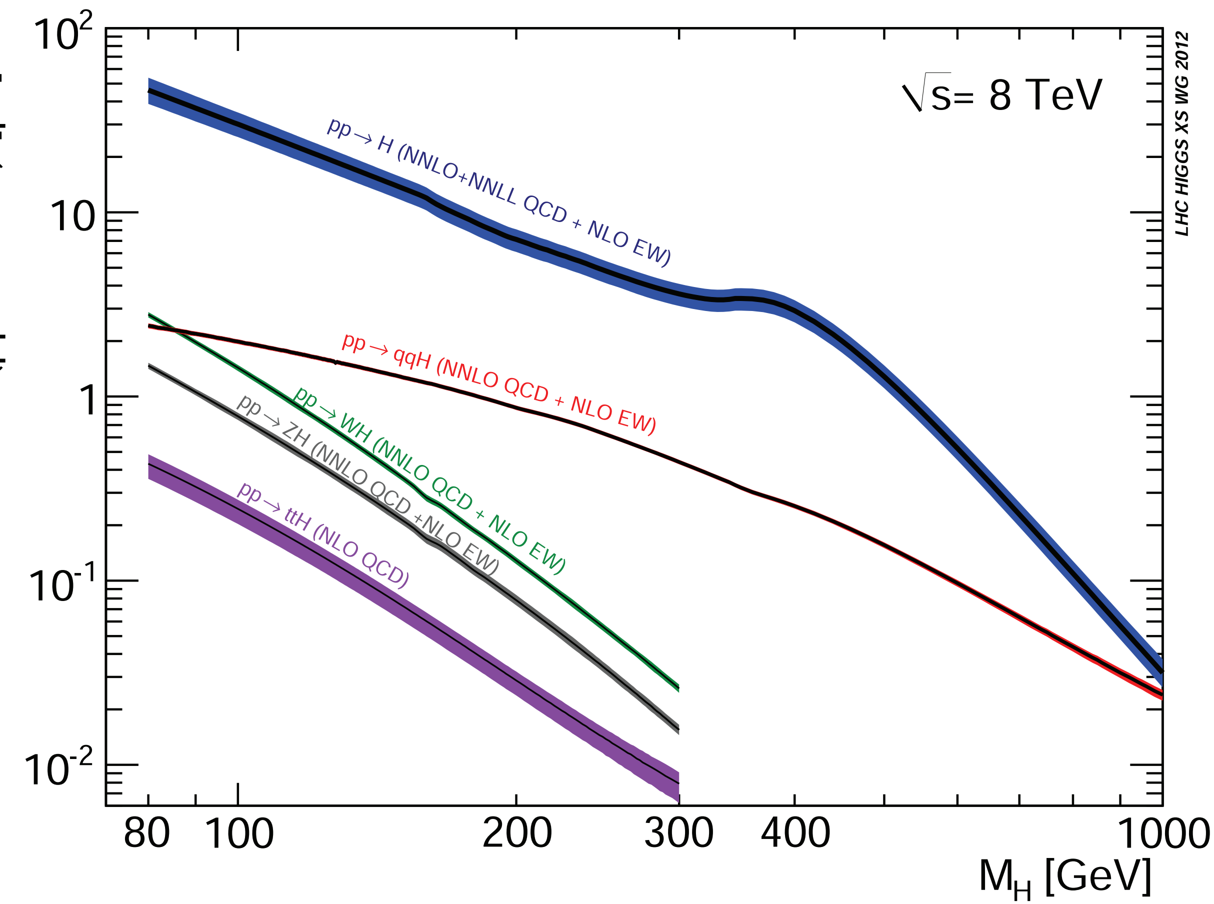
Peter W. Higgs

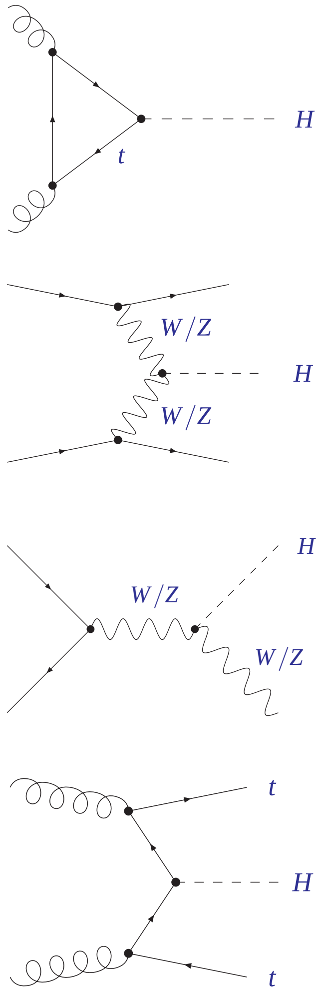
The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*

Higgs cross section

Cross section for Higgs production at the LHC

- Dominant channels for Higgs boson production LHC Higgs XS WG '12





gg-fusion

weak boson fusion

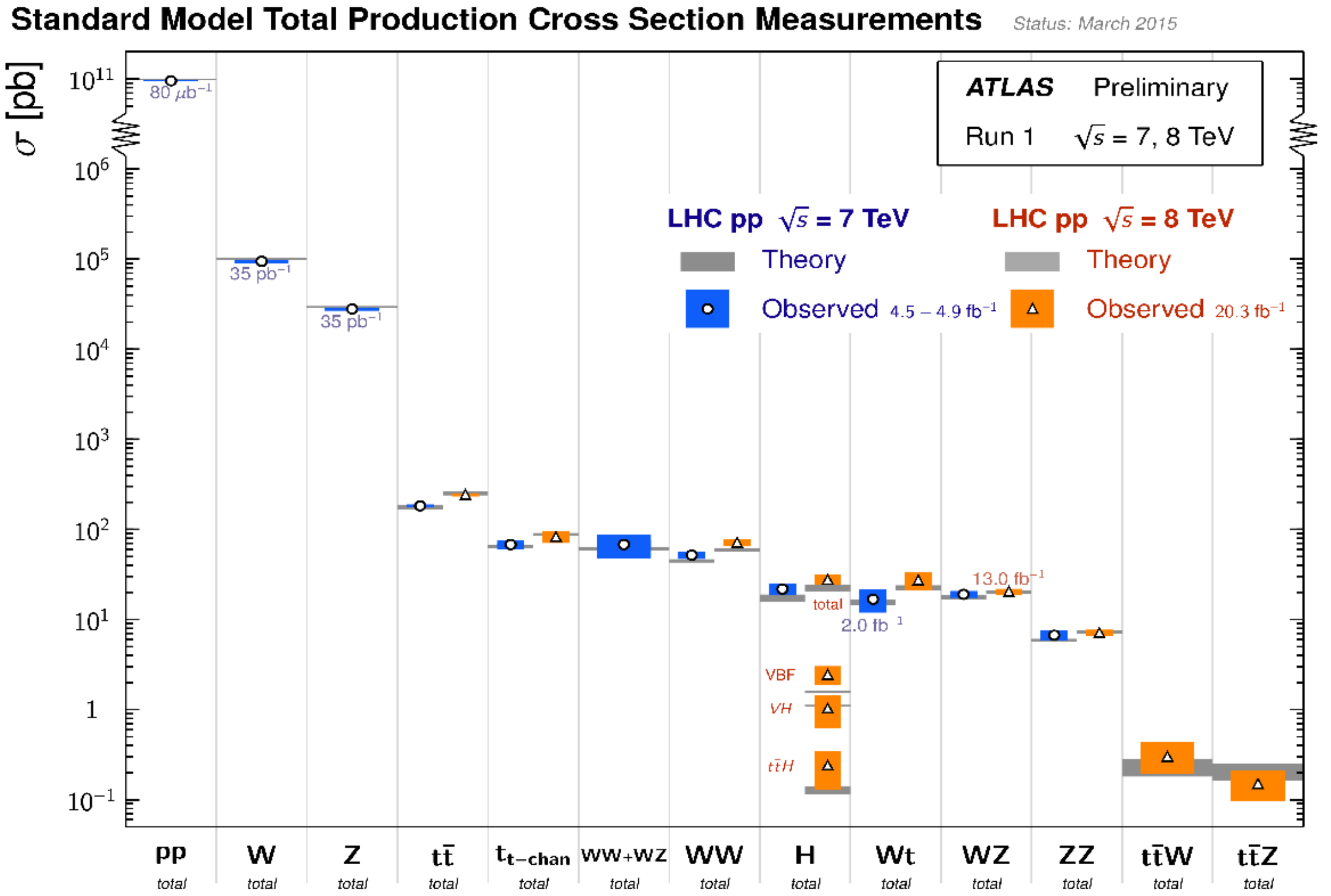
Higgs strahlung

$t\bar{t}H$-channel

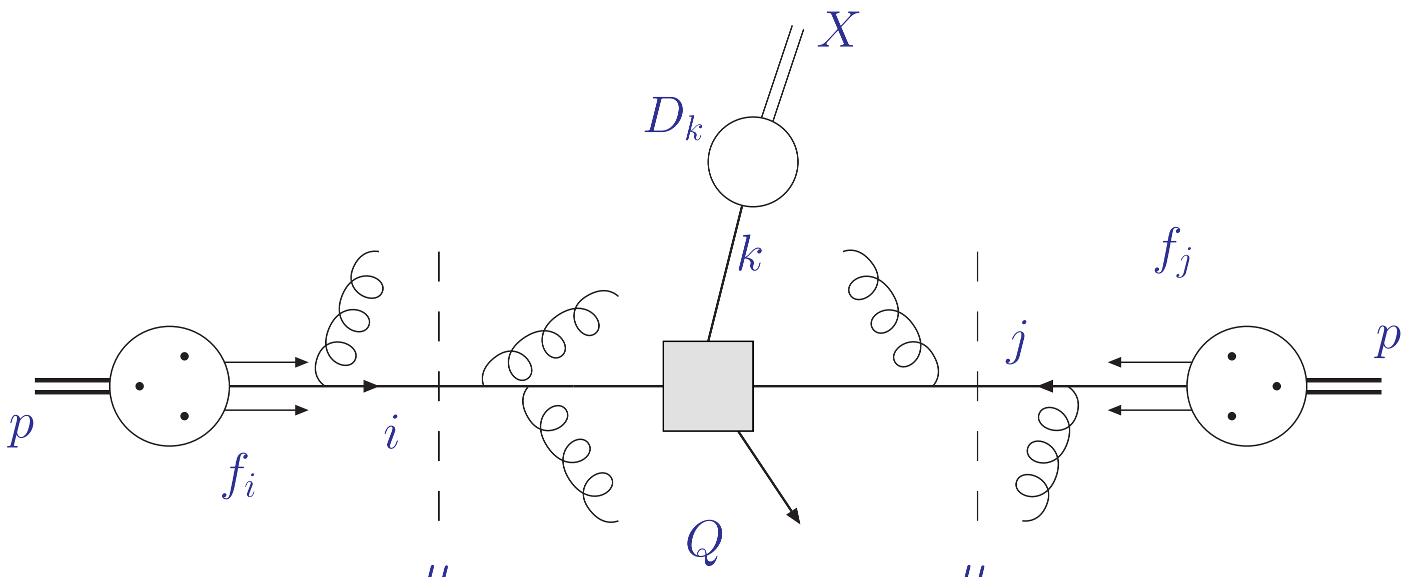
Standard Model cross sections

Cross sections for Standard Model processes at the LHC

- All dominant channels for Higgs boson production observed Atlas coll. '15



QCD factorization

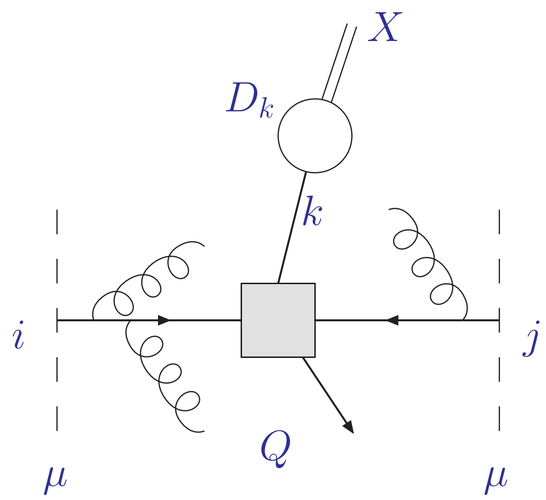


$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij \to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij \to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Hard scattering cross section

- Parton cross section $\hat{\sigma}_{ij\to k}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, ... ($\mathcal{O}(\text{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50-100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10-30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - $\mathrm{N^3LO}$ (next-to-next-to-next-to-leading order)
  - ...

