

NLO+PS matching in Monte Carlo scheme – the KrkNLO method

Sebastian Sapeta

CERN

in collaboration with:

S. Jadach, W. Płaczek, A. Siódmok and M. Skrzypek

based on arXiv:1503.06849

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Outline and motivation

I will talk about **a method for NLO+PS matching applied to Drell-Yan process.**

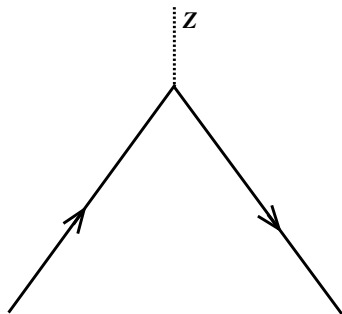
Key ingredients:

- ▶ new factorization scheme leading to new MC PDFs
- ▶ NLO correction applied to PS via reweighting of MC events

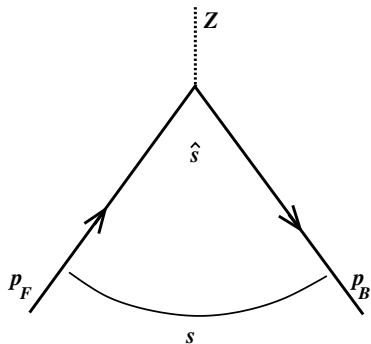
But there are two well established methods, POWHEG and MC@NLO...
...so what is the problem?

- ▶ By departing from $\overline{\text{MS}}$, the NLO+PS matching becomes very simple
→ just multiplying by a positive MC weight.
- ▶ Possibility of correcting all vertices
→ hence no need for k_T -ordered or truncated showers.
- ▶ That alone could have not been worth the effort. But, if it is so simple at NLO+LO PS, you may hope that pushing it to NNLO+NLO PS is possible.

Drell-Yan process

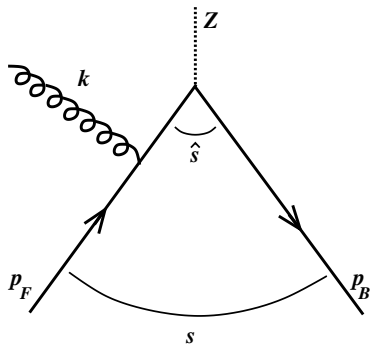


Drell-Yan process



$$s = (p_F + p_B)^2$$

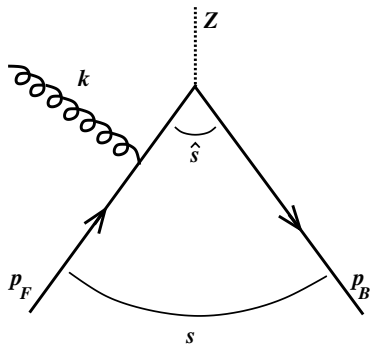
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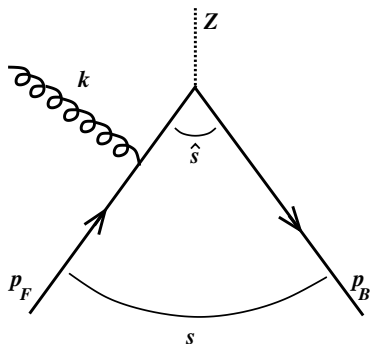
$$z = \frac{\hat{s}}{s}$$

Sudakov variables:

$$\alpha = \frac{2k \cdot p_B}{\sqrt{s}} = \frac{2k^+}{\sqrt{s}}$$

$$\beta = \frac{2k \cdot p_F}{\sqrt{s}} = \frac{2k^-}{\sqrt{s}}$$

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$$z = 1 - \alpha - \beta$$

$$k_T^2 = s\alpha\beta$$

$$y = \frac{1}{2} \ln \frac{\alpha}{\beta}$$

DY cross section at NLO in collinear $\overline{\text{MS}}$ factorization

For the $q\bar{q}$ channel:

$$d\sigma_{\text{DY}}^1 = \sigma_{\text{DY}}^B D_1^{\overline{\text{MS}}}(x_1, \hat{s}) \otimes \frac{\alpha_s}{2\pi} C_{2q}^{\overline{\text{MS}}}(z) \otimes D_2^{\overline{\text{MS}}}(x_2, \hat{s}),$$

where

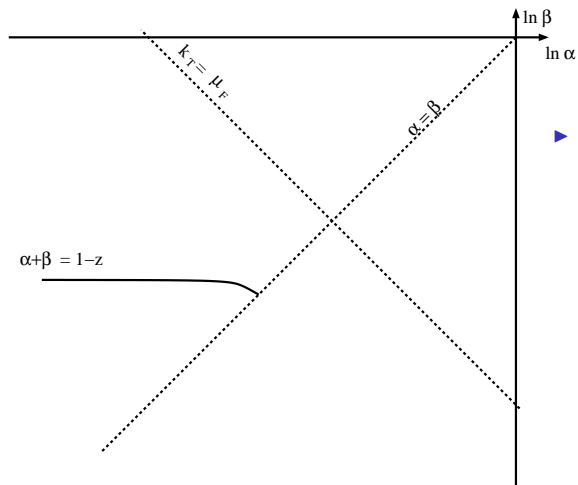
$$C_{2q}^{\overline{\text{MS}}}(z) = C_F \left[4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2 \frac{1+z^2}{1-z} \ln z + \delta(1-z) \left(\frac{2}{3} \pi^2 - 8 \right) \right].$$

We want to reproduce this with Monte Carlo, in a fully exclusive way.

