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# Nucleons and Nucleonic Systems

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International Workshop “Hadron Nuclear Physics”  
July 7-11, 2015 , Krabi, Thailand

# Content

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- Topological models and soliton
- Medium modifications
- Nucleon's structure changes in nuclear matter
- Nuclear matter
- Neutron stars
- Consistency (difference) with (from) other soliton approaches
- Summary and Outlook

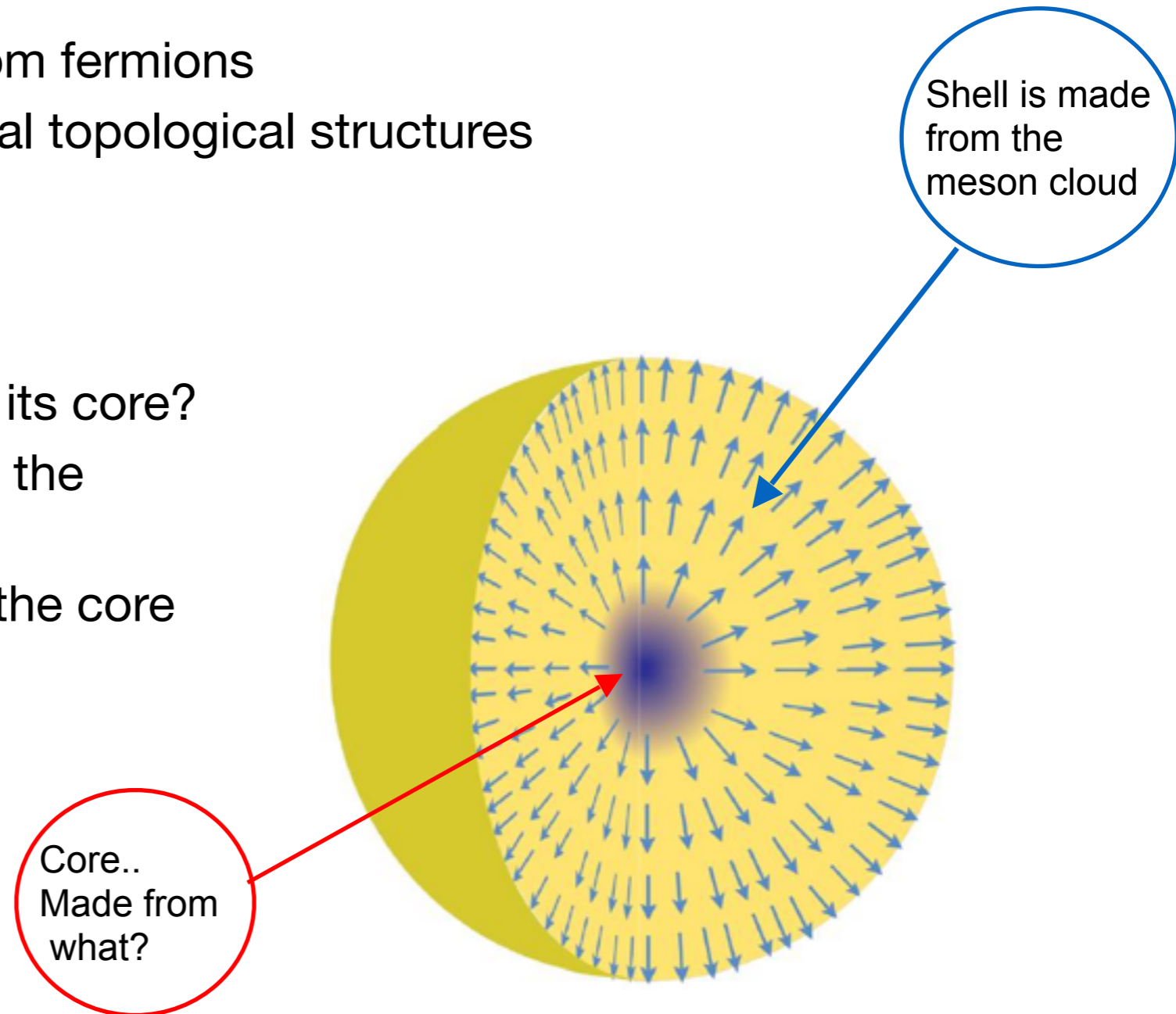
# Topological models and soliton

## Why topological models?

- At fundamental level we may have
  - fermions  $\rightarrow$  bosons are made from fermions
  - bosons  $\rightarrow$  fermions are nontrivial topological structures

## Structure

- What is a nucleon and, in particular, its core?
- The structure treatment depends on the energy scale
- At the limit of large number colours the core still has the mesonic content



# Topological models and soliton

## Stabilisation

- Soliton has the finite size and the finite energy
- One needs at least two terms in the effective (mesonic) Lagrangian