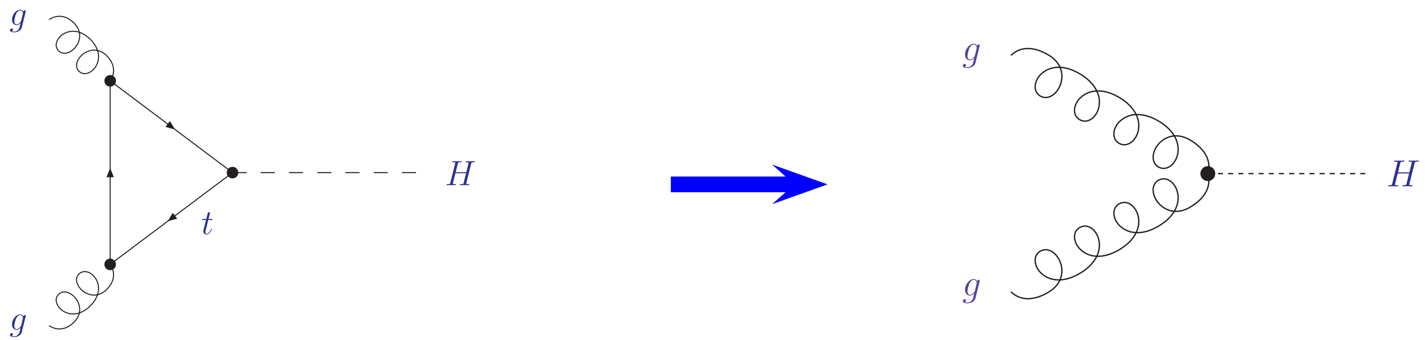
Perturbation theory at work

Particle physics

- Perturbative approach straightforward in principle
  - draw all Feynman diagrams and evaluate them,
  - use standard reduction techniques for tree/loop amplitudes
- (Extremely) hard in practice
  - intermediate expressions more complicated than final results
- Known bottlenecks
  - **many diagrams** — many diagrams are related by gauge invariance
  - **many terms in each diagram** — nonabelian gauge boson self-interactions are complicated
  - **many kinematic variables** — allowing the construction of very complicated expressions
- Computer algebra programs are a standard tool

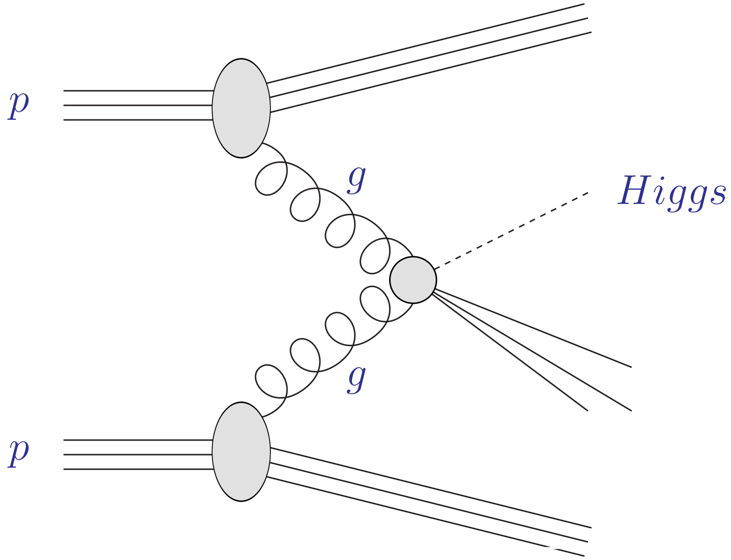
Higgs production in gg-fusion

Effective theory



- Hard scattering cross section $\hat{\sigma}_{ij\to H+X}$ dominated by gluon-gluon fusion
  - typically treated in effective theory in limit $m_t \to \infty$; Lagrangian
    $\mathcal{L} = -\frac{1}{4}\,\frac{H}{v}\,C_H\,G^{\mu\nu\,a}\,G^a_{\mu\nu}$
- QCD corrections significant
  - NNLO corrections still large
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - improvement with soft N^3LO corrections S.M., Vogt '05; Laenen, Magnea '05:
    in x-space '+'-distributions $\alpha_s^k \ln^{2k-1}(1-x)/(1-x)$
  - NNLL resummation Catani, de Florian, Grazzini, Nason '03; Ahrens et al. '10; [. . .];
    Ahmed, Mahakhud, Rana, Ravindran '14
  - exact corrections at N^3LO completed
    Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15

QCD corrections to ggF



- Hadronic cross section $\sigma_{pp\to H}$ with $\tau = m_H^2/S$
  - renormalization/factorization (hard) scale $\mu = \mathcal{O}(m_H)$

$$\sigma_{pp\to H} = \sum_{ij} \int_\tau^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i\left(\frac{x_1}{x_2}, \mu^2\right) f_j\left(x_2, \mu^2\right) \hat{\sigma}_{ij\to H}\left(\frac{\tau}{x_1}, \frac{\mu^2}{m_H^2}, \alpha_s(\mu^2)\right)$$

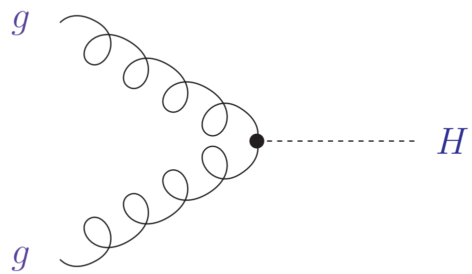
- Partonic cross section $\hat{\sigma}_{ij\to H}$

$$\hat{\sigma}_{ij\to H} \quad = \quad \underbrace{\alpha_s^2 \Big[\hat{\sigma}^{(0)}_{ij\to H} + \alpha_s\, \hat{\sigma}^{(1)}_{ij\to H}}_{} + \alpha_s^2\, \hat{\sigma}^{(2)}_{ij\to H} + \alpha_s^3\, \hat{\sigma}^{(3)}_{ij\to H} + \dots \Big]$$

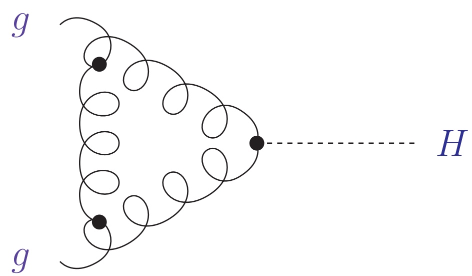
NLO: standard approximation (large uncertainties)

Radiative corrections in a nutshell

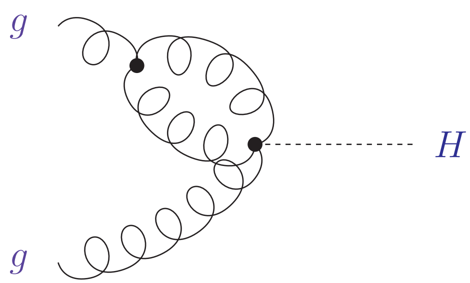
- Leading order
  - partonic cross section $x = \tau/x_1 = M_H^2/\hat{s}$
    $\hat{\sigma}^{(0)}_{gg\to H} = \delta(1-x)$



- Next-to-leading order (virtual corrections)
  kinematics proportional to Born
  - vertex correction

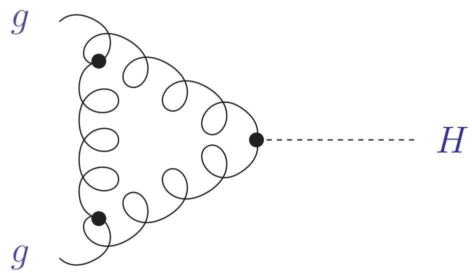


  - self-energy correction
    (massless tadpole)



NLO virtual corrections

- Contribution of vertex correction to ggF cross section
  - kinematics proportional to Born
  - gluon form factor in time-like kinematics
  - infrared divergent; $D = 4 - 2\epsilon$

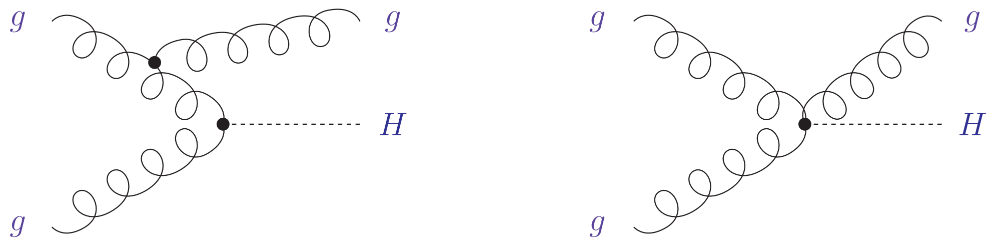


$$\hat{\sigma}^{(1),v}_{gg\to H} \quad = \quad C_A \frac{\alpha_s}{4\pi} \delta(1-x) \left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} \left(-\frac{2}{\epsilon^2} + \frac{7}{6}\pi^2 + \mathcal{O}(\epsilon)\right)$$

- Operator $H G^{\mu\nu\, a}\, G^a_{\mu\nu}$ relates to stress-energy tensor
- Effective theory requires UV renormalization
  - additional contribution from renormalization of effective operator proportional to QCD β-function Kluberg-Stern, Zuber '75; Collins, Duncan, Joglekar '77

$$\alpha_s^{\rm bare} \quad = \quad \alpha_s^{\rm ren} \left\{ 1 - \frac{\beta_0}{\epsilon} \frac{\alpha_s^{\rm ren}}{4\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

- Next-to-leading order (real emission)

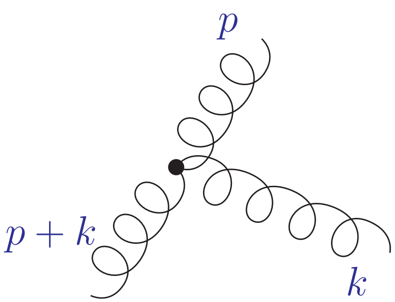


  - real corrections $\hat{\sigma}^{(1),r}_{gg\to H}$ require integration over phase space

$$\hat{\sigma}^{(1),r}_{gg\to H} \quad = \quad \int dLIPS^{(m)}\, |\mathcal{M}_{gg\to Hg}|^2$$

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



$$\frac{1}{(p+k)^2} \quad = \quad \frac{1}{2p\cdot k} = \frac{1}{2E_{g_i}E_{g_o}(1-\cos\theta_{gg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \quad \longrightarrow \quad \alpha_s \int dE_{g_o}\, d\sin\theta_{gg}\, \frac{1}{2E_{g_i}E_{g_o}(1-\cos\theta_{gg})}$$

$$\longrightarrow \quad \alpha_s \frac{1}{\epsilon^2} \times (\dots)$$ in dim. reg. $D = 4 - 2\epsilon$

- Complete NLO corrections
  - add real and virtual corrections $\hat{\sigma}^{(1)}_{gg\to H} = \hat{\sigma}^{(1),r}_{gg\to H} + \hat{\sigma}^{(1),v}_{gg\to H}$
  - collinear divergence remains: splitting functions $P^{(0)}_{gg}$

$$\hat{\sigma}^{(1)}_{gg\to H} = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} \Bigg\{$$

$$\frac{1}{\epsilon} C_A \left(\frac{8}{1-x} + \frac{8}{x} - 8(2 - x + x^2) + \frac{22}{3}\delta(1-x)\right) - \frac{1}{\epsilon} n_f \frac{4}{3}\delta(1-x)$$

$$+\mathcal{O}(1)\Bigg\}$$

- Complete NLO corrections
  - add real and virtual corrections $\hat{\sigma}^{(1)}_{gg\to H} = \hat{\sigma}^{(1),r}_{gg\to H} + \hat{\sigma}^{(1),v}_{gg\to H}$
  - large threshold logarithms for $x = M_H^2/s \simeq 1$: resummation

$$
\begin{aligned}
\hat{\sigma}^{(1)}_{gg\to H} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} \Bigg\{ & \\
& \frac{1}{\epsilon} C_A \left(\frac{8}{1-x} + \frac{8}{x} - 8(2-x+x^2) + \frac{22}{3}\delta(1-x)\right) - \frac{1}{\epsilon} n_f \frac{4}{3}\delta(1-x) \\
& + C_A \left(16\frac{\ln(1-x)}{1-x} + \left(\frac{22}{3} + \frac{4}{3}\pi^2\right)\delta(1-x) - 16x(2-x+x^2)\ln(1-x)\right. \\
& \left. \quad -8\frac{(1-x+x^2)^2}{1-x}\ln(x) - \frac{22}{3}(1-x)^3\right) + \mathcal{O}(\epsilon)\Bigg\}
\end{aligned}
$$

