Higgs Production at N3LO

PSR 2015 Krakow

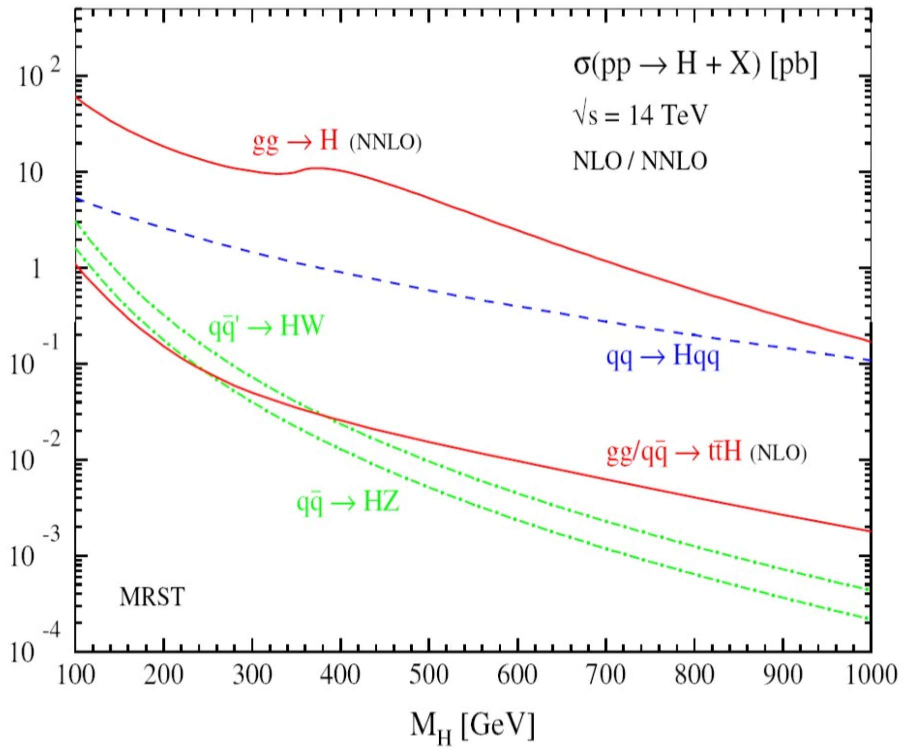
Franz Herzog (Nikhef)

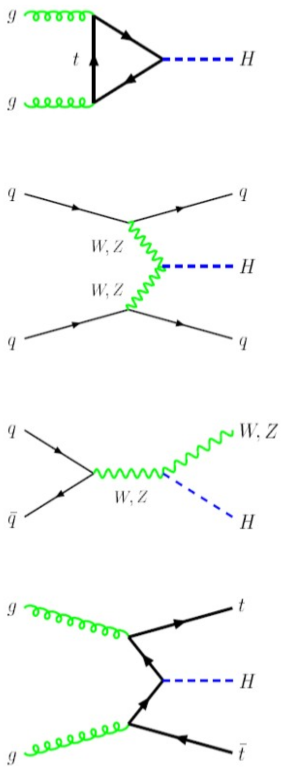
Collaborators:
C Anastasiou, C Duhr, F Dulat, E Furlan, T Gehrmann,
A Lazopoulos, B Mistlberger

Overview

- LHC Higgs Boson Production
- N3LO QCD
- Soft Expansion
- Results

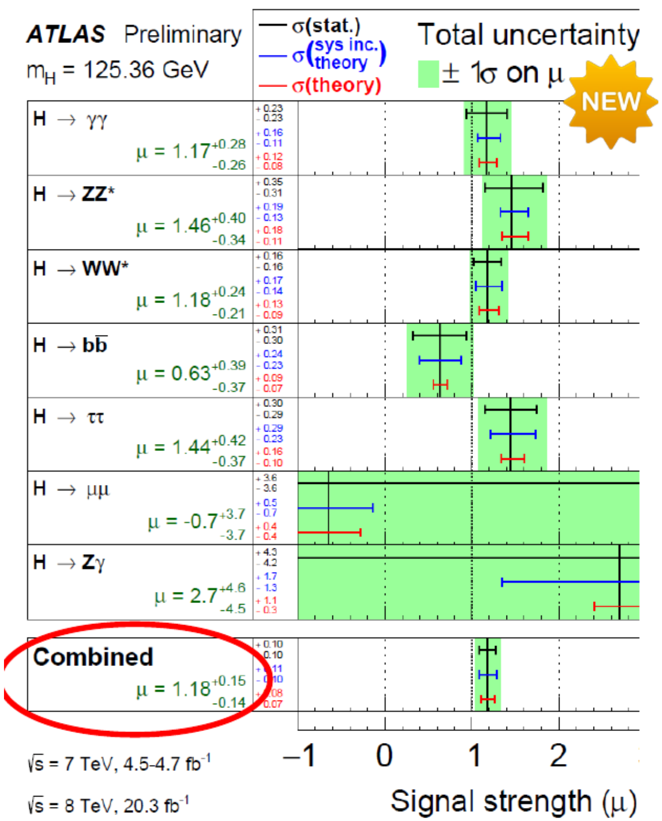
LHC Higgs Production

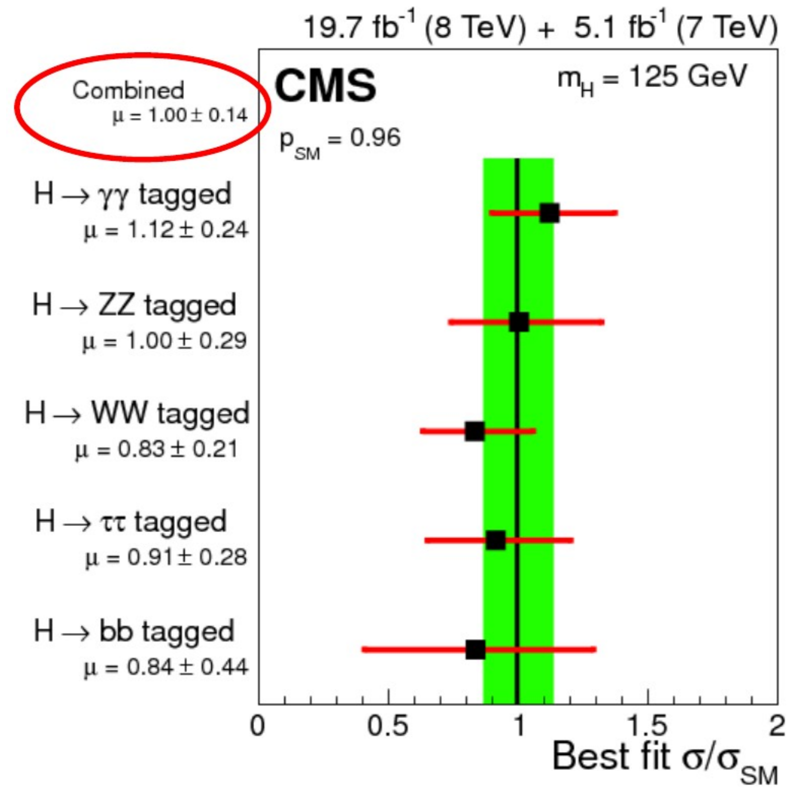




The dominant Higgs production mechanism is guon fusion.

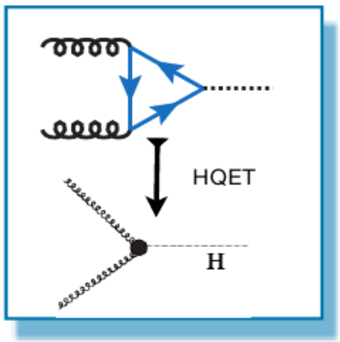
LHC Coupling measurements require precise knowledge of the Higgs Boson Cross Section

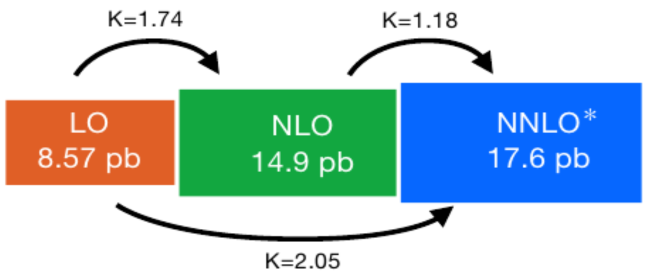


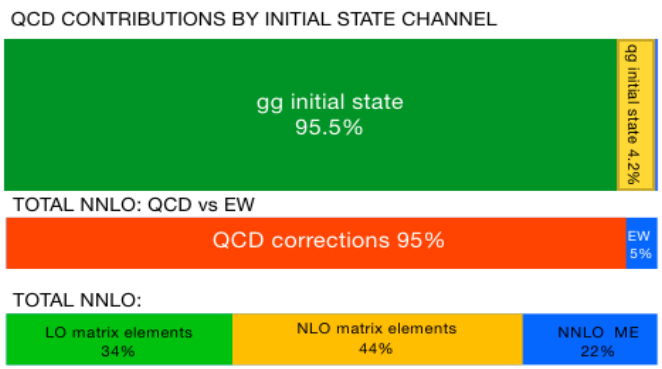


[Duehrssen at EW Moriond 2015]

Total Higgs Cross section as of 2014





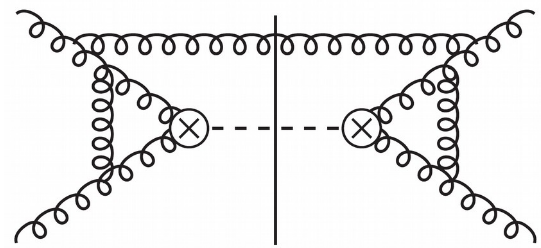


[graphics by A.Lazopoulos]

- NLO QCD corrections known exactly (with top-bottom interference) [Graudenz et al 93, Spira et al 95,Harlander et al 05,Anastasiou 06, Aglietti 06]
- NNLO QCD in HQET [Harlander et al 02, Anastasiou et al 02, Ravindran et al 03]
- Subleading terms in the heavy top expansion [Pak et al 09, Harlander et al 09]
- EW corrections [Actis et al 08+09,Aglietti et al 04,Degrassi et al 04]
- mixed QCD EW corrections [Anastasiou et al 09]
- Soft gluon NNLL [Catani et al 03] SCET NNLL [Ahrens et al 08]
- Approximate N3LO [Moch et al 05, Ball et al 13]
- N3LL resummation of threshold logs [Bonvini et al 14, Catani et al 14]
- Soft Virtual and next to soft Approximation for N3LO [Anastasiou et al 14]

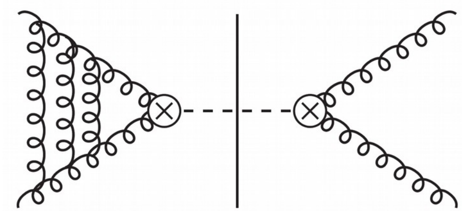
$\%\delta_{PDF}$	$\%\delta_{\mu}$
$^{+7.79}_{-7.53}$	$^{+8.37}_{-9.26}$

Meaningful Interpretation of future LHC Higgs data demand N3LO QCD Corrections!