## Prototype: Skyrme model

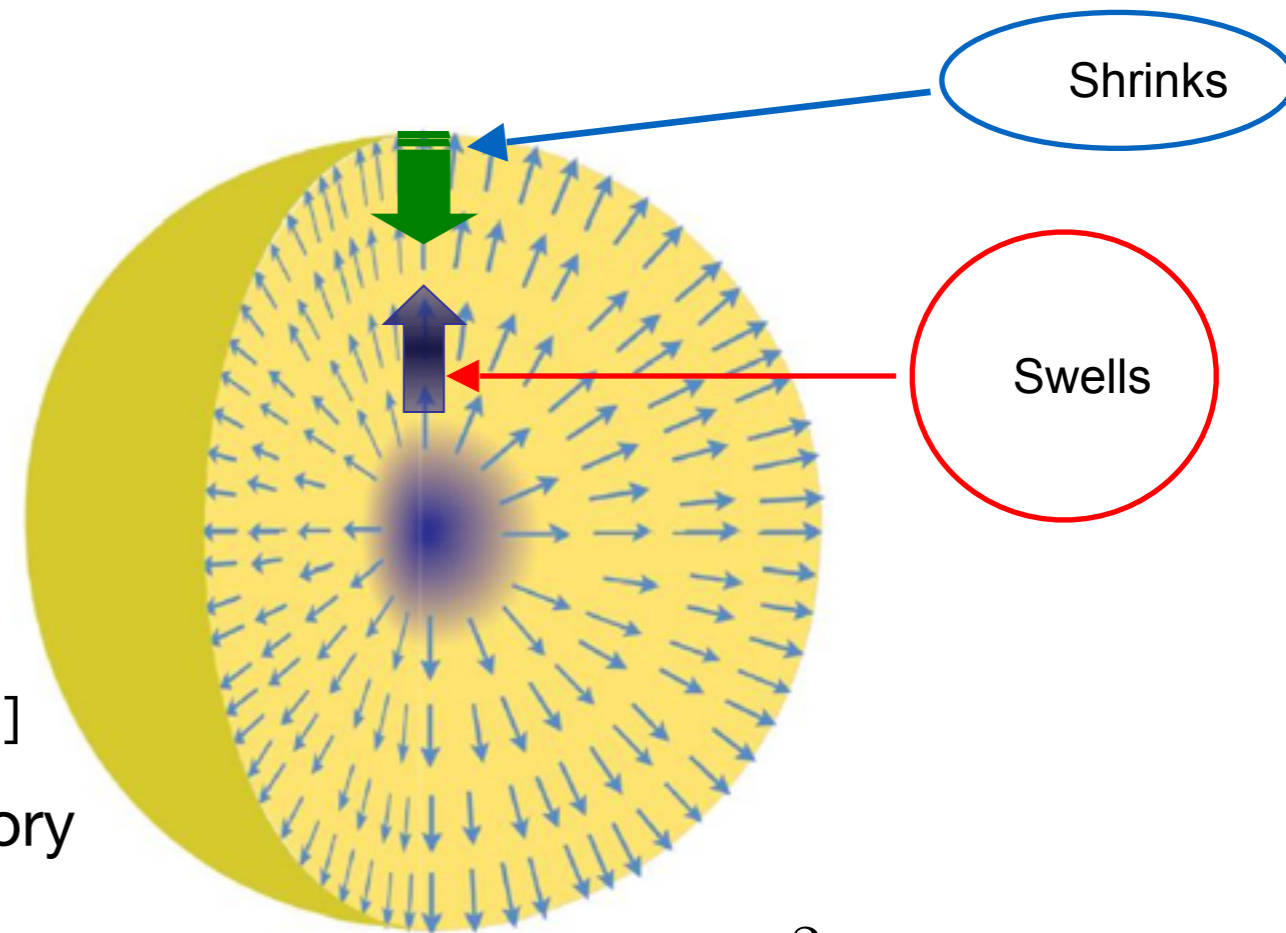
[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260 (1961)]

- Nonlinear chiral effective meson (pionic) theory

$$\mathcal{L} = \frac{F_\pi^2}{16} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) - \frac{1}{16e^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

← Shrink

→ Swells



- Hedgehog solution (nontrivial mapping)

$$U = \exp \left\{ \frac{i\vec{\tau} \cdot \vec{\pi}}{2F_\pi} \right\} = \exp \{ i\vec{\tau} \cdot \vec{n} F(r) \}$$

# Topological models and soliton

## The free space Lagrangian in use

[G.S.Adkins et al. Nucl.Phys. B228 (1983)]

$$\mathcal{L} = \frac{F_\pi^2}{16} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) - \frac{1}{16e^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 + \frac{F_\pi^2 m_\pi^2}{16} \text{Tr} (U + U^\dagger - 2)$$

- Nontrivial structure: topologically stable solitons with the corresponding conserved topological number (baryon number) **A**

$$A = \int d^3 r B^0$$

- Nucleon is quantised state of the classical soliton-skyrmion

$$U = \exp \{ i \bar{\tau} \cdot \bar{\pi} / 2F_\pi \} = \exp \{ i \bar{\tau} \cdot \bar{n} F(r) \}$$

$$B^\mu = \frac{1}{24\pi^2} \varepsilon^{\mu\nu\alpha\beta} \text{Tr} (L_\nu L_\alpha L_\beta) \quad L_\alpha = U^\dagger \partial_\alpha U$$

$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

$$|S = T, s, t \rangle = (-1)^{t+T} \sqrt{2T+1} D_{-t,s}^{S=T}(A)$$

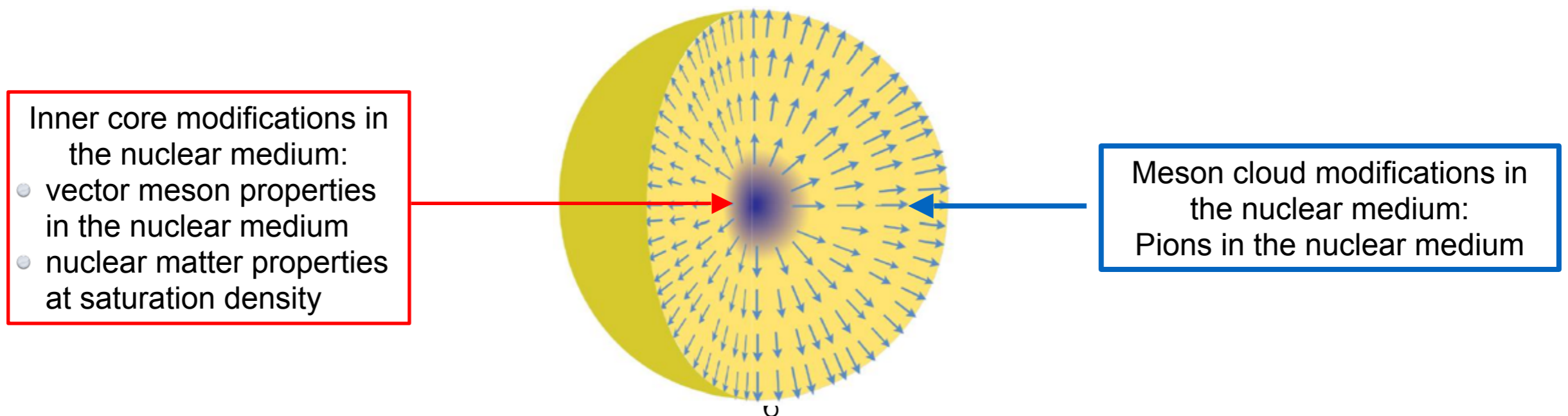
# Medium modifications

## What happens in the nuclear medium?

- The medium effects
  - Deformations (swelling or shrinking, multipole deformations) of nucleons
  - Possible characteristic changes: effective mass, charge distributions, form factors
  - NN interactions may change
  - etc.
- One should be able to describe all those phenomena