- Structure of NLO correction
  - absorb collinear divergence $P_{gg}^{(0)}$ in renormalized parton distributions

$$\hat{\sigma}^{(1),\text{bare}}_{gg\to H} = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}\left\{\frac{1}{\epsilon}\,2\,P_{gg}^{(0)}(x) + \hat{\sigma}^{(1)}_{gg\to H}(x) + \mathcal{O}(\epsilon)\right\}$$

$$g^{\text{ren}}(\mu_F^2) = g^{\text{bare}} - \frac{\alpha_s}{4\pi}\,\frac{1}{\epsilon}\,P_{gg}^{(0)}(x)\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}$$

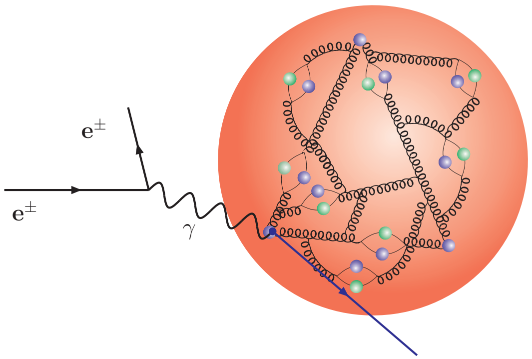
  - partonic (physical) structure function at factorization scale μ_F

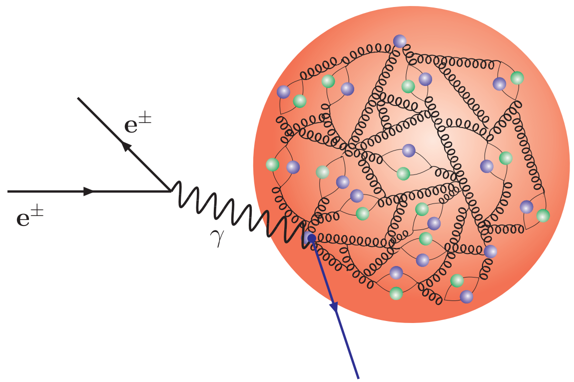
$$\hat{\sigma}_{gg\to H} = \delta(1-x) + \frac{\alpha_s}{4\pi}\left\{\hat{\sigma}^{(1)}_{gg\to H}(x) - \ln\left(\frac{m_H^2}{\mu_F^2}\right)\,2\,P_{gg}^{(0)}(x)\right\}$$

Physics picture

- Constituent partons from proton interact at short distance
  - low resolution

high resolution





Physical kernel

- Cross section for Higgs boson production function of $x = m_H^2/s$

$$\sigma(H) \quad = \quad \sigma_0 \sum_{a,b} f_a \otimes f_b \otimes c_{ab}$$

  - coefficient functions for hard scattering $c_{ab}(\alpha_s) = \alpha_s^k\, c_{ab}^{(k)}$:
    LO $c_{gg}^{(0)} = \delta(1-x)$, **NLO** $c_{ab}^{(1)}$, **NNLO** $c_{ab}^{(2)}$, **N^{3}LO** $c_{ab}^{(3)}$, ...
  - $c_{ab}(\alpha_s)$ in x-space show $x \to 1$ double logarithmic enhancement:
    '$+$'-distributions $\alpha_s^k \ln^l(1-x)/(1-x)$ with $0 \leq l \leq 2k-1$

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- Alternative factorization: partonic structure functions $\sigma(H) = \sum_{a,b} \sigma_0 \mathcal{F}_{ab}$
  - gg-channel dominant for $x \to 1$
  - evolution equation for $\mathcal{F}_{gg}$ defines physical kernel K_{gg}

$$\frac{d}{d\ln m_H^2} \mathcal{F}_{gg} \quad = \quad \underbrace{\left\{ 2P_{gg}(\alpha_s) + \beta(\alpha_s) \, \frac{dc_{gg}(\alpha_s)}{d\alpha_s} \otimes (c_{gg}(\alpha_s))^{-1} \right\}}_{\text{scheme invariant}} \otimes \mathcal{F}_{gg}$$

$$\equiv \quad K_{gg}(\alpha_s) \otimes \mathcal{F}_{gg}$$

  - $K_{gg}(\alpha_s)$ for $x \to 1$ exhibits single logarithmic enhancement
    $\alpha_s^k \ln^l(1-x)/(1-x)$ with $0 \le l \le k \quad \longrightarrow \quad$ constraints on $c_{ab}(\alpha_s)$

Coefficient functions

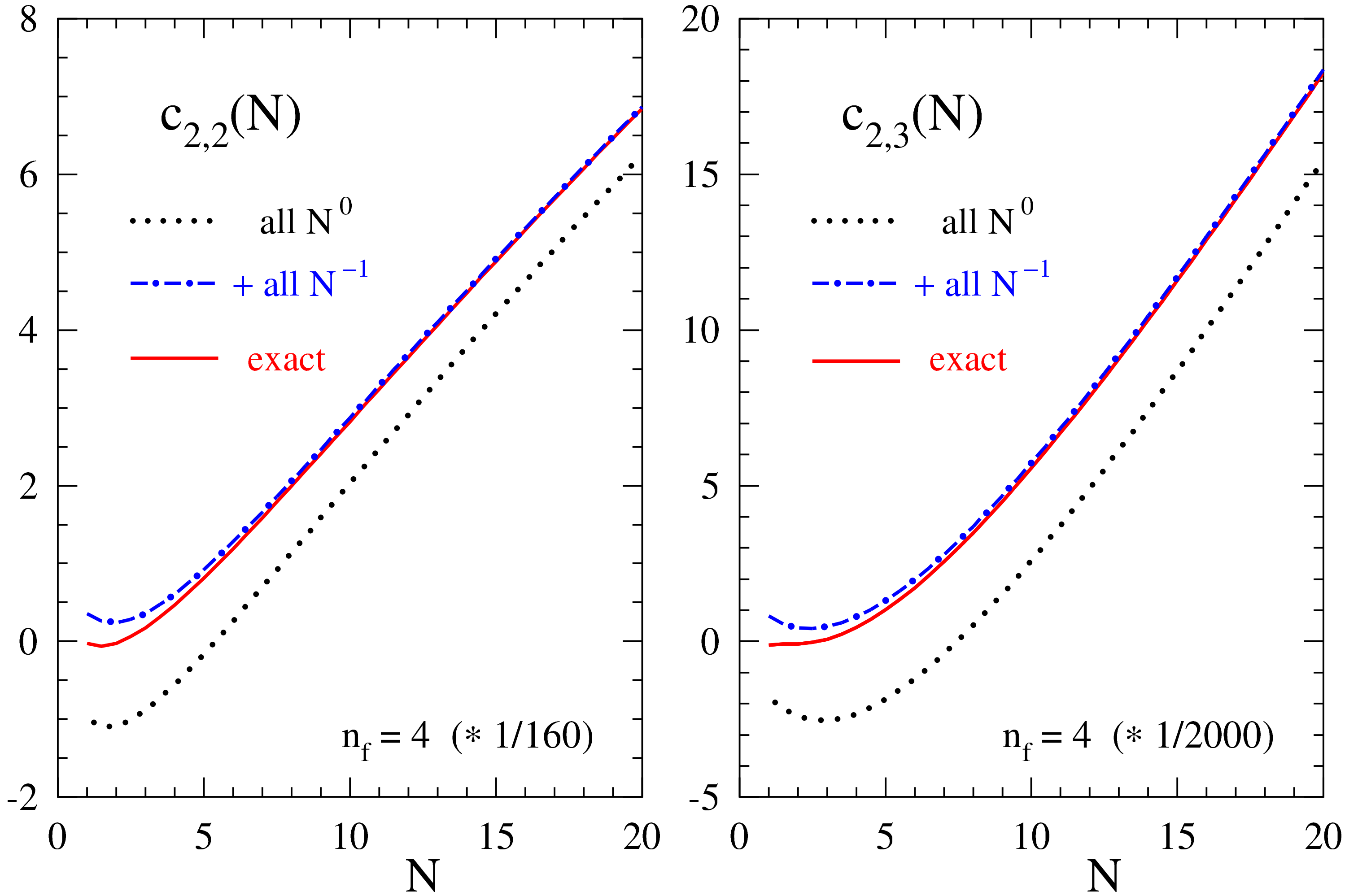
- Constraints from physical kernel $K_{gg}(\alpha_s)$ exact to all powers in $(1-x)^n$
  - access to power suppressed terms with logarithmic enhancement $\alpha_s^k \ln^l(1-x)$ with $1 \leq l \leq 2k$
- Coefficient function at **N^{3}LO** $c_{gg}^{(3)}$

$$c_{gg}^{(3)}(x) = c_{gg}^{(3)}(x)\Big|_{\mathcal{D}_k,\delta(1-x)} - 512 C_A^3 \ln^5(1-x) + \left\{ 1728 C_A^3 + \frac{640}{3} C_A^2 \beta_0 \right\} \ln^4(1-x)$$

$$+ \left\{ \left(-\frac{1168}{3} + 3584\zeta_2 \right) C_A^3 - \left(\frac{2512}{3} + \frac{1}{3}\xi_H \right) C_A^2 \beta_0 - \frac{64}{3} C_A \beta_0^2 \right\} \ln^3(1-x) + \ldots$$

  - unknown coefficient for $\ln^3(1-x)$ estimated $\xi_H = 300$
  - exact computation $\xi_H = 896/3$
    Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger arXiv:1411.3584
- Space of Mellin moments N:
  - leading contributions (SV): $\alpha_s^k \ln^l N$ with $1 \leq l \leq 2k$
  - sub-leading contributions (SV+1/N): $\alpha_s^k (\ln^l N)/N$ with $1 \leq l \leq 2k$

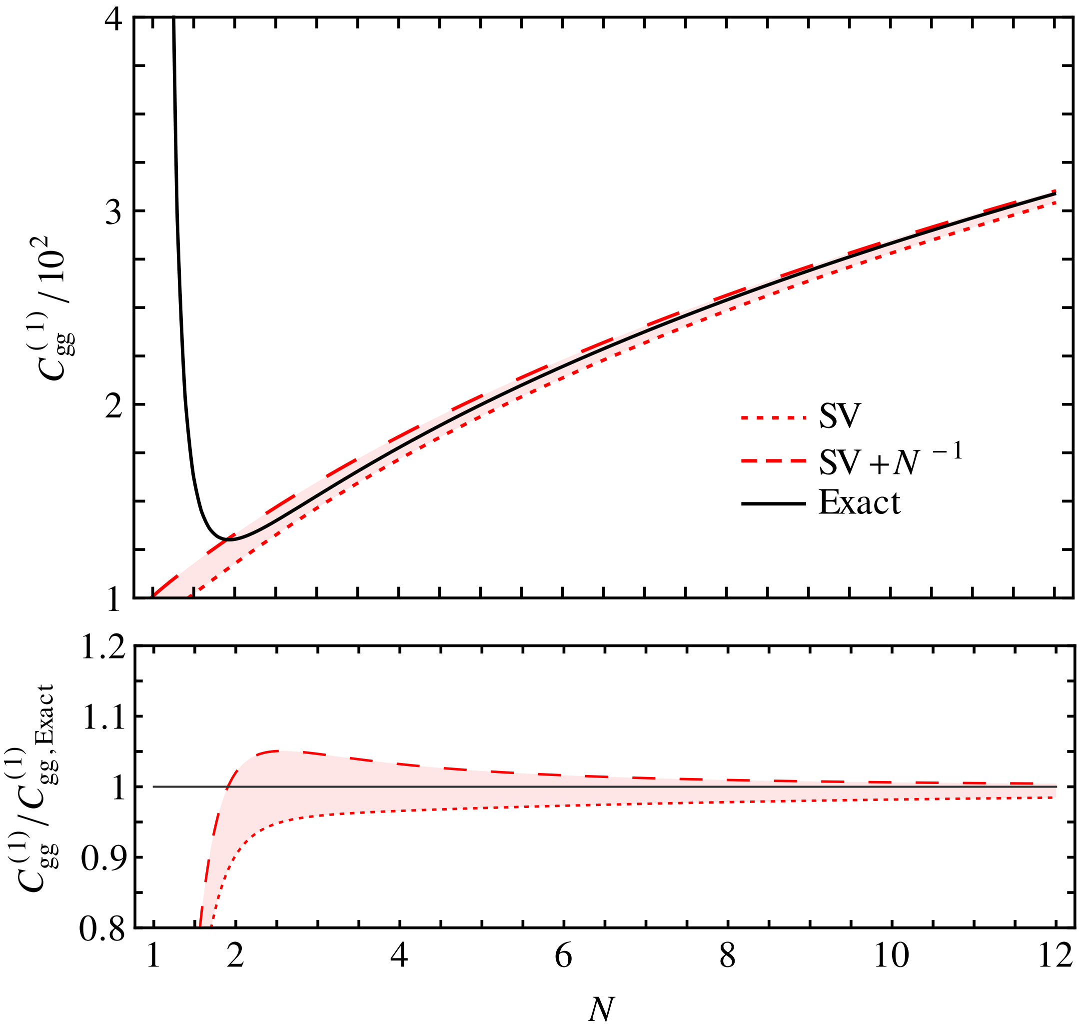
Quality check



- SV and SV+1/N approximations provide uncertainty band S.M., Vogt '09
  - tested for large number of observables
  - γ-DIS and ϕ-DIS known exactly to N^3LO
    S.M., Vermaseren, Vogt '05, Soar, S.M., Vermaseren, Vogt '09