If we decide to use $\overline{\text{MS}}$ PDFs, we need to generate terms of the type $4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+$ that are technical artefacts of $\overline{\text{MS}}$ scheme.

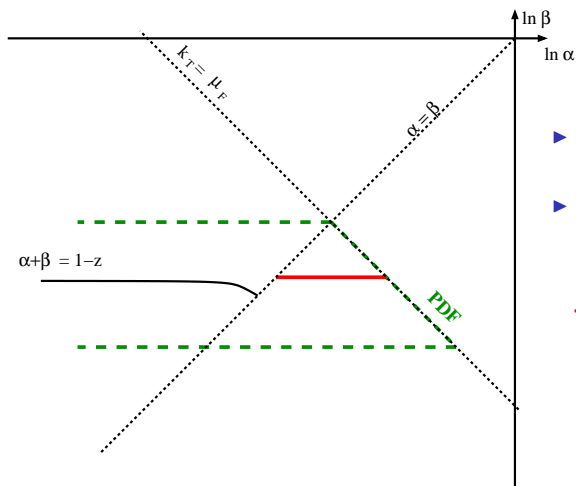
This is problematic since those terms correspond to the collinear limit but Monte Carlo lives in 4 dimensions and not in the phase space restricted by $\delta(k_T^2)$.

Origin of $4 \frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



- Integration extends up to a fixed $k_T = \mu_F$.

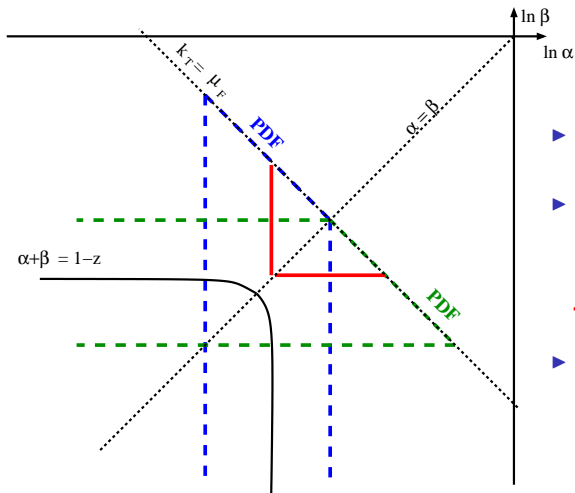
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- ▶ Integration extends up to a fixed $k_T = \mu_F$.
- ▶ For one PDF we get

$$\int \frac{1}{1-z} \frac{d\beta}{\beta} = 2 \frac{\ln(1-z)}{1-z}$$

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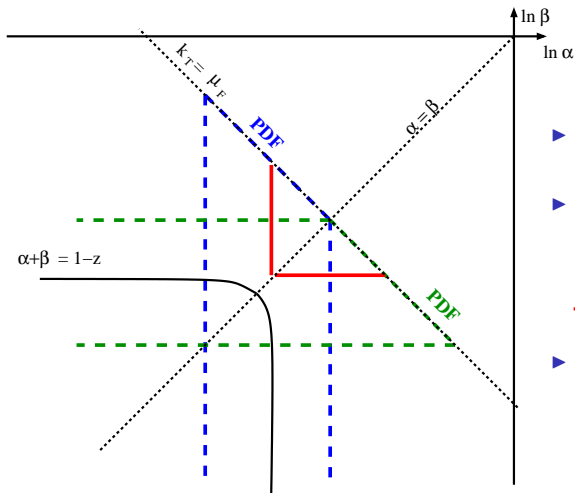


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Origin of $4 \frac{\ln(1-z)}{1-z}$ in \overline{MS}



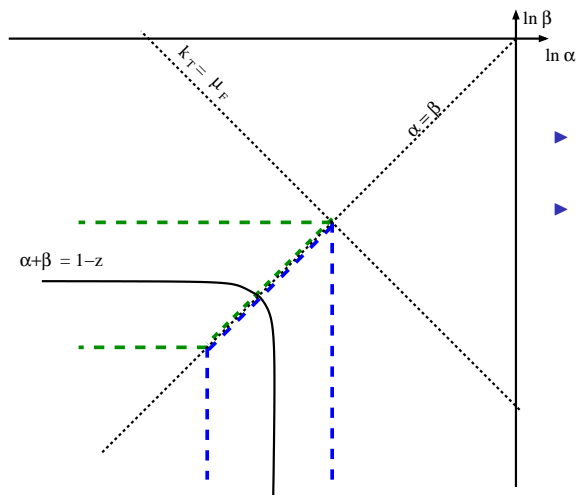
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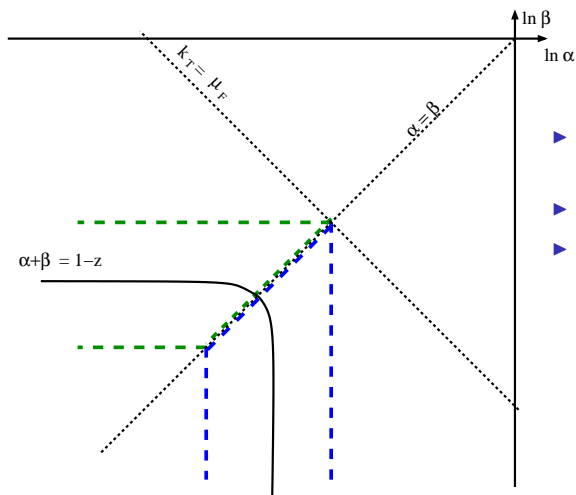
Could we reorganize phase space integration to remove the oversubtraction?