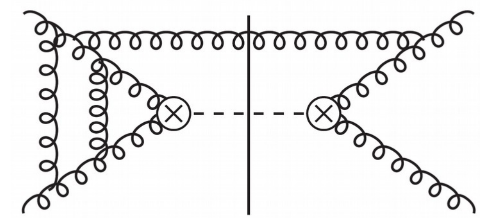


Real-Virtual Squared

+UV and IR counter terms

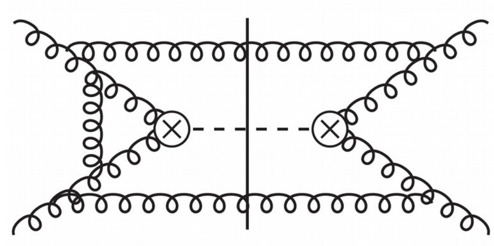


Triple Virtual

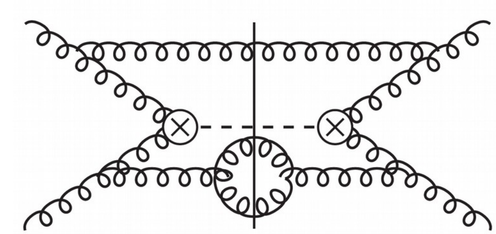


Double Virtual- Real

N3LO



Double Real - Virtual



Triple Real

Towards analytic evaluation at N3LO

- The Higgs Cross section depends only on a single dimensionless variable:

$$z = \frac{m_H^2}{\hat{s}}$$

- Analytic evaluation likely in terms of harmonic polylogarithms, or at least multiple polylogarithms.

However..

- Number of cut-diagrams to be evaluated is around 100.000
- Infra-red divergences up to

$$\mathcal{O}\left(\frac{1}{\epsilon^6}\right)$$

- Phase Space Integrals “were” completely unknown from other processes

Reverse Unitarity, IBPs and Differential Equations

- Write cut-propagators as difference of Feynman Propagators

$$2\pi i\delta^{+}(p^2) \to \left(\frac{1}{p^2}\right)_c = \frac{i}{p^2+i0} - \frac{i}{p^2-i0}$$

- Cut Propagators can be differentiated just like normal propagators

$$\frac{\partial}{\partial p^\mu}\left(\frac{1}{p^2}\right)_c = -2p_\mu \left(\frac{1}{p^2}\right)_c^2$$

- This allows to derive IBP identities and reduce integrals to a smaller set of independent master integrals, which satisfy a linear system of differential equations

$$\frac{\partial}{\partial z}\mathcal{M}_i(z,\epsilon) = \sum_j C_{ij}(z,\epsilon)\mathcal{M}_j(z,\epsilon)$$

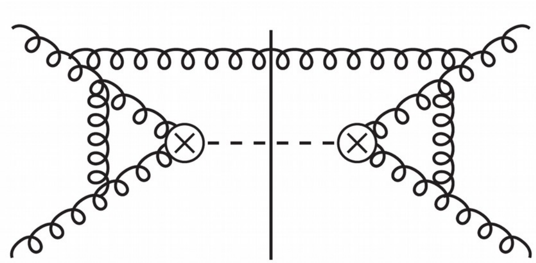
- If the system can be decoupled (or brought into canonical form) a solution can be found given a suitable boundary condition.

Integral Statistics

	NNLO	N3LO
# diagrams	~1.000	~100.000
# integrals	~50.000	517.531.178
# masters	27	1.028

- Even after IBP reduction the number of integrals to be computed is large!
- Some of the systems of differential equation are very large and decoupling them is very non-trivial.

Status of Analytic Evaluation

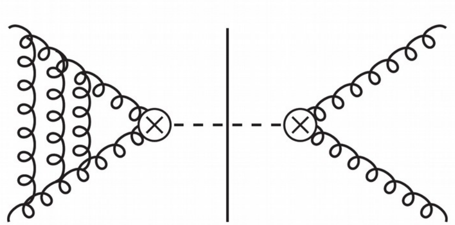


Real-Virtual Squared

Known [Anastasiou, Duhr, Dulat, FH, Mistlberger; Kilgore]

+UV and IR counter terms

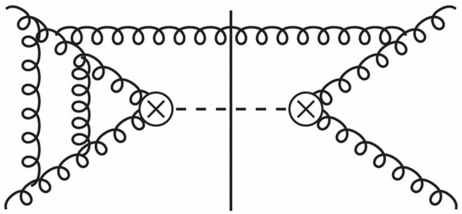
Known[Pak, Rogal, Steinhauser; Anastasiou, Buehler, Duhr, FH; Höschele, Hoff, Pak, Steinhauser, Ueda; Buehler, Lazopoulos]



Triple Virtual

Known from QCD Form Factor

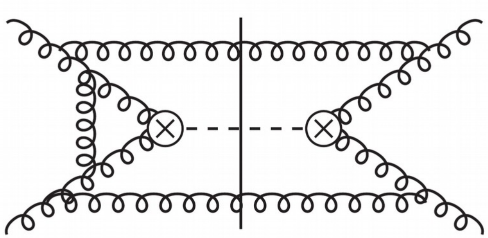
[Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]



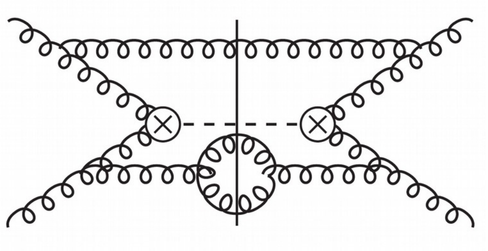
Double Virtual- Real

Known [Dulat, Mistlberger; Duhr, Gehrmann]