Higgs boson production: coefficient function

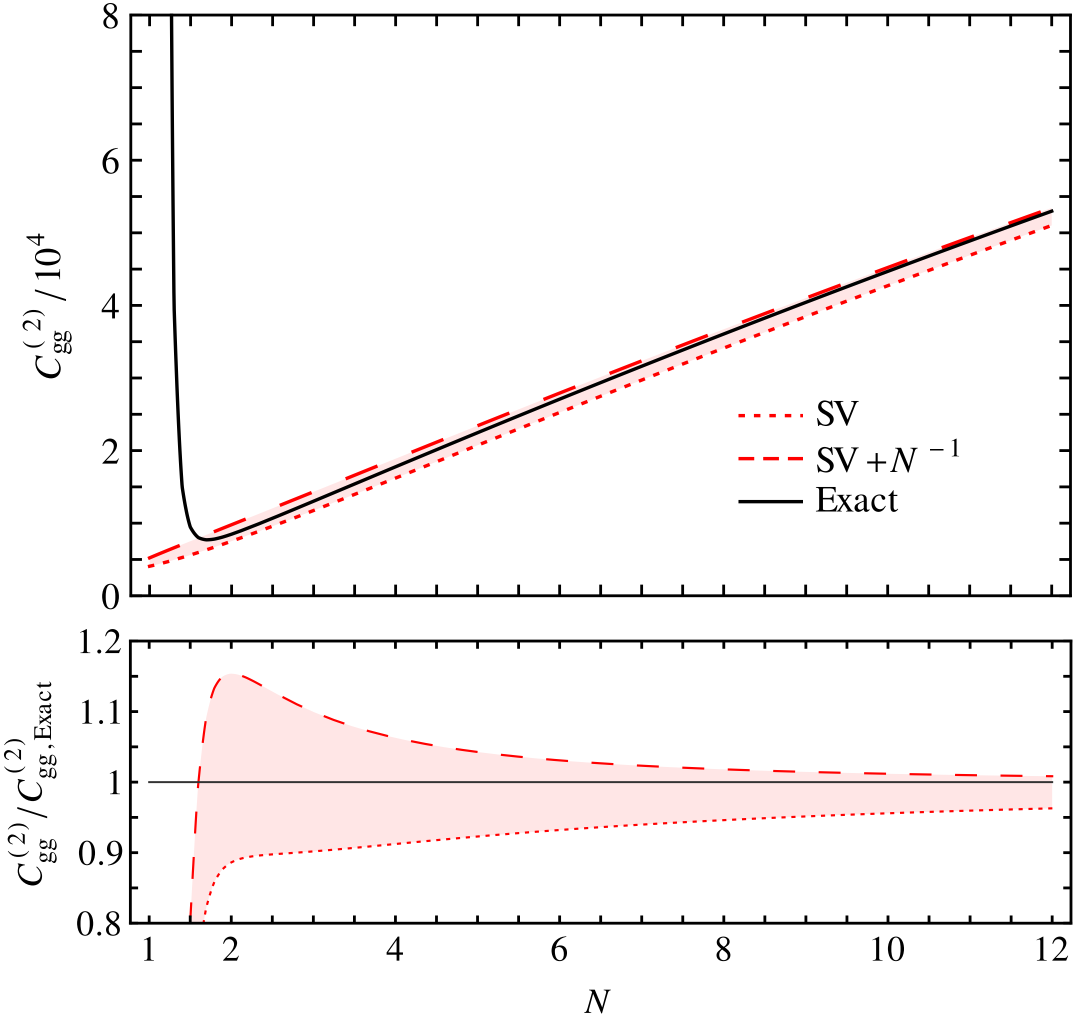
Coefficient function at NLO



- SV and SV+1/N approximations work even for N not so large

Higgs boson production: coefficient function

Coefficient function at NNLO



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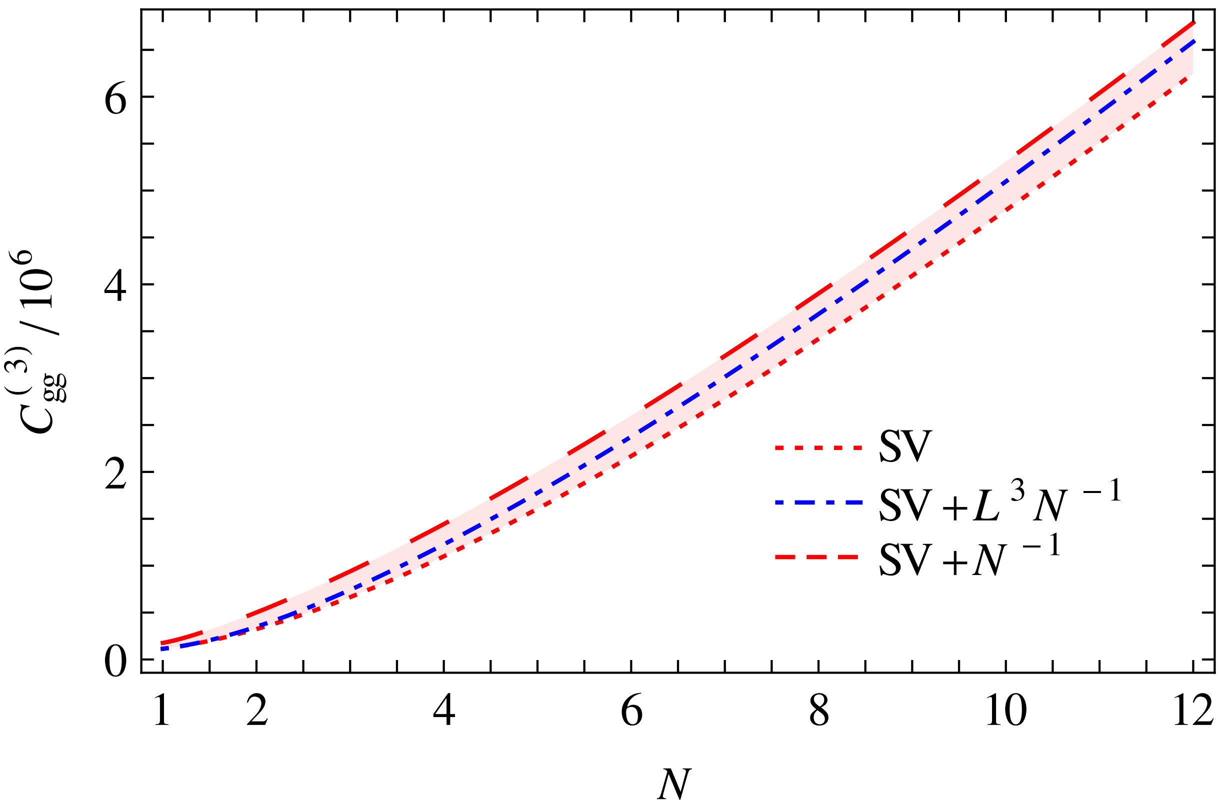
Approximate N^{3}LO: coefficient function



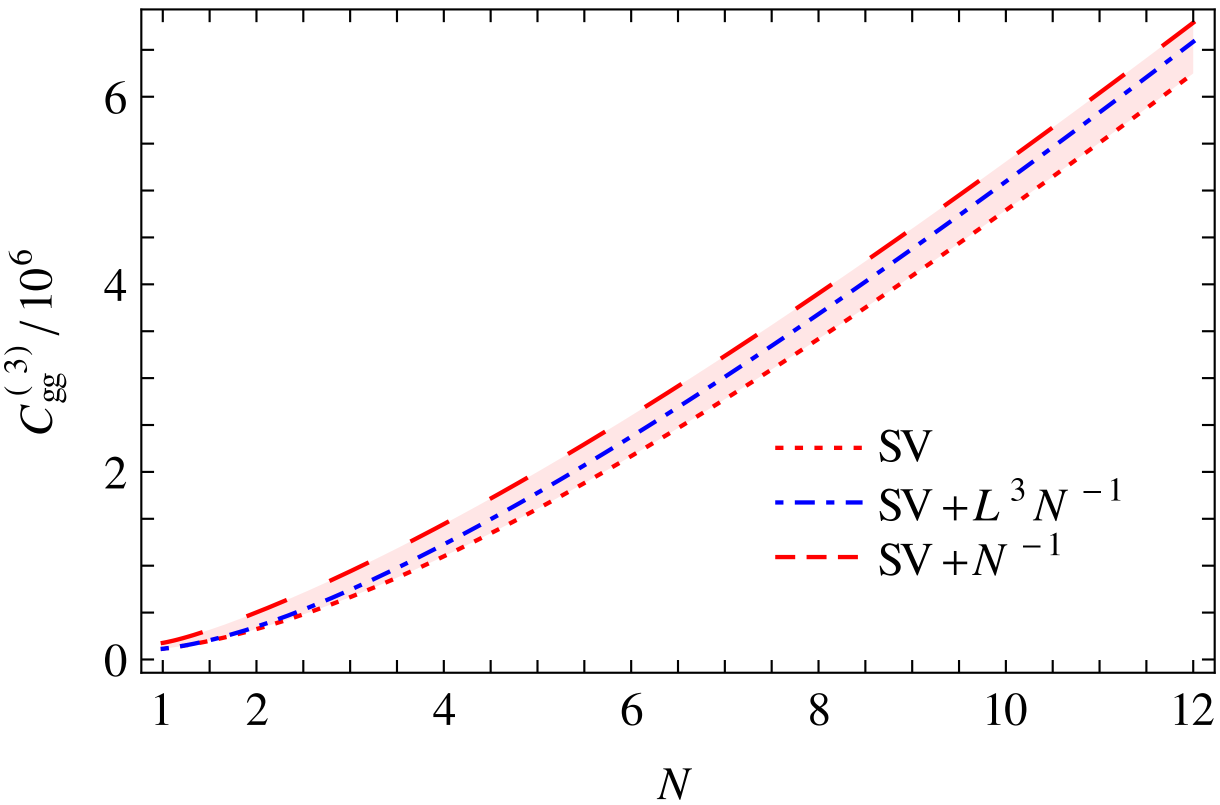
- Coefficient function at N^{3}LO (only subleading $1/N$ terms displayed)

$$c_{gg}^{(3)}(N) \sim (108 \ln^5 N + 615.7 \ln^4 N + 2041.2 \ln^3 N + \underbrace{4968 \ln^2 N + 1944 \ln N + 972}_{\text{estimate from } \phi\text{-DIS}})/N$$

- Leading logarithm agrees with estimate by factorization of splitting function $P_{gg}^{(0)}$ $\longrightarrow$ $c_{gg}^{(3)}(N) \sim 108 \ln^5 N/N$