Alternative factorization scheme



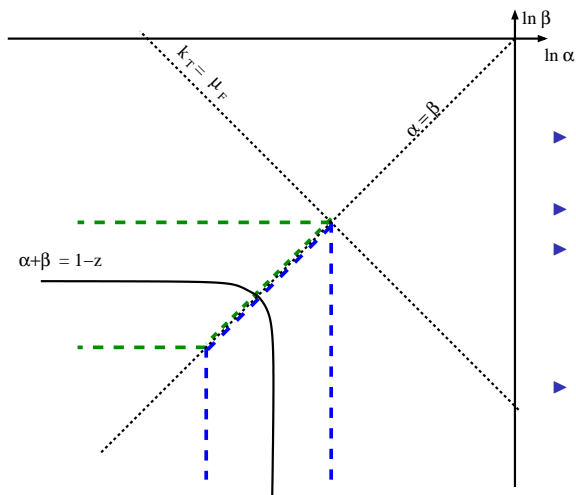
- ▶ Integration in angle rather than k_T .
- ▶ No overcounting.

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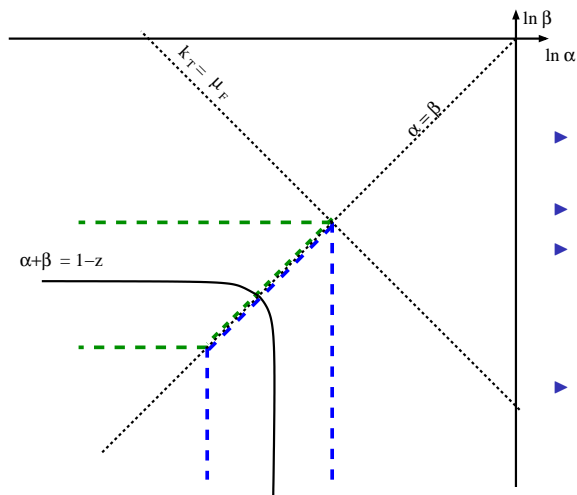
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Could the change of factorization scheme help us to simplify NLO+PS matching?

The KrkNLO method

Two essential steps

1. Change the factorization scheme from $\overline{\text{MS}}$ to MC

- ▶ produce new MC PDFs
- ▶ differences at LO
- ▶ universality: recovering $\overline{\text{MS}}$ NLO result

2. Reweight parton shower

- ▶ correct hardest emission by 'real' weight
- ▶ upgrade the cross section/distributions to NLO by multiplicative, constant 'soft+virtual' weight

The KrkNLO method

[Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13]

↔ Developed and validated in the context of a toy model.

- ▶ Take a parton shower that covers the (α, β) phase space completely and produces emissions according to approx. matrix element $K \simeq R$.
- ▶ Upgrade the real emissions to exact ME by **reweighting** with R/K .
- ▶ Upon integration over transverse d.o.f. this upgraded PS will give an extra contribution $C_2(z) = \int (R - K)$ to the cross section.
- ▶ Redefine PDFs by subtracting the above $C_2(z)$ together with all the z -dependent terms from \overline{MS} coefficient function. This means transforming PDFs to a new **MC factorization scheme**.
- ▶ Virtual+soft correction, Δ_{S+V} , is just a constant now. Multiply the whole result by $1 + \Delta_{S+V}$ to achieve complete NLO accuracy.

Implementation on top of the Catani-Seymour shower

↔ We used [Sherpa 2.0.0](#) implementation of the CS shower.

Phase space measure of emitted gluon

$$\frac{d\alpha}{\alpha} \frac{d\beta}{\beta} = \frac{d\alpha d\beta}{\beta(\alpha + \beta)} + \frac{d\alpha d\beta}{\alpha(\alpha + \beta)}$$

- ▶ The evolution variable:

$$q_F^2 = s(\alpha + \beta)\beta, \quad q_B^2 = s(\alpha + \beta)\alpha,$$

hence

$$\frac{d\alpha d\beta}{\alpha\beta} = \frac{dq_F^2}{q_F^2} \frac{dz}{1-z} + \frac{dq_B^2}{q_B^2} \frac{dz}{1-z}.$$

- ▶ The CS shower covers all space of (α, β) .

$$\alpha + \beta \leq 1 \quad \Rightarrow \quad z \geq 0 \quad \text{and} \quad q_{F,B}^2 \leq s$$

$$\alpha, \beta > 0 \quad \Rightarrow \quad (1-z)^2 > q_F^2/s \quad \text{or} \quad (1-z)^2 > q_B^2/s$$

Implementation on top of the Catani-Seymour shower

↪ It turns out that coefficient functions of the CS shower equal to those from the MC scheme of Jadach et al. arXiv:1103.5015. Hence, $CS \equiv MC$.