N3LO



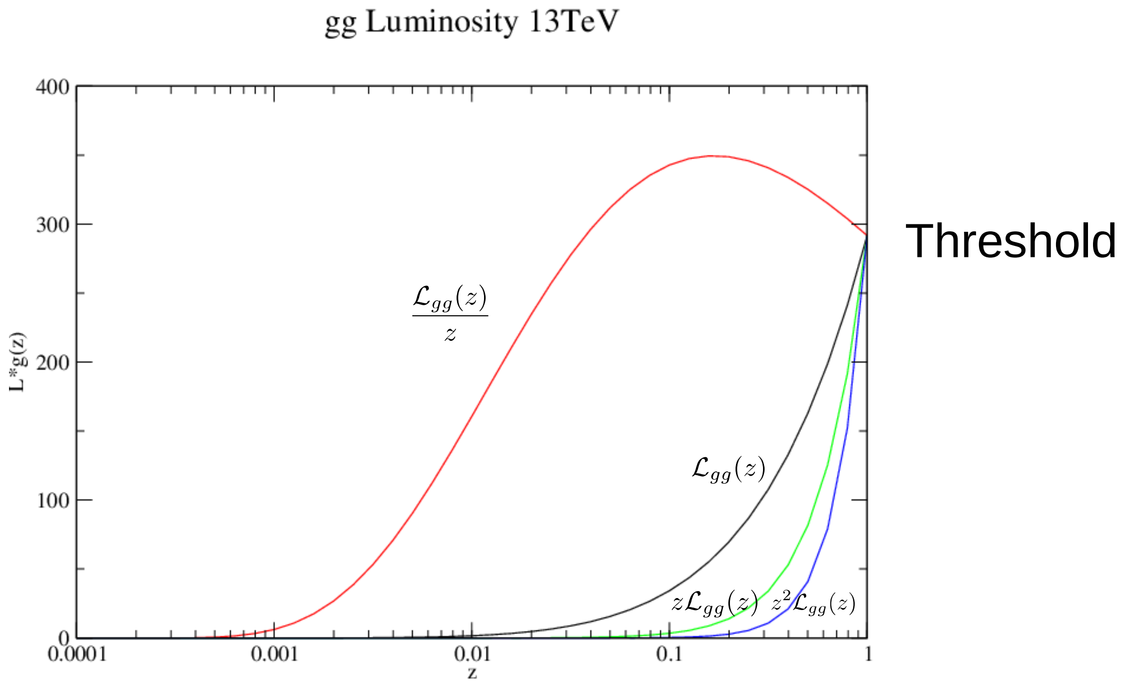
Double Real - Virtual



Triple Real

Alternative Approach: Series Expansion around Threshold

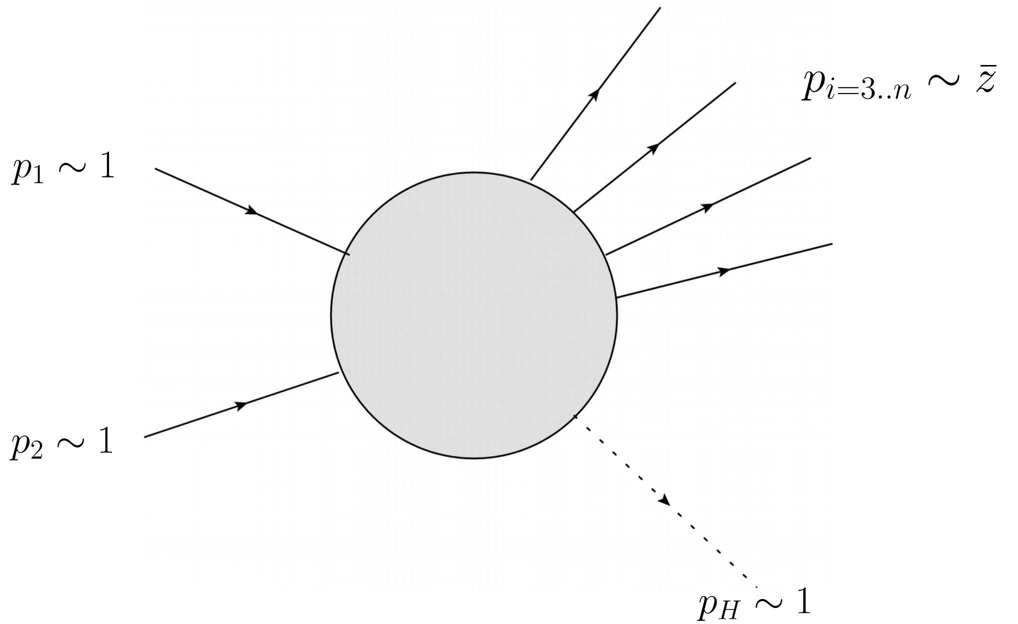
$$\sigma_{PP\to HX}(\tau) \sim \int_\tau^1 \frac{dz}{z}\,\mathcal{L}_{gg}(z)\,\sigma_{gg\to HX}(z)$$



Gluon Luminosity enhances the Threshold contribution at $z = 1$.

Threshold Expansion for Phase Space Integrals

$$\bar{z} = 1 - z$$



Given the scalings of all momenta, Cut-diagrams can be Taylor-expanded around Threshold in momentum space.

Example: Soft Expansion of the Double Real Phase Space Volume

$$\Phi_3(z;\epsilon) = \int d^Dp_3\delta^+(p_3^2)\ d^Dp_4\delta^+(p_4^2)\ d^Dp_H\delta^+(p_H^2-m_H^2)\ \delta^D(p_1+p_2-p_3-p_4-p_H)$$

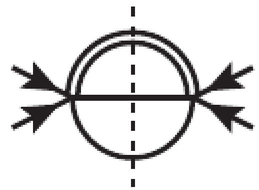
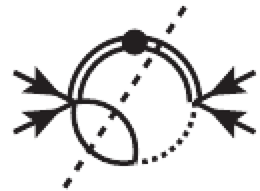
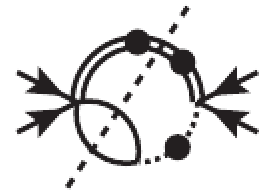
has the following cut propagator representation

$$= \int d^Dp_3 d^Dp_4 \left(\frac{1}{p_3^2}\right)_c \left(\frac{1}{p_4^2}\right)_c \left(\frac{1}{s_{12}\bar{z} - \underbrace{2p_{12}.p_{34}}_{\sim \bar{z}} + \underbrace{2p_3.p_4}_{\sim \bar{z}^2}}\right)_c$$

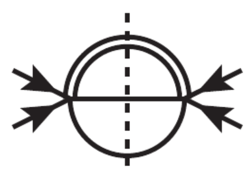
Expanding the Cut propagators around Threshold we obtain

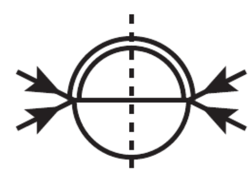
$$= \sum_{n=0}^{\infty} \int d^Dp_3 d^Dp_4 \left(\frac{1}{p_3^2}\right)_c \left(\frac{1}{p_4^2}\right)_c \left(\frac{1}{\bar{z}s_{12} - 2p_{12}.p_{34}}\right)_c^{1+n} (-2p_3.p_4)^n$$

Diagrammatically this can be written as

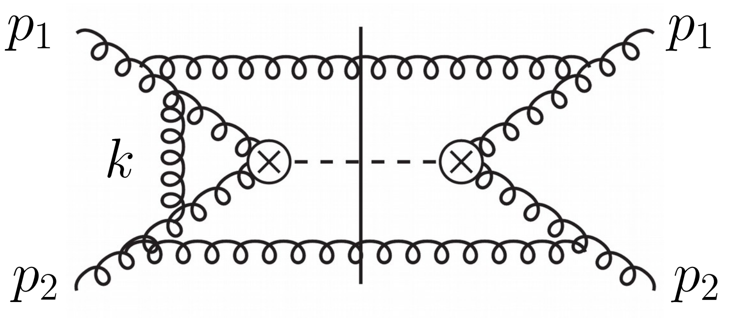
$\Phi_3(\bar{z};\epsilon) = \bar{z}^{3-4\epsilon}\Big[$  $-\,\bar{z}$  $+\,\bar{z}^2$  $+\,\mathcal{O}(\bar{z}^3)\Big]$

Since the soft expansion just raises or lowers powers of denominators, soft IBPS can be used to reduce to write everything back in terms of the first term in the expansion:

 $= -\dfrac{1-\epsilon}{2}$  ,

 $= \dfrac{(1-\epsilon)(2-\epsilon)(3-2\epsilon)}{4(5-4\epsilon)}$ 

Soft Expansion for combined Loop and Phase Space Integrals



$$k = \alpha p_1 + \beta p_2 + k^{\perp}$$

- Several soft scalings or regions are possible for Loop Momenta.
- We find that these regions can be classified according to:
  - Hard $k \sim 1$
  - Soft $k \sim \bar{z}$
  - Collinear $(\alpha, k^{\perp}) \sim \bar{z}$ or $(\beta, k^{\perp}) \sim \bar{z}$
- Although more complicated for the collinear regions we have managed to find soft Master Integrals for all regions in the double real virtual.

Integral Statistics II

	NNLO	N3LO
#diagrams	~1.000	~100.000
#integrals	~50.000	517.531.178
#masters	27	1.028
#soft masters	5	78

- The number of soft masters is much smaller than the number of full masters.
- Soft Master Integrals can be used to express the Higgs Cross section and/or the full kinematic Master Integrals to all orders in the soft expansion by direct Integrand expansion.
- Soft Master Integrals can be used as a boundary condition to solve the differential equations.
- Soft Master Integrals can be used to construct an Ansatz for the full Master Integrals, which can be fixed by the differential equations.

Truncated Series Solution via Differential Equation

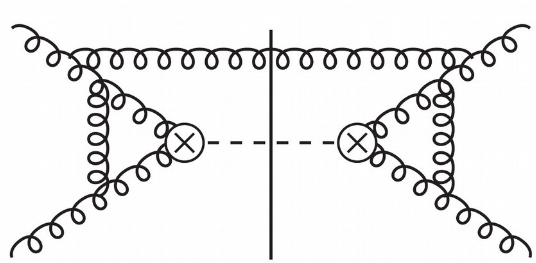
Ansatz: $$\mathcal{M}_i(z,\epsilon) = \sum_{k=0}^{6} \bar{z}^{-k\epsilon} \sum_{l=l_0}^{n} \mathcal{M}_i^{(k,l)}\, \bar{z}^l + \mathcal{O}(\bar{z}^{n+1})$$

$$\downarrow$$

$$\frac{\partial}{\partial z}\mathcal{M}_i(z,\epsilon) = \sum_j C_{ij}(z,\epsilon)\mathcal{M}_j(z,\epsilon)$$

- Substituting the Ansatz into the differential equations yields a linear system which can be solved order by order in zbar.
- Solved for the first 38 coefficients of the full Masters in terms of Soft Masters from knowledge of boundary.
- First few terms checked by explicit computation via Integrand Expansion.

Status of Evaluation



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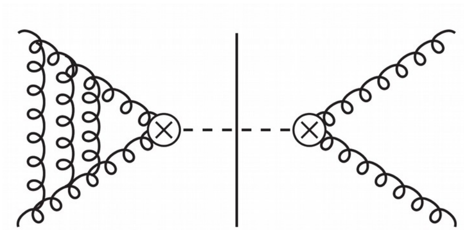

Triple Virtual

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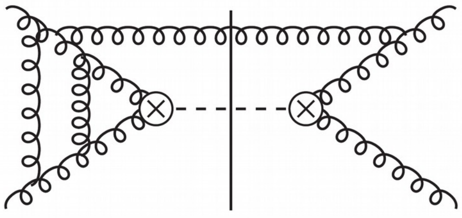
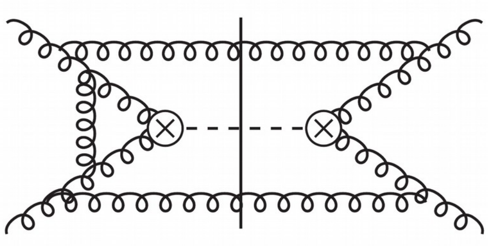


Double Virtual- Real

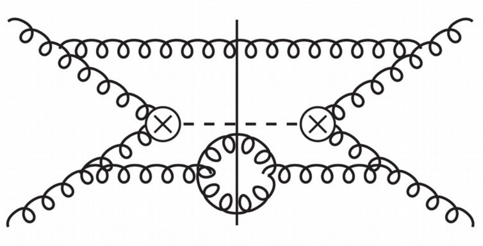
Known [Dulat, Mistlberger; Duhr, Gehrmann]