  Krämer, Laenen, Spira ‘96; Ball, Bonvini, Forte, Marzani, Ridolfi ´13

Approximate N^{3}LO: coefficient function



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- Exact computation

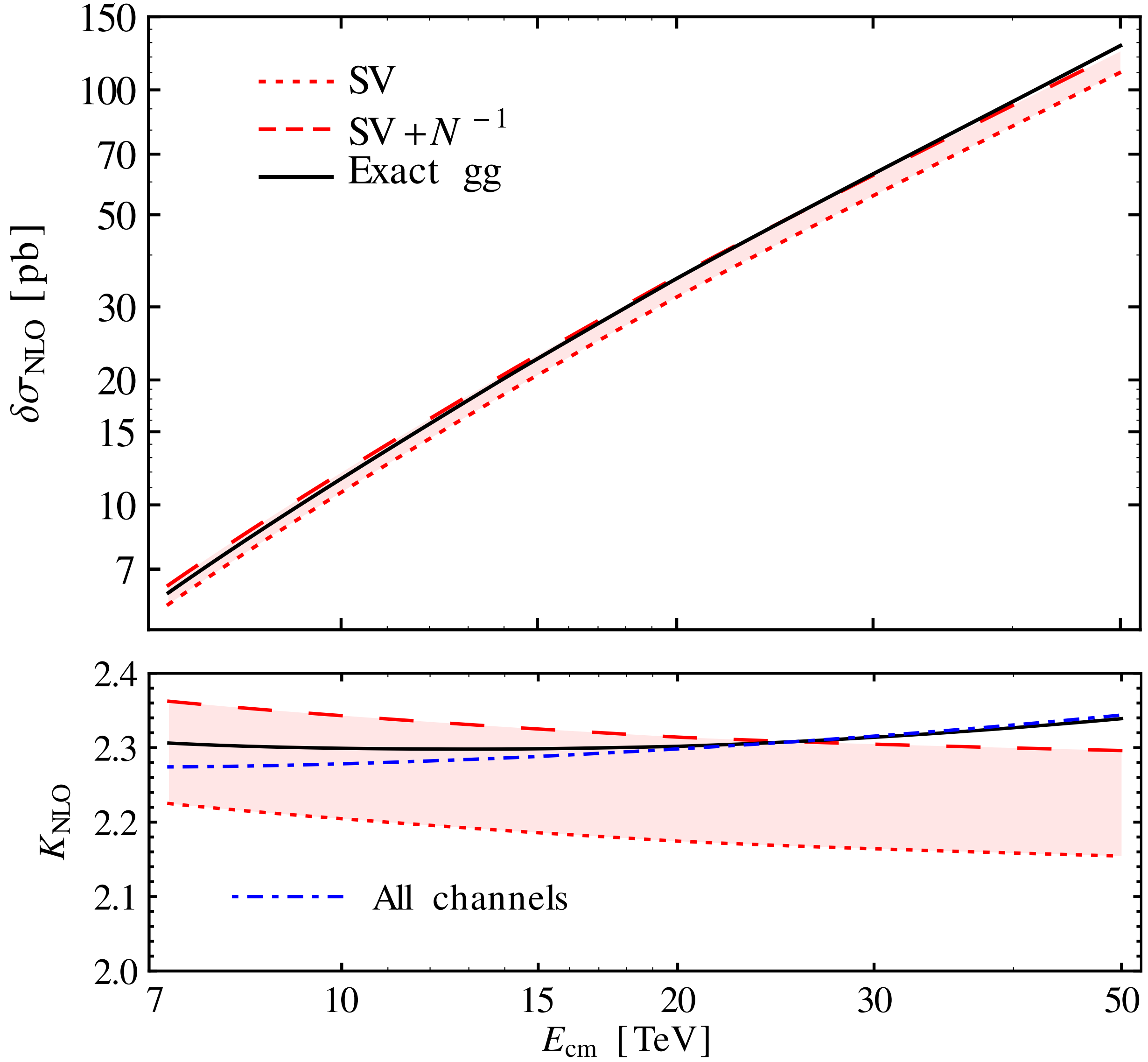
  Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger arXiv:1411.3584

$$c^{(3)}_{gg}(N) \sim (108 \ln^5 N + 615.7 \ln^4 N + 2036.4 \ln^3 N + 3305 \ln^2 N + 3459 \ln N + 703)/N$$

Higgs boson production: cross section

Cross section at NLO

- Approximation based on SV and SV+1/N works for current LHC energies $\sqrt{s} = 13$ TeV very well



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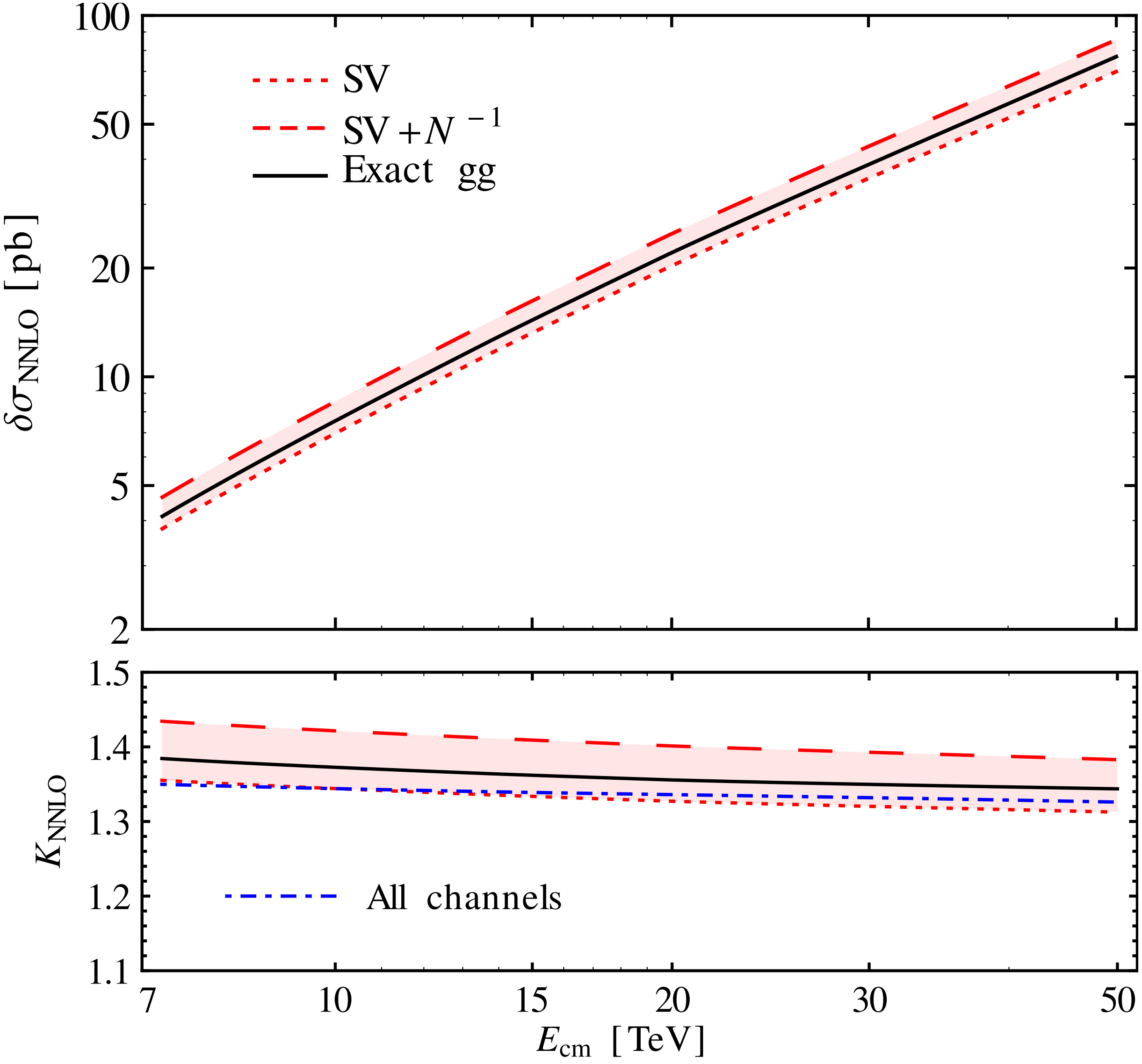
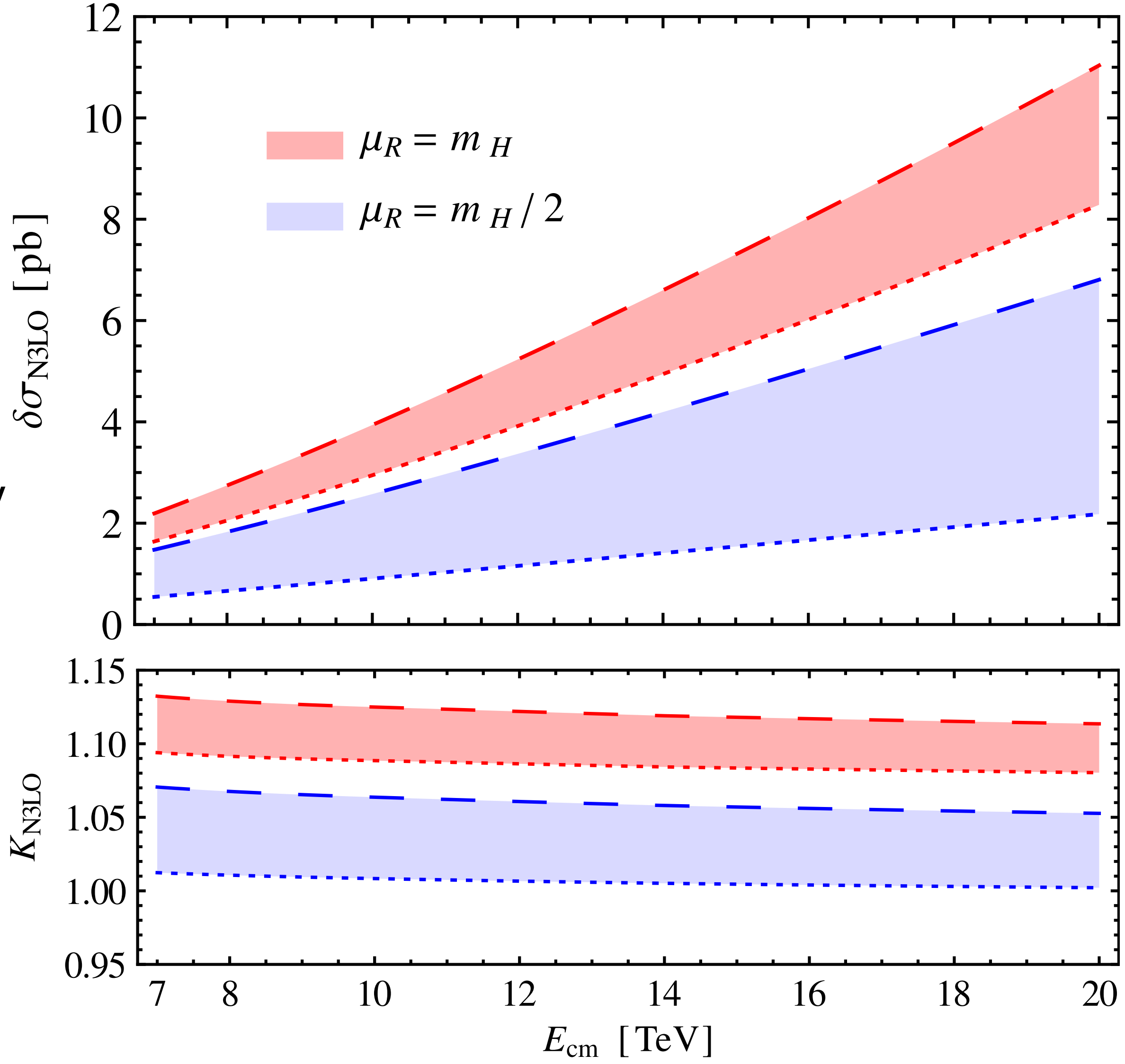
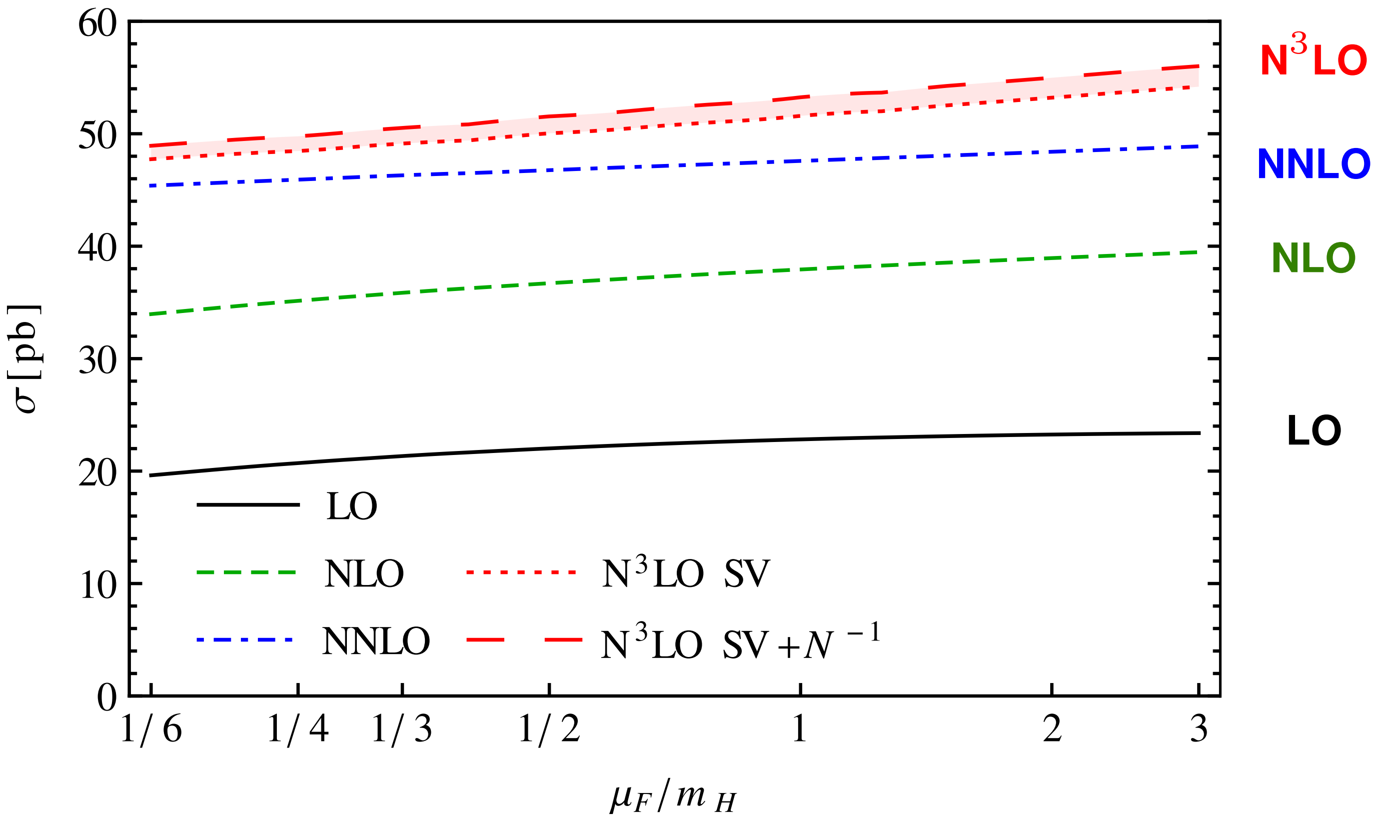


