Large-N_c QCD and the chiral extrapolation of baryon masses

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- ✔ **Large-N_c, chiral SU(3) and χPT for baryons**
- ✔ **Chiral extrapolation of baryon masses at N^2LO and N^3LO**
- ✔ **Summary and outlook**

Accidental symmetries of QCD with up and down quarks

$$
\begin{aligned}
\mathcal{L}_{\rm QCD}(x) \;&=\; \bar q_R(x)\, i\,\gamma^\mu \left(\frac{\partial}{\partial x_\mu} - i\,g \sum_{a=1}^{N_c{}^2-1} A^a_\mu(x)\, \frac{\lambda_a}{2} \right) q_R(x) \\
&+\; \bar q_L(x)\, i\,\gamma^\mu \left(\frac{\partial}{\partial x_\mu} - i\,g \sum_{a=1}^{N_c{}^2-1} A^a_\mu(x)\, \frac{\lambda_a}{2} \right) q_L(x) - \frac{1}{4} \sum_{a=1}^{N_c{}^2-1} G^{\mu\nu}_a(x)\, G_{\mu\nu,a}(x) \\
&-\; \underbrace{\bar q_R(x) \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} q_L(x) - \bar q_L(x) \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} q_R(x)}_{\text{breaks the chiral SU(2) symmetry explicitly}}
\end{aligned}
$$

✔ consider $m_{u,d} \simeq 5$ MeV to be small

- approximate $SU(2)_L \otimes SU(2)_R$ chiral symmetry
- parity doublets in hadron spectrum (if not broken spontaneously !)

✔ consider number of colors $N_c = 3$ to be large

- contracted spin-flavor symmetry SU(4)

$$g \to \frac{g}{\sqrt{N_c}}$$

The chiral Lagrangian with baryon fields

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\,\eta \end{pmatrix}$$

Goldstone boson octet ($J^P = 0^-$)

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\,\Lambda \end{pmatrix}$$

baryon octet ($J^P = \frac{1}{2}^+$)

✔ Leading order terms

$$\begin{aligned} \mathcal{L} \;=\;& \mathrm{tr}\left\{\bar B(i\,D\cdot\gamma - M_{[8]})\,B\right\} + F\,\mathrm{tr}\left\{\bar B\,\gamma^\mu\gamma_5\,[i\,U_\mu, B]\right\} + D\,\mathrm{tr}\left\{\bar B\,\gamma^\mu\gamma_5\,\{i\,U_\mu,\,B\}\right\} \\ -\;& \mathrm{tr}\left\{\bar B_\mu\cdot\big((i\,D\cdot\gamma - M_{[10]})\,g^{\mu\nu} - i\,(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + \gamma^\mu(i\,D\cdot\gamma + M_{[10]})\gamma^\nu\big)\,B_\nu\right\} \\ +\;& C\left(\mathrm{tr}\left\{(\bar B_\mu\cdot i\,U^\mu)\,B\right\} + h.c.\right) + H\,\mathrm{tr}\left\{(\bar B^\mu\cdot\gamma_\nu\gamma_5\,B_\mu)\,i\,U^\nu\right\}, \end{aligned}$$

- from $B \to B' + e + \bar\nu_e$: $F \simeq 0.45$ and $D \simeq 0.80$
- from large-N_c: $H = 9\,F - 3\,D$ and $C = 2\,D$

Chiral symmetry breaking terms

$$
\begin{aligned}
\mathcal{L}_\chi^{(2)} \;&=\; 2\,b_0\,\mathrm{tr}\,\left(\bar B\,B\right)\mathrm{tr}\,(\chi_+) + 2\,b_D\,\mathrm{tr}\,\left(\bar B\,\{\chi_+,\,B\}\right) + 2\,b_F\,\mathrm{tr}\,\left(\bar B\,[\chi_+,B]\right)\\
&-\; 2\,d_0\,\mathrm{tr}\,\left(\bar B_\mu\cdot B^\mu\right)\mathrm{tr}\,(\chi_+) - 2\,d_D\,\mathrm{tr}\,\left(\left(\bar B_\mu\cdot B^\mu\right)\chi_+\right)
\end{aligned}
$$

$$
\chi_+ = \chi_0 - \frac{1}{8\,f^2}\left\{\Phi,\left\{\Phi,\,\chi_0\right\}\right\} + \mathcal{O}\left(\Phi^4\right)
$$

$$
\chi_0 \sim \begin{pmatrix} m_u & 0 & 0\\ 0 & m_d & 0\\ 0 & 0 & m_s \end{pmatrix} + \underbrace{s}_{scalar\ source\ function}
$$

✔ **Relevance of low-energy parameters**

- quark-mass dependence of the baryon masses ↔ lattice QCD
- meson-baryon scattering ↔ resonances in QCD

Large-N_c at work

✔ **Axial vector current** : $A_\mu^{(a)}(x) = \bar{q}(x)\,\gamma_\mu\,\gamma_5\,\frac{\lambda^{(a)}}{2}\,q(x)$

$$\langle B'(\boldsymbol{p}')|A_i^{(a)}(0)|B(\boldsymbol{p})\rangle = (\mathcal{B}'|\,\mathcal{O}_i^{(a)}(\boldsymbol{p}',\boldsymbol{p})\,|\mathcal{B})$$
$$= (\mathcal{B}'|\,g(\boldsymbol{p}',\boldsymbol{p})\,G_i^a + h(\boldsymbol{p}',\boldsymbol{p})\,J_i\,T^a$$
$$+ f(\boldsymbol{p}',\boldsymbol{p})\,(J\cdot\boldsymbol{p}')\,G_i^a\,(J\cdot\boldsymbol{p}) + \cdots|\mathcal{B})$$

- effective baryon state $|\mathcal{B})$ depends on spin and flavour only
- spin (J_i), flavour (T^a) and spin-flavour (G_i^a) operators

✔ **Any operator $\mathcal{O}$** that is of order N_c^r in the large N_c limit can be written as a polynomial of J_i, $G_i^{(a)}$ and $T^{(a)}$

$$\frac{\mathcal{O}_{\rm QCD}}{N_c^r} = \text{Polynomial}\left(\frac{J_i}{N_c}, \frac{T^a}{N_c}, \frac{G_i^a}{N_c}\right)$$

Large-N_c sum rules and the chiral Lagrangian

✔ Axial-vector coupling constant of the baryons

$$F\,\mathrm{tr}\,\bar B_{[8]}\,\gamma_5\,\gamma^\mu\,\Big[a_\mu, B_{[8]}\Big]_- + D\,\mathrm{tr}\,\bar B_{[8]}\,\gamma_5\,\gamma^\mu\,\Big[a_\mu, B_{[8]}\Big]_+$$

$$C\,\mathrm{tr}\,\Big\{\Big(\bar B^\mu_{[10]}\cdot a_\mu\Big) B_{[8]} + \mathrm{h.c.}\Big\} \qquad H\,\mathrm{tr}\,\Big(\bar B^\alpha_{[10]}\cdot\gamma_5\,\gamma^\mu\,g_{\alpha\beta}\,a_\mu\Big) B^\beta_{[10]}$$

✔ At subleading order in the large-N_c analysis

$$\langle B'(\boldsymbol{p}')|A_i^{(a)}(0)|B(\boldsymbol{p})\rangle = (\mathcal{B}'|\,g\,G_i^a + h\,J_i\,T^a|\mathcal{B}) + \cdots$$

- consider the limit $\boldsymbol{p}$ and $\boldsymbol{p}' \to 0$
- matching with the result of the chiral Lagrangian leads to

$$D = g\,, \quad F = \frac{2}{3}\,g + h\,, \quad C = 2\,g\,, \qquad H = 3\,g + 9\,h$$