The $C_2(z)$ function:

$$C_2^{\text{MC}}(z) \Big|_{\text{real}} = \int (R - K)$$

- ▶ For the $q\bar{q}$ channel:

$$C_{2q}^{\text{MC}}(z) \Big|_{\text{real}} = \frac{\alpha_s}{2\pi} C_F [-2(1-z)]$$

- ▶ For the qg channel:

$$C_{2g}^{\text{MC}}(z) \Big|_{\text{real}} = \frac{\alpha_s}{2\pi} T_R \frac{1}{2} (1-z)(1+3z)$$

- ▶ Quark and anti-quark PDFs are redefined by:
 - ▶ subtracting $C_{2q}^{\text{MC}}(z)$ and $C_{2g}^{\text{MC}}(z)$ from $\overline{\text{MS}}$ PDFs
 - ▶ absorbing all z -dependent terms from $\overline{\text{MS}}$ coefficient functions
- ▶ The virtual correction:

$$C_{2q} \Big|_{\text{virt}} = \delta(1-z) \left(\frac{4}{3}\pi^2 - \frac{5}{2} \right)$$

is applied multiplicatively.

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Simple form of the coefficient functions with no singular terms!

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MC PDFs

Transformation from $\overline{\text{MS}}$ to MC. The (anti-)quark PDF:

$$f_{q(\bar{q})}^{\text{MC}}(x, Q^2) = f_{q(\bar{q})}^{\overline{\text{MS}}}(x, Q^2) + \int_x^1 \frac{dz}{z} f_{q(\bar{q})}^{\overline{\text{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2q}(z) + \int_x^1 \frac{dz}{z} f_g^{\overline{\text{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2g}(z)$$

where

$$\Delta C_{2q}(z) = C_{2q}^{\overline{\text{MS}}}(z) - C_{2q}^{\text{MC}}(z) = \frac{\alpha_s}{2\pi} C_F \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z} + 1-z \right]_+$$

$$\Delta C_{2g}(z) = C_{2g}^{\overline{\text{MS}}}(z) - C_{2g}^{\text{MC}}(z) = \frac{\alpha_s}{2\pi} T_R \left\{ \left[z^2 + (1-z)^2 \right] \ln \frac{(1-z)^2}{z} + 2z(1-z) \right\}$$

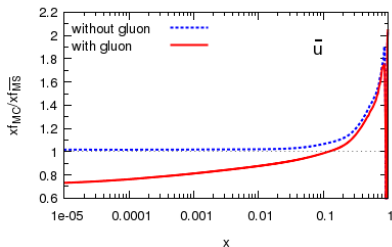
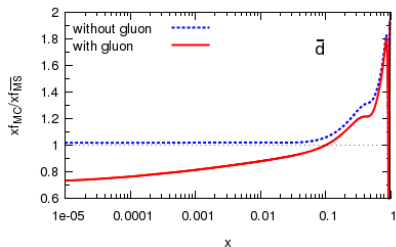
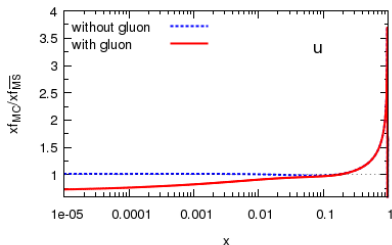
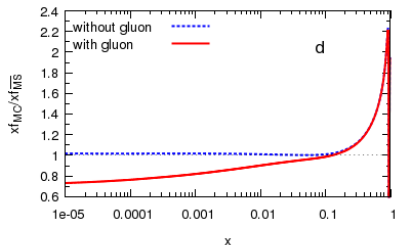
The formula is valid for any process up to $\mathcal{O}(\alpha_s^2)$.

The gluon PDF for DY

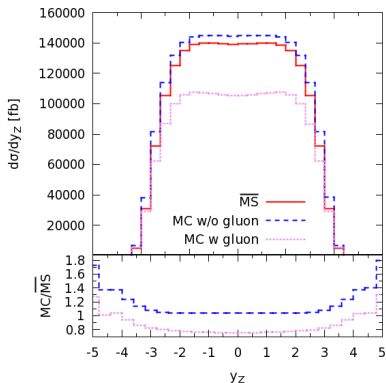
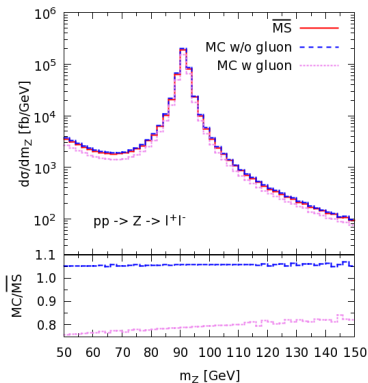
$$f_g^{\text{MC}}(x, Q^2) = f_g^{\overline{\text{MS}}}(x, Q^2)$$

MC PDFs

- ▶ Ratios with respect to standard $\overline{\text{MS}}$ PDFs for light quarks.