## Soliton in the nuclear medium (phenomenological way)

- Outer shell modifications (informations from pionic atoms)
- Inner core modifications, in particular, at large densities (nuclear matter properties)



# Medium modifications

## “Outer shell” modifications

- In free space three types of pions can be treated separately: isospin breaking
- In nuclear matter: three types of polarization operators
- Optic potential approach: parameters from the pion-nucleon scattering, including **isospin dependents**

$$(\partial^\mu \partial_\mu + m_\pi^2) \vec{\pi}^{(\pm,0)} = 0$$

$$(\partial^\mu \partial_\mu + m_\pi^2 + \hat{\Pi}^{(\pm,0)}) \vec{\pi}^{(\pm,0)} = 0$$

$$\hat{\Pi}^0 = 2\omega U_{\text{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0) \vec{\nabla}$$

	$\pi$ -atom	$T_\pi = 50$ MeV
$b_0 [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1 [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0 [m_\pi^{-3}]$	0.23	0.25
$c_1 [m_\pi^{-3}]$	0.15	0.16
$g'$	0.47	0.47

# Medium modifications

“Outer shell” modifications [U.Meissner *et al.*, EPJ A36 (2008)]

$$\mathcal{L}_2^* = \frac{F_\pi^2}{16} \alpha_\tau \text{Tr} (\partial_0 U \partial_0 U^\dagger) - \frac{F_\pi^2}{16} \alpha_s \text{Tr} (\partial_i U \partial_i U^\dagger)$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16} \alpha_m \text{Tr} (2 - U - U^\dagger)$$

- Due to the nonlocality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters following parts of the kinetic term is modified in different form:
  - Temporal part
  - Space part

$$\hat{\Pi}^0 = 2\omega U_{\text{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0) \vec{\nabla}$$

	$\pi$ -atom	$T_\pi = 50$ MeV
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$g'$	0.47	0.47

# Medium modifications

## “Inner core” modifications

[ UY & H.Ch. Kim, PRC83 (2011); UY, JKPS62 (2013); UY, PRC88 (2013) ]

$$\mathcal{L}_4^* = -\frac{1}{16e^2\zeta_\tau} \text{Tr} [U^\dagger \partial_0 U, U^\dagger \partial_i U]^2 + \frac{1}{32e^2\zeta_s} \text{Tr} [U^\dagger \partial_i U, U^\dagger \partial_j U]^2$$

- May be related to

- Vector meson properties in nuclear matter
- Nuclear matter properties

$$\zeta_{\tau,s} = \zeta_{\tau,s}(\rho, \delta\rho, \text{parameters})$$

# Medium modifications

## Final Lagrangian

[ UY, JKPS62 (2013); UY, PRC88 (2013) ]

- Separated into two parts

$$\mathcal{L}^* = \mathcal{L}_{\text{sym}}^* + \mathcal{L}_{\text{asym}}^*$$

- Isoscalar part

$$\mathcal{L}_{\text{sym}}^* = \mathcal{L}_2^* + \mathcal{L}_4^* + \mathcal{L}_m^*$$

- Isovector part

$$\mathcal{L}_{\text{asym}}^* = \mathcal{L}_{\delta m}^* + \mathcal{L}_{\delta \rho}^*$$

- Nuclear matter stabilisation**

- Asymmetric matter properties**

$$\mathcal{L}_2^* = \frac{F_\pi^2}{16} \alpha_\tau \text{Tr} (\partial_0 U \partial_0 U^\dagger) - \frac{F_\pi^2}{16} \alpha_s \text{Tr} (\partial_i U \partial_i U^\dagger)$$

$$\mathcal{L}_4^* = -\frac{1}{16e^2 \zeta_\tau} \text{Tr} [U^\dagger \partial_0 U, U^\dagger \partial_i U]^2 + \frac{1}{32e^2 \zeta_s} \text{Tr} [U^\dagger \partial_i U, U^\dagger \partial_j U]^2$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16} \alpha_m \text{Tr} (2 - U - U^\dagger)$$

$$\mathcal{L}_{\delta m}^* = -\frac{F_\pi^2}{32} \sum_{a=1}^2 (m_{\pi^\pm}^2 - m_{\pi^0}^2) \text{Tr} (\tau_a U) \text{Tr} (\tau_a U^\dagger)$$

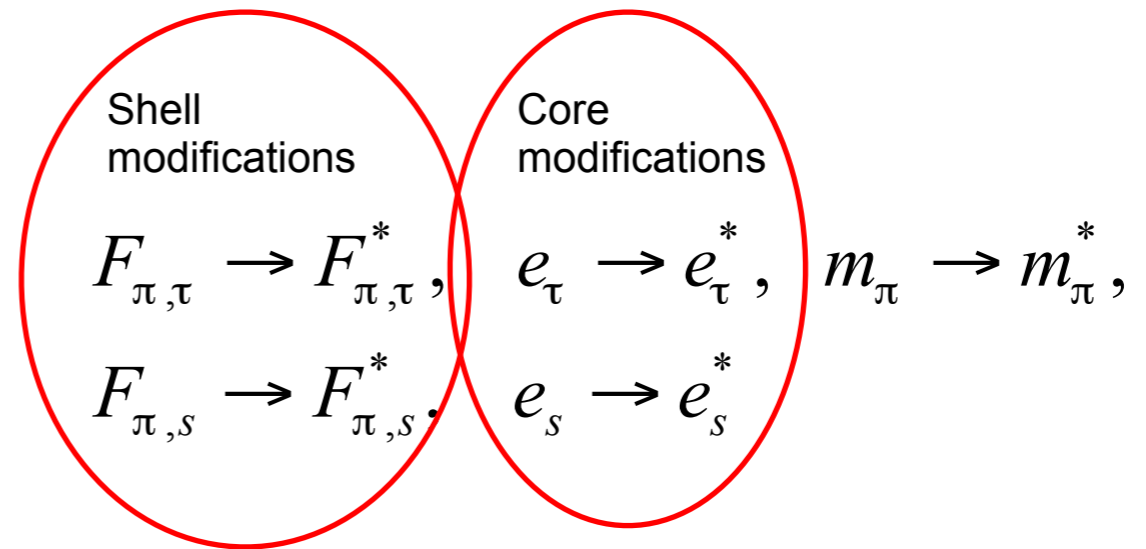
$$\mathcal{L}_{\delta \rho}^* = -\frac{F_\pi^2}{16} m_\pi \alpha_e \varepsilon_{ab3} \text{Tr} (\tau_a U) \text{Tr} (\tau_b \partial_0 U^\dagger)$$