N3LO



Double Real - Virtual

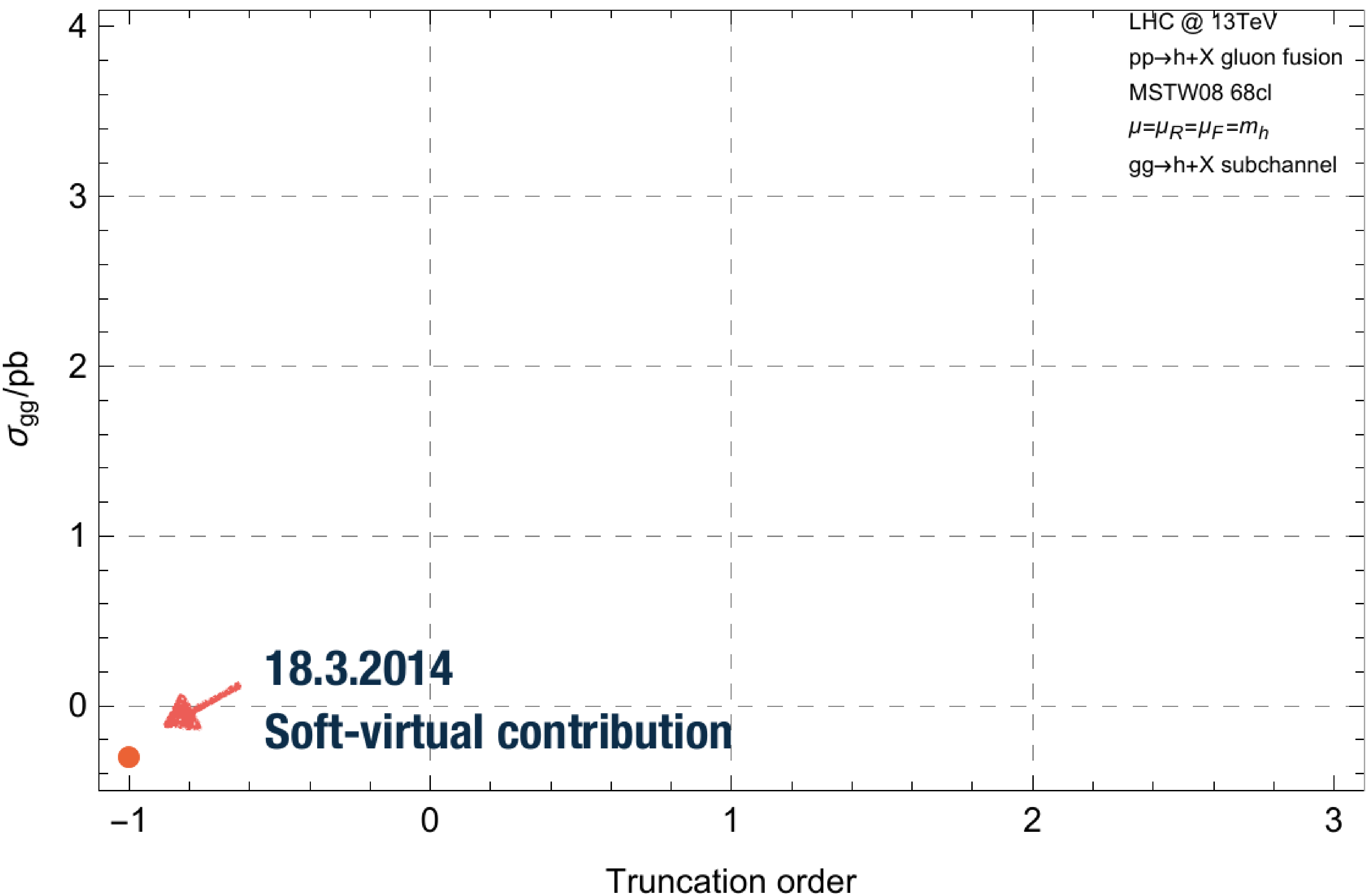
2 terms in soft expansion [Anastasiou, Duhr, Dulat, FH, Mistlberger,Furlan; Li, Mantueffel, Schabinger, Zhu] 30 terms [Anastasiou, Duhr, Dulat, FH, Mistlberger]

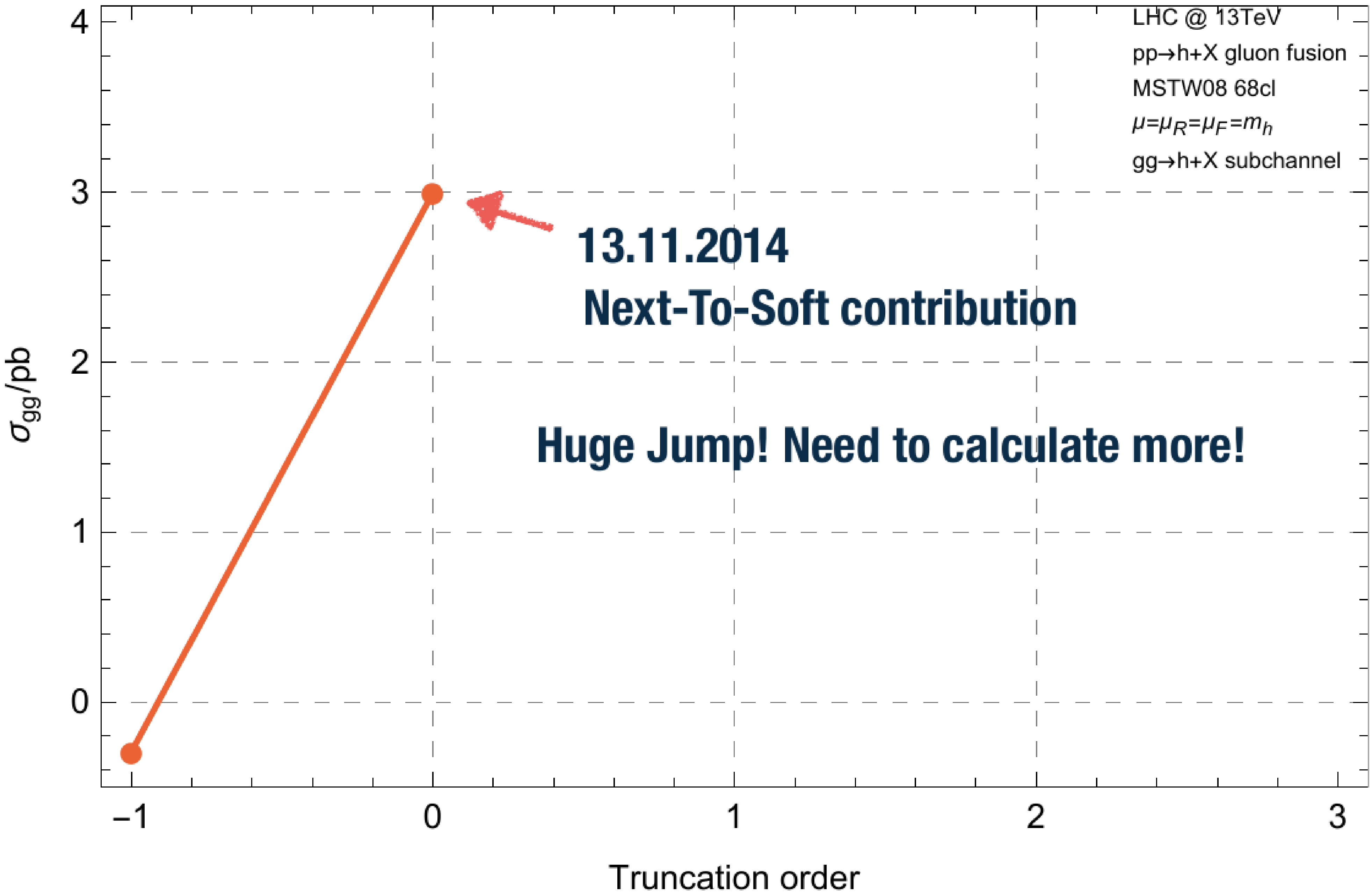


Triple Real

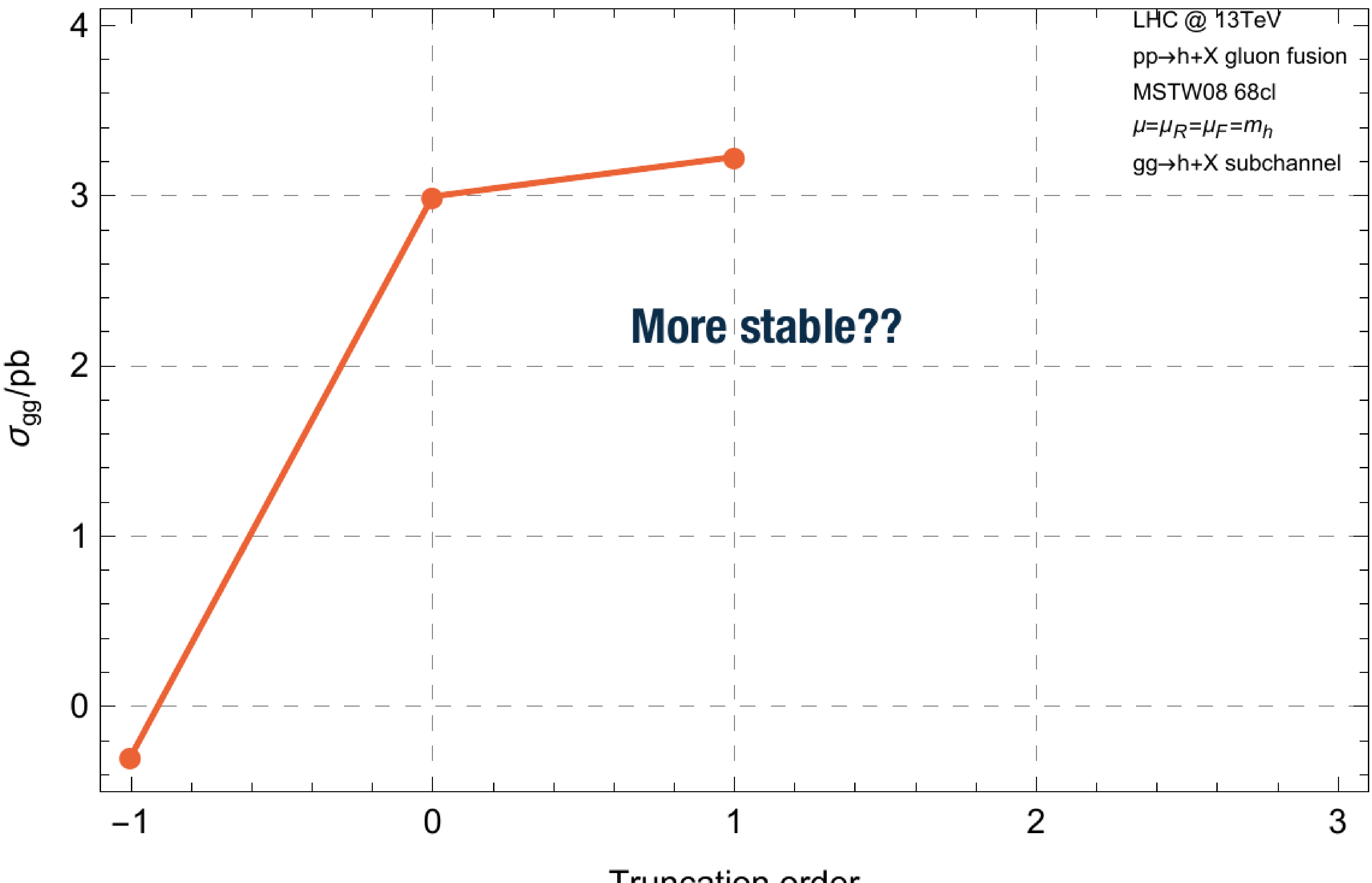
2 terms in soft expansion [Anastasiou, Duhr, Dulat, Mistlberger; Zhu]
30 terms [Anastasiou, Duhr, Dulat, FH, Mistlberger]