✔ Two sum rules are predicted

$$C = 2\,D \qquad H = 9\,F - 3\,D$$

Quark-mass dependence of the baryon masses

✔ A challenge

- 'poor' convergence in the heavy-baryon formulation of χPT

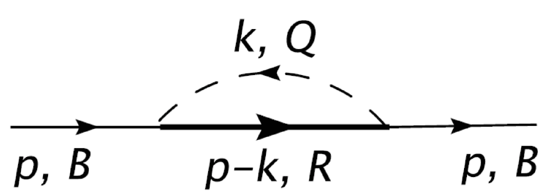
$$\text{e.g.} \qquad M_\Xi = (1018 + 1311 - 1007)\,\text{MeV} = 1322\,\text{MeV}$$

- χPT results are inconsistent with three-flavor QCD lattice simulations

✔ Self consistent resummation

$$M_B - \Sigma_B(M_B) = \begin{cases} M_{[8]} & \text{for} \quad B \in [8] \\ M_{[10]} & \text{for} \quad B \in [10] \end{cases}$$

$$\Sigma_B(p) = \sum_{Q\in[8]} \sum_{R\in[8],[10]}$$



$$+ \cdots$$

- one-loop expressions depend sensitively on the internal masses

Pion-mass dependence of the baryon octet masses

✔ Perform fits at N^{2}LO: exclude physical baryon masses in the fit

- determine 9 parameters relevant at N^{2}LO from lattice data
- consider data from PACS, HSC, LHPC, QCDSF, NPLQCD (about 220 data points)
- best fit leads to $F \sim D \sim 0$

  $\leftrightarrow$ the physical baryon masses are off by about 50 MeV
- a further solution is compatble with $F \simeq 0.45$ and $D \simeq 0.80$

  $\leftrightarrow$ the physical baryon masses are off by about 10 MeV only

Pion-mass dependence of the baryon octet masses

✔ Perf … es in the fit

- de …
- co … 220 data points)
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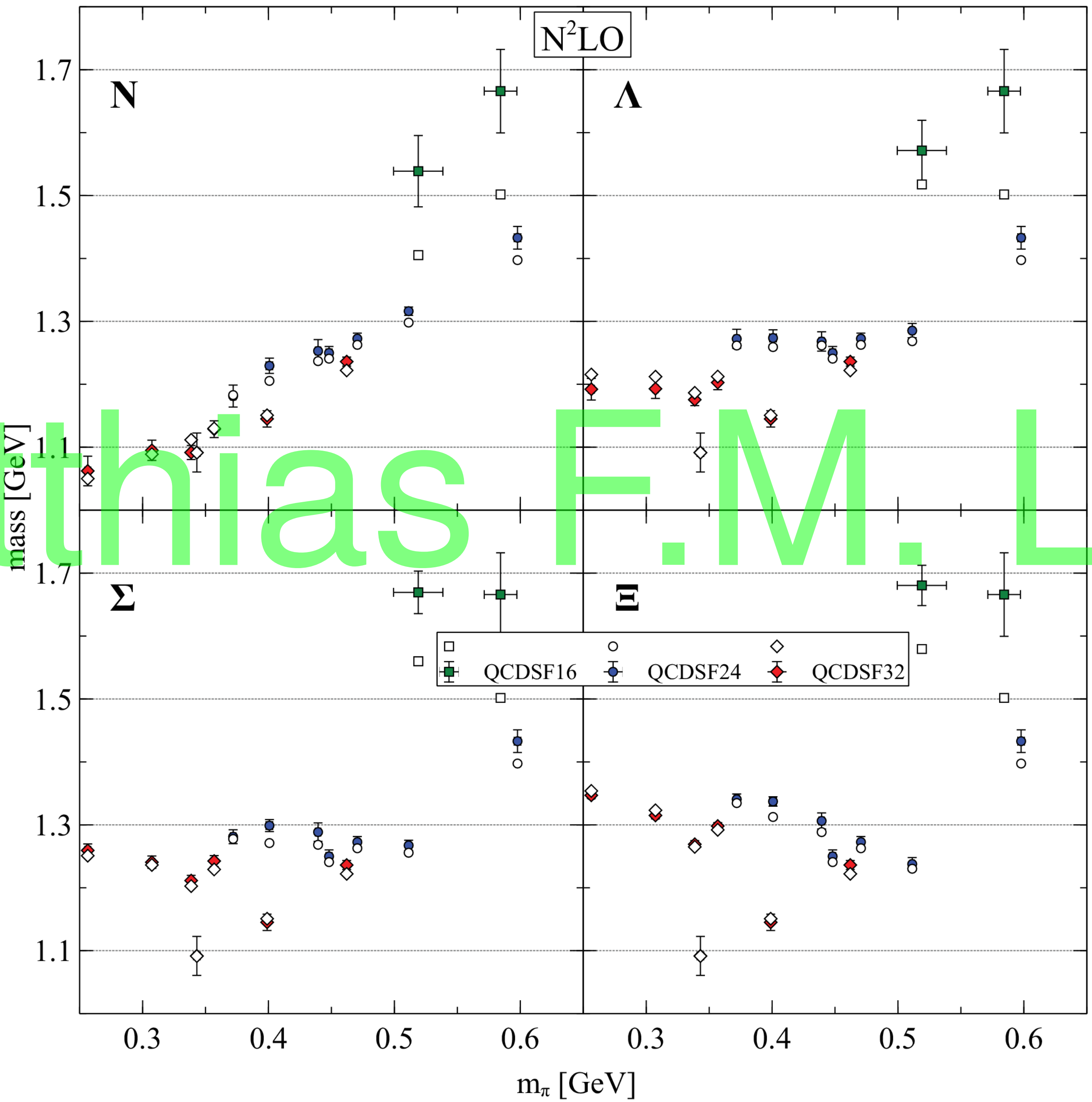