# Medium modifications

## Reparametrization

[ UY, PRC88 (2013) ]

- Five density dependent parameters



- Rearrangment (technical simplification)

$$1 + C_1 \frac{\rho}{\rho_0} = f_1\left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2\left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$1 + C_3 \frac{\rho}{\rho_0} = f_3\left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right) \frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

# Nucleon properties in nuclear matter

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## Structure: Energy momentum tensor

- It allows to address the questions like:
  - How are the total angular momentum and angular momentum of the nucleon shared among its constituents?
  - How are the strong forces experienced by its constituents distributed inside the nucleon?
- EMT form factors studied in lattice QCD, ChPT and in different models (chiral quark soliton model, Skyrme model, etc.)
- We made further step studying EMT form factors in nuclear matter

# Nucleon properties in nuclear matter

**Structure: Pressure distribution inside the nucleon in free space and in symmetric matter** [H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

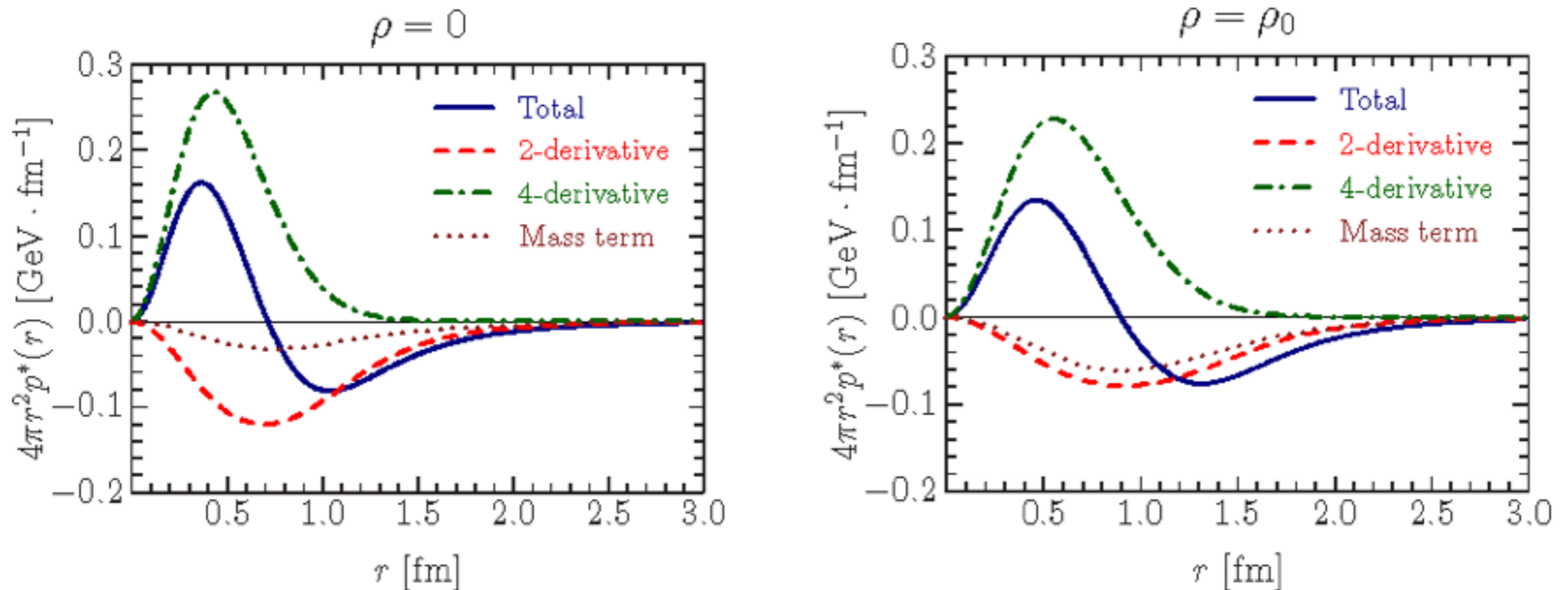


FIG. 3: (Color online) The decomposition of the pressure densities  $4\pi r^2 p^*(r)$  as functions of  $r$ , in free space ( $\rho = 0$ ) and at  $\rho = \rho_0$ , in the left and right panels, respectively. The solid curves denote the total pressure densities, the dashed ones represent the contributions of the 2-derivative (kinetic) term, the long-dashed ones are those of the 4-derivative (stabilizing) term, and the dotted ones stand for those of the pion mass term.