Approximate N^{3}LO: cross section

- N^{3}LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$
- Point of minimal sensitivity around $\mu = m_H/2$ with K-factor $1 \dots 5\%$ at $\sqrt{s} = 13$ TeV

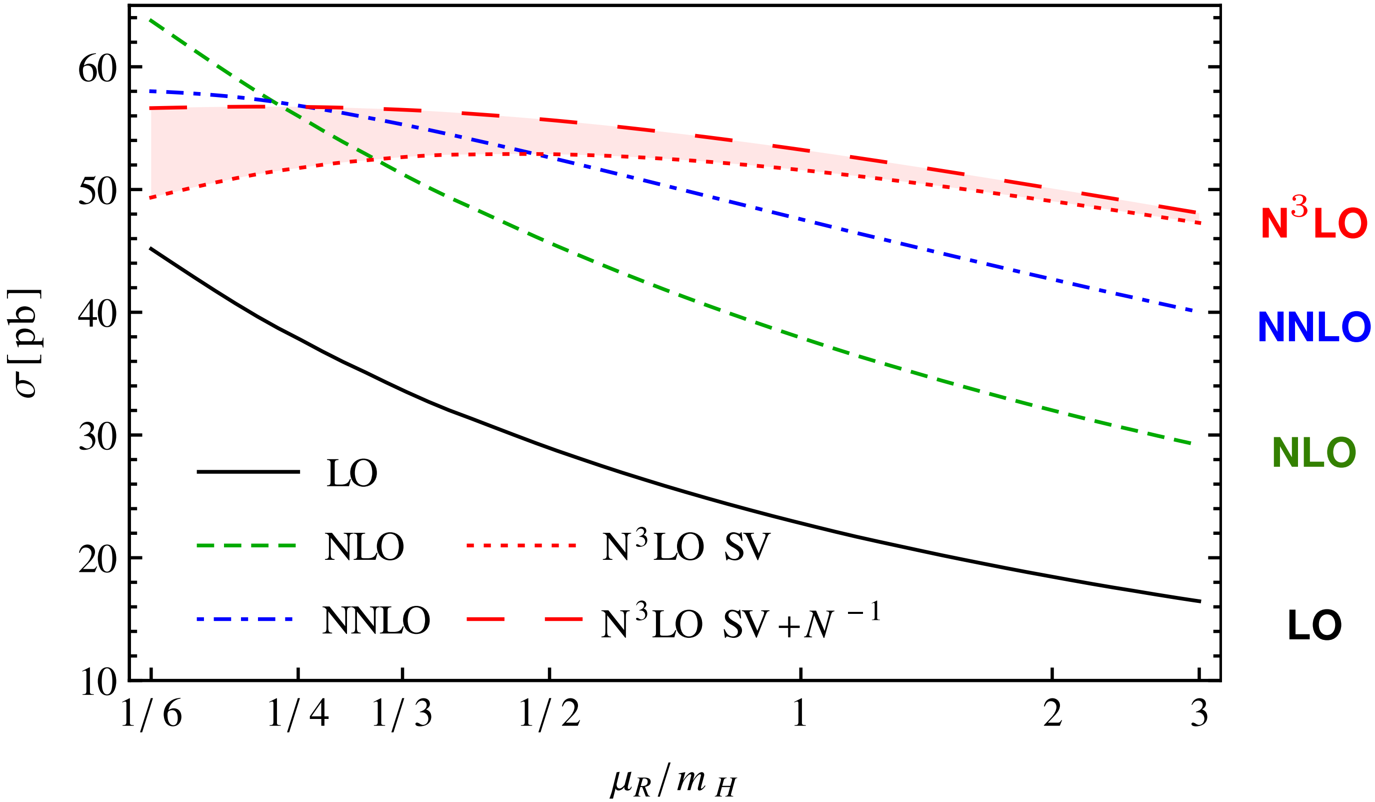


Factorization scale variation



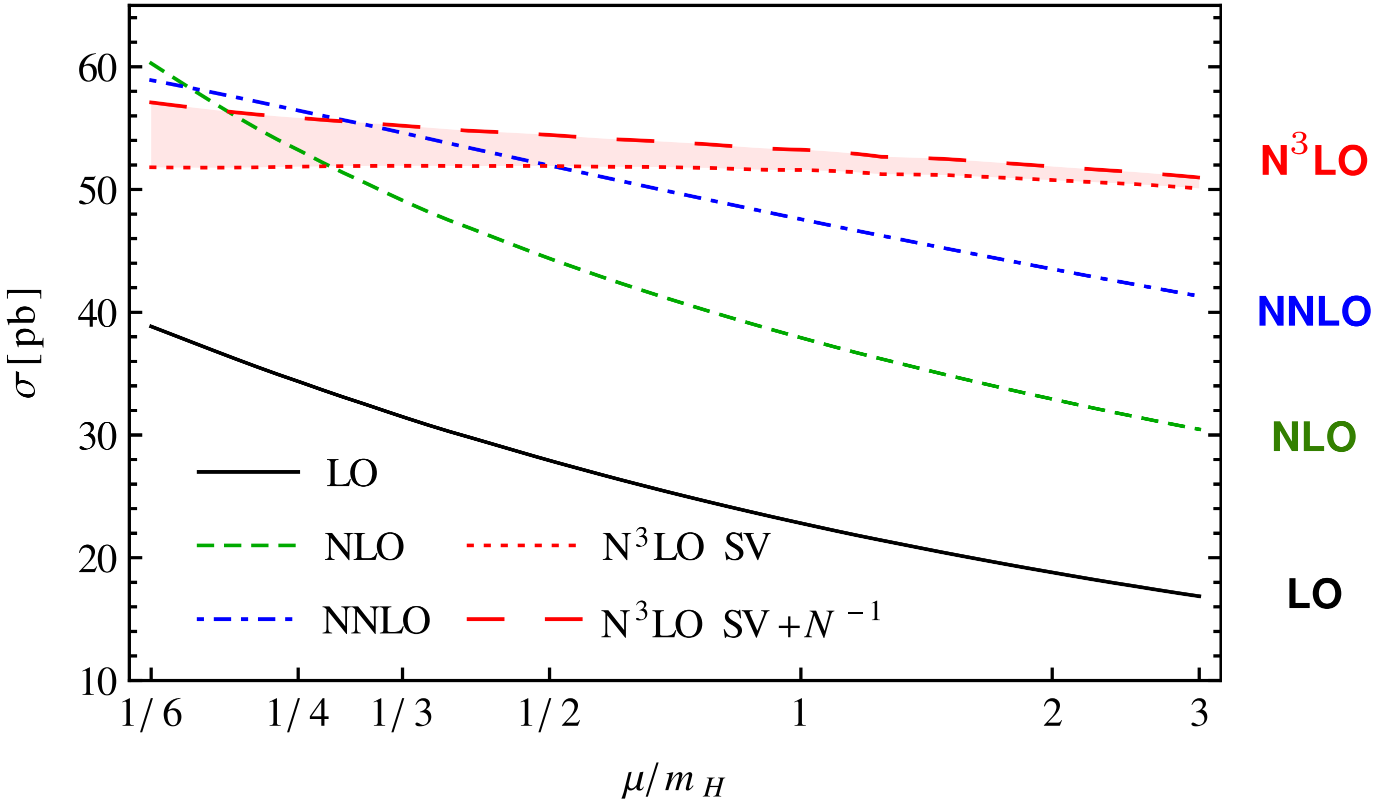
- Variation of factorization scale μ_F for fixed $\mu_R = m_H$ at $\sqrt{s} = 14$ TeV
- Dependence on μ_F at **N^{3}LO** larger than at **NNLO**
  - missing four-loop splitting functions $P_{ij}^{(3)}$
  - gluon PDF only known up to **NNLO**
  - omission of the quark-gluon and quark-(anti)quark channels

Renormalization scale variation



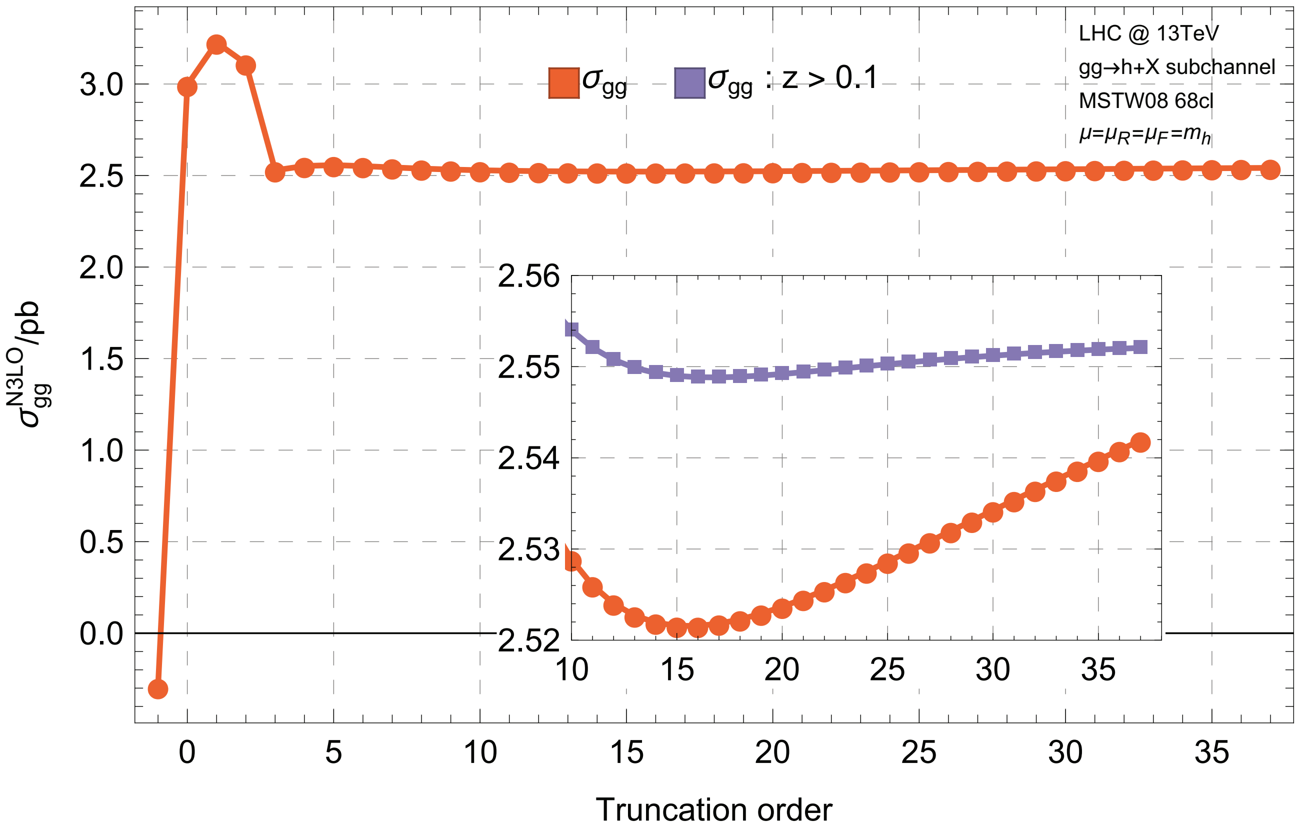
- Variation of renormalization scale μ_R for fixed $\mu_F = m_H$ at $\sqrt{s} = 14$ TeV
- Perturbative stability under renormalization scale variation
  - dependence on μ_R flattens off at **N^{3}LO**
  - four-loop beta function β_3 known
- Point of minimal sensitivity around $\mu_R = m_H/2$