Pion-mass dependence of the baryon octet masses

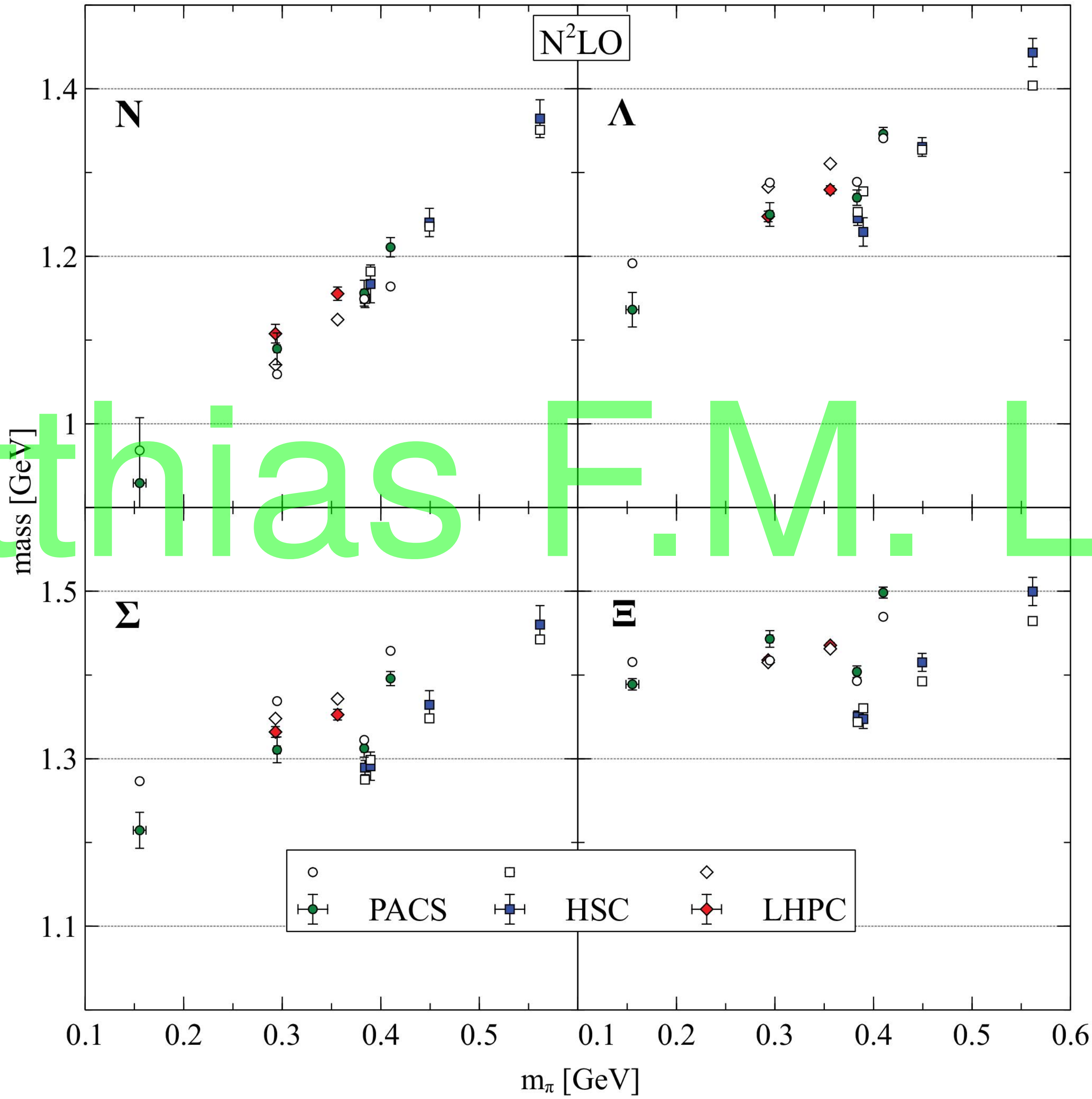
✔ QCD Lattice simulations

- consider data from PACS, HSC, LHPC, QCDSF, NPLQCD
- finite volume effects are considered
- lattice spacing effects are ignored in this work
- data are not always at physical strange-quark mass

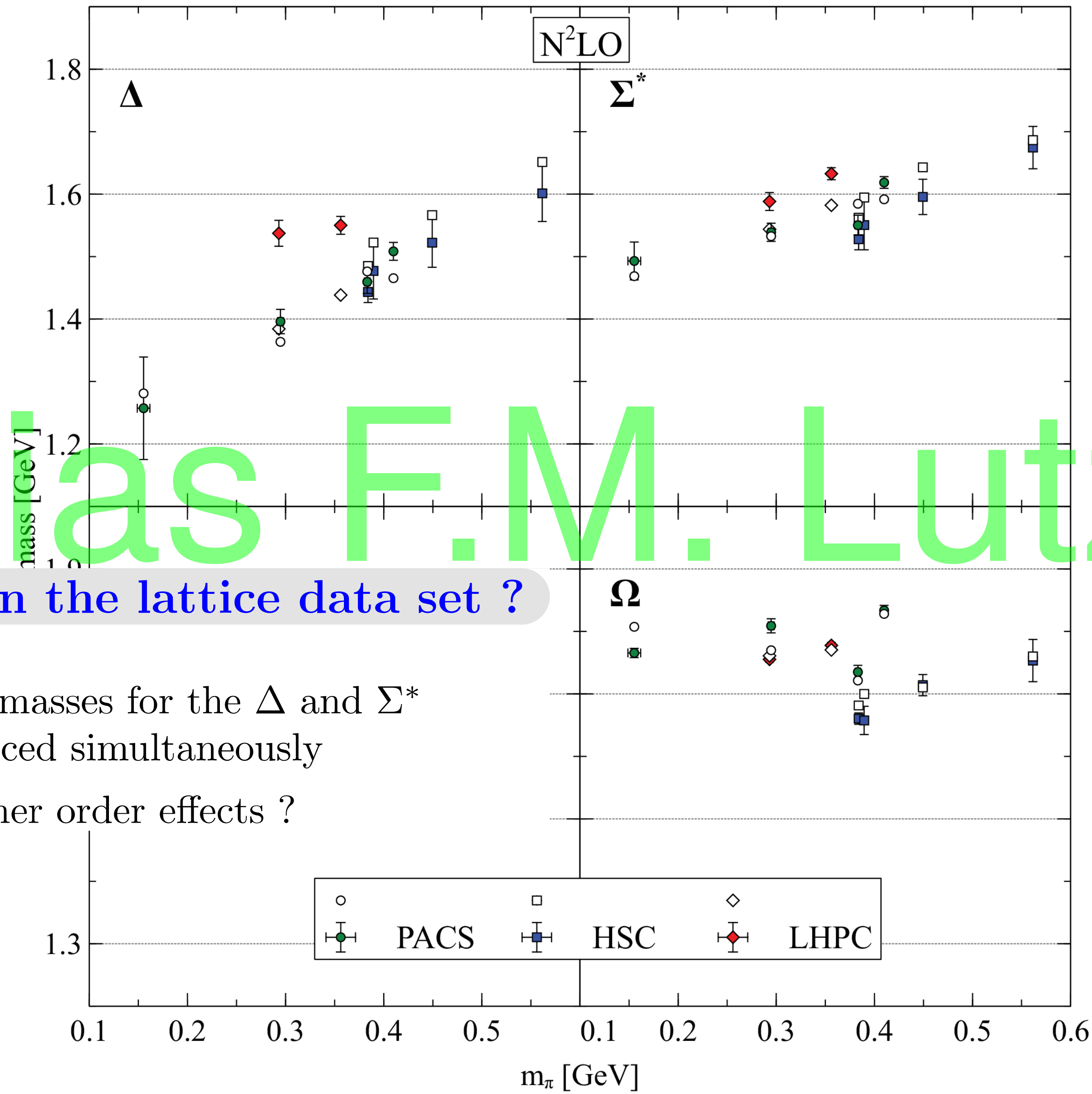
Pion-mass dependence of the baryon octet masses

✔ QCD

- consi
- finite
- lattic
- data



Pion-mass dependence of the baryon decuplet masses



✔ **Inconsistencies in the lattice data set ?**

- PACS and LHPC masses for the Δ and Σ^* cannot be reproduced simultaneously
- significance of higher order effects ?

Chiral extrapolation at N^{3}LO

✔ **Chiral symmetry breaking terms** : altogether 12+5 terms

$$
\begin{aligned}
\mathcal{L}_\chi^{(3)} &= \zeta_0\,\mathrm{tr}\big(\bar B\,(i\,D\cdot\gamma - M_{[8]})\,B\big)\,\mathrm{tr}(\chi_+) + \zeta_D\,\mathrm{tr}\big(\bar B\,(i\,D\cdot\gamma - M_{[8]})\,[B,\chi_+]\big) \\
&+ \zeta_F\,\mathrm{tr}\big(\bar B\,(i\,D\cdot\gamma - M_{[8]})\,\{B,\chi_+\}\big) \\
&- \xi_0\,\mathrm{tr}\big(\bar B_\mu\,(i\,D\cdot\gamma - M_{[10]})\,B^\mu\big)\,\mathrm{tr}(\chi_+) - \xi_D\,\mathrm{tr}\big(\bar B_\mu\,(i\,D\cdot\gamma - M_{[10]})\,B^\mu\,\chi_+\big)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_\chi^{(4)} &= c_0\,\mathrm{tr}\left(\bar B\,B\right)\mathrm{tr}\left(\chi_+^2\right) + c_1\,\mathrm{tr}\left(\bar B\,\chi_+\right)\mathrm{tr}\left(\chi_+ B\right) \\
&+ c_2\,\mathrm{tr}\left(\bar B\,\{\chi_+^2,\,B\}\right) + c_3\,\mathrm{tr}\left(\bar B\,[\chi_+^2,\,B]\right) \\
&+ c_4\,\mathrm{tr}\left(\bar B\,\{\chi_+,\,B\}\right)\mathrm{tr}(\chi_+) + c_5\,\mathrm{tr}\left(\bar B\,[\chi_+,\,B]\right)\mathrm{tr}(\chi_+) \\
&+ c_6\,\mathrm{tr}\left(\bar B\,B\right)\left(\mathrm{tr}(\chi_+)\right)^2 \\
&- e_0\,\mathrm{tr}\left(\bar B_\mu\cdot B^\mu\right)\mathrm{tr}\left(\chi_+^2\right) - e_1\,\mathrm{tr}\left(\left(\bar B_\mu\cdot\chi_+\right)\left(\chi_+\cdot B^\mu\right)\right) \\
&- e_2\,\mathrm{tr}\left(\left(\bar B_\mu\cdot B^\mu\right)\cdot\chi_+^2\right) - e_3\,\mathrm{tr}\left(\left(\bar B_\mu\cdot B^\mu\right)\cdot\chi_+\right)\mathrm{tr}(\chi_+) \\
&- e_4\,\mathrm{tr}\left(\bar B_\mu\cdot B^\mu\right)\left(\mathrm{tr}(\chi_+)\right)^2
\end{aligned}
$$

