High multiplicity QCD at NLO

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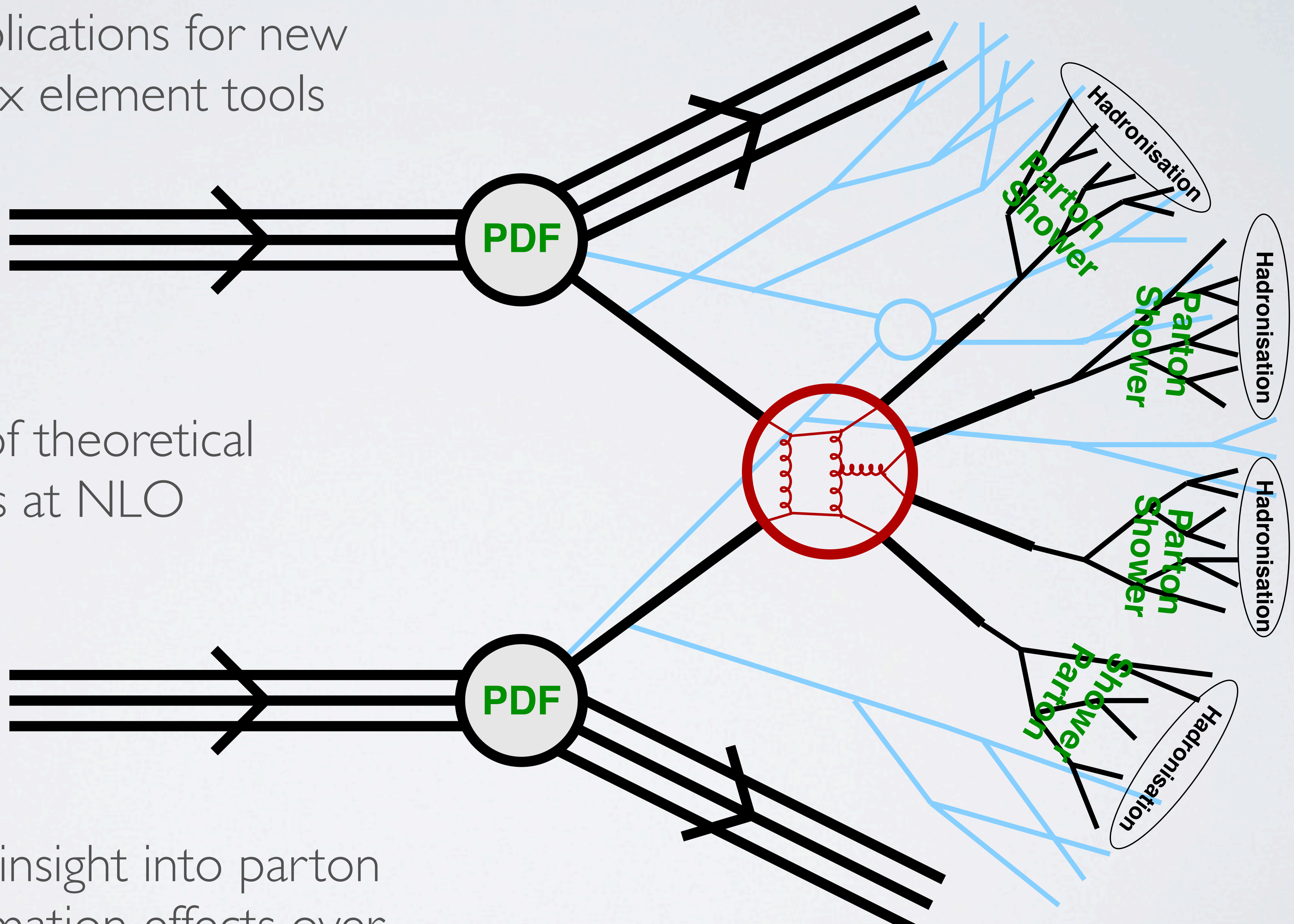
Parton showers, Event generators and Resummation 2015
Kraków, Poland

Modelling hadron collisions

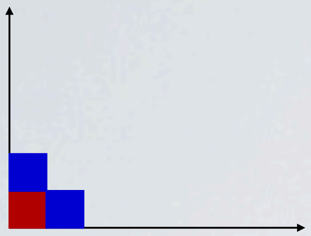
wide range of applications for new automated matrix element tools

better control of theoretical uncertainties at NLO

gain improved insight into parton shower/resummation effects over many differential observables

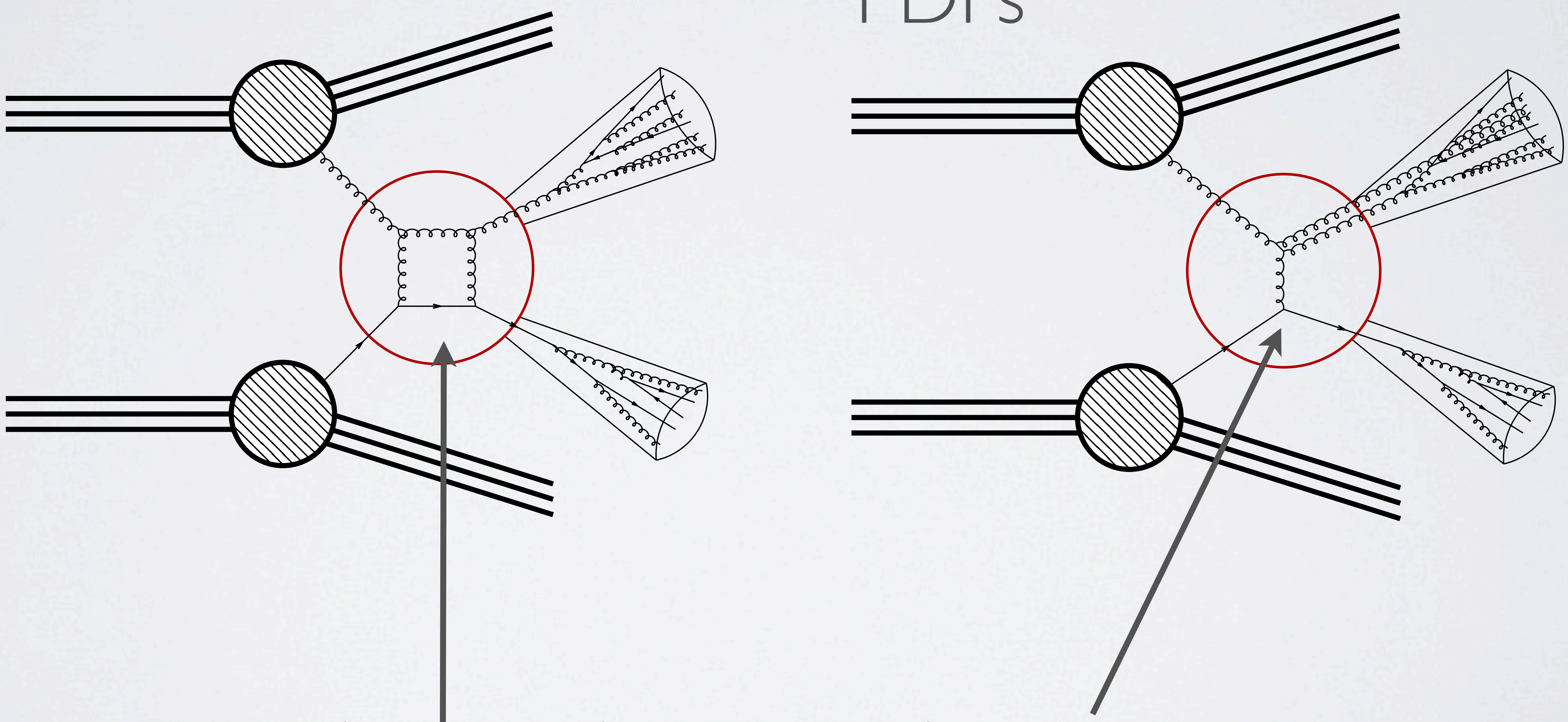


QCD at NLO



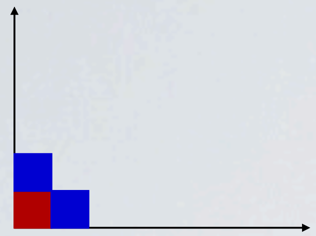
$$\sigma_{pp\to X} = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, Q)\, f_j(x_2, Q)\, \widehat{\sigma}^{NLO}_{ij\to n}$$

PDFs



$$\widehat{\sigma}^{\delta NLO}_{ij\to n} = \int_n \left(d\sigma^V + \int_1 d\sigma^{S_1(R)} \right) + \int_{n+1} \left(d\sigma^R - d\sigma^{S_1(R)} \right)$$

QCD at NLO



$$\sigma_{pp\to X} = \sum \int dx_1 dx_2 f_i(x_1, Q)\, f_j(x_2, Q)\, \widehat{\sigma}^{NLO}_{ij\to n}$$

challenges at high multiplicity

real radiation phase-space
extremely large - many channels

complicated matrix elements - slow/inaccuracte

large colour space

$\Rightarrow$ **sample dominant channels more frequently**

$\Rightarrow$ **re-use MC events as much as possible**

$\Rightarrow$ **reliable determination of numerical accuracy**

$$\widehat{\sigma}^{\delta NLO}_{ij\to n} = \int_n \left(d\sigma^V + \int_1 d\sigma^{S_1(R)} \right) + \int_{n+1} \left(d\sigma^R - d\sigma^{S_1(R)} \right)$$

State-of-the-art

BLACKHAT+SHERPA : W + 5 jets (leading colour)

Weinzierl et al. : Z + 5 jets (leading colour - total cross-section

NJET+SHERPA: 5 jets and di-photon + 3 jets (full colour)

Challenges are to a large extent solved

2→4

masses BSM

QCD/EW

HELAC-NLO

GOSAM

OPENLOOPS

AMC@NLO

RECOLA

Loop amplitudes

Efficient tree level generators well established

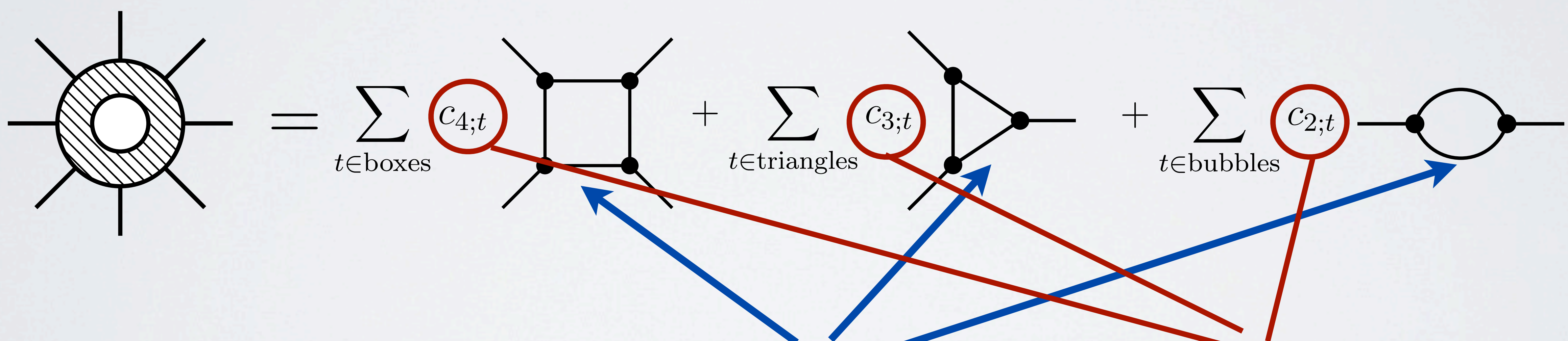
e.g. MadGraph, Alpgen, Comix, Helac,...

Tree Methods

Feynman diagrams

off-shell recursion

on-shell recursion (BCFW)



integral basis separates **analytic** and **algebraic** parts

known functions at one-loop

process independent

e.g. QCDLoop, OneLoop

tree-like

complex momenta

calculate numerically

Dealing with colour

$SU(N_c)$ colour matrices

$$\mathcal{A}_n^{(L)} = \sum_j c_j^{(L)} A_{n;j}^{(L)}$$

partial amplitude

$$\sum_{\text{colours}} \left(c_i^{(L_1)}\right)^T c_j^{(L_2)} = \mathcal{C}_{ij}^{(L_1,L_2)}(N_c)$$

$$d\sigma^{\mathrm{B}}(s) = d\Phi \sum_{i,j} [A_{n;j}^{(0)}(s)]^\dagger \mathcal{C}_{ij}^{(0,0)} A_{n;j}^{(0)}(s) \qquad d\sigma^{\mathrm{V}}(s) = d\Phi \sum_{i,j} [A_{n;j}^{(0)}(s)]^\dagger \mathcal{C}_{ij}^{(0,1)} A_{n;j}^{(1)}(s)$$

partial amplitudes are a linear combination of primitive amplitudes

$$A_{n;j}^{(L)} = \sum_k a_{k,j} A_n^{[m]} + b_{k,j} A_n^{[f]}$$

Dealing with colour

Feynman diagram matching algorithm

Ellis, Kunszt, Melnikov, Zanderighi [1105.4319]

Ita, Ozeren [1111.4193]

SB, Biedermann, Uwer, Yundin [1209.0100]

diagrams

$$A_{n;j}^{(L)} = \sum_{i=1}^{N} D_i = \sum_{i=1}^{\widehat{N}} C_i K_i$$

invert to find independent set of primitives

$$P_i = \sum_{j=1}^{\widehat{N}} M_{ij} K_i$$

matching matrix {0,+1,-1}

match topology only

4-gluon vertex not needed

diagrams symmetries reduce independent set of primitives (e.g. Furry's theorem)

combinatorical approaches: Melia [1304.7809,1312.0599]; Schuster [1311.6296]; Weinzierl, Reuschle [1310.0413]

Dealing with colour

Process	$N_{\text{pri}}^{[0]}$	$N_{\text{pri}}^{[m]}$	$N_{\text{pri}}^{[f]}$
$8\,g$	720	2520	2520
$\overline{u}u + 6\,g$	720	5040	1800
$\overline{u}u\overline{d}d + 4\,g$	360	3360	671
$\overline{u}u\overline{d}d\overline{s}s + 2\,g$	120	1344	194
$\overline{u}u\overline{d}d\overline{s}s\overline{c}c$	30	384	65

large numbers of primitive amplitudes for high multiplicity

use phase space symmetry to reduce computational cost

$$\sigma^V_{gg\to n(g)} = \int \mathrm{d}PS_n \, A^{(0)\dagger} \cdot \mathcal{C}_{n!\times(n+1)!/2} \cdot A^{(1)}$$

$$= (n-2)! \int \mathrm{d}PS_n \, A^{(0)\dagger} \cdot \mathcal{C}^{\text{dsym}}_{n!\times(n+1)} \cdot A^{(1),\text{dsym}}$$

factor of **(final state gluons)!/2**

retain full colour information

NJET framework

Numerical implementation in C++

Trees	off-shell recursion (Berends-Giele)
Loops	generalized unitarity / analytic formulae
Colour	full (via primitive matching), de-symmetrized, leading/sub-leading
Interface	Binoth Les Houches Accord (Python)

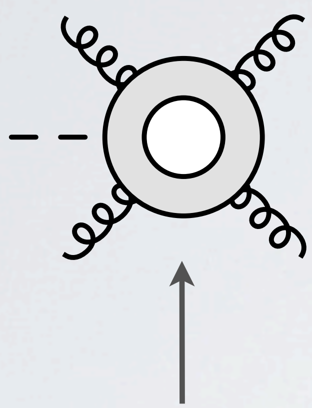
accuracy: switch to higher precision

building on NGLUON [1011.2900]

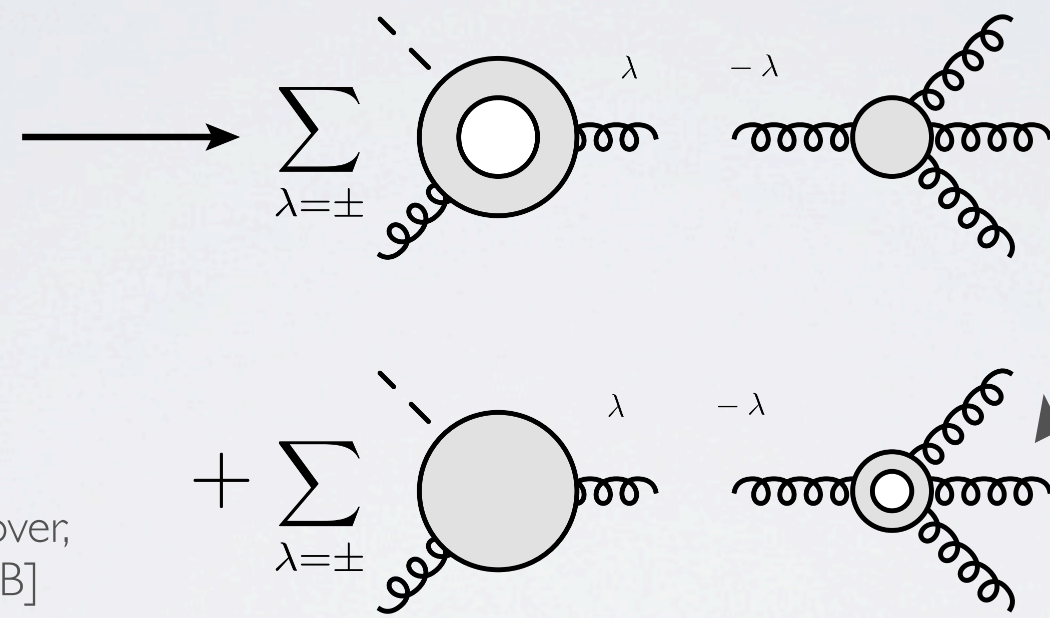
SB, Biedermann, Uwer, Yundin [1209.0100]

Application: triple collinear limits

[SB, Buciuni, Gardi, Peraro (in prep.)]