$\overline{\text{MS}}$ vs MC at LO



- ▶ +5% effect at central rapidities in $q\bar{q}$ and -20% for both channels
- ▶ pronounced difference at large y coming from the $x \sim 1$ region

$$x_{1,2} = \frac{m_Z}{\sqrt{s}} e^{\pm y_Z}$$

$\overline{\text{MS}}$ scheme vs MC scheme at NLO

$$\begin{aligned}\sigma_{\text{tot}}^{\overline{\text{MS}}} &= f_q \otimes (1 + \alpha_s C_q^{\overline{\text{MS}}}) \otimes f_{\bar{q}} \\ \sigma_{\text{tot}}^{\text{MC}} &= (f_q + \alpha_s \Delta f_q) \otimes (1 + \alpha_s C_q^{\text{MC}}) \otimes (f_{\bar{q}} + \alpha_s \Delta f_{\bar{q}}) \\ &= f_q f_{\bar{q}} + \alpha_s \left(\Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}} \right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3)\end{aligned}$$

At $\mathcal{O}(\alpha_s)$:

$$C_q^{\overline{\text{MS}}} f_q f_{\bar{q}} = \Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}}$$

Drell-Yan, $q\bar{q}$ channel, $\alpha_s = \alpha_s(m_Z)$, MCFM, MSTW2008LO

$$(336.36 \pm 0.09) \text{ pb} = \underbrace{25.79 \text{ pb} + 25.79 \text{ pb} + 284.77 \text{ pb}}_{(336.35 \pm 0.09) \text{ pb}}$$

- ▶ Final result is scheme independent up to $\mathcal{O}(\alpha_s)$.
- ▶ A nontrivial test as $\Delta f_{q,\bar{q}}$ and C_2 come from totally different places. In particular, the former includes convolutions.
- ▶ Terms $\mathcal{O}(\alpha_s^2) \simeq 16 \text{ pb}$, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2 \text{ pb}$.

↔ Identical validation performed with both $q\bar{q}$ and qg channels.

Reweighting procedure

The “Sudakov” form factor for the CS shower

$$S(Q^2, \Lambda^2, x) = \int_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz K(q^2, z, x),$$

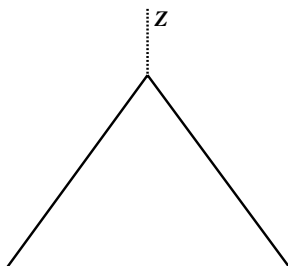
where

$$K(q^2, z, x) = \frac{C_F \alpha_s}{2\pi} \frac{1+z^2}{1-z} \frac{D(q^2, x/z)/z}{D(q^2, x)}.$$

- ▶ z, q^2 - internal variables of the shower
- ▶ $D(q^2, x)$ - parton distribution functions

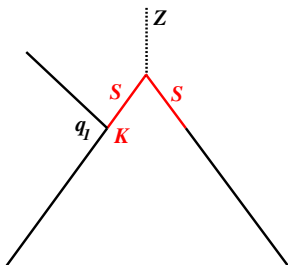
The kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution.

Upgrading to NLO: the hardest emission



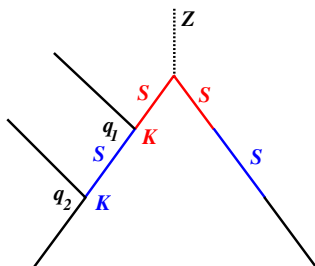
$$\sigma^{\text{LO}} = \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus})$$

Upgrading to NLO: the hardest emission



$$\sigma_{1+}^{\text{PS}} = \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus}) \\ \otimes \left\{ S_{\oplus}(q_1^2, Q^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(q_1^2, Q^2) + S_{\ominus}(q_1^2, Q^2) K_{\ominus}(q_1^2, z_1) S_{\oplus}(q_1^2, Q^2) \right\}$$

Upgrading to NLO: the hardest emission



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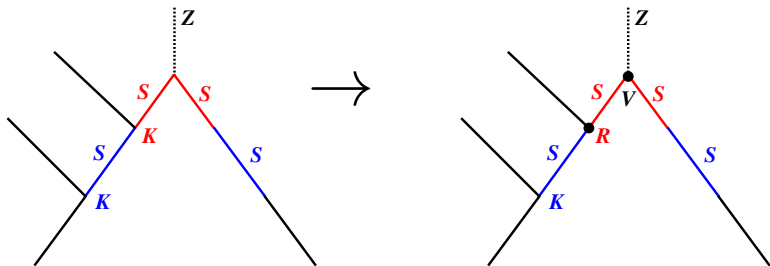
$$\otimes \left\{ S_{\oplus}(q_1^2, Q^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(q_1^2, Q^2) \right.$$

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$$+ S_{\ominus}(q_1^2, Q^2) \otimes K_{\ominus}(q_1^2, z_1) \otimes S_{\oplus}(q_1^2, Q^2)$$

$$\otimes \left\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \right\}$$

Upgrading to NLO: the hardest emission



$$\begin{aligned}
 \sigma_{2+}^{\text{NLO+PS}} &= \sigma_B (1 + V) \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus}) \\
 &\otimes \left\{ S_{\oplus}(q_1^2, Q^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(q_1^2, Q^2) R_{\oplus}(q_1^2, z_1) / K_{\oplus}(q_1^2, z_1) \right. \\
 &\quad \otimes \left\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \right\} \\
 &\quad + S_{\ominus}(q_1^2, Q^2) \otimes K_{\ominus}(q_1^2, z_1) \otimes S_{\oplus}(q_1^2, Q^2) R_{\ominus}(q_1^2, z_1) / K_{\ominus}(q_1^2, z_1) \\
 &\quad \left. \otimes \left\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \right\} \right\}
 \end{aligned}$$

The MC weight for CS shower

↔ Averaged over angles of decay products.

Real part:

$$W_R^{q\bar{q}}(\alpha, \beta) = 1 - \frac{2\alpha\beta}{1 + (1 - \alpha - \beta)^2}$$
$$W_R^{qg}(\alpha, \beta) = 1 + \frac{\beta(\beta + 2z)}{(1 - z)^2 + z^2}$$

Virtual + soft:

$$W_{V+S}^{q\bar{q}} = \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2} \right]$$
$$W_{V+S}^{qg} = 0$$

- ▶ **Real weight is a simple function of kinematic variables.**

One can compute it on the fly, inside an MC, or outside, using information from event record.