✔ **Chiral symmetry conserving terms** : altogether 25 terms

Large-N_c sum rules and the chiral Lagrangian

✔ Consider two axial-vector currents in the baryon ground states

$$C^{(ab)}_{\mu\nu}(q) = i \int d^4x\, e^{-i\,q\cdot x}\, \mathcal{T}\, A^{(a)}_\mu(x)\, A^{(b)}_\nu(0)\,.$$

✔ At leading order in the large-N_c analysis

$$\begin{aligned}\langle B'(\boldsymbol{p}')|C^{(ab)}_{ij}(q)|B(\boldsymbol{p})\rangle &= -\delta_{ij}\,(\mathcal{B}'|\,g_1\left(\tfrac{1}{3}\,\delta_{ab} + d_{abc}\,T^c\right) + \tfrac{1}{2}\,g_2\,\{T^a,\,T^b\}\,|\mathcal{B})\\ &\quad + (\bar{p}+p)_i\,(\bar{p}+p)_j\,(\mathcal{B}'|\,g_3\left(\tfrac{1}{3}\,\delta_{ab} + d_{abc}\,T^c\right) + \tfrac{1}{2}\,g_4\,\{T^a,\,T^b\}\,|\mathcal{B})\\ &\quad + \epsilon_{ijk}\,f_{abc}\,(\mathcal{B}'|\,g_5\,G^c_k\,|\mathcal{B}) + (\mathcal{B}'|\,\tfrac{1}{2}\,g_6\,\{G^a_i,\,G^b_j\}\,|\mathcal{B}) + (\mathcal{B}'|\,\tfrac{1}{2}\,g_7\,\{G^a_j,\,G^b_i\}\,|\mathcal{B}) + \cdots\end{aligned}$$

- consider the limit $\boldsymbol{p}$ and $\boldsymbol{p}' \to 0$
- matching with the result of the chiral Lagrangian leads to

✔ 18 sum rules are predicted

- 7 unknown symmetry conserving parameters only

Number of parameters relevant at N^{3}LO

✔ Symmetry conserving parameters

- two Q^0 parameters $M_{[8]}$ and $M_{[10]}$
- four Q^1 parameters F, D, C and H
- twenty five Q^2 parameters (see PR D83 (2011))

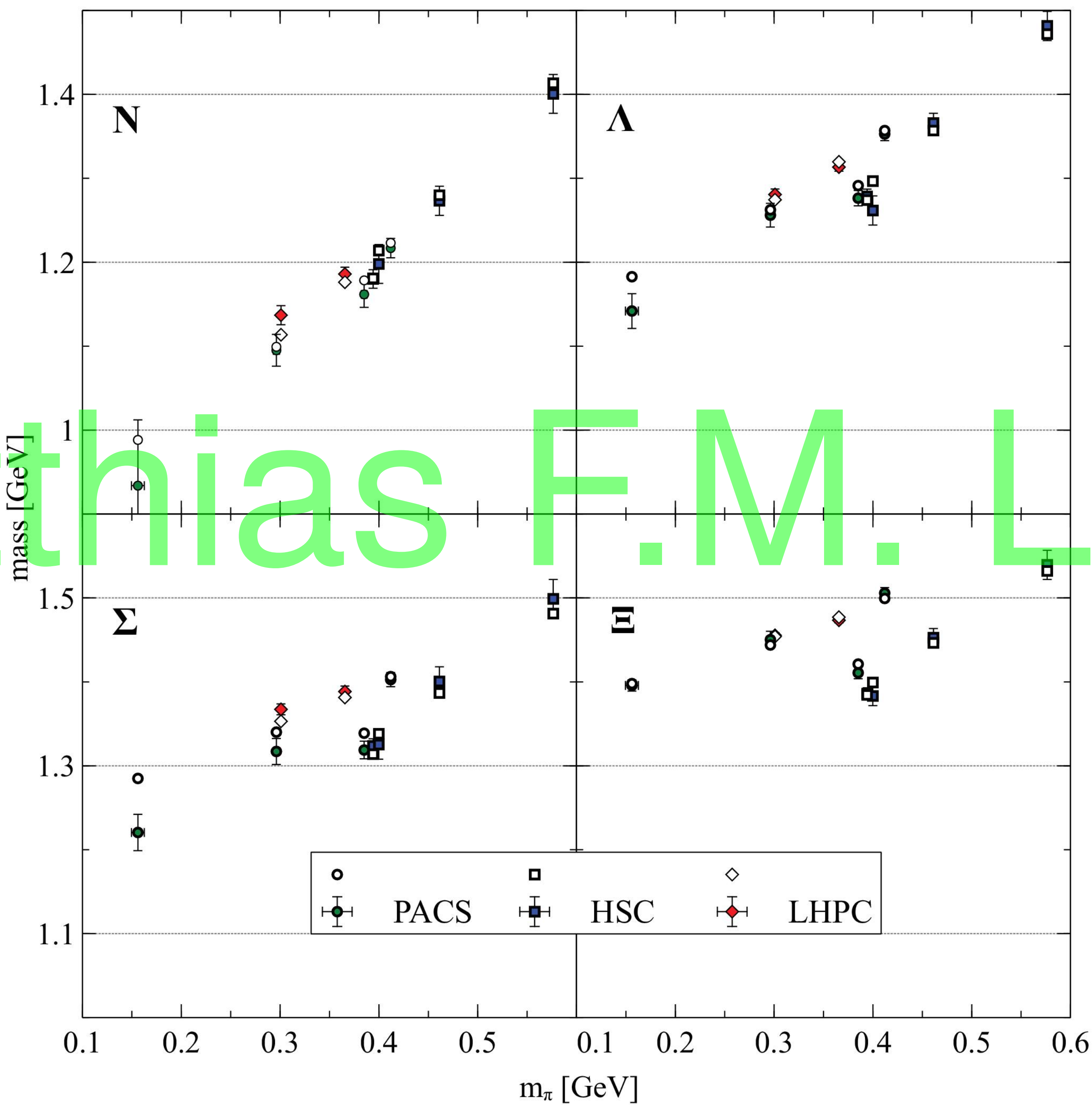
✔ Symmetry breaking parameters

- five Q^2 parameters b_0, b_D, b_F and d_0, d_D
- five Q^3 parameters $\zeta_0, \zeta_D, \zeta_F$ and ξ_0, ξ_D
- twelve Q^4 parameters c_i and e_i

✔ Sum rules from QCD in the limit of large N_c

- significant parameter reduction
- e.g. only five symmetry conserving Q^2 parameters relevant at large-N_c
- all together we adjust 20=8+12 parameters to the physical masses and lattice data