[Berger, Dixon, Del Duca, Sofianatios, Ellis, Campbell, Glover, Williams, Mastrolia, Risager, SB]



double unresolved virtual splitting functions contribute at N^3LO

Check universal factorization properties with NJET

Application: triple collinear limits

[qd Bailey, Hida, Li]

[OneLoop van Hameren]

$$\lim_{1||\cdots||m} A_n^{(L)}(\{p_i^{\lambda_i}\}) = \lim_{\delta\to 0} A_n^{(L)}(\{p_i^{\lambda_i}(\delta)\})$$

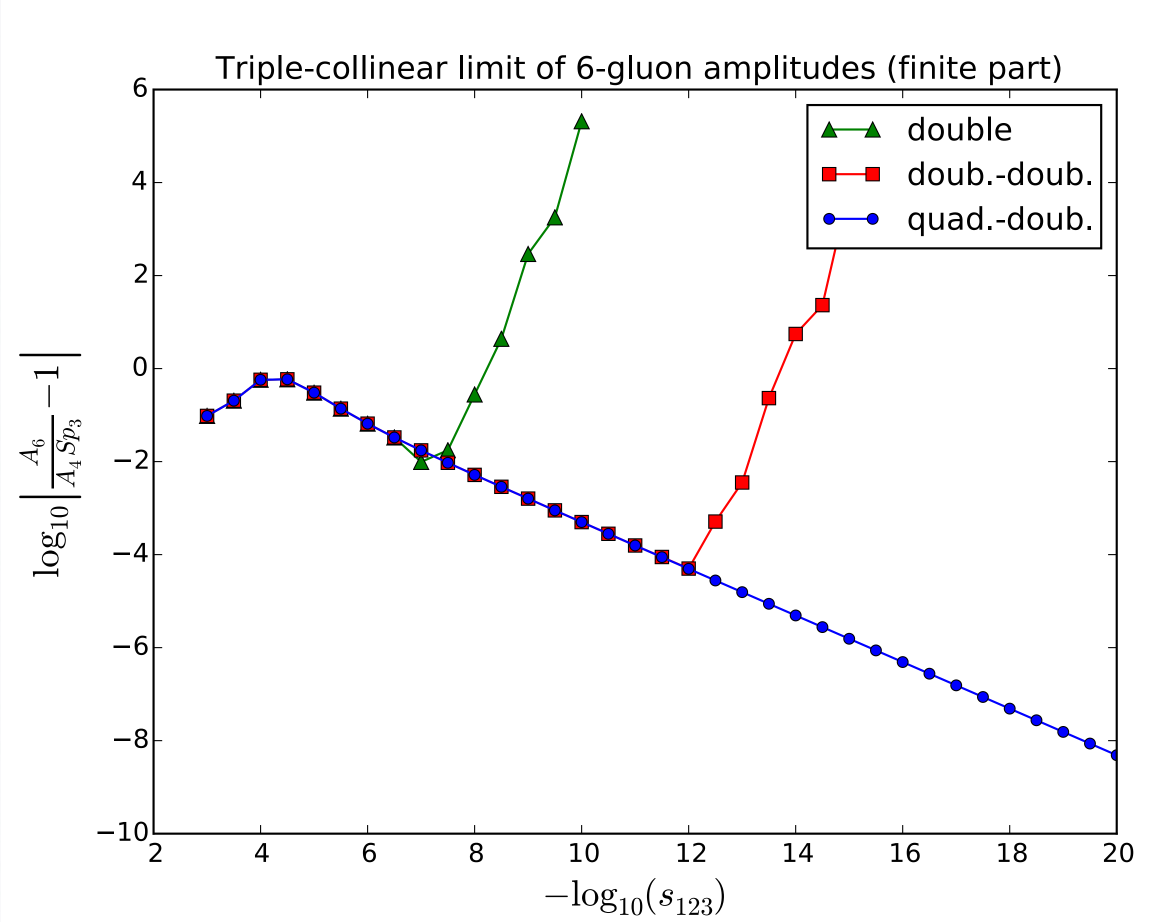
$$= \frac{1}{\delta^{m-1}} \sum_{k=0}^{L} \sum_{\lambda_P} \mathrm{Sp}_m^{(L-k)}(-P^{-\lambda_P}, \{p_i^{\lambda_i}\}_{i=1}^m) A_{n-m+1}^{(k)}(P^{\lambda_P}, \{p_i^{\lambda_i}\}_{i=m+1}^n) + \mathcal{O}\left(\frac{1}{\delta^{m-2}}\right)$$

Switch precision on the fly: explicit vectorization of scaling test with Vc

[Vc Kretz, Lindenstruth (2011)]

Numerically stable limit when switching to high precision

verified full colour limit of 6/7 gluon one-loop amplitudes



NJET + Sherpa

$$\sigma_n^{\mathrm{NLO}} = \sigma^{\mathrm{LO}} + \int_n d\sigma_n^{\mathrm{V}} + \int_n d\sigma_n^{\mathrm{I+fac.}} + \int_{n+1} \left(d\sigma_{n+1}^R - d\sigma_{n+1}^S\right)$$

NJET v2.0

leading/sub-leading colour
de-symmetrized colour sums

$pp \to \leq 5j$

$pp \to W^{[\to l^\pm \nu_l]} + \leq 5j$

$pp \to Z/\gamma^{*\ [\to l^+ l^-]} + \leq 5j$

$pp \to \gamma + \leq 4j$

$pp \to \gamma\gamma + \leq 4j$

Sherpa MC v2.0.0

Comix [Gleisberg, Hoeche (2008)]

CS subtraction [Gleisberg, Krauss (2007)]

ROOT Ntuple event generation

also:

FastJet [Caccari, Salam, Soyez (2008)]

LHAPDF [Whalley, Bourilkov, Group (2005)]

NJet + Sherpa

$$\sigma_n^{\text{NLO}} = \sigma^{\text{LO}} + \int_n d\sigma_n^{\text{V}} + \int_n d\sigma_n^{\text{I+fac.}} + \int_{n+1} \left(d\sigma_{n+1}^{R} - d\sigma_{n+1}^{S}\right)$$

NJE…

leading/su…

de-symm…

$pp \to \leq$ …

$pp \to W^{[\ldots]} + \leq 5j$

$pp \to Z/\gamma^{*\,[\to l^+ l^-]} + \leq 5j$

$pp \to \gamma + \leq 4j$

$pp \to \gamma\gamma + \leq 4j$

…0

…08)]

…007)]

…ration

pre-generated NTuples on EOS at CERN

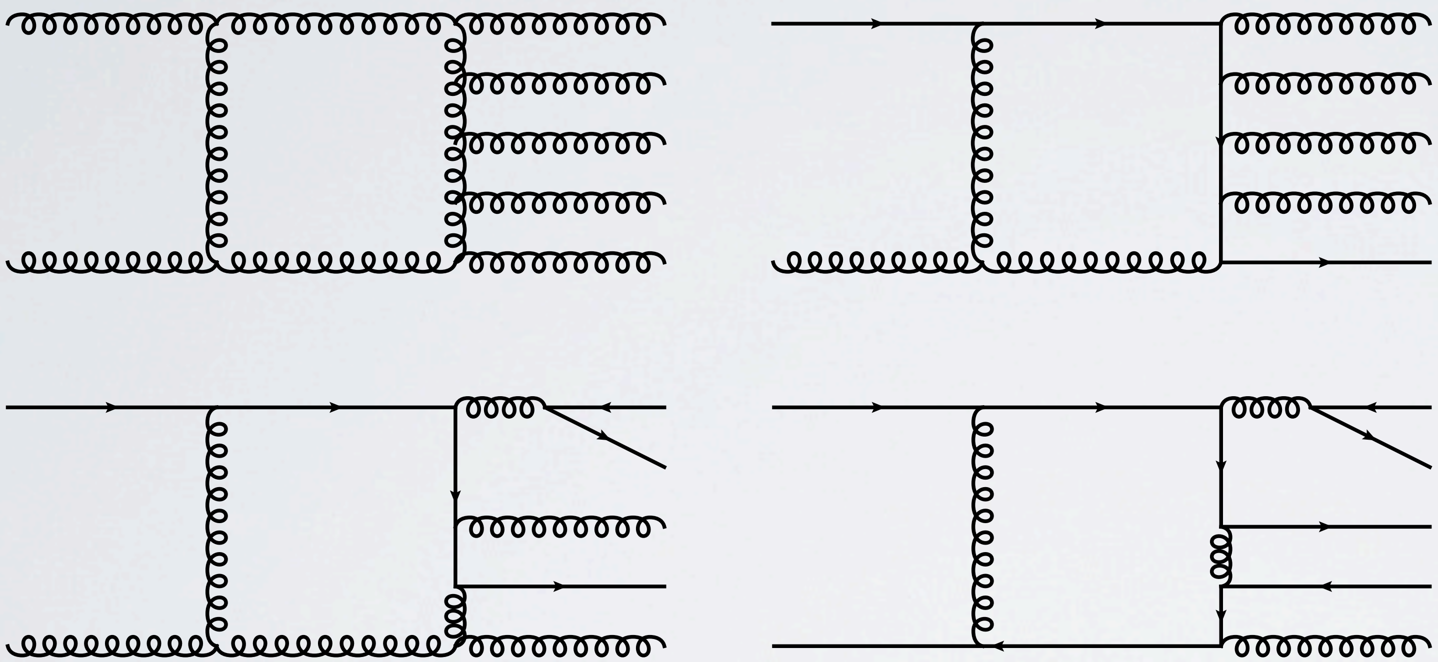
/eos/theory/user/b/badger

also:

FastJet [Caccari, Salam, Soyez (2008)]

LHAPDF [Whalley, Bourilkov, Group (2005)]

Multi-jet production at the LHC



full colour

$N_f = 5$ flavour scheme

no top loops (<1%)

ATLAS cuts [1107.2092]

$p_{T,j_1} > 80$ GeV $\quad$ $p_{T,j} > 60$ GeV $\quad$ $|\eta_j| < 2.8$

anti-kt $R = 0.4$ $\quad$ $\mu_R = \mu_F = \widehat{H}_T/2$

NLO QCD corrections

$pp \to \leq 3j$ $\quad$ Nagy (NLOJet++) [hep-ph/0307268]

$pp \to \leq 4j$ $\quad$ Bern et al. (BlackHat) [1112.3940]

SB, Biedermann, Uwer, Yundin [1209.0098]

$pp \to \leq 5j$ $\quad$ SB, Biedermann, Uwer, Yundin [1309.6585]

Virtual matrix elements

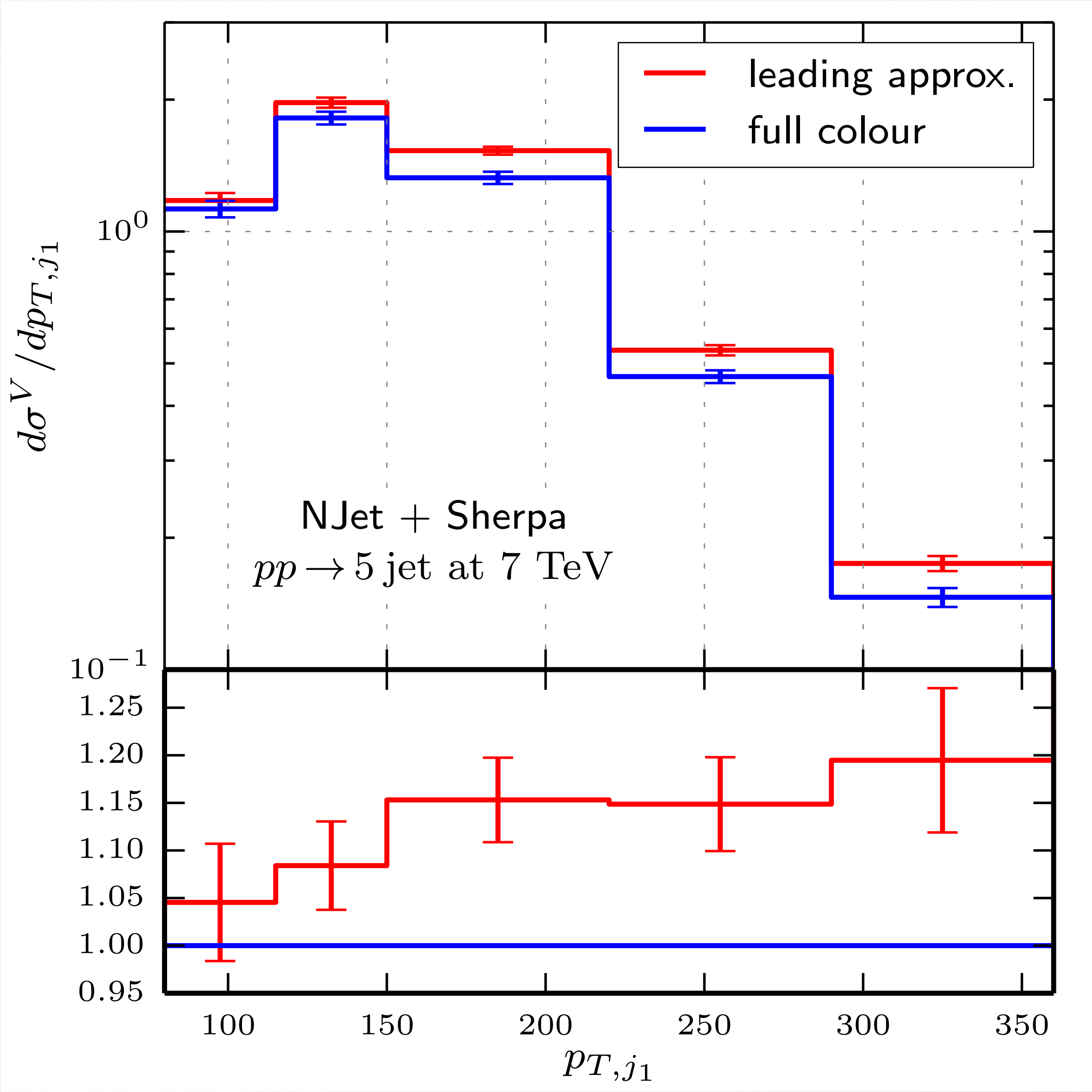
leading part: best case of de-sym. and leading colour