# Nucleon properties in nuclear matter

## Stability and applicability [H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

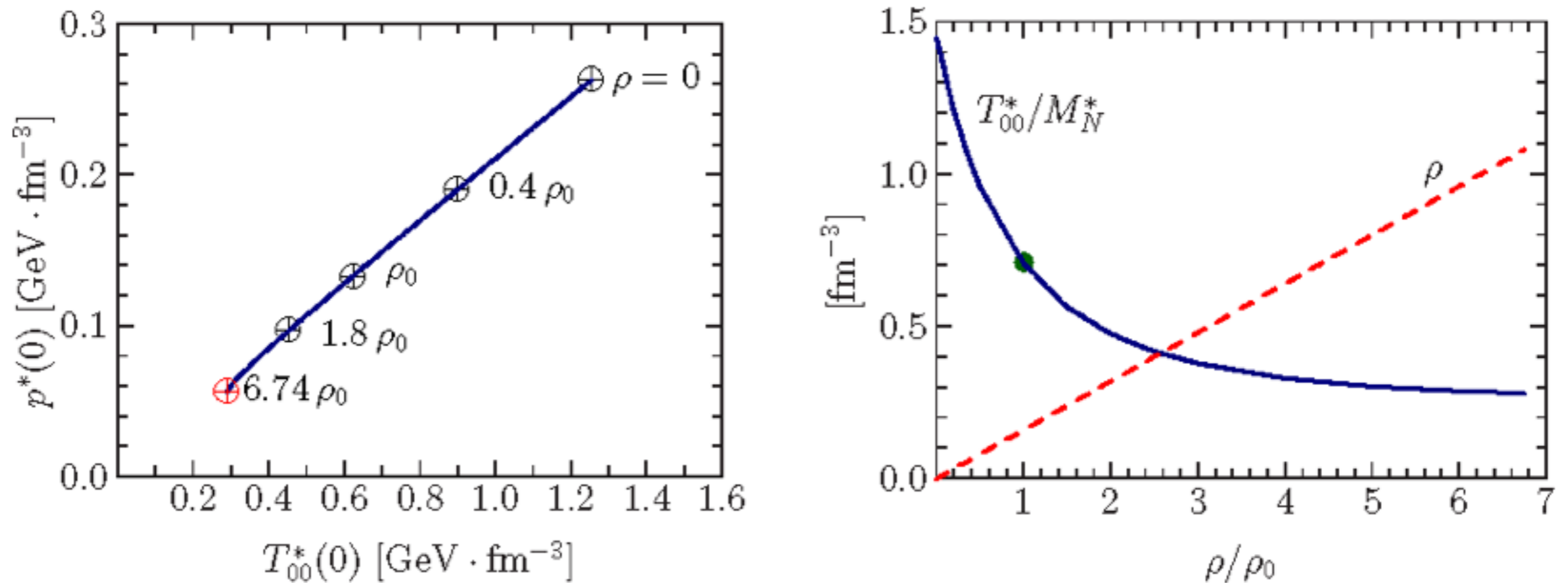


FIG. 5: (Color online) In the left panel, the correlated change of  $p^*(0)$  and  $T_{00}^*(0)$  drawn with  $\rho$  varied. In the right panel, the  $T_{00}^*/M_N^*$  and  $\rho$  depicted as a function of  $\rho/\rho_0$ . The maximal density is given as about  $6.74\rho_0$ , above which the Skyrmion does not exist anymore. The filled circle on the solid curve represents the value of  $T_{00}^*/M_N^*$  at normal nuclear matter density.

# Nucleon properties in nuclear matter

## Nucleon in nuclear matter

[ UY, PRC88 (2013) ]

- Isovector mass

$$m_{N,s}^* = M_S^* + \frac{3}{8\Lambda^*} + \frac{\Lambda^*}{2} \left( a^{*2} + \frac{\Lambda_{\text{env}}^{*2}}{\Lambda^{*2}} \right)$$

- Isovector mass

$$\Delta m_{np}^* = a^* + \frac{\Lambda_{\text{env}}^*}{\Lambda^*}$$

- Mass of the nucleon

$$m_{n,p}^* = m_{N,s}^* - \Delta m_{np}^* T_3$$

# Nuclear matter

The binding-energy-formula terms in the framework of present model

$$\varepsilon(A, Z) = -a_V + a_S \frac{(N - Z)^2}{A^2} + \text{[Watermark]}$$

**We are ready  
to reproduce**

- Volume term
  - Infinite and asymmetric nuclear matter
- Asymmetry term
  - Isospin asymmetric environment
- Surface and Coulomb terms
  - Nucleons in a finite volume
- Finite nuclei properties
  - Local density approximation

# Nuclear matter

## The volume term and Symmetry energy

- At infinite nuclear matter approximation the binding energy per nucleon takes the form

$$\varepsilon(\lambda, \delta) = \varepsilon_V(\lambda) + \varepsilon_S \delta^2 + O(\delta^4) \equiv \varepsilon_V(\lambda) + \varepsilon_A(\lambda, \delta)$$

- $\lambda$  is normalised nuclear matter density
  - $\delta$  is asymmetry parameter
  - $\varepsilon_S$  is symmetry energy
- In our model
    - Symmetric matter
    - Asymmetric matter

$$\varepsilon_V(\lambda) = m_{N,s}^*(\lambda, 0) - m_N^{\text{free}}$$

$$\begin{aligned} \varepsilon_A(\lambda, \delta) &= \varepsilon(\lambda, \delta) - \varepsilon_V(\lambda) \\ &= m_{N,s}^*(\lambda, \delta) - m_{N,s}^*(\lambda, 0) + m_{N,v}^*(\lambda, \delta) \delta \end{aligned}$$

# Nuclear matter

## Nuclear matter properties

- Symmetric matter properties (pressure, compressibility and third derivative)

$$p = \rho_0 \lambda^2 \left. \frac{\partial \varepsilon_V(\lambda)}{\partial \lambda} \right|_{\lambda=1}, \quad K_0 = 9\rho^2 \left. \frac{\partial^2 \varepsilon_V(\lambda)}{\partial \rho^2} \right|_{\rho=\rho_0}, \quad Q = 27\lambda^3 \left. \frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3} \right|_{\lambda=1}$$

- Symmetry energy properties (coefficient, slop and curvature)

$$\varepsilon_s(\lambda) = \varepsilon_s(1) + \frac{L_s}{3}(\lambda - 1) + \frac{K_s}{18}(\lambda - 1)^2 + \boxed{\text{W}}$$