- ▶ **Virtual+soft weight is a constant.**

No need to generate strictly collinear contributions (like $d\Sigma^{c\pm}$ terms in MC@NLO).

NLO accuracy of KrkNLO results

$$\begin{aligned}d\sigma^{\text{KrkNLO}} &= \frac{d\sigma}{d\Omega}(s_{X_F X_B}, \hat{\theta})(1 + \Delta_{VS})e^{-S_F(s, q_s^2)}e^{-S_B(s, q_s^2)} \\ &\quad \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B) \\ &+ \int d^3\rho_1^F(s_{X_F X_B})(1 + \Delta_{VS})\frac{d\sigma}{d\Omega}(s_{X_F X_B} z_{1F}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2) \\ &+ \int d^3\rho_1^F(s_{X_F X_B})W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(s_{X_F X_B} z_{1F}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\end{aligned}$$

NLO accuracy of KrkNLO results

$$\begin{aligned}d\sigma^{\text{KrkNLO}} &= \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1 + \Delta_{VS})e^{-S_F(s, q_s^2)}e^{-S_B(s, q_s^2)} \\ &\quad \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B) \\ &+ \int d^3\rho_1^F(sx_F x_B)(1 + \Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2) \\ &+ \int d^3\rho_1^F(sx_F x_B)W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\end{aligned}$$

NLO accuracy of KrkNLO results

$$\begin{aligned}d\sigma^{\text{KrkNLO}} &= \frac{d\sigma}{d\Omega}(s_{x_F x_B}, \hat{\theta})(1 + \Delta_{VS})e^{-S_F(s, q_s^2)}e^{-S_B(s, q_s^2)} \\ &\quad \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B) \\ &+ \int d^3\rho_1^F(s_{x_F x_B})(1 + \Delta_{VS})\frac{d\sigma}{d\Omega}(s_{x_F x_B z_{1F}}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2) \\ &+ \int d^3\rho_1^F(s_{x_F x_B})W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(s_{x_F x_B z_{1F}}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\end{aligned}$$

- ▶ $D_{\text{MC}}^{F,B}(q_{1F}^2, x_{F,B}) \rightarrow D_{\text{MC}}^{F,B}(s, x_{F,B})$
- ▶ $e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)} \rightarrow 1$

NLO accuracy of KrkNLO results

$$\begin{aligned}d\sigma^{\text{KrkNLO}} &= \frac{d\sigma}{d\Omega}(s_{x_F x_B}, \hat{\theta})(1 + \Delta_{VS})e^{-S_F(s, q_s^2)}e^{-S_B(s, q_s^2)} \\ &\quad \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B) \\ &+ \int d^3\rho_1^F(s_{x_F x_B})(1 + \Delta_{VS})\frac{d\sigma}{d\Omega}(s_{x_F x_B z_{1F}}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2) \\ &+ \int d^3\rho_1^F(s_{x_F x_B})W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(s_{x_F x_B z_{1F}}, \hat{\theta}) \\ &\quad \times D_{\text{MC}}^F(s, x_F)D_{\text{MC}}^B(s, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\end{aligned}$$

NLO accuracy of KrkNLO results

$$\begin{aligned}d\sigma^{\text{KrkNLO}} &= \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1 + \Delta_{VS})e^{-S_F(s, q_s^2)}e^{-S_B(s, q_s^2)} \\ &\quad \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B) \\ &+ \int d^3\rho_1^F(sx_F x_B)(1 + \Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2) \\ &+ \int d^3\rho_1^F(sx_F x_B)W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \\ &\quad \times D_{\text{MC}}^F(s, x_F)D_{\text{MC}}^B(s, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\end{aligned}$$

NLO accuracy of KrkNLO results

$$\begin{aligned}d\sigma^{\text{KrkNLO}} &= \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1 + \Delta_{VS})e^{-S_F(s, q_s^2)}e^{-S_B(s, q_s^2)} \\ &\quad \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B) \\ &+ \int d^3\rho_1^F(sx_F x_B)(1 + \Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)\left[J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\right. \\ &\quad \left.+ J_{\text{LO}}(x_F, x_B) - J_{\text{LO}}(x_F, x_B)\right] \\ &+ \int d^3\rho_1^F(sx_F x_B)W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \\ &\quad \times D_{\text{MC}}^F(s, x_F)D_{\text{MC}}^B(s, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\end{aligned}$$

NLO accuracy of KrkNLO results

$$\begin{aligned}d\sigma^{\text{KrkNLO}} &= \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1 + \Delta_{VS})e^{-S_F(s, q_s^2)}e^{-S_B(s, q_s^2)} \\ &\quad \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B) \\ &+ \int d^3\rho_1^F(sx_F x_B)(1 + \Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \\ &\quad \times e^{-S_F(s, q_{1F}^2)}e^{-S_B(s, q_{1F}^2)}D_{\text{MC}}^F(q_{1F}^2, x_F)D_{\text{MC}}^B(q_{1F}^2, x_B)\left[J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\right. \\ &\quad \left.+ J_{\text{LO}}(x_F, x_B) - J_{\text{LO}}(x_F, x_B)\right] \\ &+ \int d^3\rho_1^F(sx_F x_B)W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \\ &\quad \times D_{\text{MC}}^F(s, x_F)D_{\text{MC}}^B(s, x_B)J_{\text{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)\end{aligned}$$