Virtual part	Time per event	QP	QP2	OP
leading	17 s	2%	0.5%	0.01%
subleading	112 s	2.5%	1%	0.05%

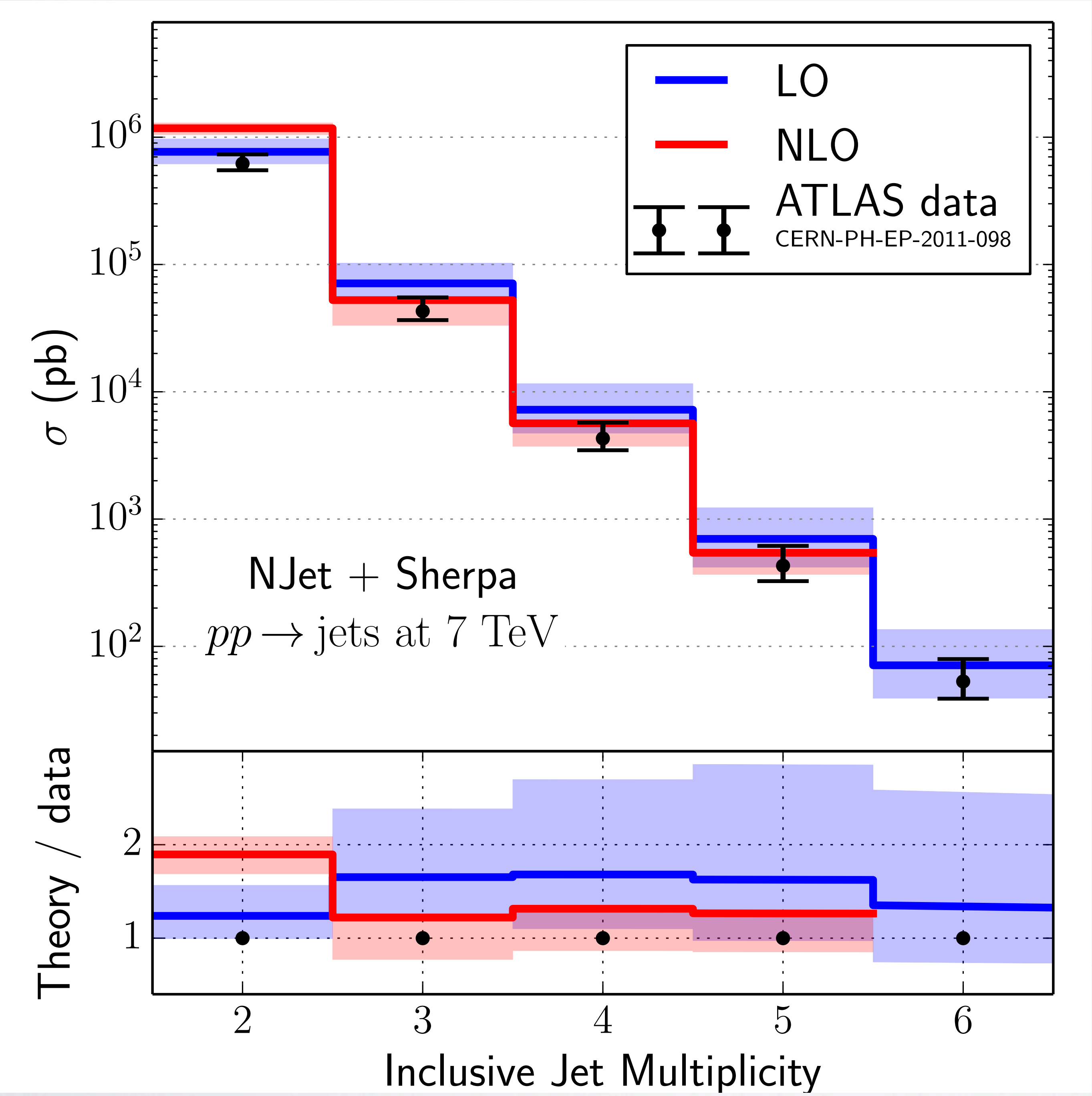
average evaluation time on cluster

possibility to switch to octuple precision - not necessary in practice

fermion loops included in sub-leading part



Total cross-sections



$$\sigma_5^{\text{7TeV-LO}}(\mu = \widehat{H}_T/2) = 0.699(0.004)^{+0.530}_{-0.280}\ \text{nb},$$
$$\sigma_5^{\text{7TeV-NLO}}(\mu = \widehat{H}_T/2) = 0.544(0.016)^{+0.0}_{-0.177}\ \text{nb}.$$

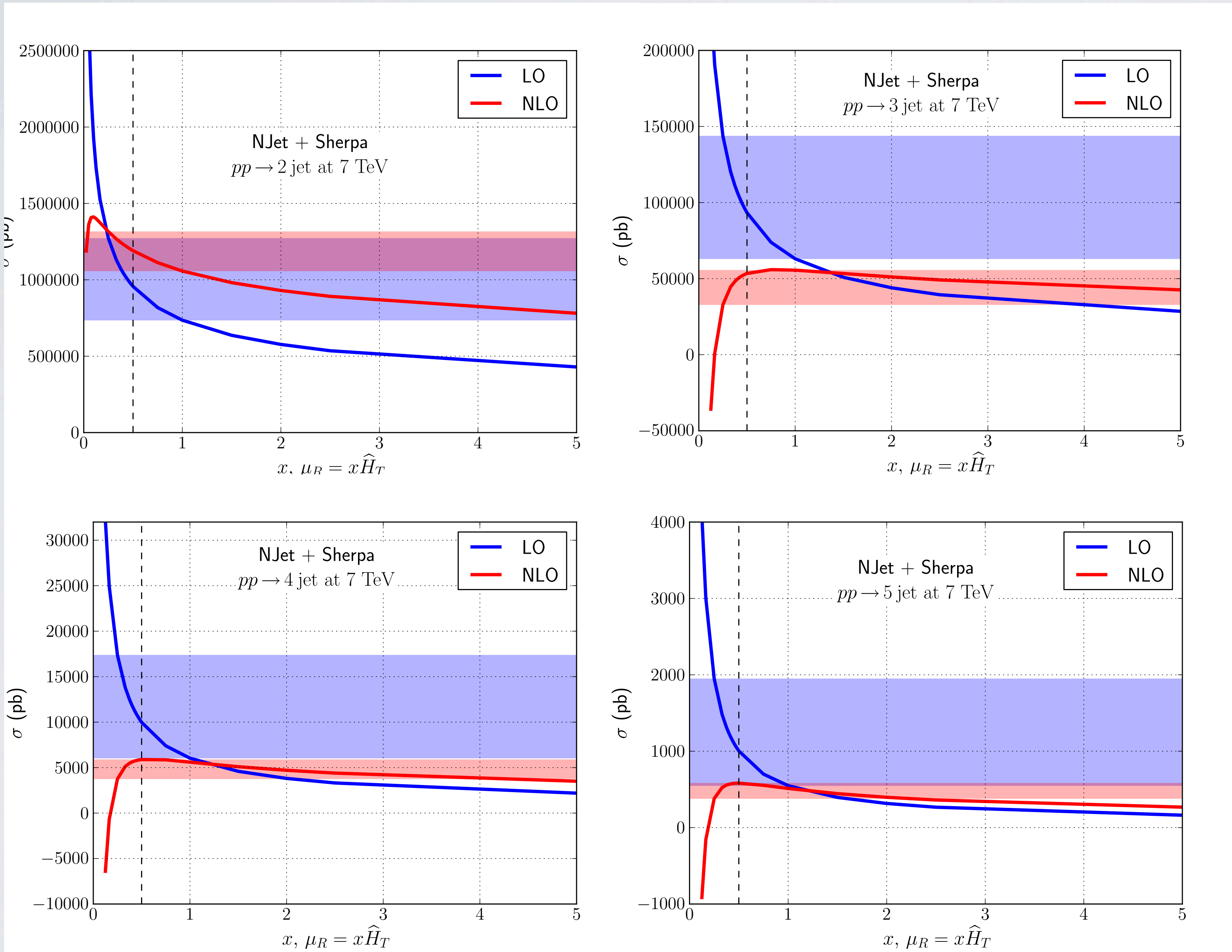
NNPDF2.1 LO

$\alpha_s(M_Z) = 0.119$

NNPDF2.3 NLO

$\alpha_s(M_Z) = 0.118$

Scale dependence



re-weighting ROOT Ntuples

LO PDF: MSTW2008lo

$$\alpha_s(M_Z) = 0.139$$

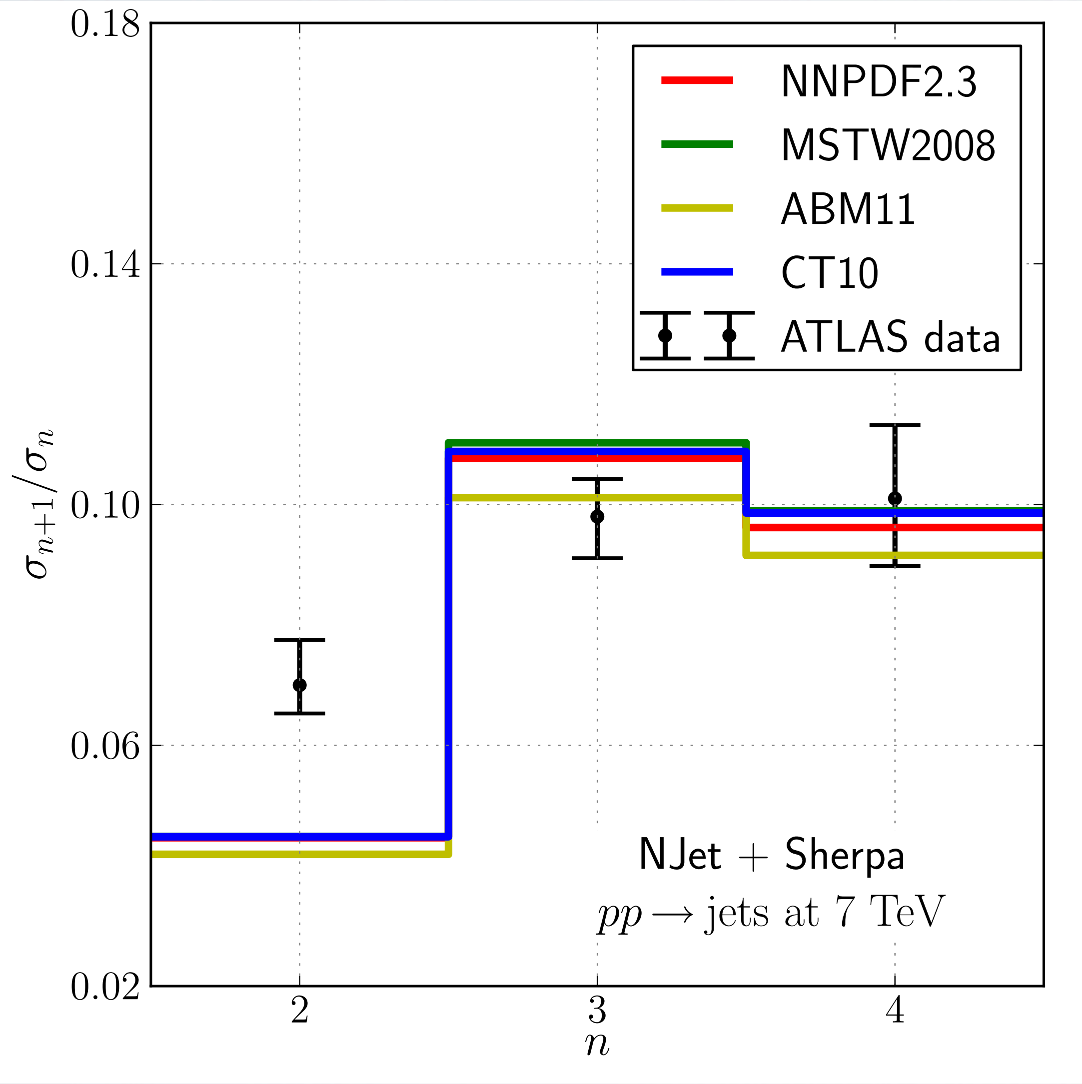
NLO PDF: MSTW2008nlo

$$\alpha_s(M_Z) = 0.120$$

Dynamical scale attempting include large logarithms

Jet Ratios

Reliable quantities for both theory and experiment



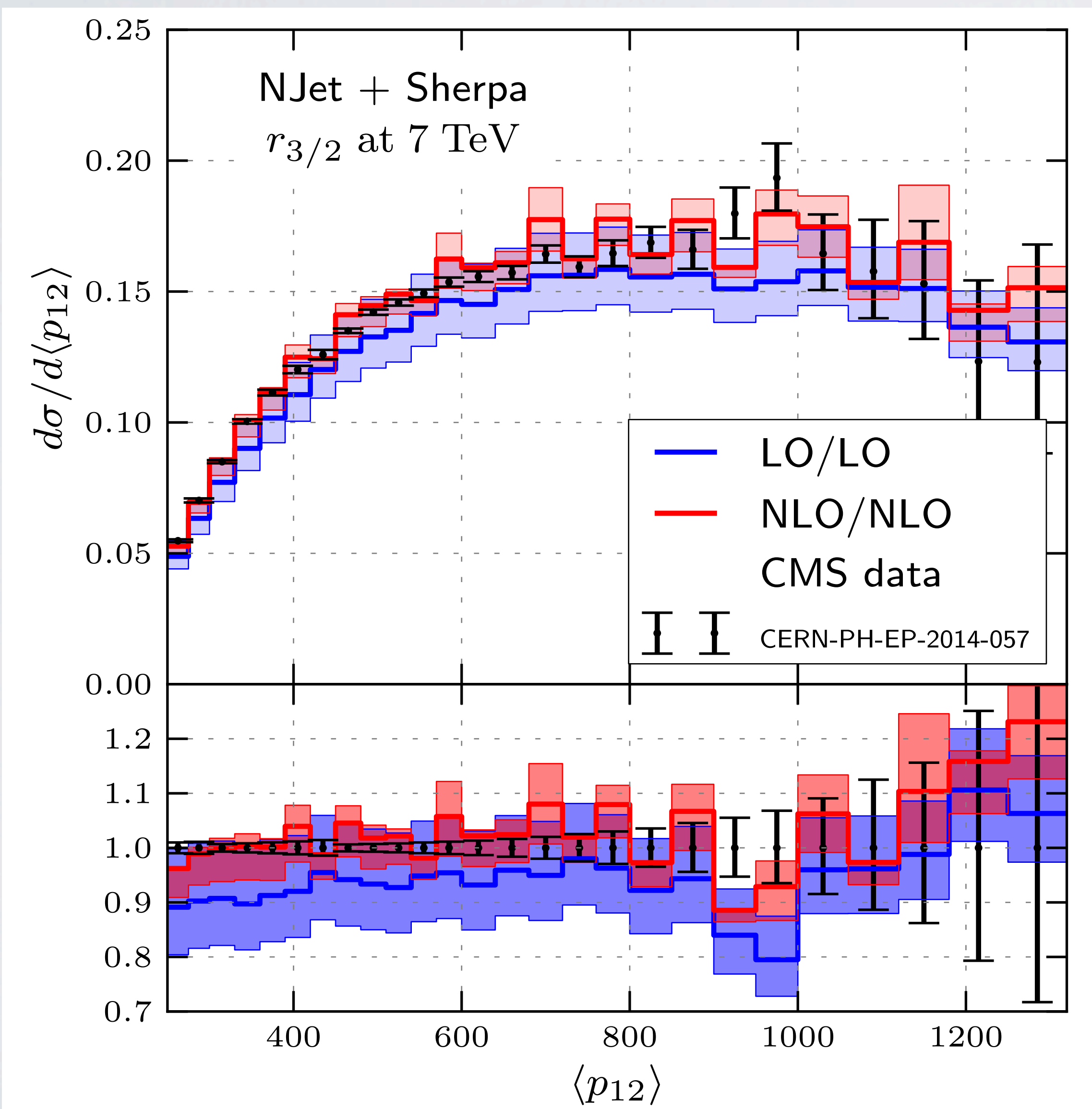
fixed order NLO not good for di-jets with asymmetric cuts