Pion-mass dependence of the baryon octet masses



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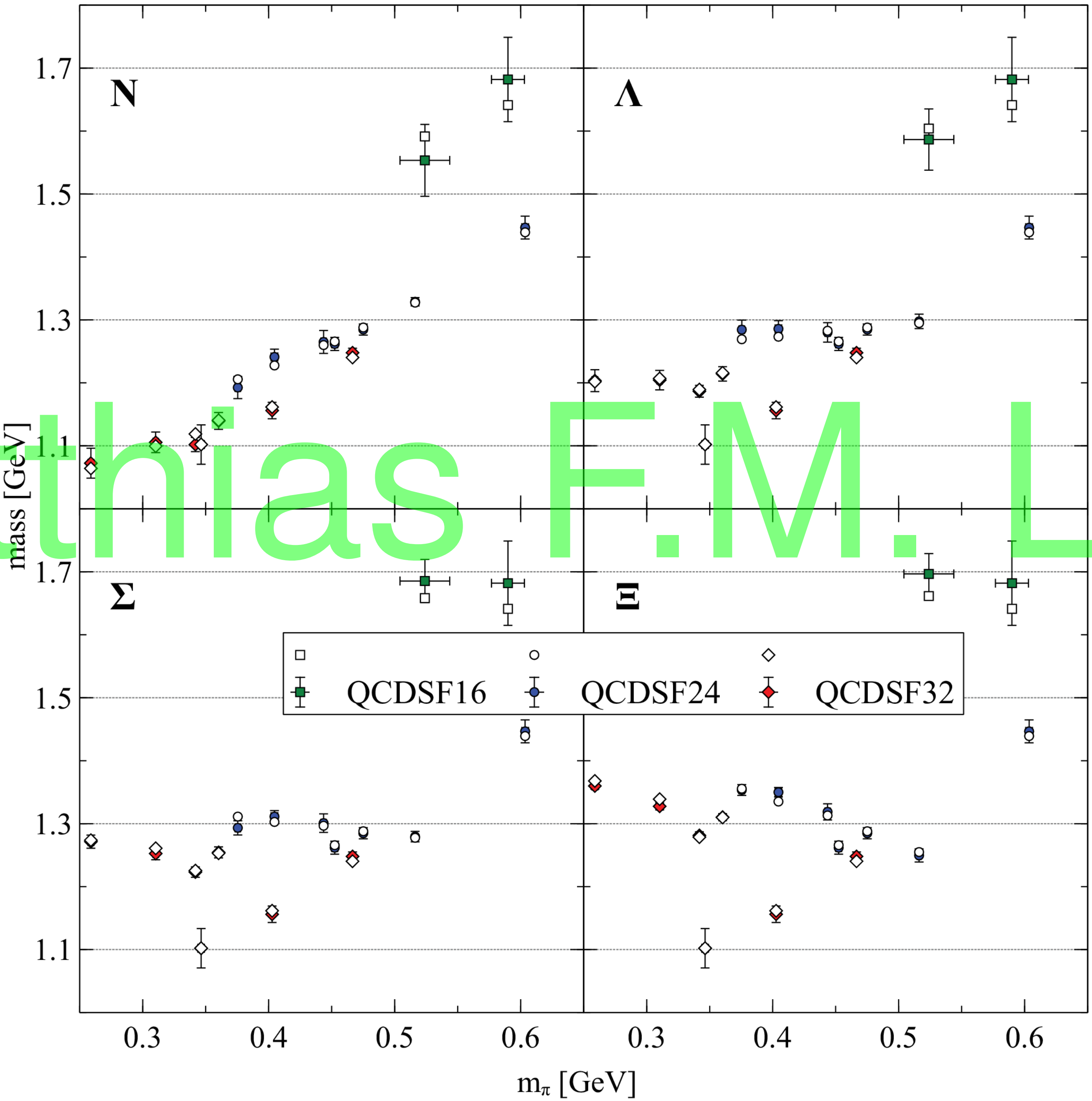

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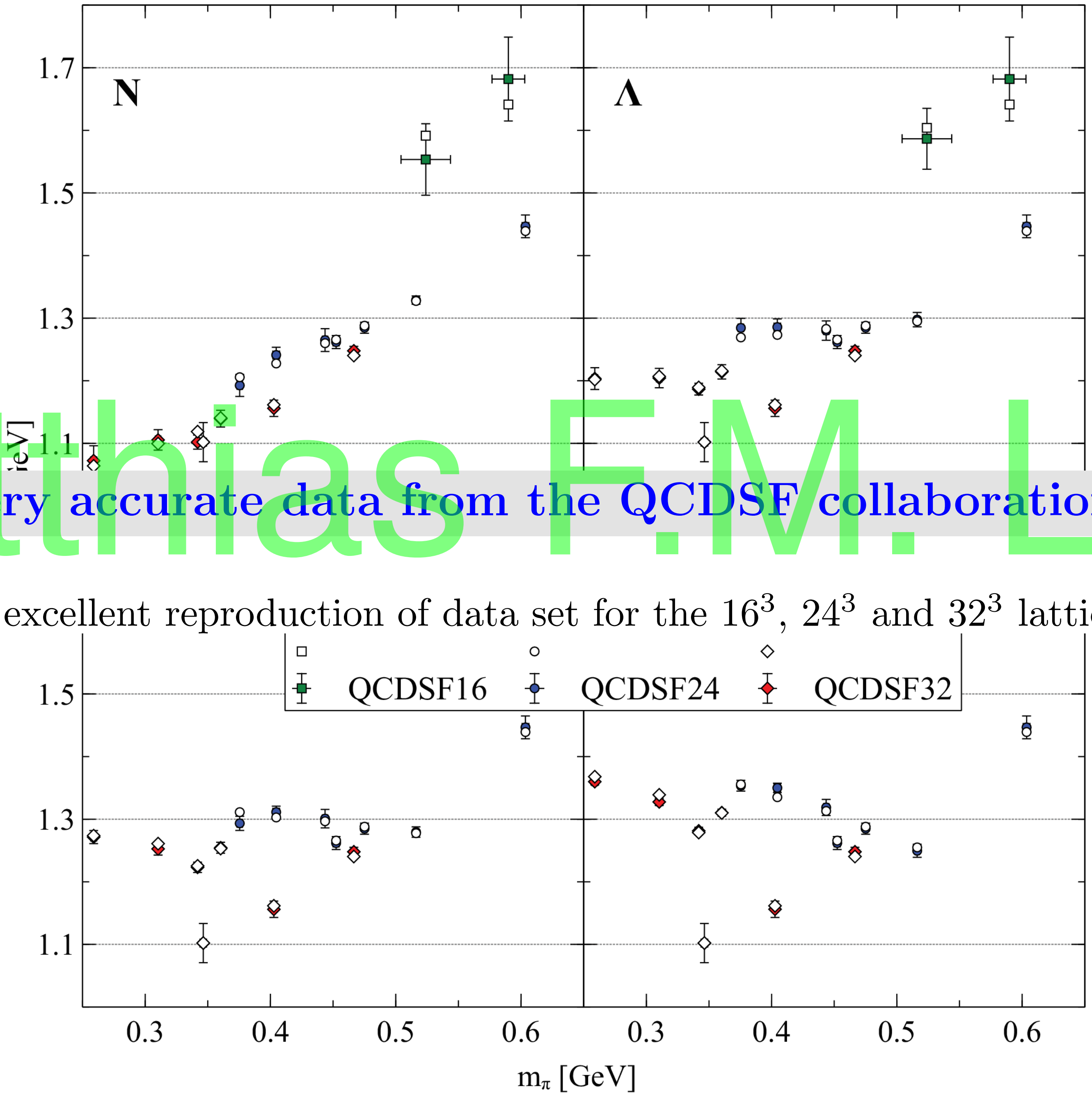
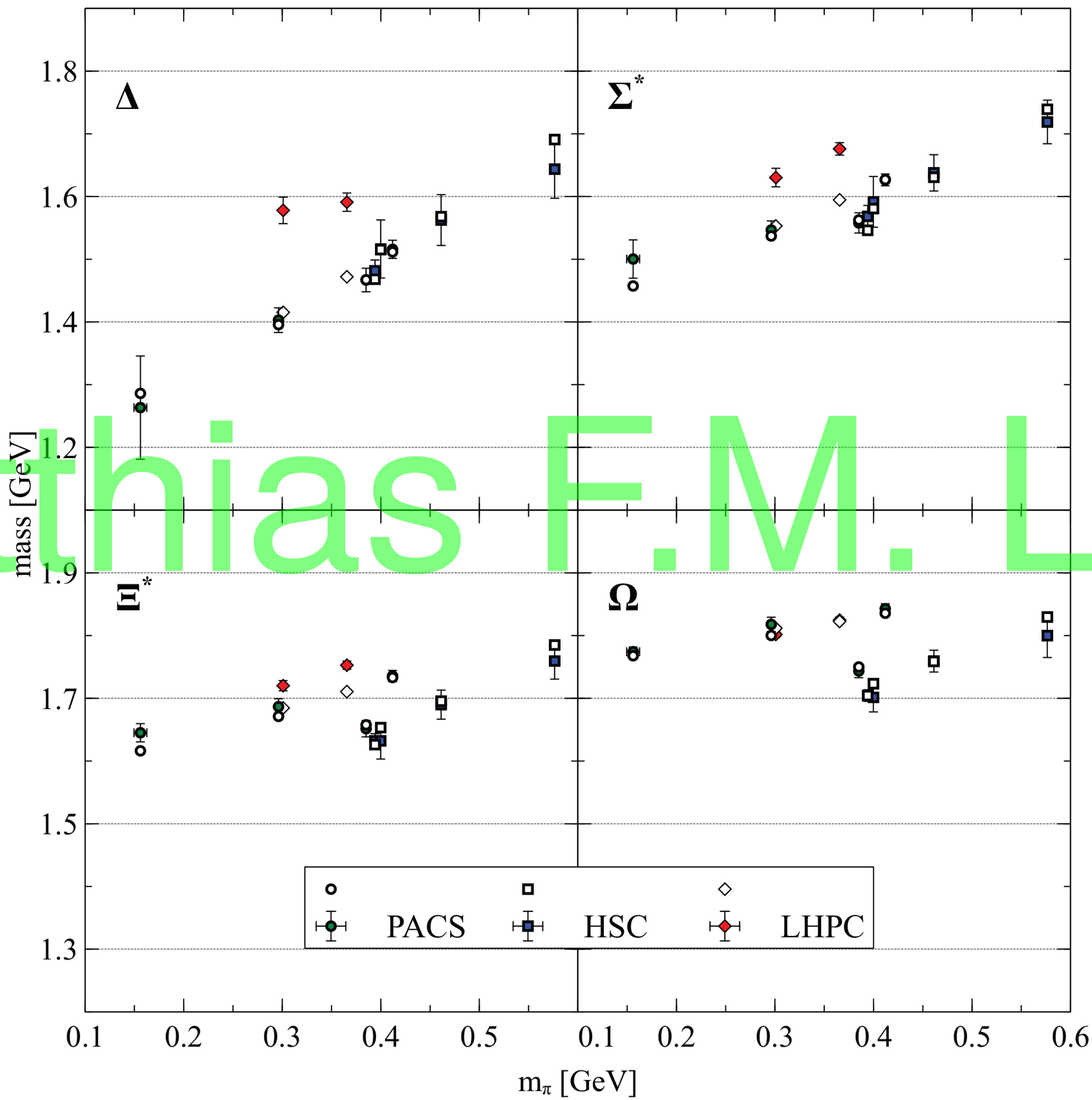