Combined scale variation



- Simultaneous variation of scales $\mu_R = \mu_F$ around m_H at $\sqrt{s} = 14$ TeV
- Small improvement with respect μ_R variation for fixed μ_F
  - should not be trusted quantitatively
- Conservative error estimate: variation of μ_R for fixed μ_F
  - overall theoretical uncertainty $\Delta\sigma(H)$ less than 5%

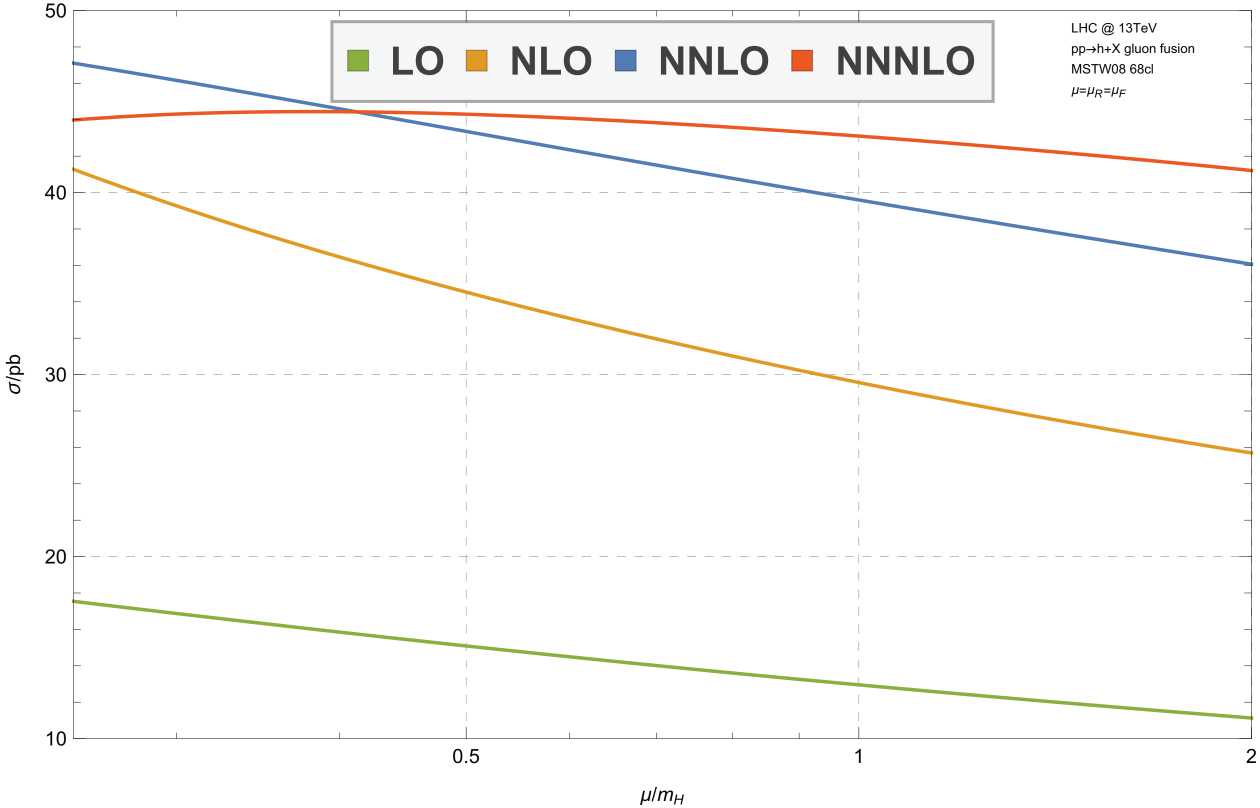
Exact N^{3}LO: cross section



Anastasiou, Duhr, Dulat, Herzog, Mistlberger ‘15

- N^{3}LO prediction from expansion around threshold in x-space
- Numerical convergence slow due to well-known oscillation of successive terms in x-space

Exact N^{3}LO: cross section

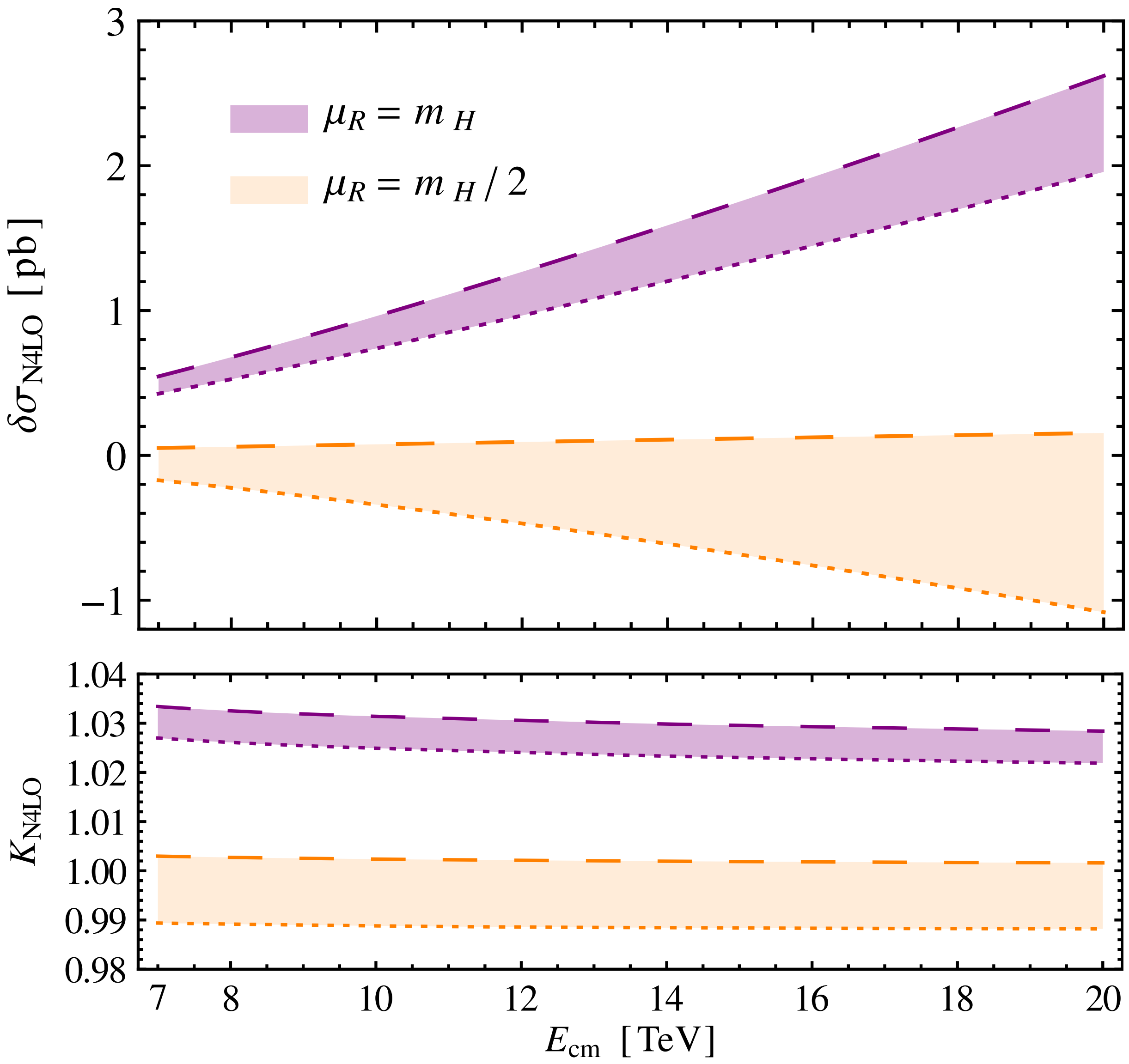


Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15

- Scale dependence of exact N^{3}LO prediction confirms the patterns previously observed
  - minimal sensitivity at scale $\mu = m_H/2$
  - residual scale uncertainty 3%

Approximate N^4LO: cross section

- Consistency check with approximate N^4LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$
- K-factor $\simeq 1\%$ for $\mu = m_H/2$ with at $\sqrt{s} = 13$ TeV



Parton luminosity

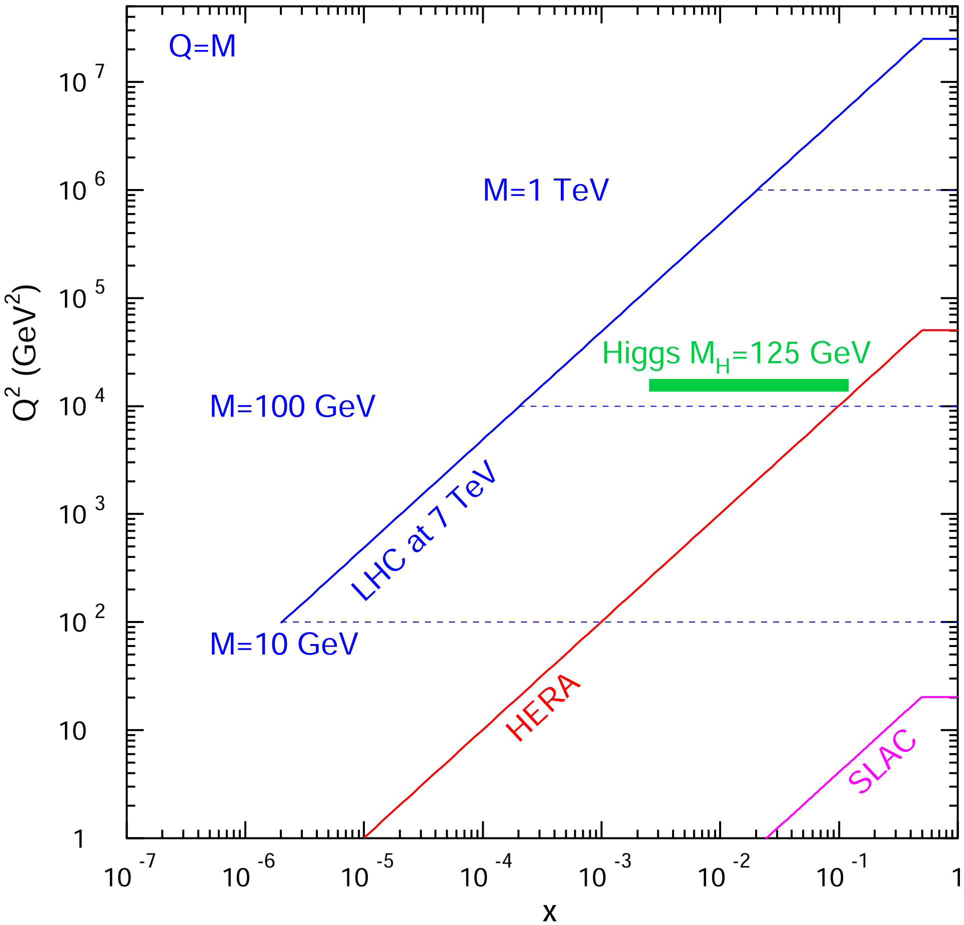
- Long distance dynamics due to proton structure

- Cross section depends on parton distributions f_i

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \Big[\ldots \Big]$$

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO
  - information on proton structure depends on kinematic coverage

Parton kinematics at LHC



- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics at effective $\langle x \rangle = M/\sqrt{S}$
  - 100 GeV physics: small-x, sea partons
  - TeV scales: large-x

Parton distribution fits

Global PDF fits

- PDF sets currently available
  - ABM12 Alekhin, Blümlein, S.M. '13
  - CT10 Gao et al. '13
  - MSTW Martin, Stirling, Thorne, Watt '09
    (now: MMHT Martin, Motylinski, Harland-Lang, Thorne '15)
  - NNPDF (NN23) Ball et al. '12 (now: NN30 Ball et al. '14)

Iterative cycle of PDF fits

i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of the non-perturbative parameters
  - parton distributions
  - strong coupling $\alpha_s(M_Z)$
  - heavy quark masses

LHC measurements

General remarks

- QCD corrections important
  - require theory predictions to NNLO accuracy
- PDF fits with 3-flavors for DIS, 5-flavors for LHC data (matching from 3 to 5-flavors)
  - QCD evolution over large range

Benchmark processes

- Complete NNLO QCD corrections available for
  - $W^{\pm}$- and Z-boson production
    Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - hadro-production of top-quark pairs Czakon, Fiedler, Mitov '13
  - single top-quark production (t-channel) Brucherseifer, Caola, Melnikov '14
- Jet data from Tevatron and LHC
  - QCD corrections only NLO known
  - possible impact of jet definition and algorithm
  - ongoing effort towards NNLO (corrections expected to be as big as $\mathcal{O}(15-20\%)$) Gehrmann-De Ridder, Gehrmann, Glover, Pires '13