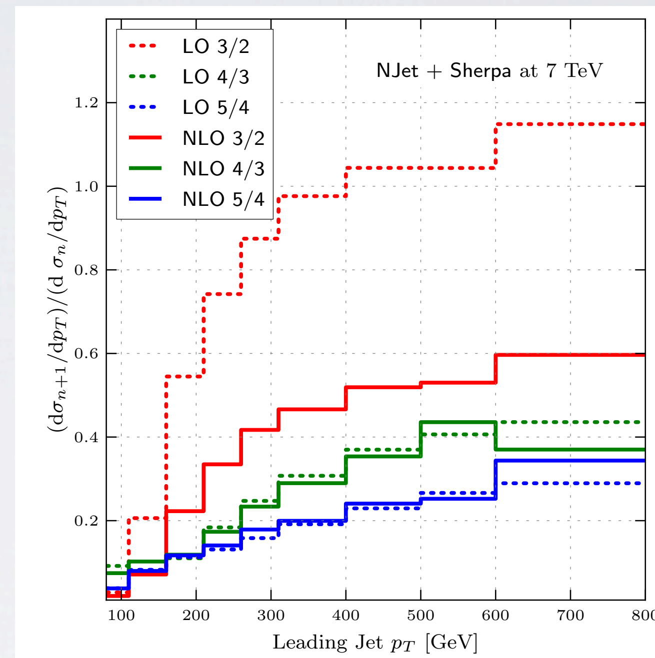
c.f. large NLO K-factors Rubin, Salam, Sapeta [1006.2144]

e.g. α_s determination [1304.7498; CMS-QCD-11-003]

$$\alpha_S(M_Z) = 0.1148 \pm 0.0014\,(\text{exp.}) \pm 0.0018\,(\text{PDF}) \pm 0.0050\,(\text{theory})$$

Jet Ratios





$R_{3/2}$ in leading jet p_T has large NLO corrections

$R_{4/3}$ and $R_{5/4}$ are more stable

average p_T of two leading jets quite stable to perturbative corrections

e.g. α_s determination
[1304.7498; CMS-QCD-11-003]

$$\alpha_S(M_Z) = 0.1148 \pm 0.0014\,(\text{exp.}) \pm 0.0018\,(\text{PDF}) \pm 0.0050\,(\text{theory})$$

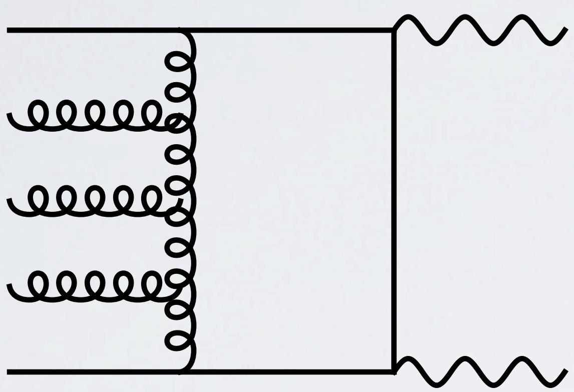
Di-photon plus jets

Backgrounds to Higgs measurements $pp \to H \to \gamma\gamma$

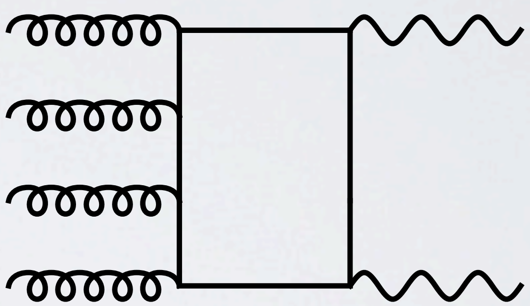
$pp \to \gamma\gamma$	NNLO	Catani, Cieri, de Florian, Ferrera, Grazzini [1110.2375]
$pp \to \gamma\gamma + 1j$	NLO	Gehrmann, Greiner, Heinrich [1303.0824] Del Duca, Maltoni, Nagy, Trocsanyi [hep-ph/0303012]
$pp \to \gamma\gamma + 2j$	NLO	Gehrmann, Greiner, Heinrich [1308.3660] Bern, Dixon, Febres Cordero, Hoeche, Ita, Kosower, Lo Presti, Maitre [1312.0592, 1402.4127]
$pp \to \gamma\gamma + 3j$	NLO	SB, Guffanti, Yundin [1312.5927]

$pp \to \gamma\gamma + \text{jets}$

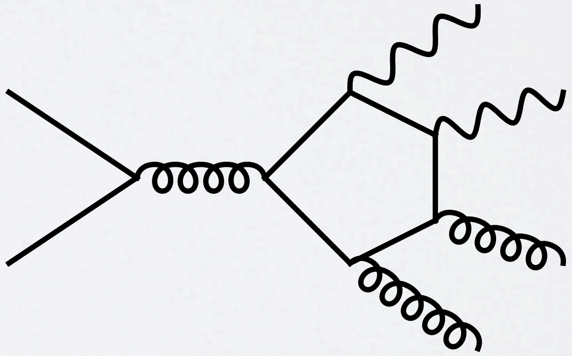
dominant channels - split into leading and sub-leading colour



gluon only channels ~ 2.5% for 2 jets [Bern et al 1402.4127]



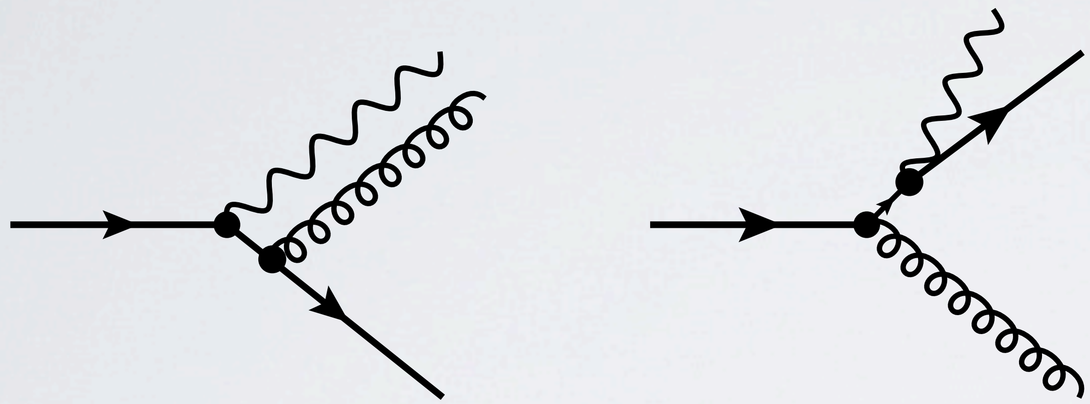
vector loops ~ 0.5% for 2 jets



cut dependent!

Isolating hard photons

[Frixione (1998)]



Infra-red safe definition of a hard photon must include QCD partons

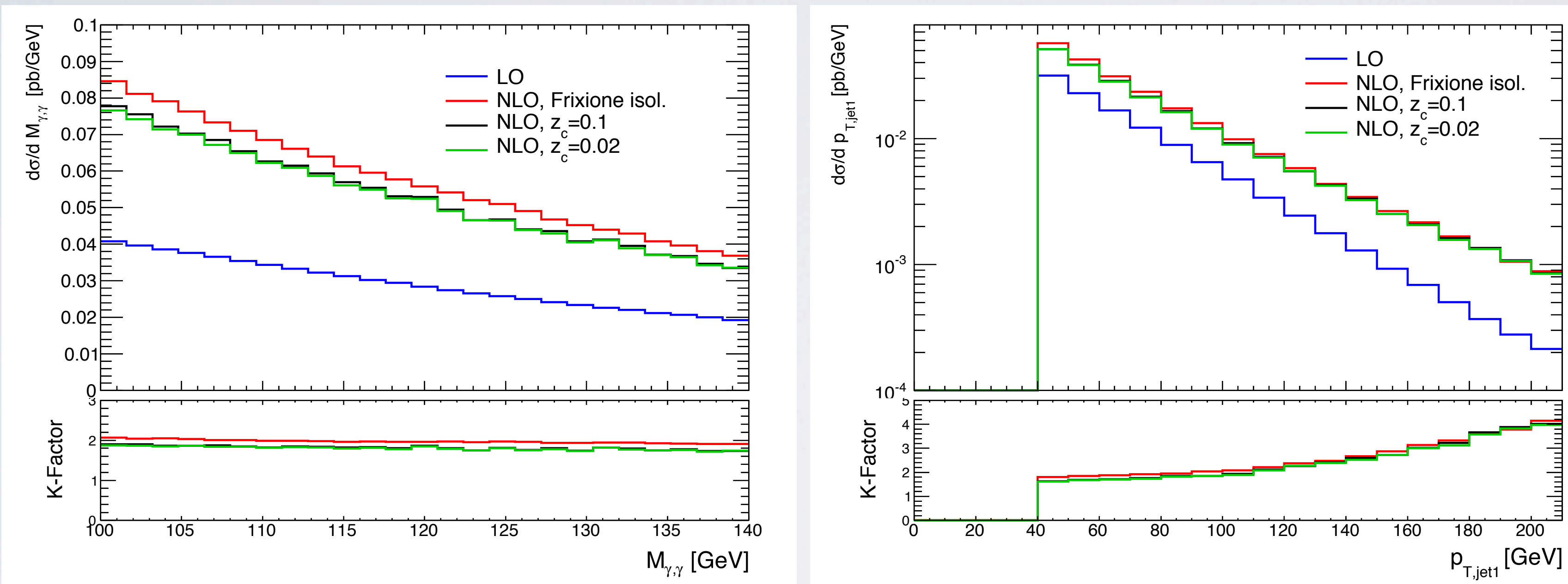
Smooth cone isolation

- keep soft gluons
- discard partons collinear to photon

$$E_{\text{hadronic}}(r_\gamma) \leq \epsilon\, p_{T,\gamma} \left(\frac{1 - \cos r_\gamma}{1 - \cos R} \right)^n$$

no need for fragmentation functions

Fragmentation vs. Smooth Cone



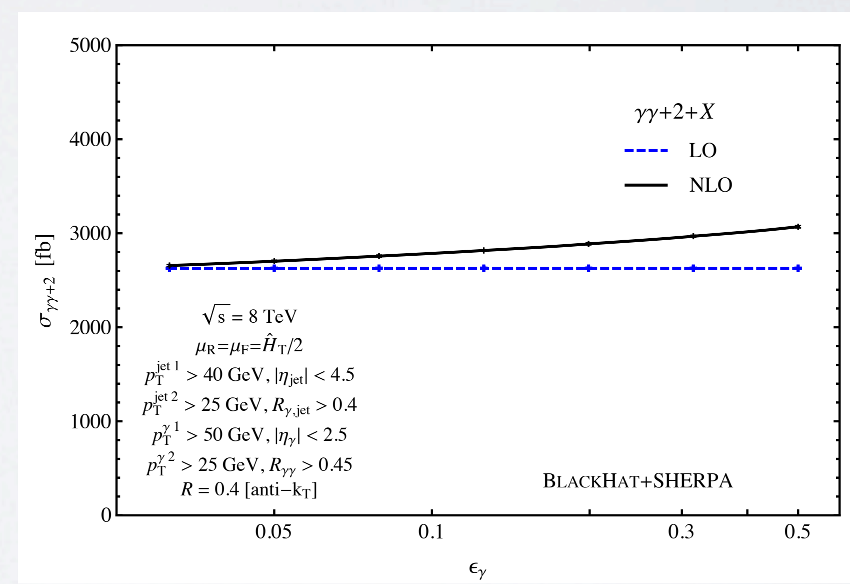
[Gehrmann, Greiner, Heinrich (2013)]

Pragmatic approach:
Tight isolation accord

$$E_T^{max} \leq 5 \text{ GeV} \;\; (\text{or } \epsilon < 0.1)$$

$$R \sim 0.4 \qquad R_{\gamma\gamma} \sim 0.4$$

[Cieri, de Florian (Les Houches 2013)]



[Bern at al. (2014)]

$pp \to \gamma\gamma + \text{jets}$ at NLO

SB, Guffanti, Yundin [1312.5927]

$p_{T,j} > 30\,\text{GeV}$ $\quad |\eta_j| \leq 4.7$

$p_{T,\gamma_1} > 40\,\text{GeV}$ $\quad p_{T,\gamma_2} > 25\,\text{GeV}$ $\quad |\eta_\gamma| \leq 2.5$

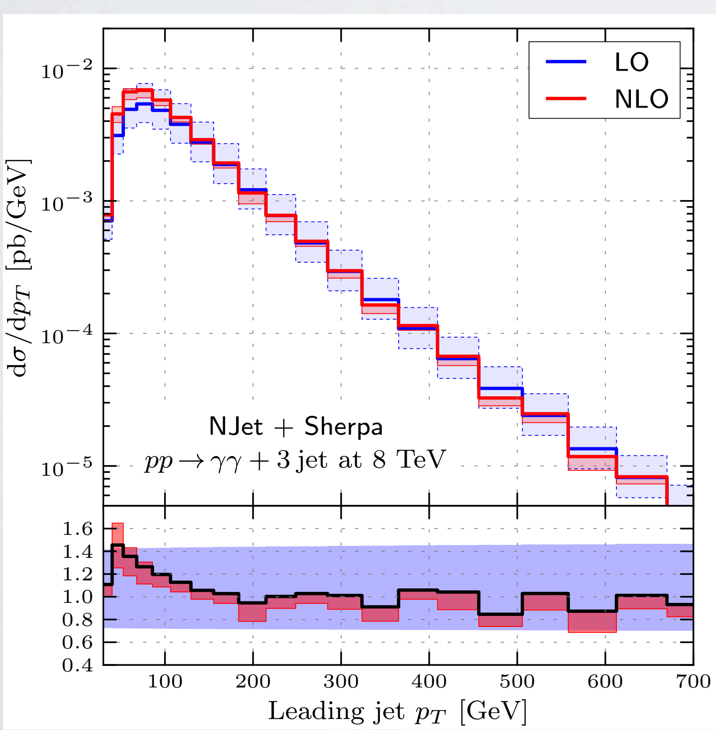
$R_{\gamma,j} = 0.5$ $\quad R_{\gamma,\gamma} = 0.45$

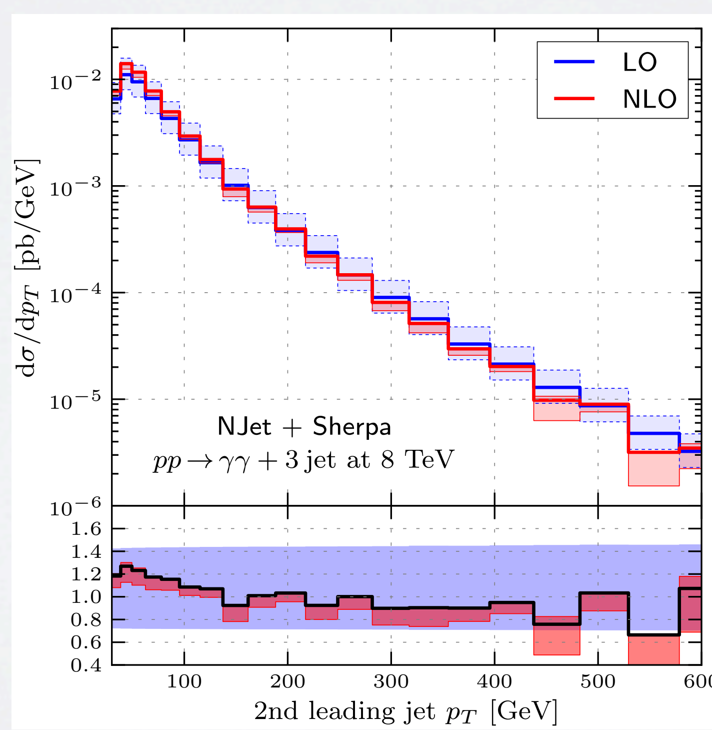
anti-k$_T$ $R = 0.5$ (FastJet)