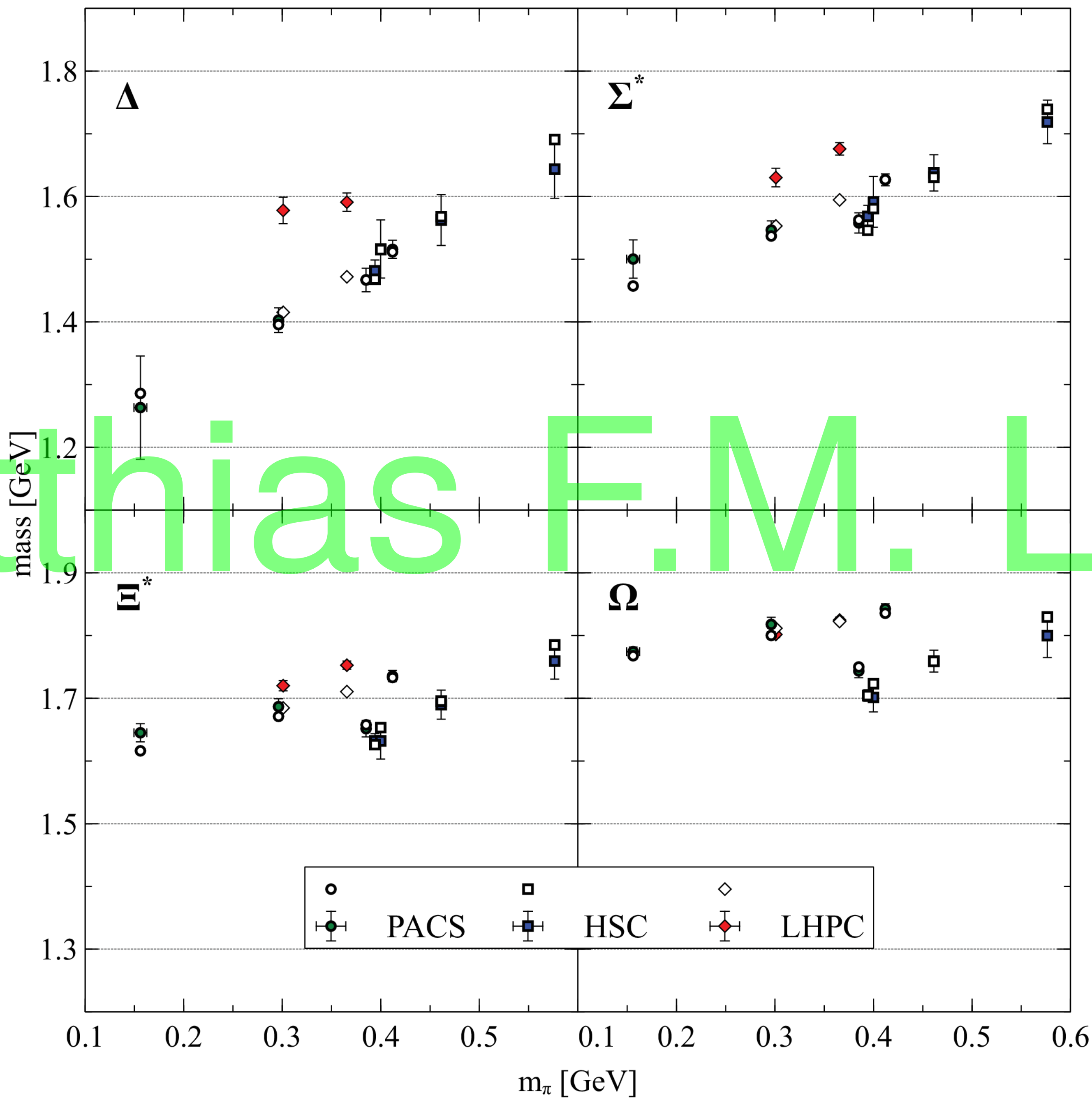
✔ Very accurate data from the QCDSF collaboration