Results

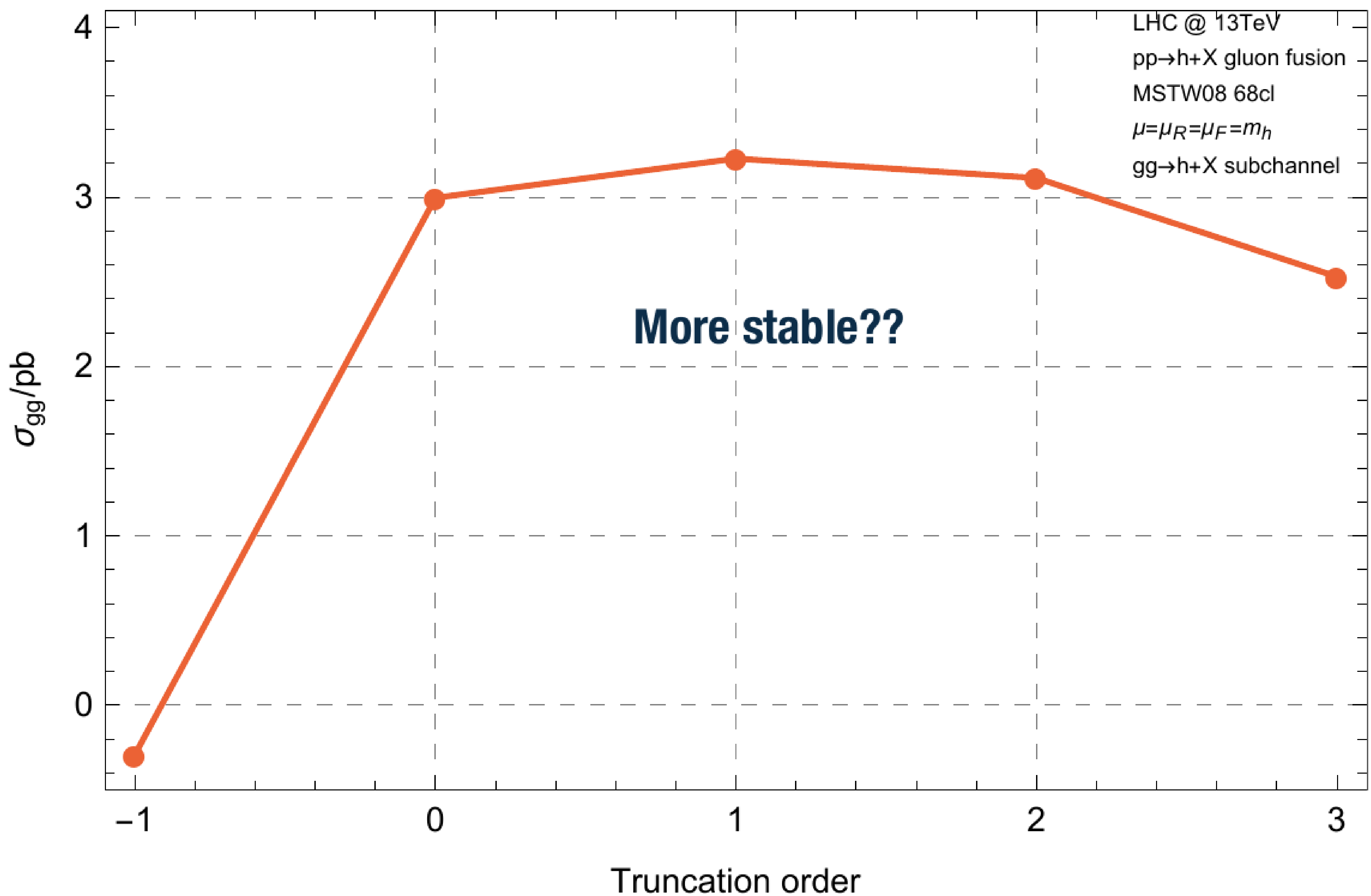

LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
gg→h+X subchannel
18.3.2014
Soft-virtual contribution
σ_{gg}/pb
Truncation order
4
3
2
1
0
−1
0
1
2
3