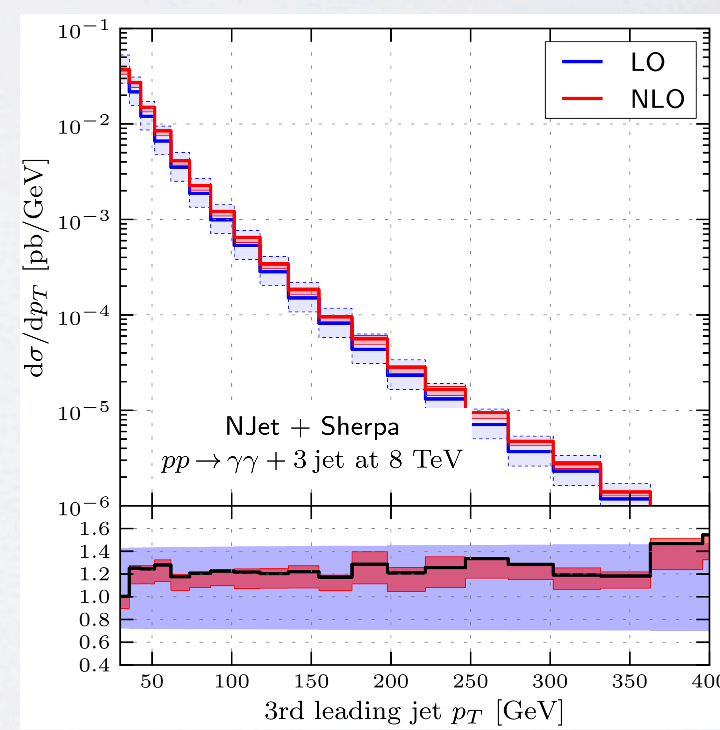
Frixione smooth cone photon isolation $\quad \epsilon = 0.05,\ R = 0.4$ and $n = 1$

CT10 NLO PDF set

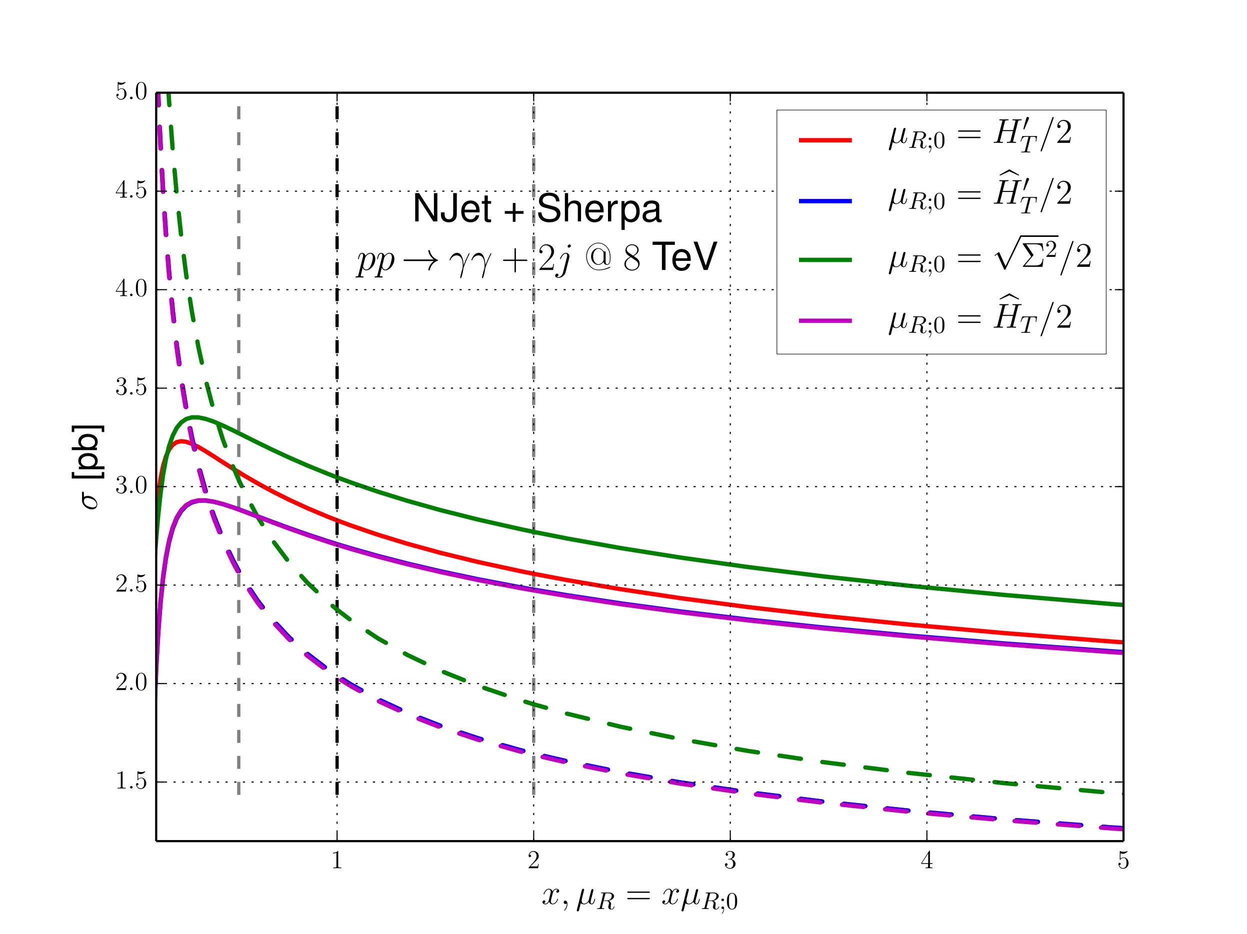
$$\sigma^{LO}_{\gamma\gamma+3j}(\widehat{H}'_T/2) = 0.643(0.003)^{+0.278}_{-0.180}\,\text{pb} \qquad \sigma^{NLO}_{\gamma\gamma+3j}(\widehat{H}'_T/2) = 0.785(0.010)^{+0.027}_{-0.085}\,\text{pb}$$

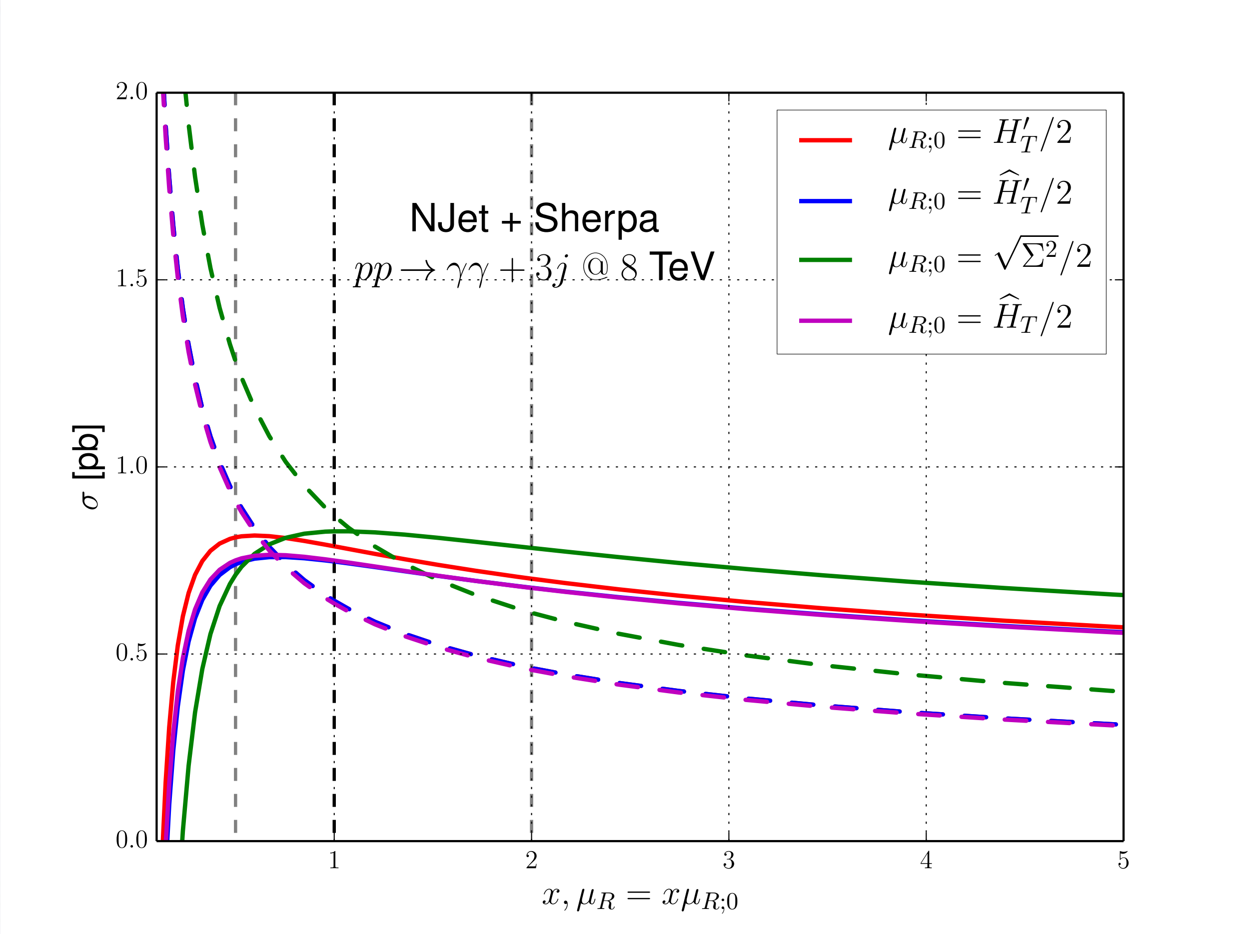






Scale dependence

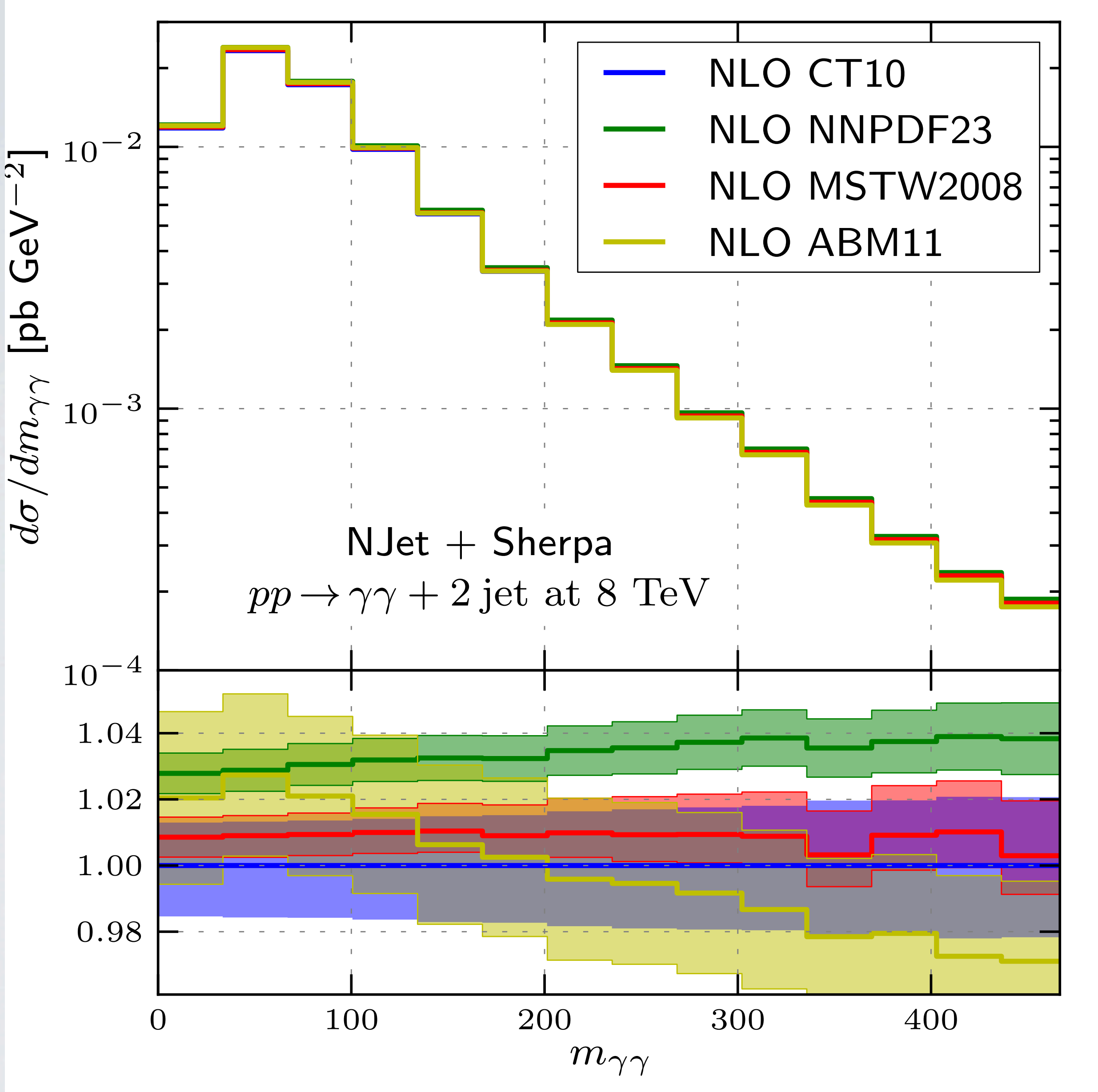




NLO predictions reduce uncertainty from 50% to ~15%

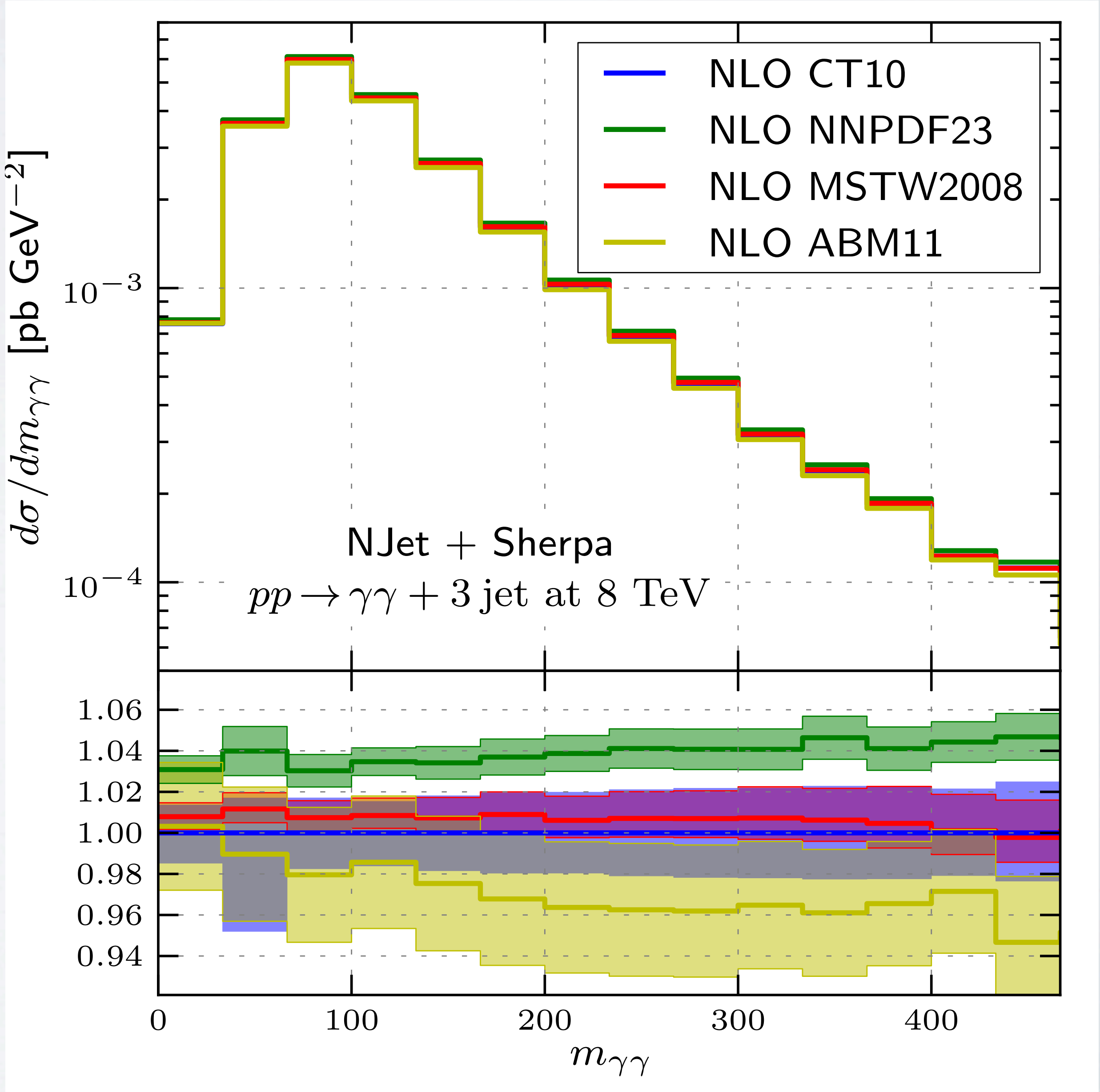
Fairly wide range of predictions with different dynamical scales