- excellent reproduction of data set for the 16^3, 24^3 and 32^3 lattice

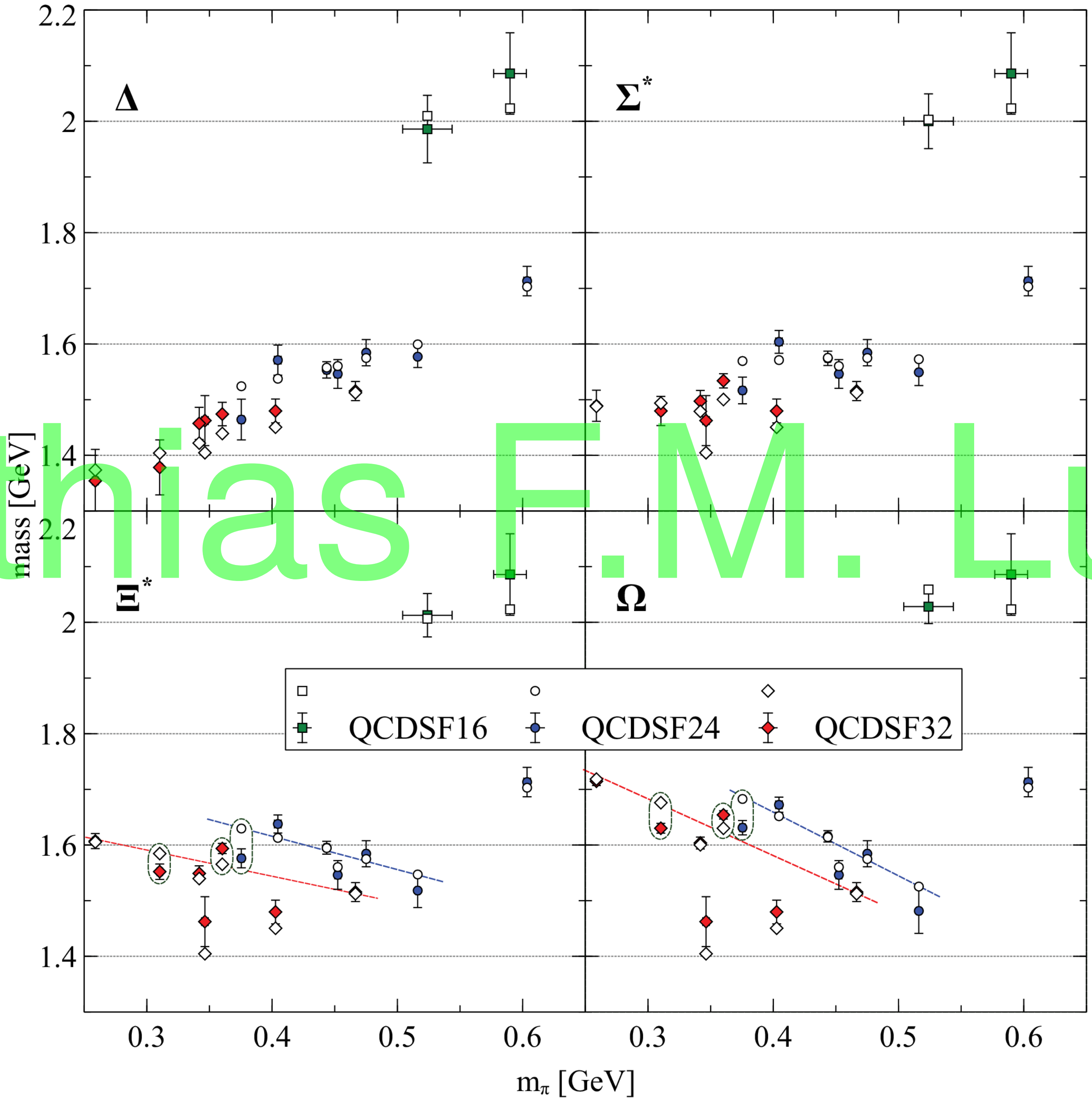
Pion-mass dependence of the baryon decuplet masses



Pion-mass dependence of the baryon decuplet masses



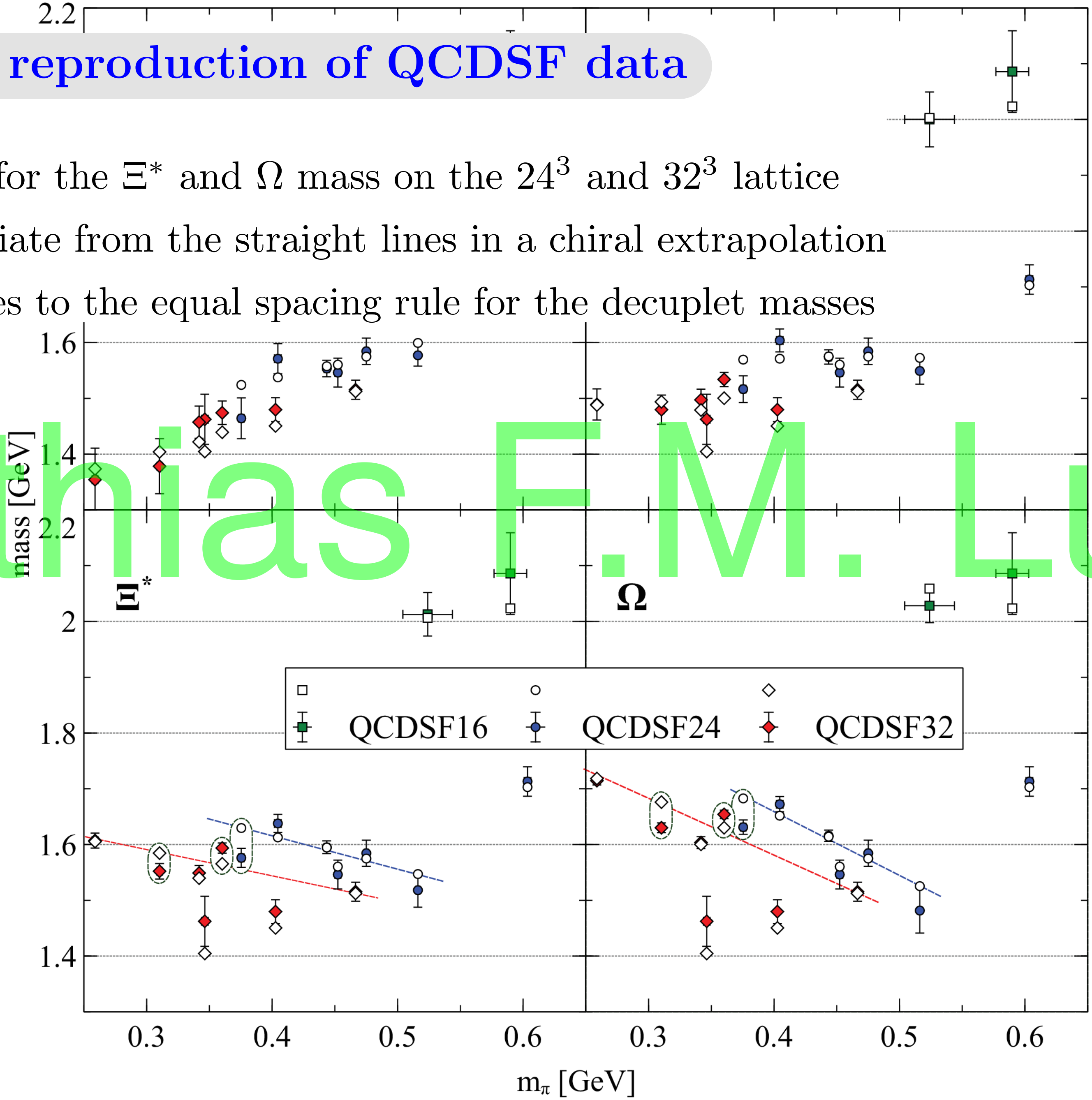
Pion-mass dependence of the baryon decuplet masses