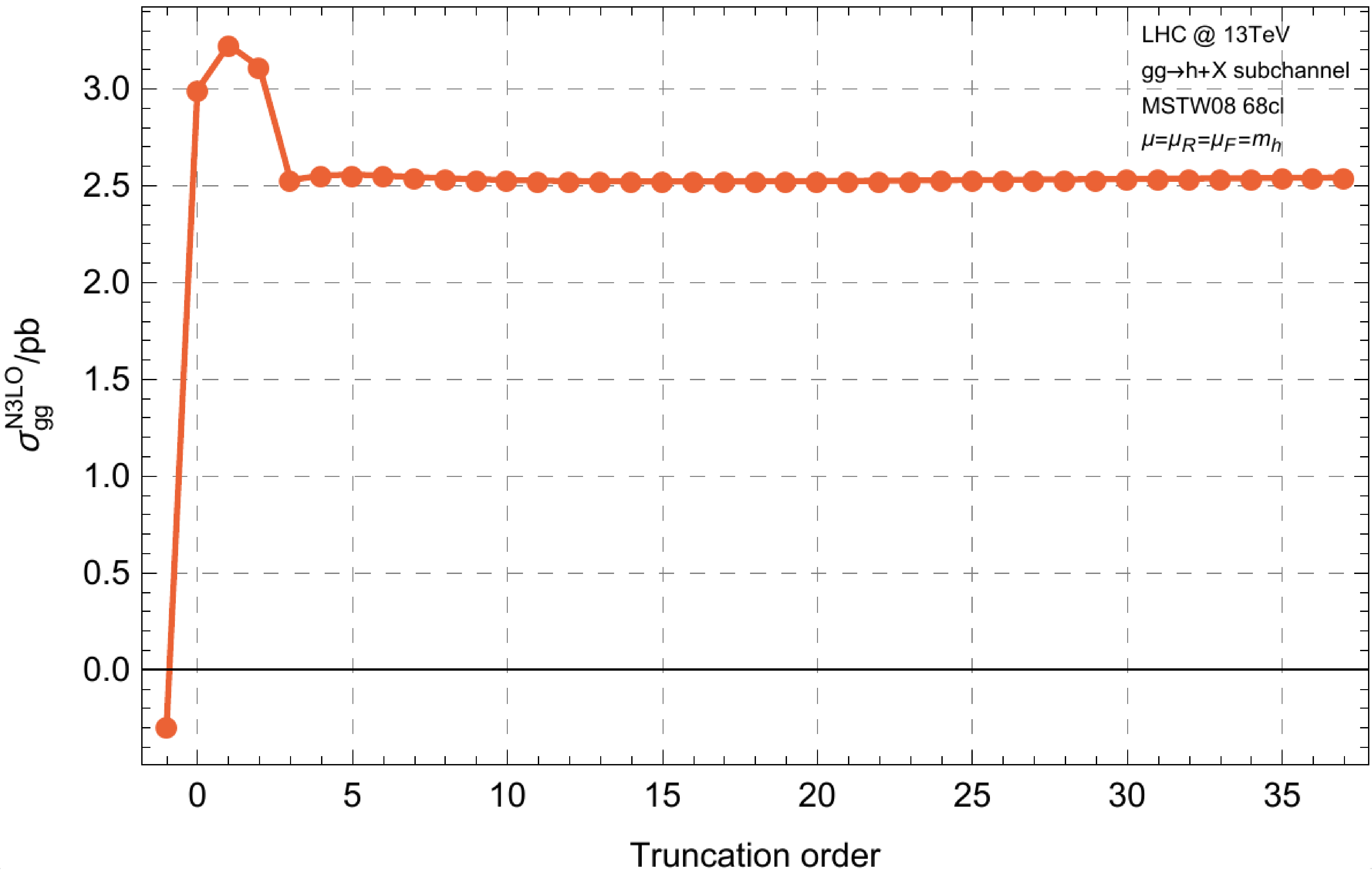


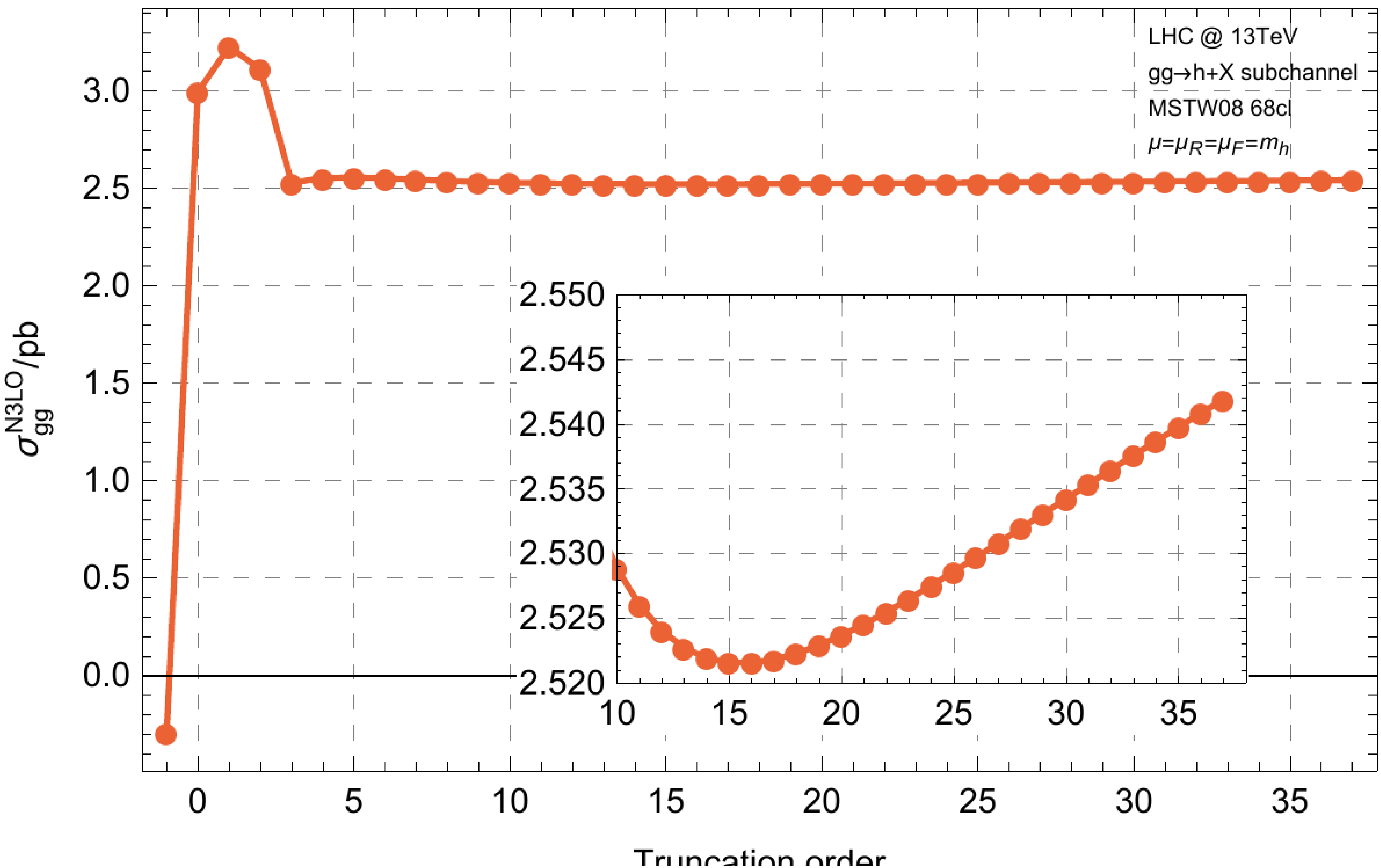
LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
gg→h+X subchannel
13.11.2014
Next-To-Soft contribution
Huge Jump! Need to calculate more!
σ_{gg}/pb
Truncation order



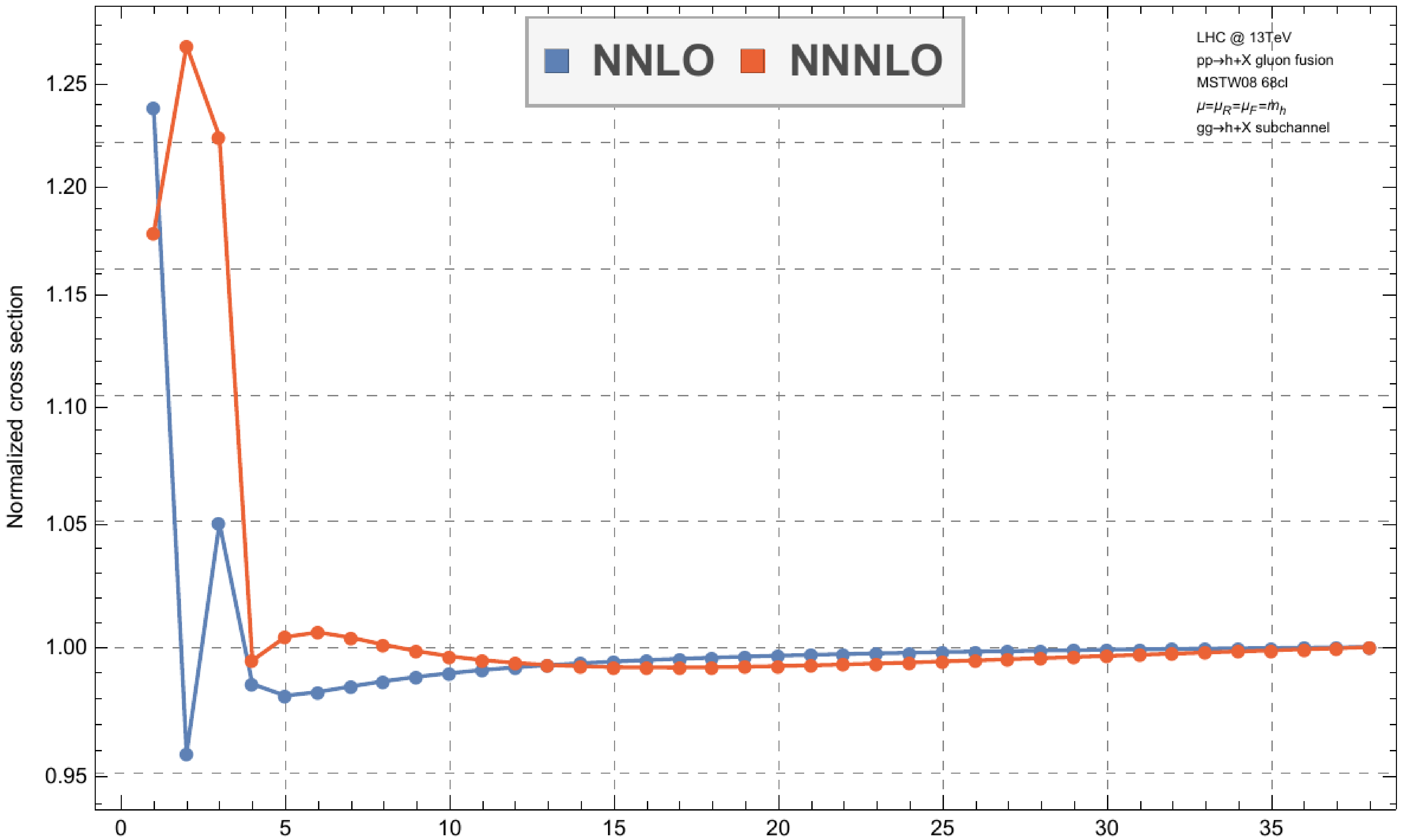
LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
μ=μR=μF=mh
gg→h+X subchannel
More stable??
σgg/pb
Truncation order