PDF dependence



comparison using all sets with

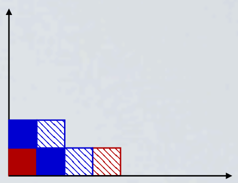
$$\alpha_s(M_Z) = 0.118$$



APPLgrids produced from ROOT Ntuples

(APPLgrid framework Carli et al. [0911.2985])

Beyond fixed order

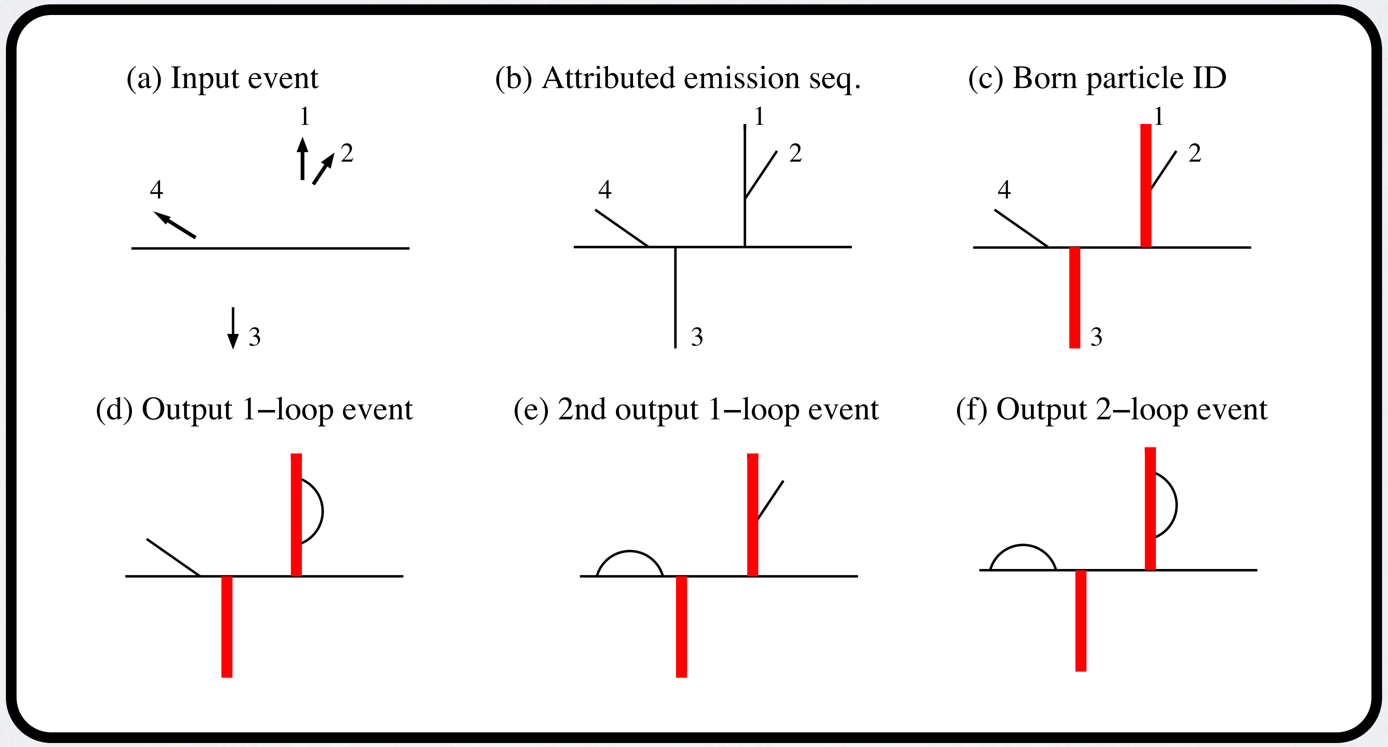


- LoopSim offers a fixed order alternative to NLO merging but without shower matching

  [Rubin, Sapeta, Salam (2010)]

- predictions at nNLO include some NNLO ingredients - double real and real-virtual

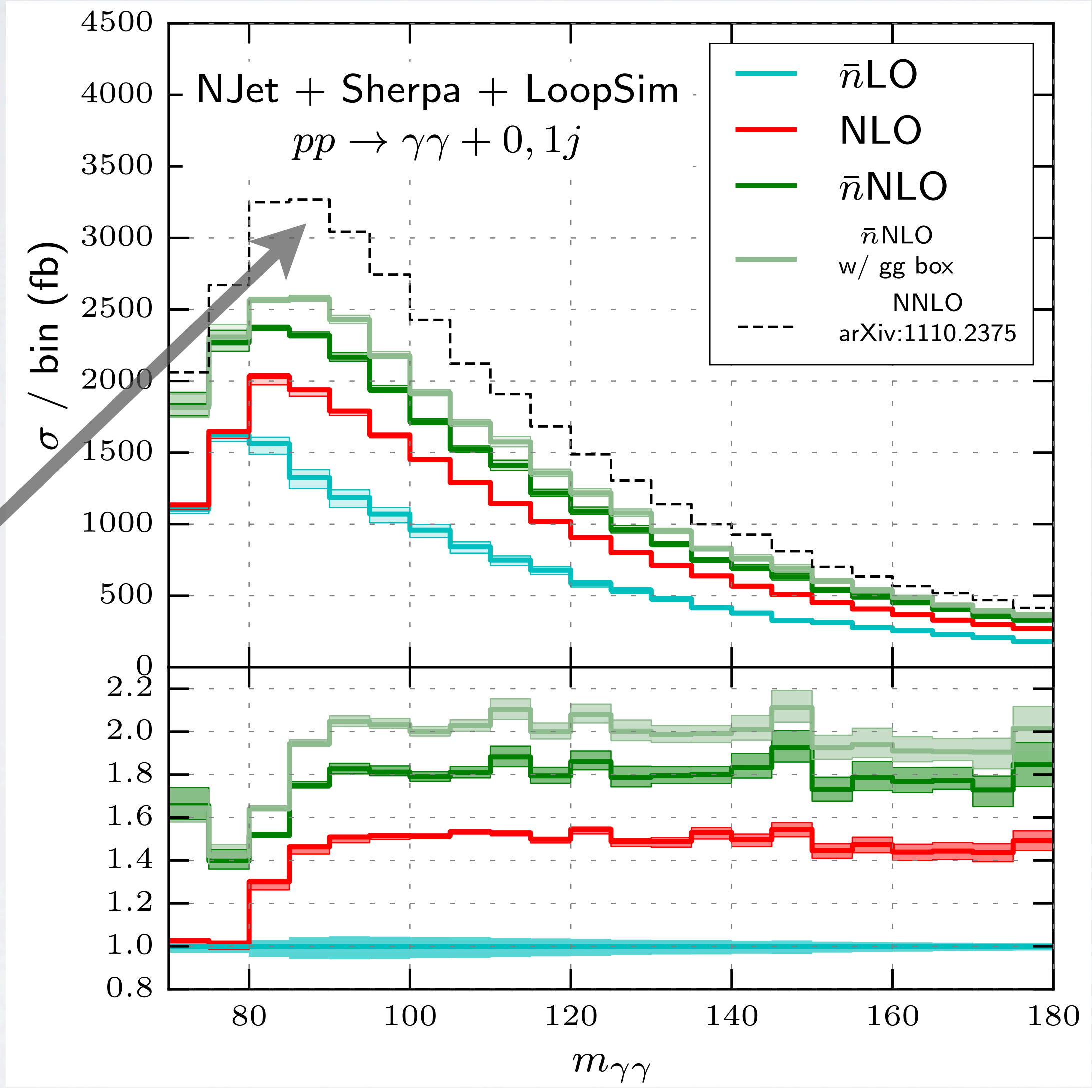
- fixed order Root Ntuples can be merged using a modified analysis



$pp \to \gamma\gamma + \mathrm{jets}$ beyond NLO

very preliminary!

[SB, Sapeta]

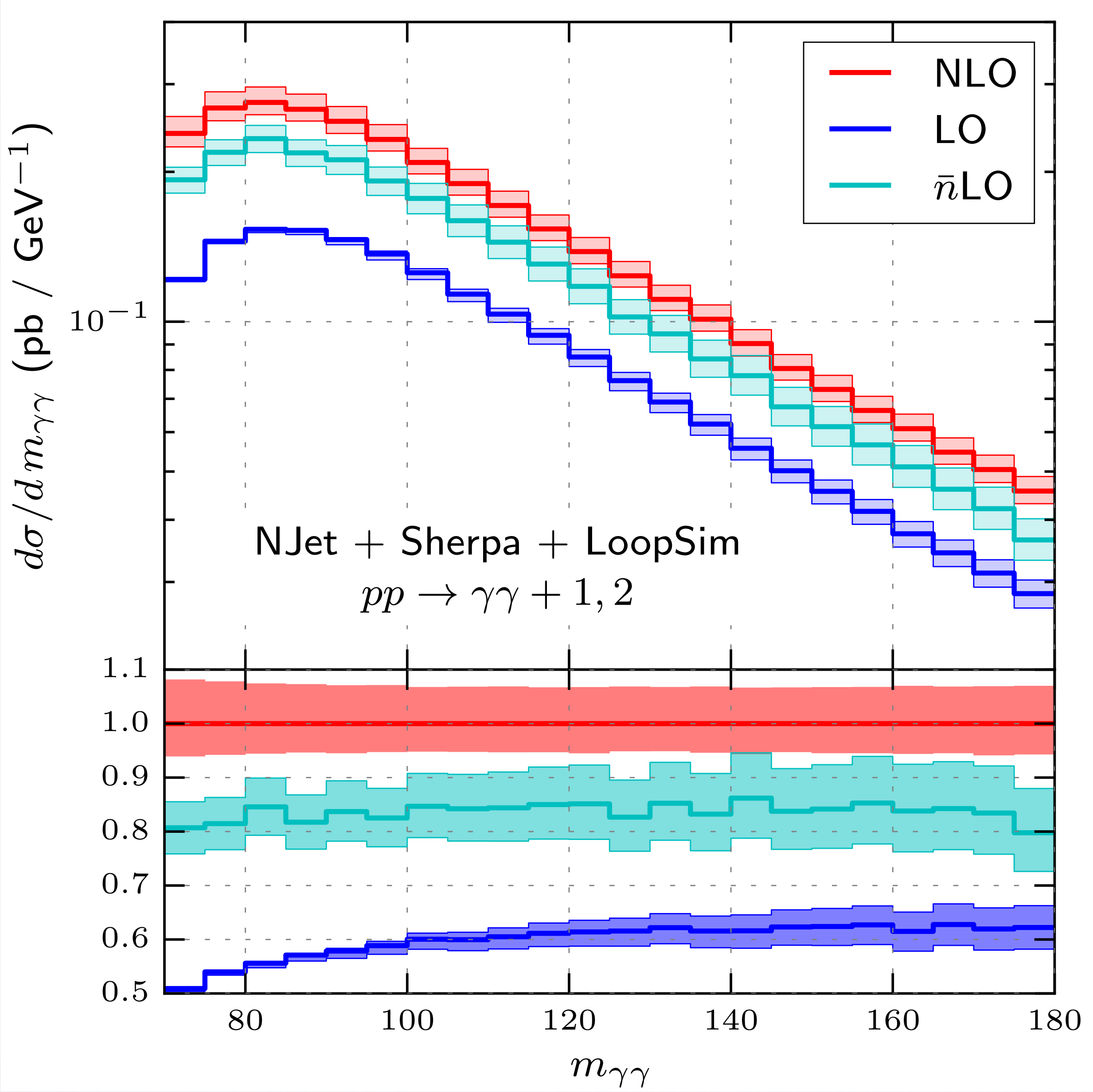


full NNLO
[Catani et al. (2011)]

Large k-factors for di-photons due to new partonic channels

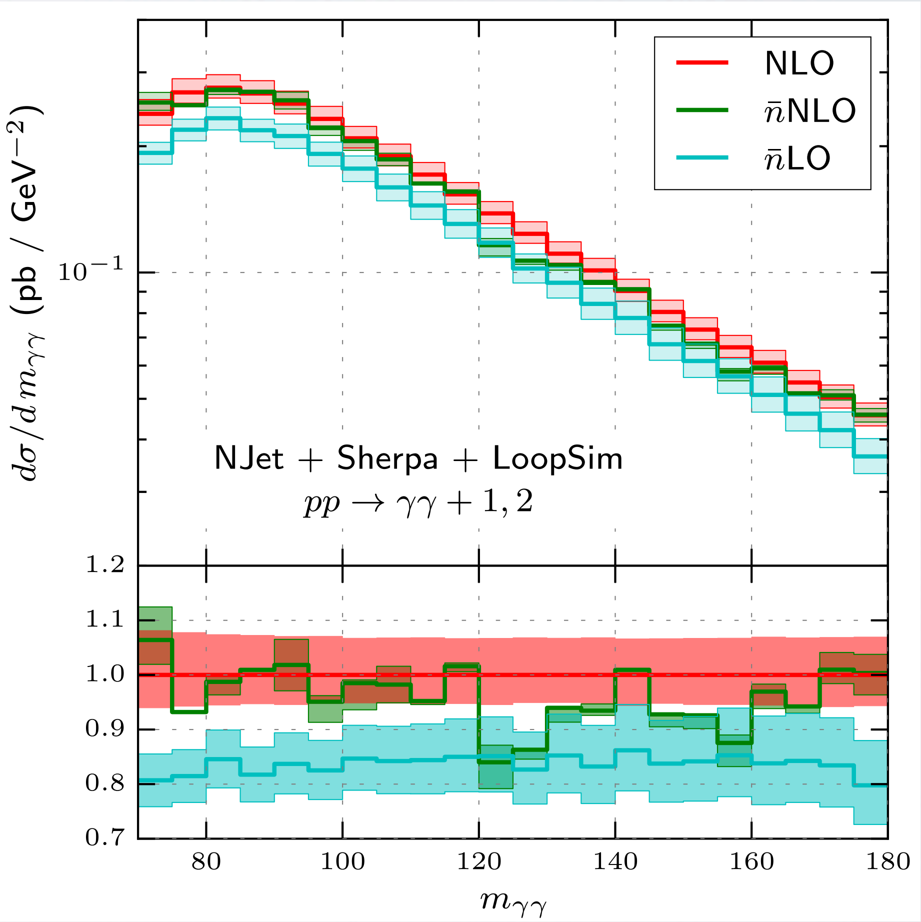
$pp \to \gamma\gamma + \text{jets}$ beyond NLO

very preliminary!