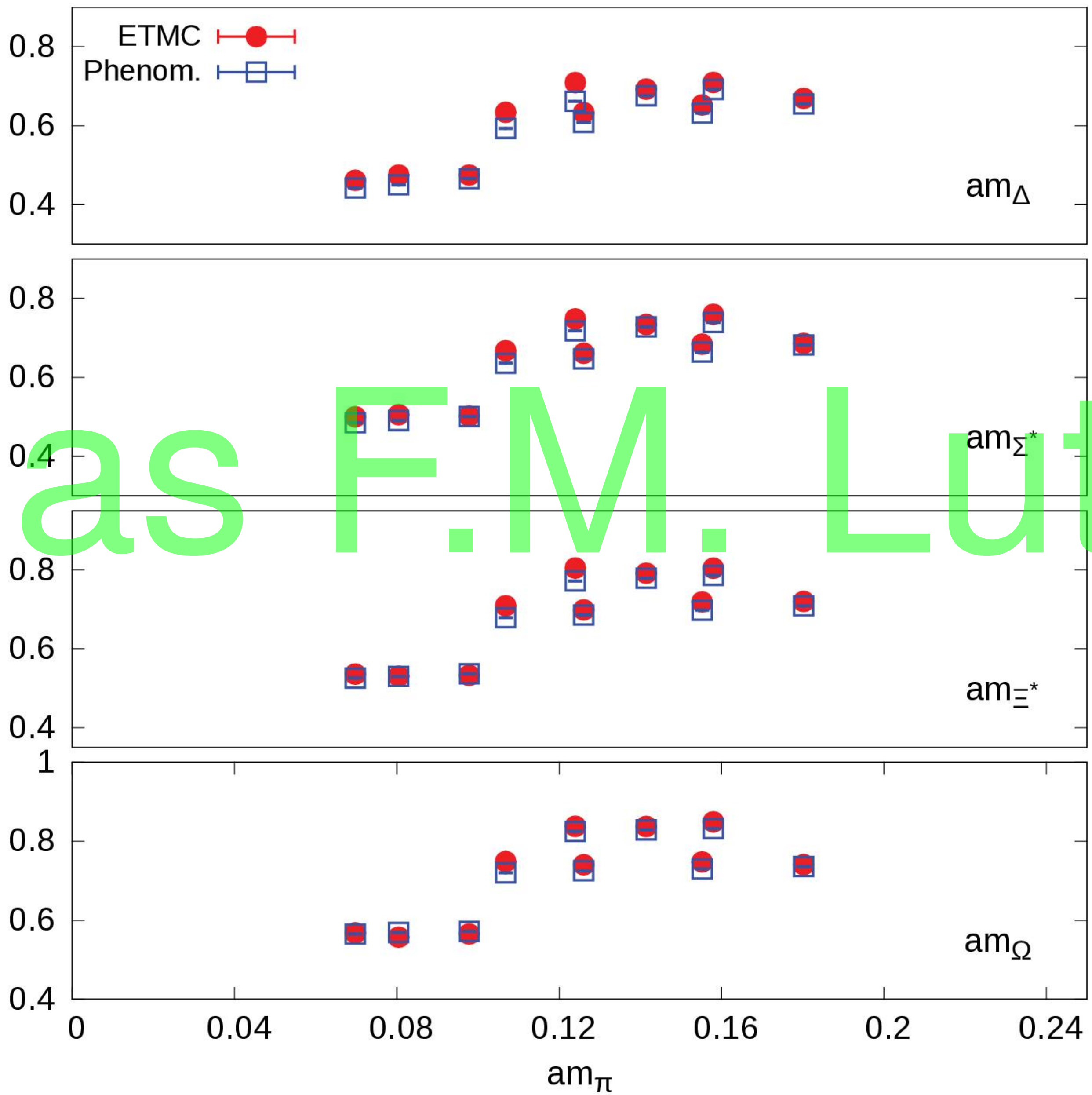
Pion-mass dependence of the baryon decuplet masses

✔ **Overall good reproduction of QCDSF data**

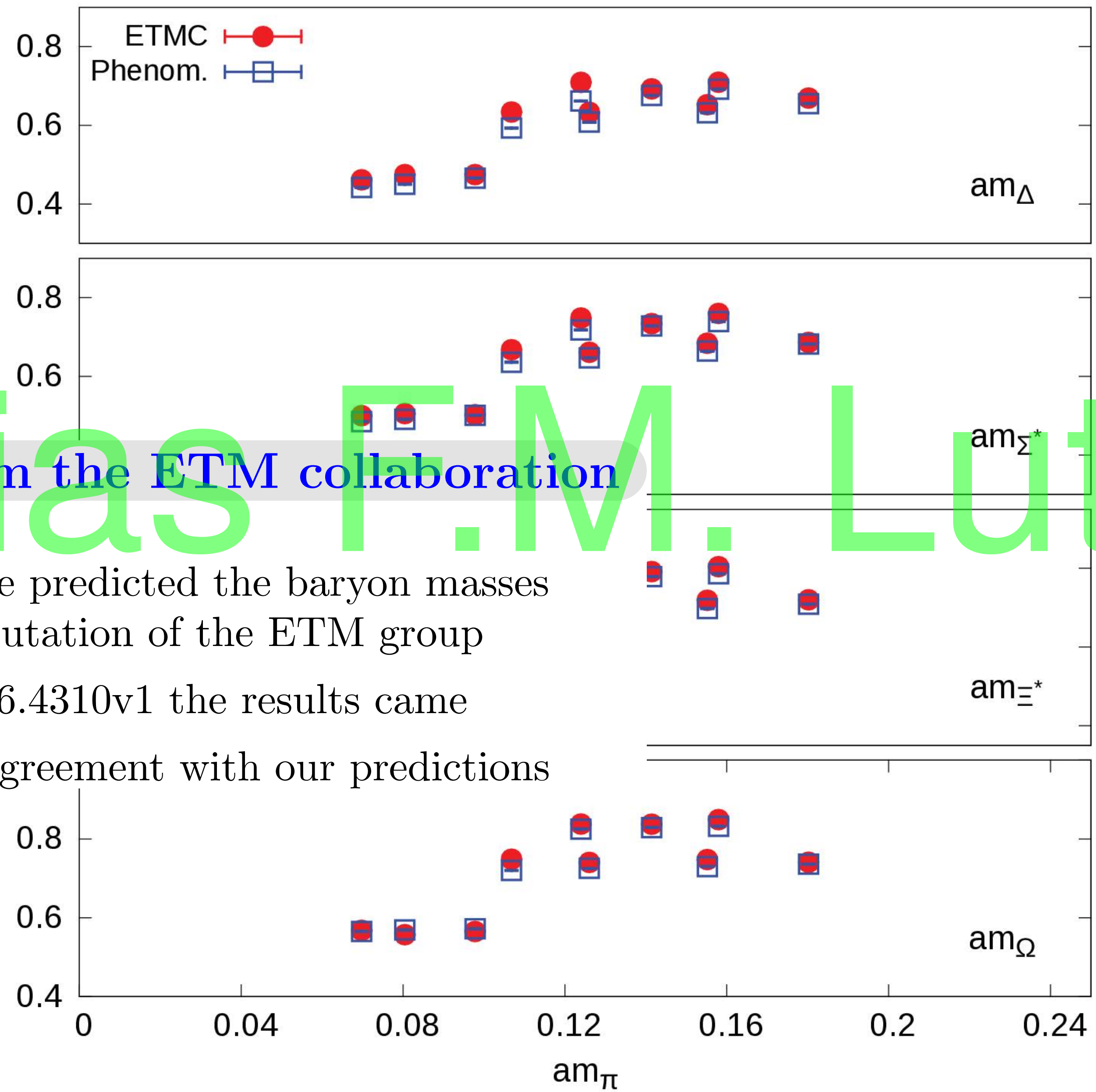
- some outliers for the Ξ^* and Ω mass on the 24^3 and 32^3 lattice
- no way to deviate from the straight lines in a chiral extrapolation
- linearity relates to the equal spacing rule for the decuplet masses



Predictions for simulations of the ETM group



Predictions for simulations of the ETM group



✔ Recent results from the ETM collaboration

- in arXiv:1401.7805 we predicted the baryon masses for the ongoing computation of the ETM group
- recently in arXiv:1406.4310v1 the results came
- amazingly accurate agreement with our predictions

Summary & Outlook

✔ Chiral extrapolation of baryon octet and decuplet masses

- resummed χPT : use physical masses in the loops
- use large-N_c sum rules for counter terms
- 12 parameter description at N^{3}LO of data from BMW, LHPC, PACS, HSC, NPLQ, QCDSF, ETMC groups (more than 260 data points)
- predict baryon sigma terms, flavor symmetric baryon masses etc
- combined lattice data determine the axial coupling constants

✔ QCD spectroscopy with coupled-channel dynamcis

- lattice QCD provides the counter terms for hadron-hadron scattering
- use as input in systematic coupled-channel computations
- analze and predict the pion mass dependence of hadron resonances in QCD

thanks to: A. Semke, K. Schwarz, R. Bavontaweepanya , C. Kobdaj