Example: ABM PDFs

Data considered in the fit

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC
  - semi-inclusive DIS charm production data (HERA)
  - Drell-Yan data (fixed target) E-605, E-866
  - neutrino-nucleon DIS data (di-muon production) CCFR/NuTeV
  - LHC data for $W^{\pm}$- and Z-boson production

Theory considerations

- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Determination of PDFs and strong coupling constant α_s to NNLO QCD
- Consistent scheme for treatment of heavy quarks
  - fixed-flavor number scheme for $n_f = 3, 4, 5$
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
- Full account of error correlations

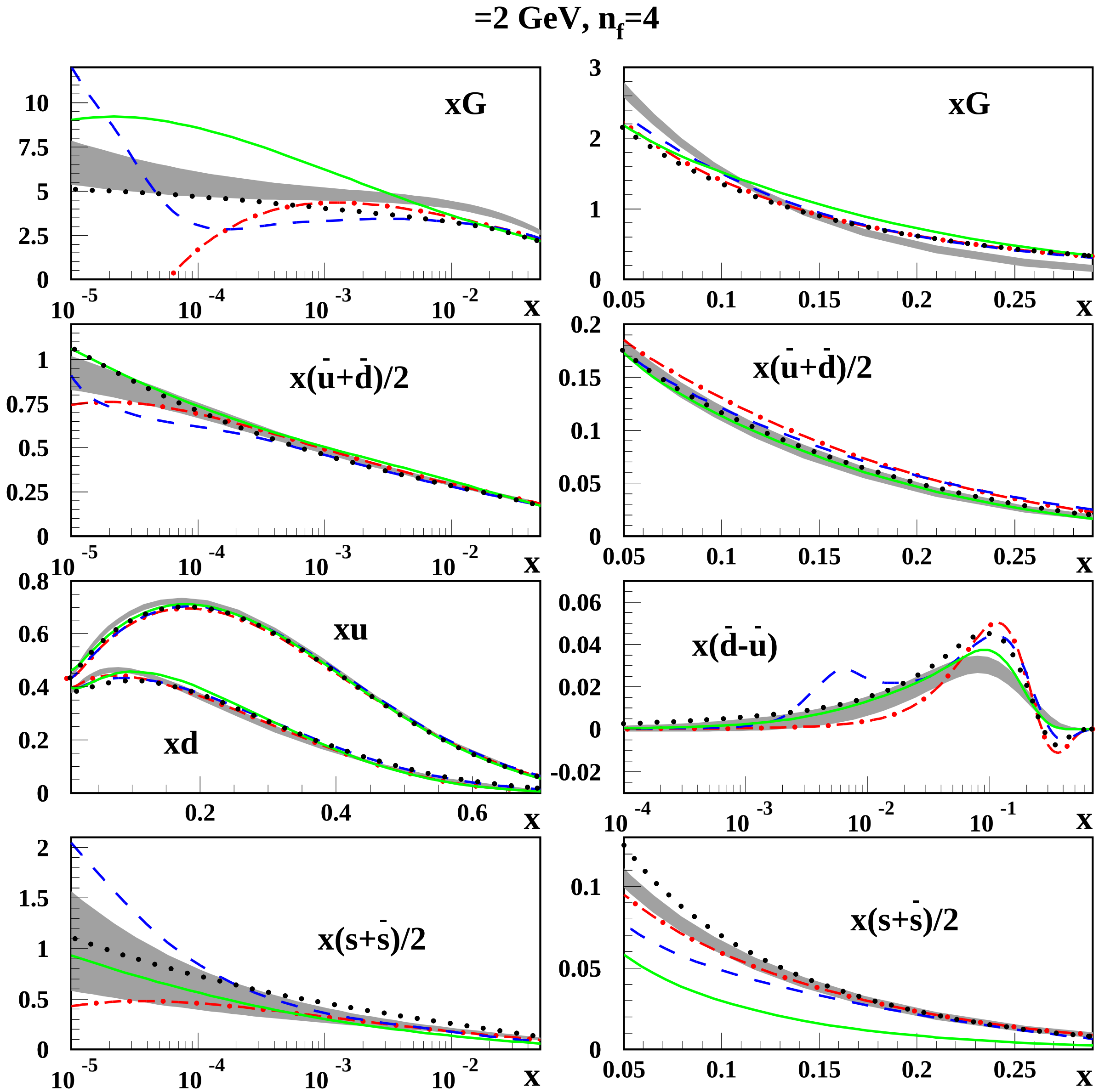
ABM PDF ansatz

- PDFs parameterization at scale $Q_0 = 3\mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M. '12
  - ansatz for valence-/sea-quarks, gluon with polynomial $P(x)$
  - strange quark is taken in charge-symmetric form
  - 24 parameters in polynomials $P(x)$
  - 4 additional fit parameters: $\alpha_s^{(n_f=3)}(\mu = 3$ GeV), m_c, m_b and deuteron correction
  - simultaneous fit of higher twist parameters (twist-4)

$$
\begin{aligned}
xq_v(x, Q_0^2) &= \frac{2\delta_{qu} + \delta_{qd}}{N_q^v} x^{a_q} (1-x)^{b_q} x^{P_{qv}(x)} \\
xu_s(x, Q_0^2) = x\bar{u}_s(x, Q_0^2) &= A_{us} x^{a_{us}} (1-x)^{b_{us}} x^{a_{us} P_{us}(x)} \\
x\Delta(x, Q_0^2) = xd_s(x, Q_0^2) - xu_s(x, Q_0^2) &= A_\Delta x^{a_\Delta} (1-x)^{b_\Delta} x^{P_\Delta(x)} \\
xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) &= A_s x^{a_s} (1-x)^{b_s} , \\
xg(x, Q_0^2) &= A_g x^{a_g} (1-x)^{b_g} x^{a_g P_g(x)}
\end{aligned}
$$

- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

Parton distributions tuned to LHC data



- 1σ band for ABM12 PDFs (NNLO, 4-flavors) at $\mu = 2\ \mathrm{GeV}$
  Alekhin, Blümlein, S.M.'13
- comparison with:
  JR09 (solid lines),
  MSTW (dashed dots),
  NN23 (dashes) and
  CT10 (dots)
- Some interesting observations to be made . . .

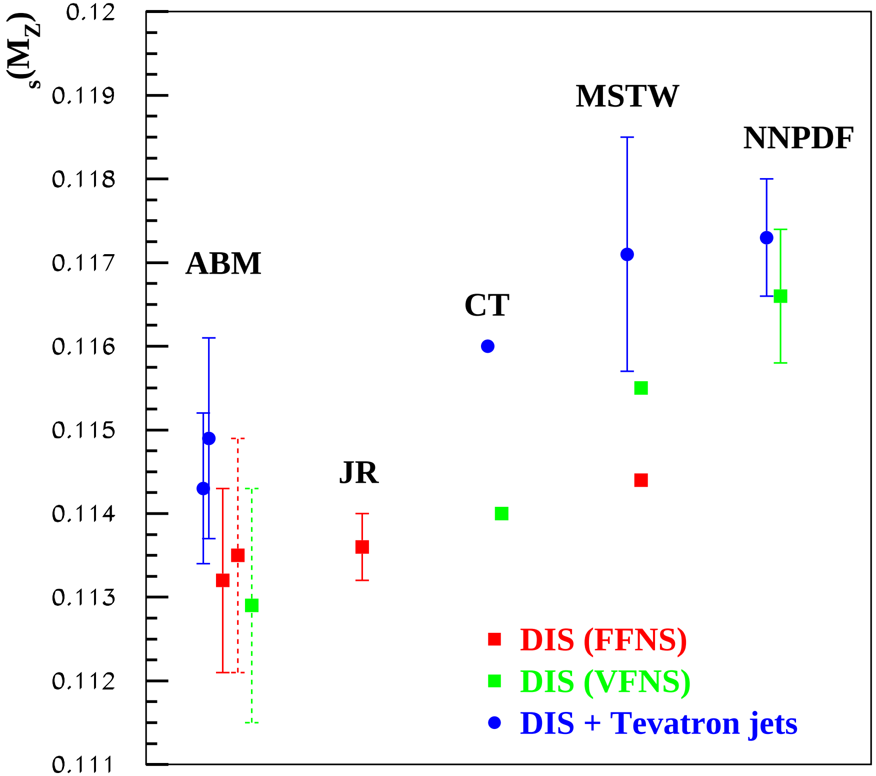
Dependence on parton distributions

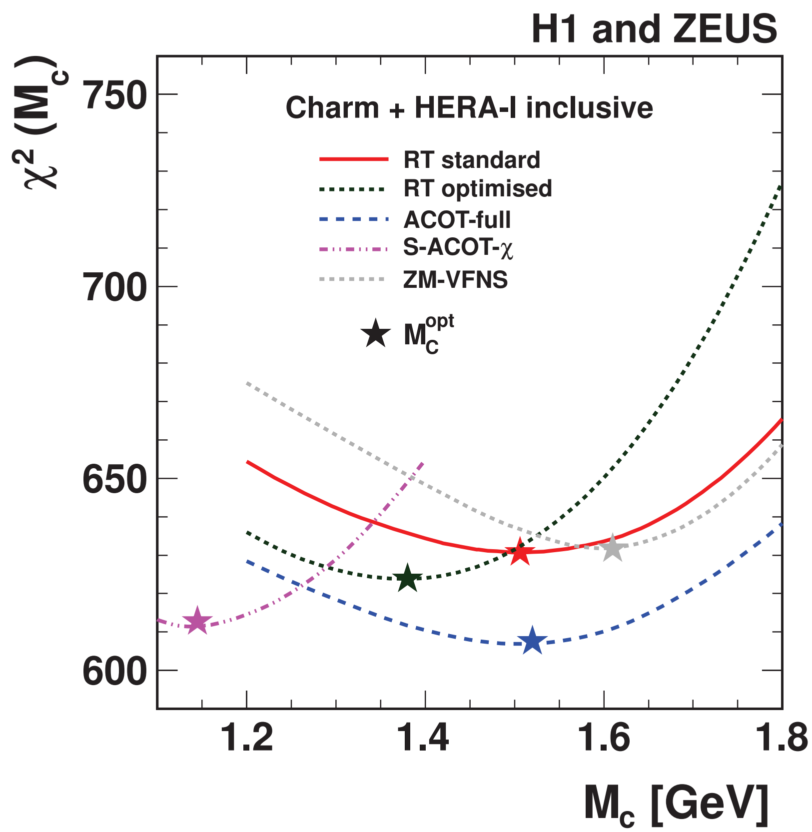
- Cross section $\sigma(H)$ at NNLO with uncertainties: $\sigma(H) + \Delta\sigma(\mathrm{PDF} + \alpha_\mathrm{s})$

ABM11	ABM12	CT10	MSTW	NN23
39.58 ± 0.77	39.70 ± 0.84	$41.84\ ^{+1.30}_{-1.69}$	$42.12\ ^{+0.44}_{-0.63}$	43.75 ± 0.41

- Comparison for PDF sets at NNLO
  - ABM11, ABM12 Alekhin, Blümlein, S.M. '13, CT10 Gao et al. '13, MSTW Martin, Stirling, Thorne, Watt '09, NNPDF (NN23) Ball et al. '12
- Large spread for predictions from different PDFs $\sigma(H) = 39.6 \ldots 43.8$
- PDF and α_s differences between sets amount to up to 10%
  - significantly larger than residual theory uncertainty due to incomplete N^{3}LO QCD corrections
- Observed spread due to differences in theory considerations and analysis procedures $\longrightarrow$ correlations between α_s, $g(x)$ and m_q
  - target mass corrections and higher twist in DIS
  - treatment of heavy quarks
  - error correlations among data sets
  - fits to compatible data sets

Non-perturbative parameters





- Differences in α_s values due to different physics models for heavy quarks
- Charm mass m_c “tuning” parameter for variable flavor number schemes
  H1 coll. arxiv:1211.1182
- Effect of Higgs cross section
  - linear rise in $\sigma(H) = 40.6 \ldots 43.8$ for $m_c = 1.05 \ldots 1.75$ with MSTW PDFs Martin, Stirling, Thorne, Watt ‘10

Summary

Inclusive Higgs cross section at the LHC

- Radiative corrections at higher orders in QCD well under control
  - N^{3}LO in QCD with overall theoretical uncertainty $\Delta\sigma(H)$ less than 4%
  - good stability around point of minimal sensitivity $\mu = m_H/2$
  - conservative error estimate: variation of μ_R for fixed μ_F

Parton distributions, α_s and all that

- Precision determinations of non-perturbative parameters is dominating source of uncertainty
  - parton content of proton (PDFs)
  - coupling constants $\alpha_s(M_Z)$
  - masses m_c, m_b, m_t, m_H, ...
- Spread in predictions amounts currently up to 10%