$\sqrt{s} = 14$ TeV $\quad p_{T,j} > 20$ GeV

$p_{T,\gamma_1} > 40$ GeV $\quad p_{T,\gamma_2} > 25$ GeV $\quad 20$ GeV $< m_{\gamma\gamma} < 250$ GeV



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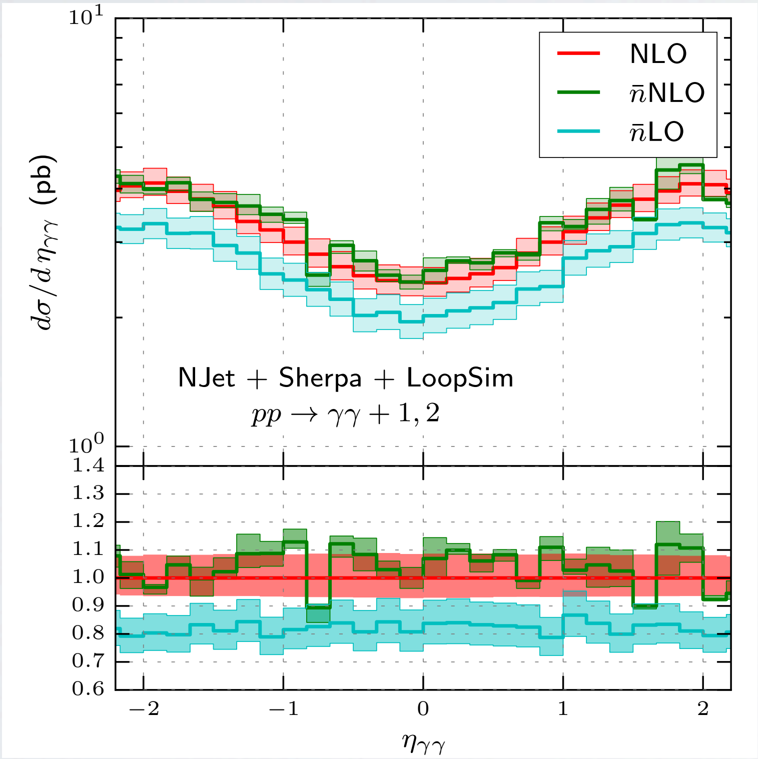
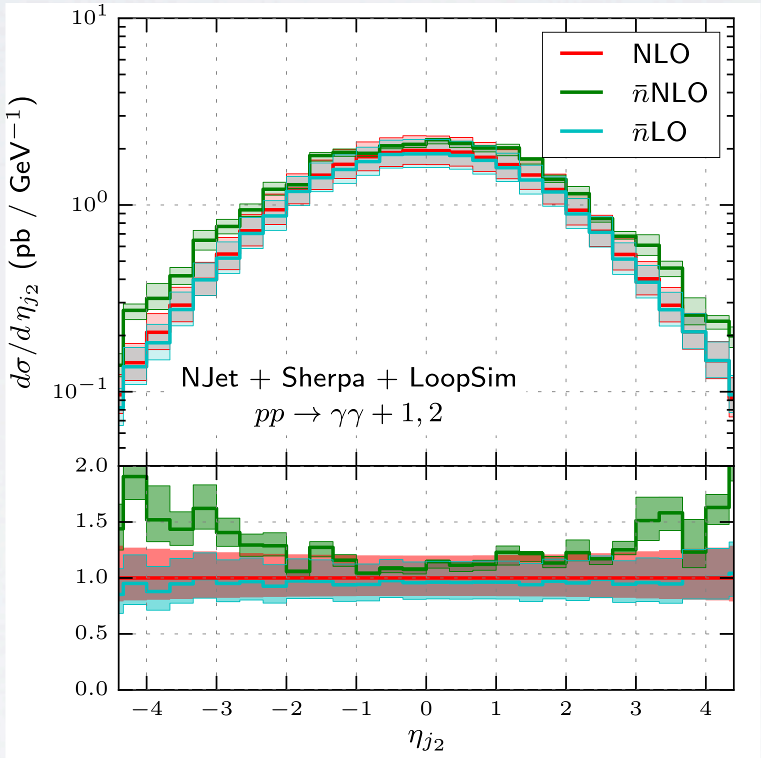
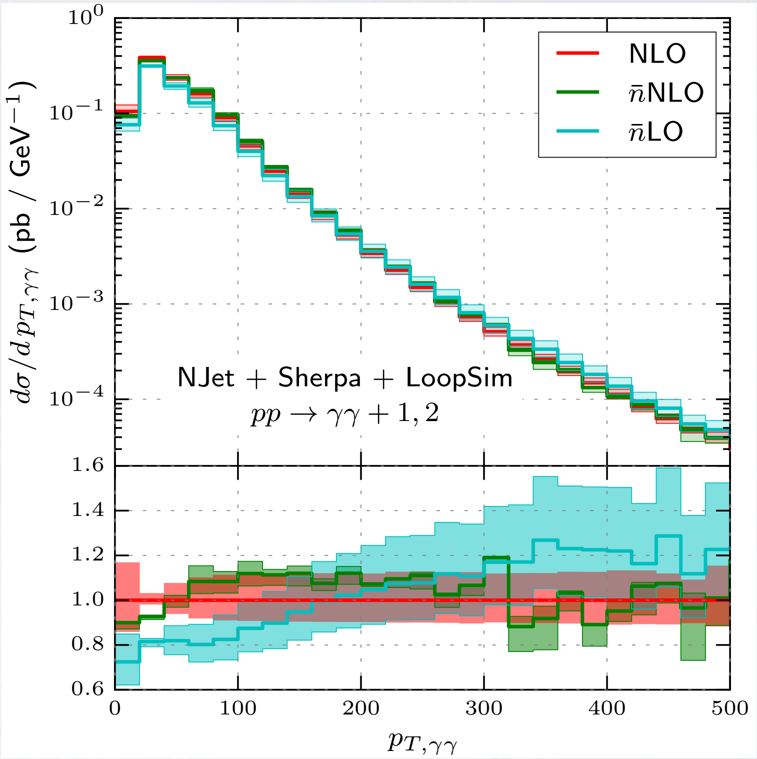
very preliminary!







Conclusions

- On-shell methods do a good job at keeping theoretical complexity of high-multiplicity amplitudes under control
- NJET available with $pp \to \leq 5j,\, W/Z/\gamma + \leq 5j, \gamma\gamma + \leq 4j$
- First computations of NLO QCD corrections to $pp \to 5j$ and $pp \to \gamma\gamma + 3j$
- Merging fixed order Ntuples for $pp \to \gamma\gamma + 1j$ with LOOPSIM

Backup Slides

Momentum twistors

$$Z_{i,a} = (\lambda_{i,a}, \mu_{i,a})$$

momentum conservation automatically satisfied for any 4 x n matrix, Z

$$W_{i,\dot{a}} = (\tilde{\mu}_{\dot{a}}, \tilde{\lambda}_{\dot{a}}) = \frac{\epsilon_{\dot{a},b,c,d} Z_{i-1,b} Z_{i,c} Z_{i+1,d}}{\langle i-1 i\rangle\langle i i+1\rangle}$$

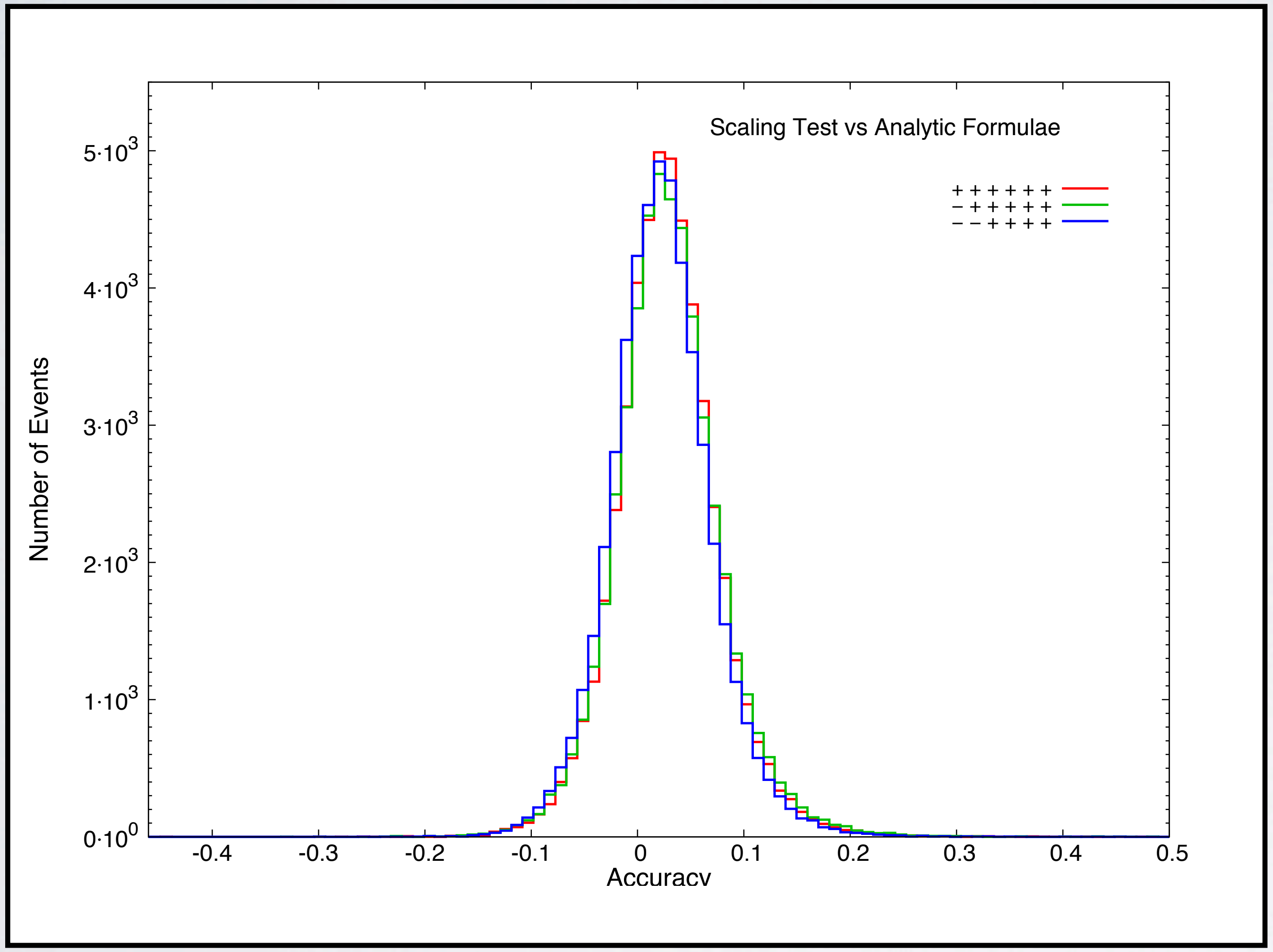
complex phase should be evaluated separately

3n-10 independent variables

$$Z = \begin{pmatrix} 1 & 0 & \frac{1}{x_1} & \frac{1}{x_1}+\frac{1}{x_2} & \frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_2} \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & x_4 & 1 \\ 0 & 0 & 1 & 1 & \frac{x_5}{x_4} \end{pmatrix}$$

$$x_1 = s_{12}, \qquad x_2 = \frac{\langle 23\rangle\langle 41\rangle[12]}{\langle 34\rangle}, \ldots$$

Accuracy



dimension scaling test

$$A(p_i, m_i, \mu_R) = x^{4-n} A(xp_i, xm_i, x\mu_R) := A_{\text{NJET}}(x)$$

$$\#\text{digits} = \log_{10}\left(\frac{A_{\text{NJET}}(s_1) + A_{\text{NJET}}(s_2)}{2(A_{\text{NJET}}(s_1) - A_{\text{NJET}}(s_2))}\right)$$

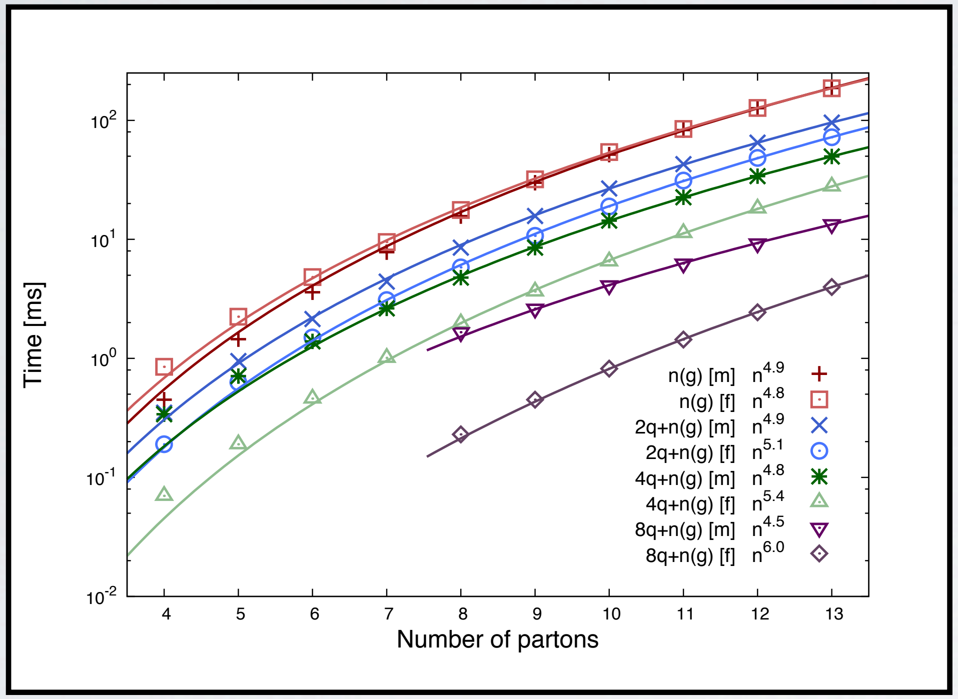
2 calls for the price of 1 using explicit vectorization with Vc

$$\text{accuracy} = \log_{10}\left(\frac{A_{\text{NJET}}(s_1) + A_{\text{NJET}}(s_2)}{2(A_{\text{NJET}}(s_1) - A_{\text{NJET}}(s_2))}\right) - \log_{10}\left(\frac{A_{\text{NJET}}(1) + A_{\text{analytic}}}{2(A_{\text{NJET}}(1) + A_{\text{analytic}})}\right)$$

reliable but statistical: add ~2 digits on min. accuracy

Performance

primitives scale ~n^6 for n $\lesssim$ 20



process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
4g	0.030	0.030	(0.00)
2u2g	0.032	0.032	(0.00)
2u2d	0.011	0.011	(0.00)
4u	0.022	0.022	(0.00)

process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
5g	0.22	0.22	(0.22)
2u3g	0.34	0.35	(0.06)
2u2d1g	0.11	0.11	(0.00)
4u1g	0.22	0.22	(0.03)