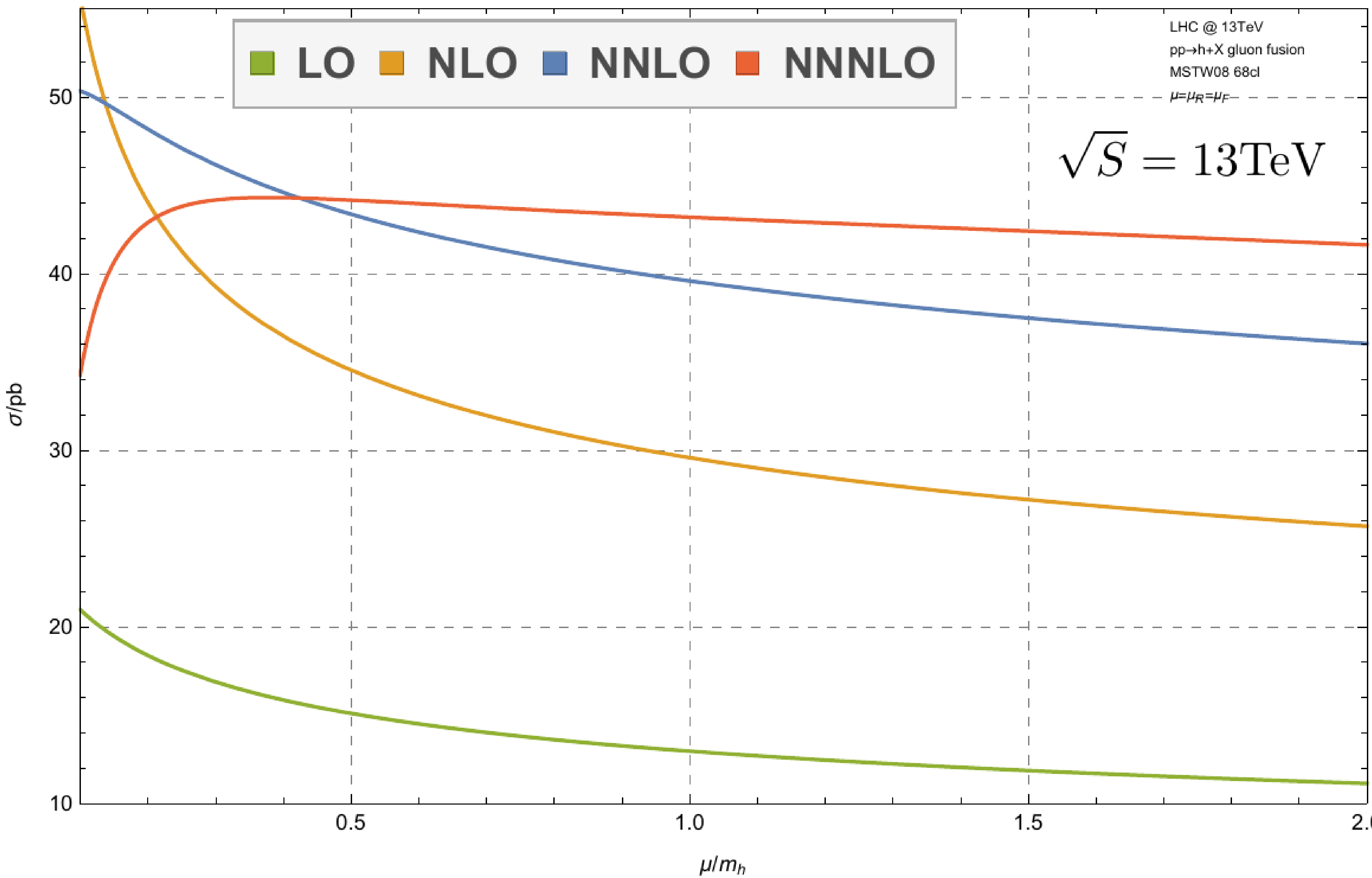

LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
gg→h+X subchannel
More stable??
σ_{gg}/pb
Truncation order
4
3
2
1
0
−1
0
1
2
3

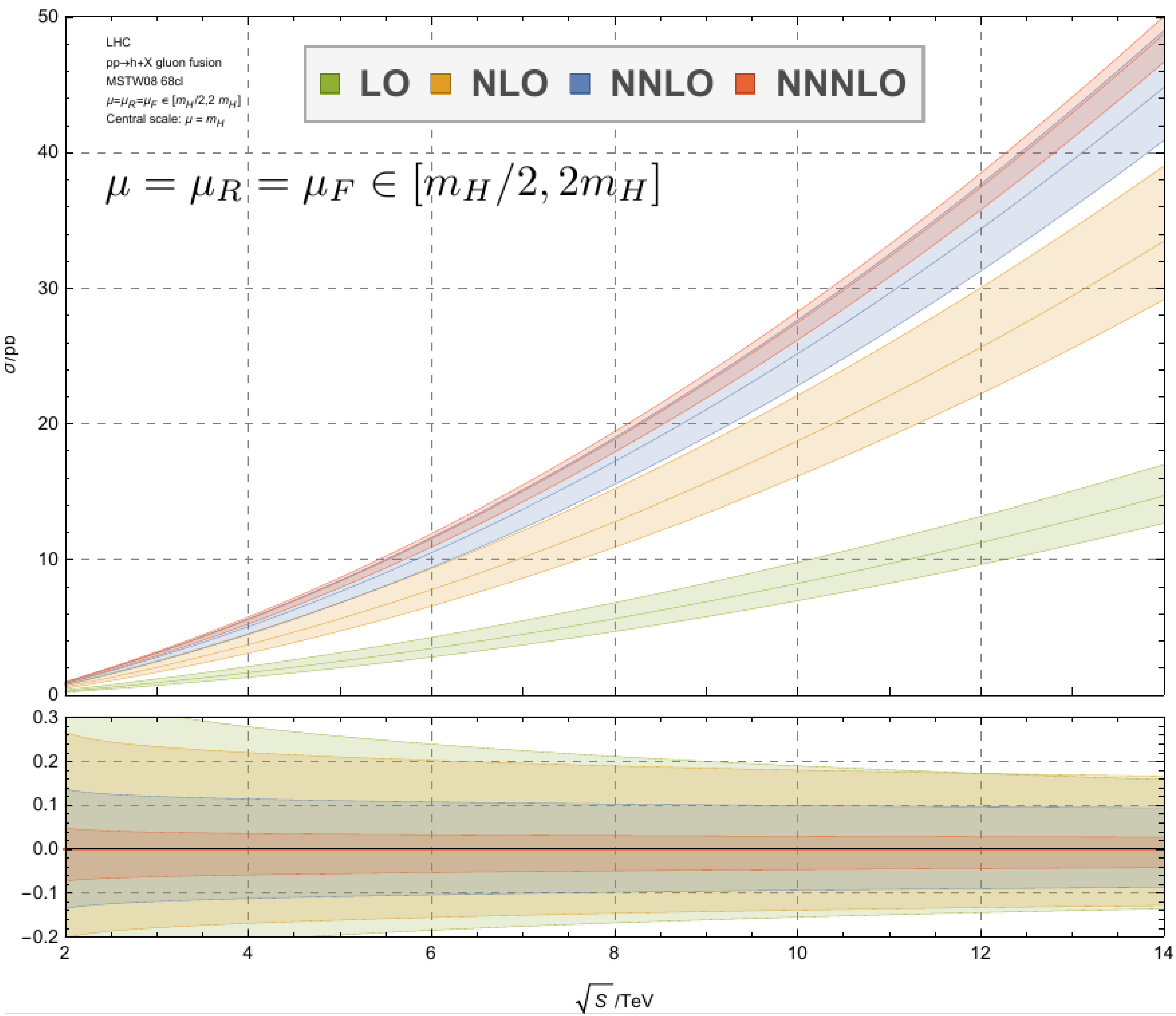

LHC @ 13TeV
gg→h+X subchannel
MSTW08 68cl
μ=μR=μF=mh
σ_gg^N3LO/pb
Truncation order
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
30
35

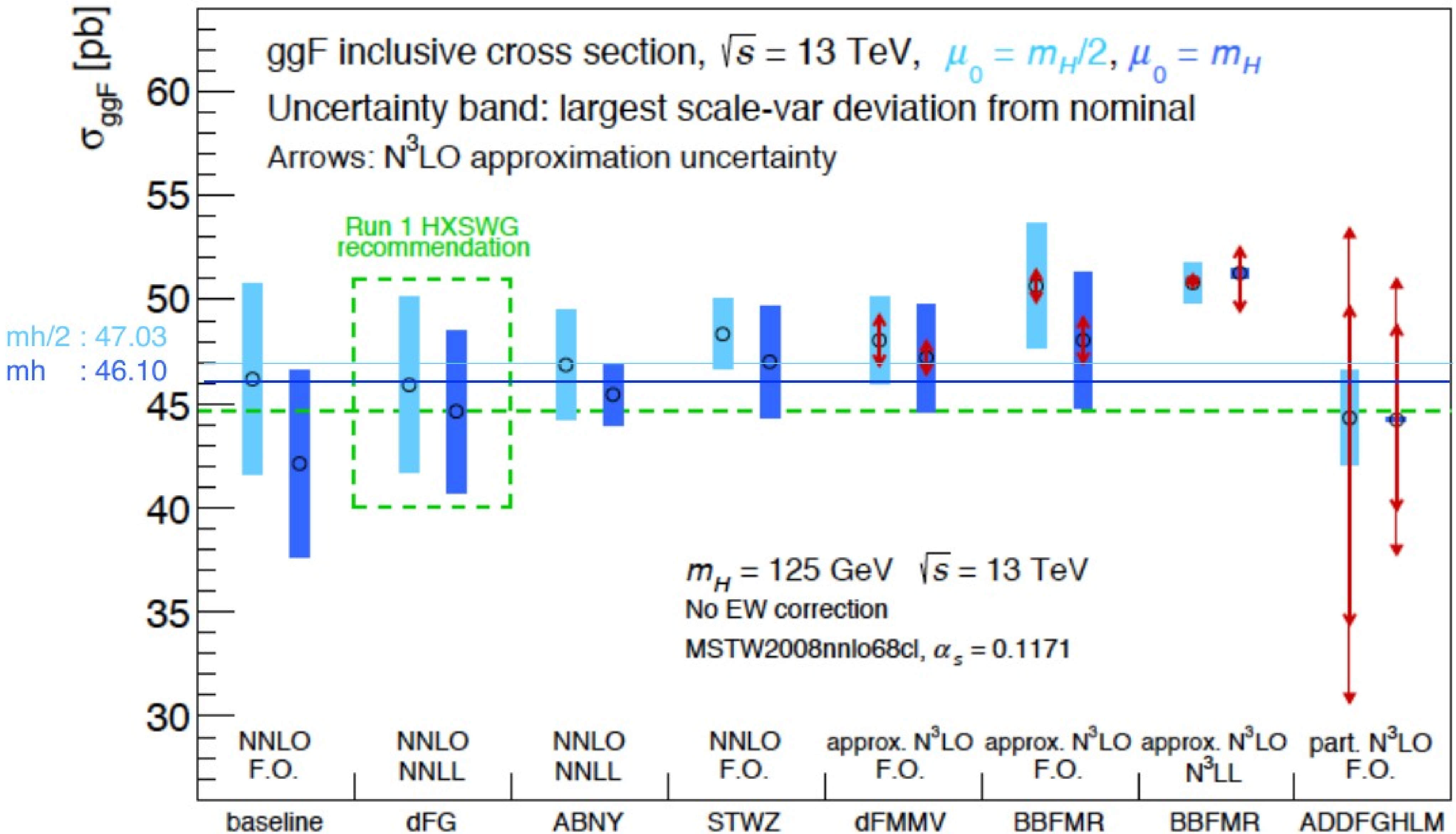


LHC @ 13TeV
gg→h+X subchannel
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
σ_{gg}^{N3LO}/pb
Truncation order