- ▶ $+J_{\text{LO}}$ term goes to the first part and moves PDFs to the scale s
- ▶ $-J_{\text{LO}}$ term makes the second part finite \Rightarrow drop Sudakovs and Δ_{VS}
- ▶ $s \rightarrow \hat{s}$ (corrections $\mathcal{O}(\alpha_s^2)$)

NLO accuracy of KrkNLO results

We get

$$\begin{aligned} d\sigma^{\text{KrkNLO}} = & (1 + \Delta_{VS}) \frac{d\sigma}{d\Omega}(s_1, \hat{\theta}) J_{\text{LO}} D_F^{\text{MC}}(\hat{s}, x_F) D_B^{\text{MC}}(\hat{s}, x_B) \\ & + \int d^3\rho_1 \left\{ \frac{d\sigma}{d\Omega}(zs_1, \hat{\theta}) (1 + W_{q\bar{q}}^{[1]}) J_{\text{NLO}} - \frac{d\sigma}{d\Omega}(zs_1, \hat{\theta}) J_{\text{LO}} \right\} \\ & \times D_F^{\text{MC}}(\hat{s}, x_F) D_B^{\text{MC}}(\hat{s}, x_B) \end{aligned}$$

to be compared with

$$\begin{aligned} d\sigma^{\text{NLO}} = & \int dz D_F^{\overline{\text{MS}}}(\hat{s}, x_F) D_B^{\overline{\text{MS}}}(\hat{s}, x_B) \\ & \times \left\{ (1 + \Delta_{VS}) \delta(1 - z) + 2\Sigma_q(z) \right\} \frac{d\sigma_0}{d\Omega}(zs_1, \theta) J(x_F, x_B, 1, 0) \\ & + \int \left\{ d\sigma_{\text{NLO}} J_{\text{NLO}} - d\sigma_{\text{NLO-ct}} J_{\text{LO}} \right\} D_F^{\overline{\text{MS}}}(\hat{s}, x_F) D_B^{\overline{\text{MS}}}(\hat{s}, x_B) \end{aligned}$$

Results

NLO+PS results

KrkNLO

- ▶ Virtual: $\mu^2 = \mu_F^2 = \mu_R^2 = m_Z^2$
 - ▶ Real: two choices
 - ▶ $\mu^2 = m_Z^2$
 - ▶ $\mu^2 = q^2$
- ↪ differences formally beyond NLO, indicative of missing higher orders

Compared to:

- ▶ **MCFM**: pure NLO, $\mu^2 = m_Z^2$
- ▶ **MC@NLO**: from Sherpa, with the evolution variable q^2
- ▶ **POWHEG**: from Herwig++, with the evolution variable k_T^2

Matched results: total cross section

$q\bar{q}$ channel

	$\sigma_{\text{tot}}^{q\bar{q}}$ [pb]
MCFM	1273.4 ± 0.1
MC@NLO	1273.4 ± 0.1
POWHEG	1272.1 ± 0.7
KrkNLO $\alpha_s(q^2)$	1282.6 ± 0.2
KrkNLO $\alpha_s(M_Z^2)$	1285.3 ± 0.2

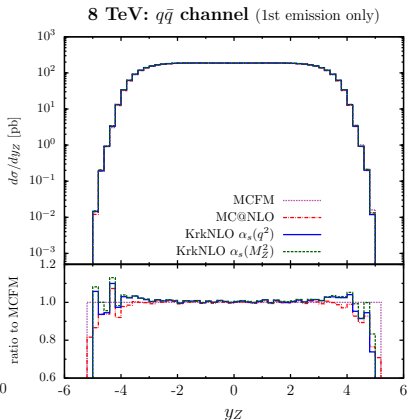
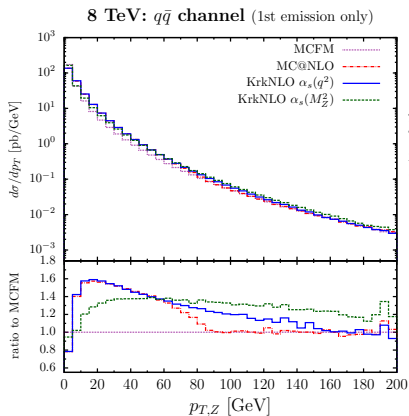
- ▶ sub-percent differences from beyond-NLO terms in the KrkNLO result (MC PDFs, mixed real-virtual)
- ▶ negligible difference between fixed and running coupling

$q\bar{q} + qg$ channels

	$\sigma_{\text{tot}}^{q\bar{q}+qg}$ [pb]
MCFM	1086.5 ± 0.1
MC@NLO	1086.5 ± 0.1
POWHEG	1084.2 ± 0.6
KrkNLO $\alpha_s(q^2)$	1045.4 ± 0.1
KrkNLO $\alpha_s(M_Z^2)$	1039.0 ± 0.1

- ▶ beyond-NLO terms reach up to 4% in the KrkNLO result
↪ resulting from large gluon luminosity leading to $f^{\text{MC}}/f^{\overline{\text{MS}}} < 1$
- ▶ small differences between fixed and running coupling choices