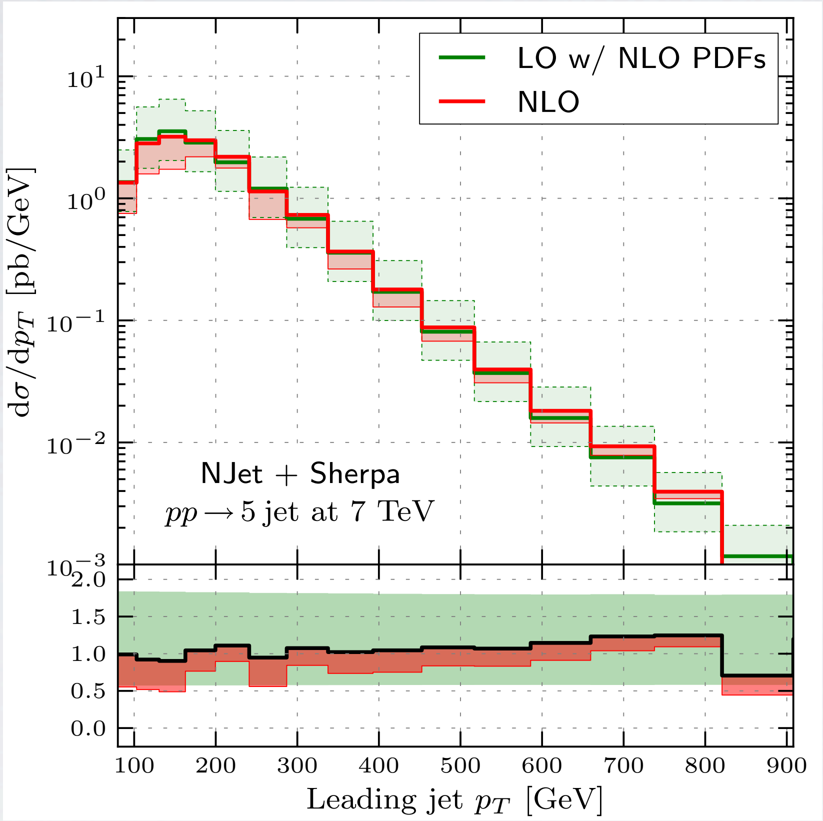
process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
6g	6.19	6.81	(1.37)
2u4g	7.19	7.40	(0.38)
2u2d2g	2.05	2.06	(0.08)
4u2g	4.08	4.15	(0.21)
2u2d2s	0.38	0.38	(0.00)
2u4d	0.74	0.74	(0.00)
6u	2.16	2.17	(0.02)

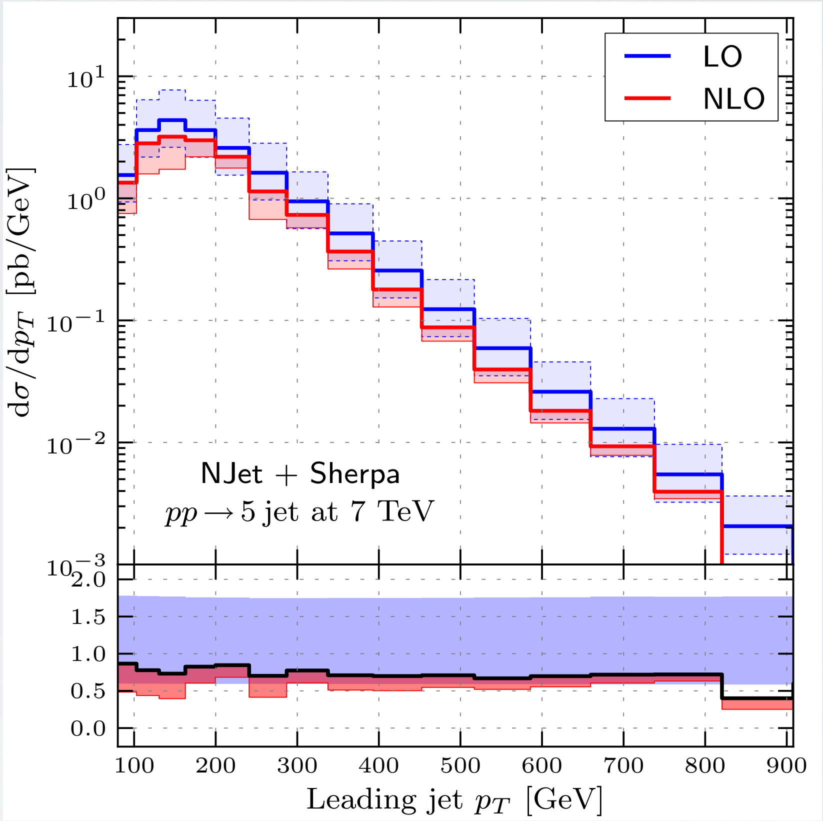
process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
7g	171.3	276.7	(8.63)
2u5g	195.1	241.2	(3.25)
2u2d3g	45.7	48.8	(0.88)
4u3g	92.5	101.5	(1.29)
2u2d2s1g	7.9	8.1	(0.23)
2u4d1g	15.8	16.2	(0.29)
6u1g	47.1	48.6	(0.41)

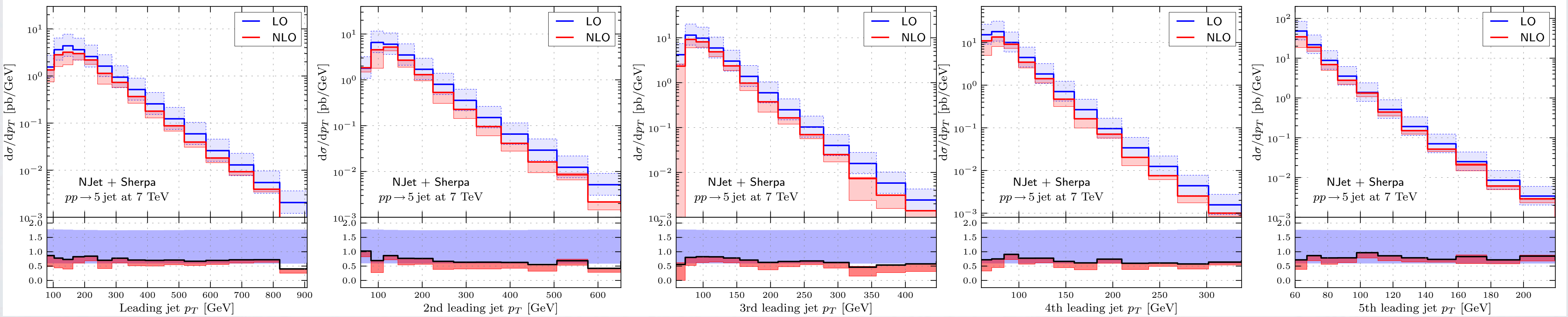
full colour sums a few seconds for 2⟶4

	$gg \to 2g$	$gg \to 3g$	$gg \to 4g$	$gg \to 5g$
standard sum	0.03	0.22	6.19	171.31
de-symmetrized	0.03	0.07	0.57	3.07

p_T distributions



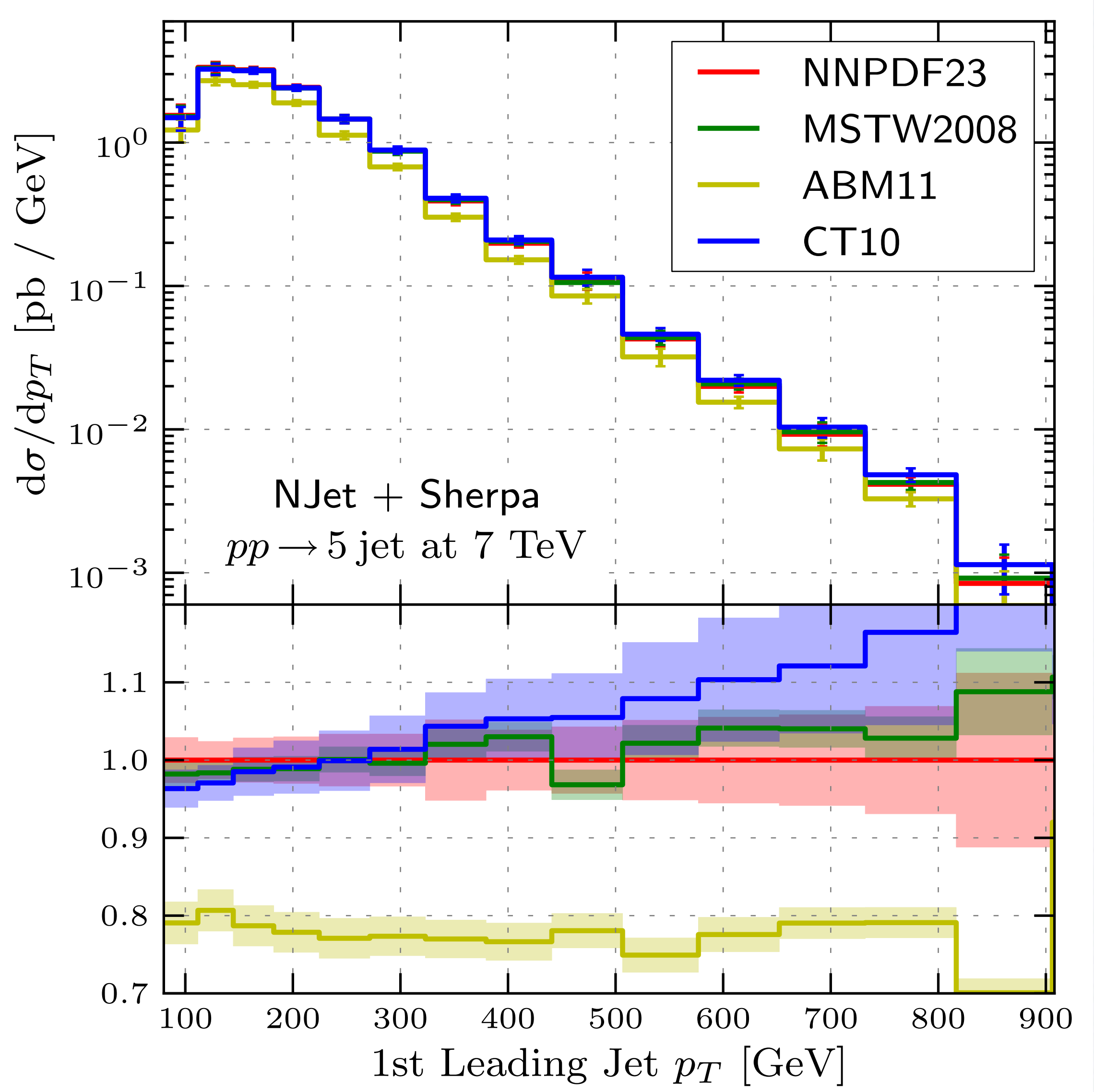




more distributions at https://bitbucket.org/njet/njet/wiki/Results/Physics

PDF dependence

Generally weak dependence on the choice of PDF fit (excluding choice for $\alpha_s(M_Z)$)



comparison using all sets with

$$\alpha_s(M_Z) = 0.118$$

some dependence on p_T

significant deviation in normalization of ABM11

Efficient Event Generation

- Leading/sub-leading expansion - sample dominant contributions more often
- Separate contributions by number of fermion lines
- ROOT Ntuples - make the most out of the integration run
  - Re-weighting PDFs and renormalization/factorization scales (also jet algorithms with suitable event generation)
- APPLgrid - extremely fast and flexible analysis (very useful for PDF error analyses)

Carli et al. [0911.2985]

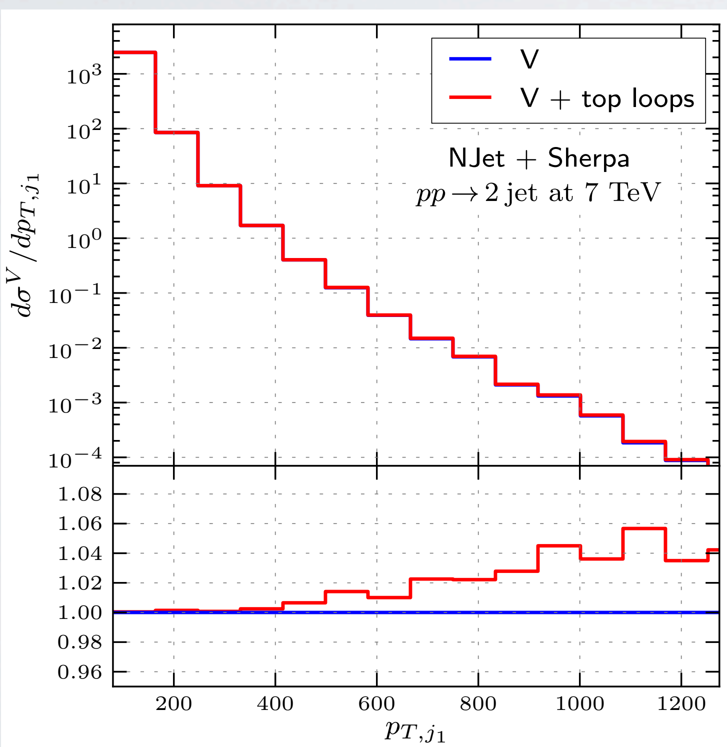
Heavy quark loops

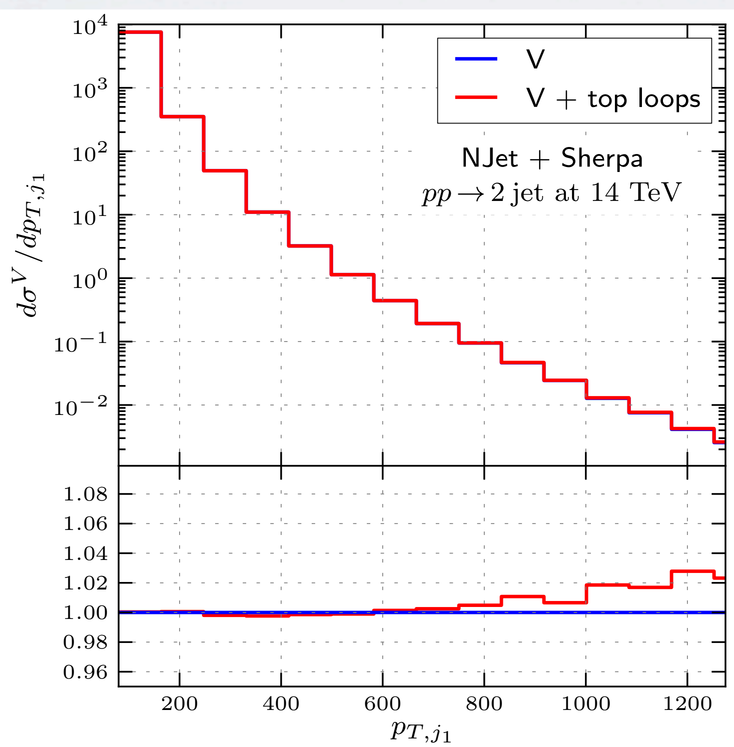
preliminary

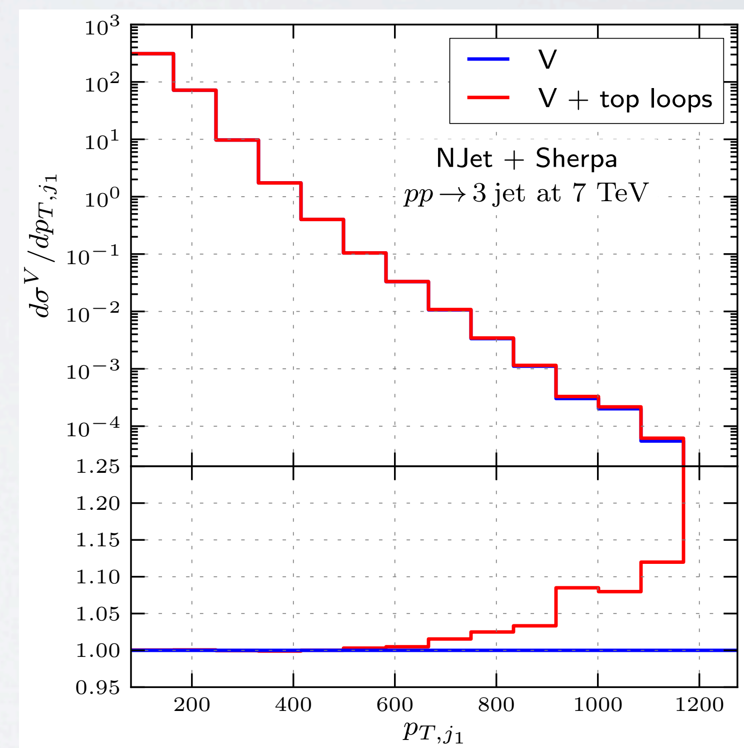
top quark loop effects are small (<1%)

di-jets seem to have additional kinematic suppression

matrix elements checked against MadLoop







corrections grow at very large p_T - still negligible