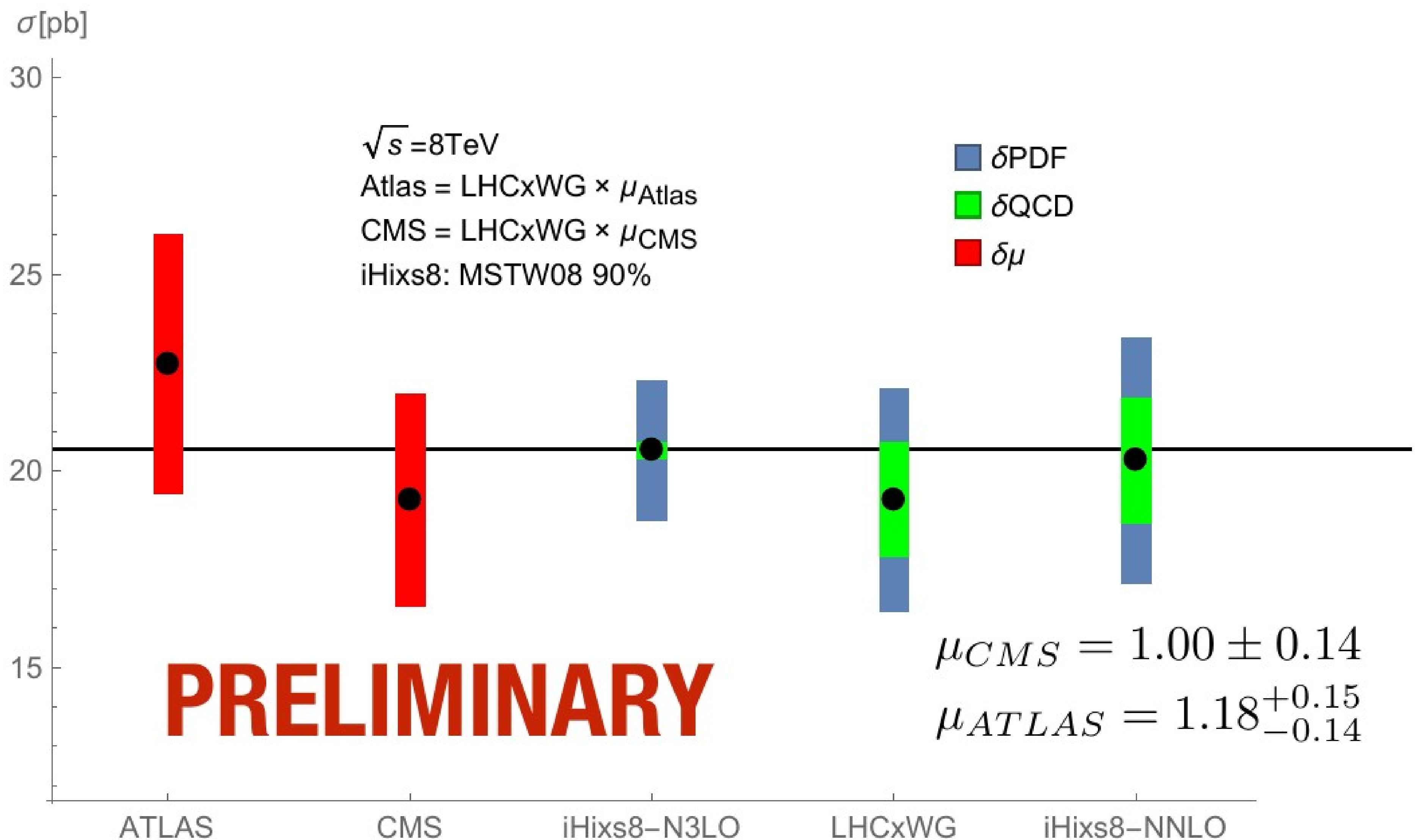

NNLO
NNNLO
LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
gg→h+X subchannel
Normalized cross section
0.95
1.00
1.05
1.10
1.15
1.20
1.25
0
5
10
15
20
25
30
35


LO
NLO
NNLO
NNNLO
LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
μ=μR=μF
$\sqrt{S} = 13\mathrm{TeV}$
σ/pb
μ/mh
10
20
30
40
50
0.5
1.0
1.5
2.0


LHC
pp→h+X gluon fusion
MSTW08 68cl
μ=μR=μF ∈ [mH/2,2 mH]
Central scale: μ = mH
LO
NLO
NNLO
NNNLO
$\mu = \mu_R = \mu_F \in [m_H/2, 2m_H]$
σ/pb
√S /TeV



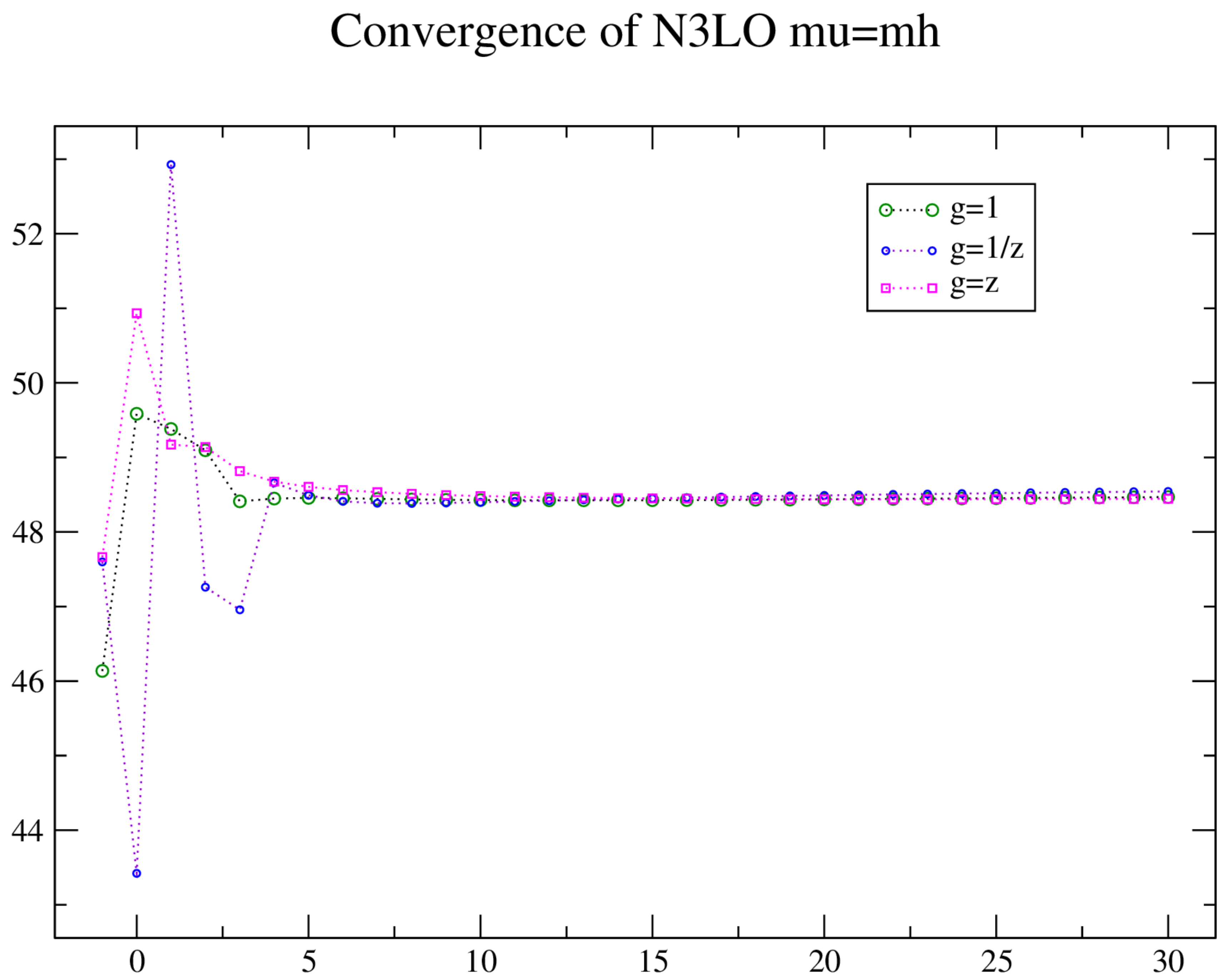
mh/2 : 47.03
mh : 46.10
σ_ggF [pb]
ggF inclusive cross section, √s = 13 TeV, μ_0 = m_H/2, μ_0 = m_H
Uncertainty band: largest scale-var deviation from nominal
Arrows: N3LO approximation uncertainty
Run 1 HXSWG recommendation
m_H = 125 GeV √s = 13 TeV
No EW correction
MSTW2008nnlo68cl, α_s = 0.1171
60
55
50
45
40
35
30
NNLO F.O.
NNLO NNLL
NNLO NNLL
NNLO F.O.
approx. N3LO F.O.
approx. N3LO F.O.
approx. N3LO N3LL
part. N3LO F.O.
baseline
dFG
ABNY
STWZ
dFMMV
BBFMR
BBFMR
ADDFGHLM



σ[pb]
30
25
20
15
√s =8TeV
Atlas = LHCxWG × μAtlas
CMS = LHCxWG × μCMS
iHixs8: MSTW08 90%
δPDF
δQCD
δμ
PRELIMINARY
μCMS = 1.00 ± 0.14
μATLAS = 1.18 +0.15 −0.14
ATLAS
CMS
iHixs8−N3LO
LHCxWG
iHixs8−NNLO

Conclusions

- A new Era of N3LO QCD Precision Physics
has begun at the LHC!
- The theory uncertainties in Higgs Production have decreased dramatically.
- It is now time to re-investigate other effects in Higgs Production:
  - PDF + α_s Uncertainties
  - Electroweak corrections
  - Top, bottom mass corrections
  - ...


Convergence of N3LO mu=mh
g=1
g=1/z
g=z
52
50
48
46
44
0
5
10
15
20
25
30