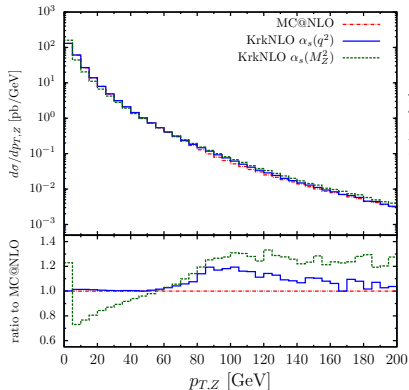
Matched results: $q\bar{q}$, 1st emission



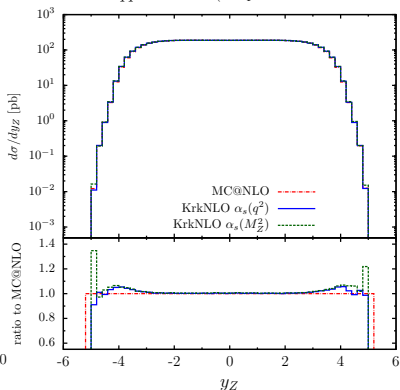
- ▶ Reproduction of y_Z distribution at NLO.
- ▶ Agreement of KrkNLO $\alpha_s(q^2)$ with MC@NLO at low $p_{T,Z}$: PS domination
- ▶ KrkNLO results above MC@NLO and MCFM at higher $p_{T,Z}$: $\mathcal{O}(\alpha_s^2)$ terms

Matched results: $q\bar{q}$, full PS

8 TeV: $q\bar{q}$ channel (full parton shower evolution)

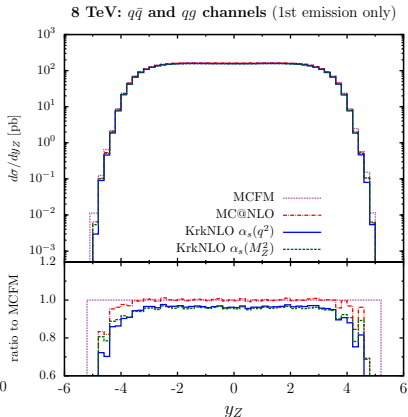
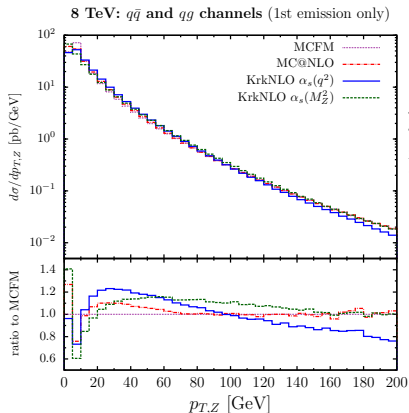


8 TeV: $q\bar{q}$ channel (full parton shower evolution)



- ▶ Low $p_{T,Z}$ part of the spectrum changes but KrkNLO $\alpha_s(q^2)$ with MC@NLO agree there because of shower domination
- ▶ KrkNLO results above pure NLO at high $p_{T,Z}$: admixture of NNLO terms
- ▶ Differs between two KrkNLO result at high $p_{T,Z}$: running coupling effects

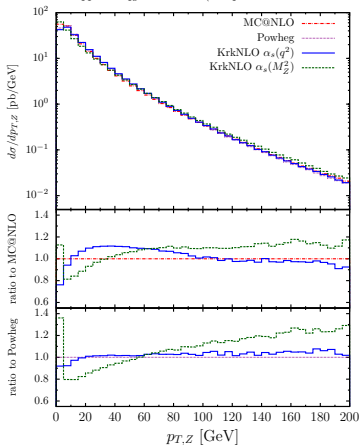
Matched results: botch channels, 1st emission



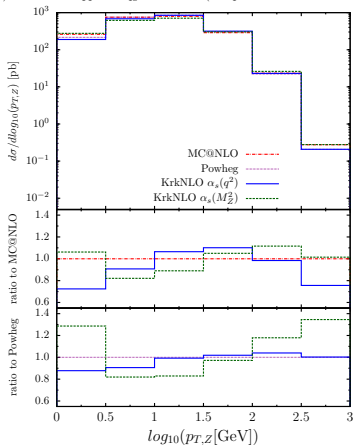
- Moderate differences between KrkNLO $\alpha_s(q^2)$ and MC@NLO in the region below M_Z and between KrkNLO $\alpha_s(M_Z^2)$ and MC@NLO in the region above M_Z

Matched results: both channels, full PS

8 TeV: $q\bar{q}$ and qg channels (full parton shower evolution)



8 TeV: $q\bar{q}$ and qg channels (full parton shower evolution)



- ▶ KrkNLO $\alpha_s(q^2)$ stays overall very close to MC@NLO
- ▶ KrkNLO $\alpha_s(q^2)$ almost coincides with POWHEG $p_{T,Z}$ distributions

Conclusions

- ▶ I have discussed a method of NLO+PS matching:
 - ▶ Real emissions are corrected by simple reweighting.
 - ▶ Collinear terms are dealt with by putting them to PDFs. This amounts to change of factorization scheme from $\overline{\text{MS}}$ to MC.
 - ▶ Virtual correction is just a constant and does not depend on Born kinematics.
- ▶ The method has been implemented on top of Catani-Seymour shower in Sherpa event generator.
- ▶ It has been validated against fixed order NLO for Drell-Yan process.
- ▶ A range of comparisons to MC@NLO and POWHEG.