# Symmetric matter

## Volume energy

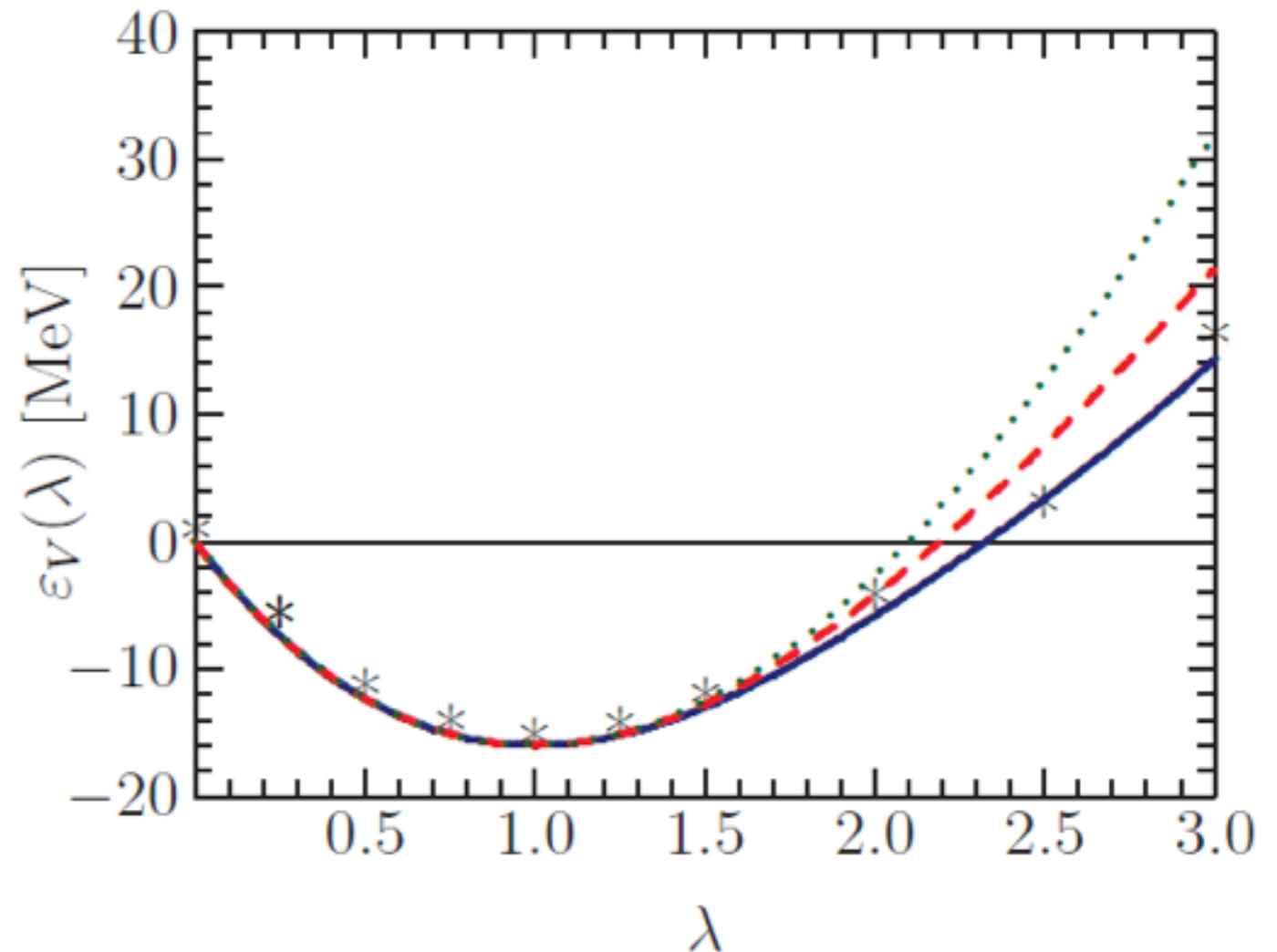
[ UY, PRC88 (2013) ]

- Set I – solid
- Set II – dashed
- Set III – dotted

For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions

[PRC 58, 1804 (1998)]  
are given by stars.

(From Arigonna 2 body interactions + 3 body interactions)

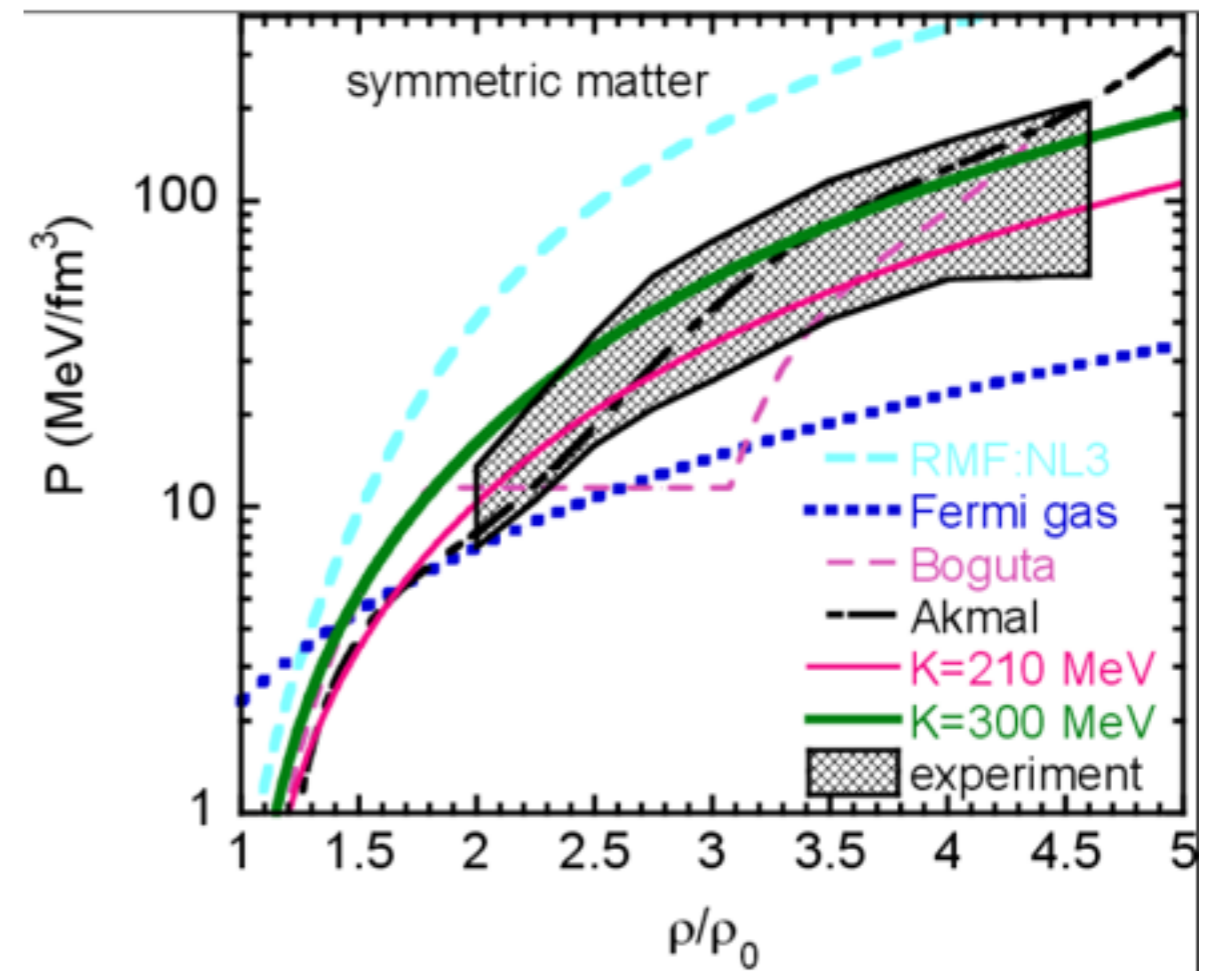
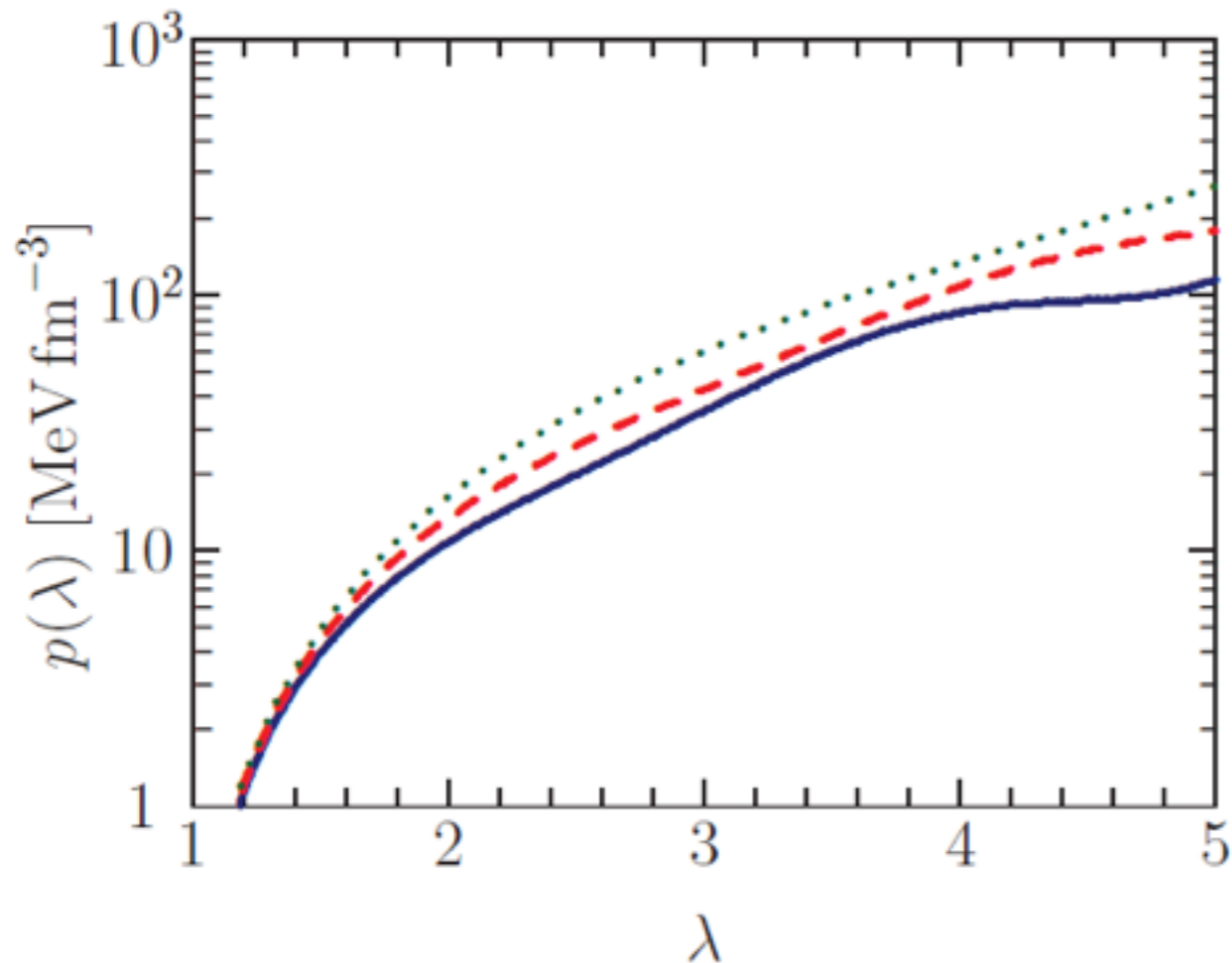


Set	$C_1$	$C_2$	$C_3$	$\varepsilon_V(\rho_0)$ (MeV)	$K_0$ (MeV)	$Q$ (MeV)
I	-0.279	0.737	1.782	-16	240	-410
II	-0.273	0.643	1.858	-16	250	-279
III	-0.277	0.486	2.124	-16	260	-178

# Symmetric matter

## Pressure

[ UY, PRC88 (2013) ]



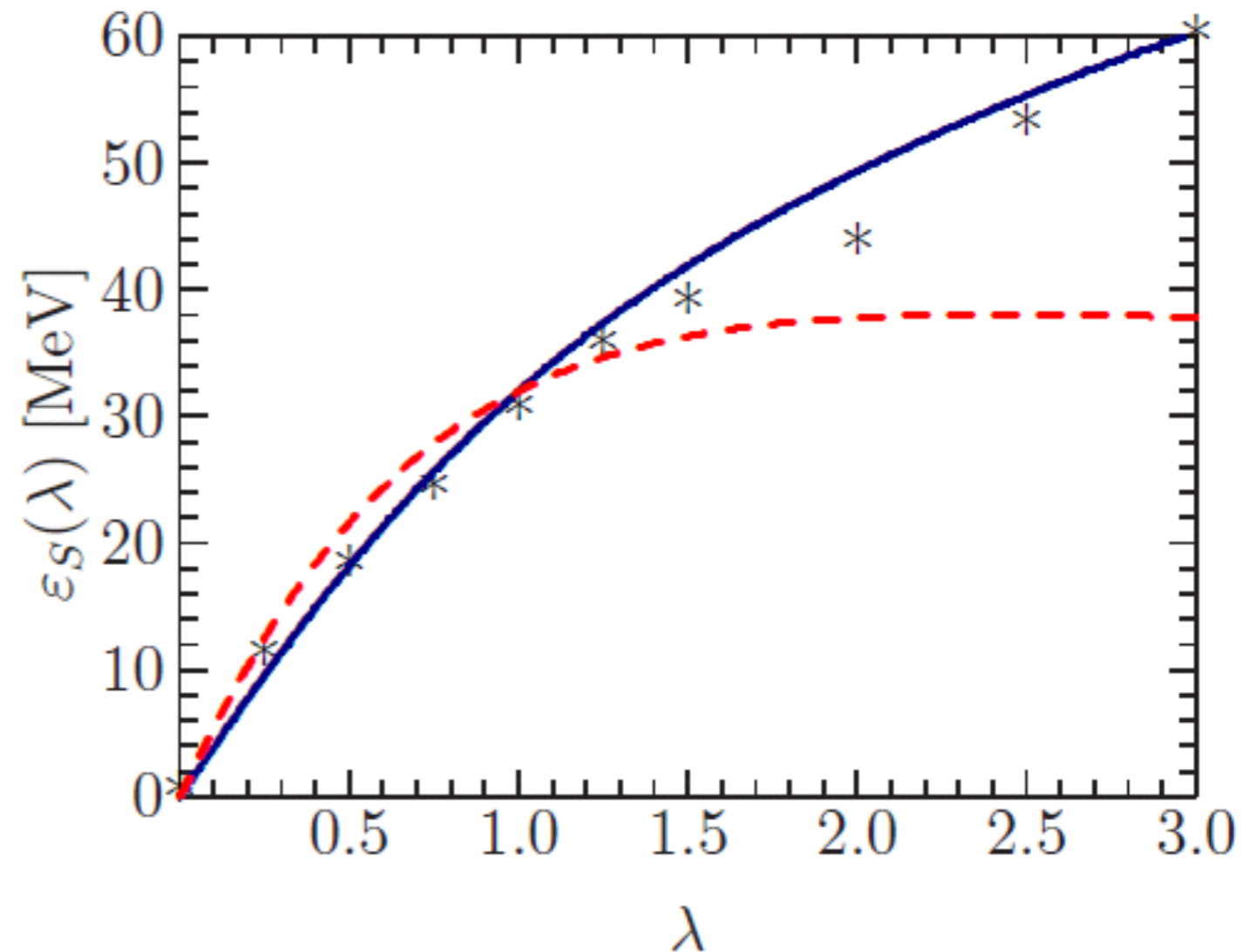
For comparison: Right figure from  
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).  
(Deduced from experimental flow data and simulations studies)

# Asymmetric matter

## Symmetry energy

- Solid  $L_s = 70$  MeV
- Dashed  $L_s = 40$  MeV

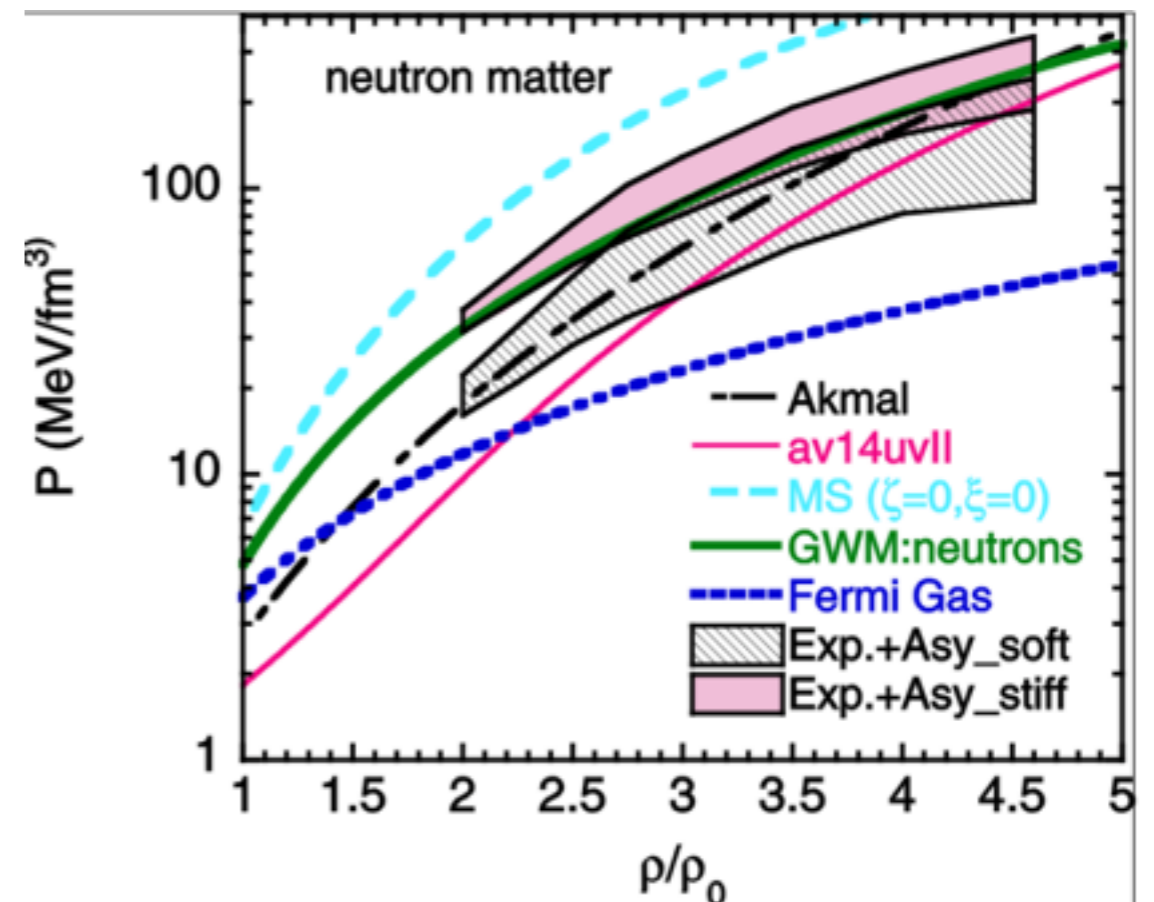
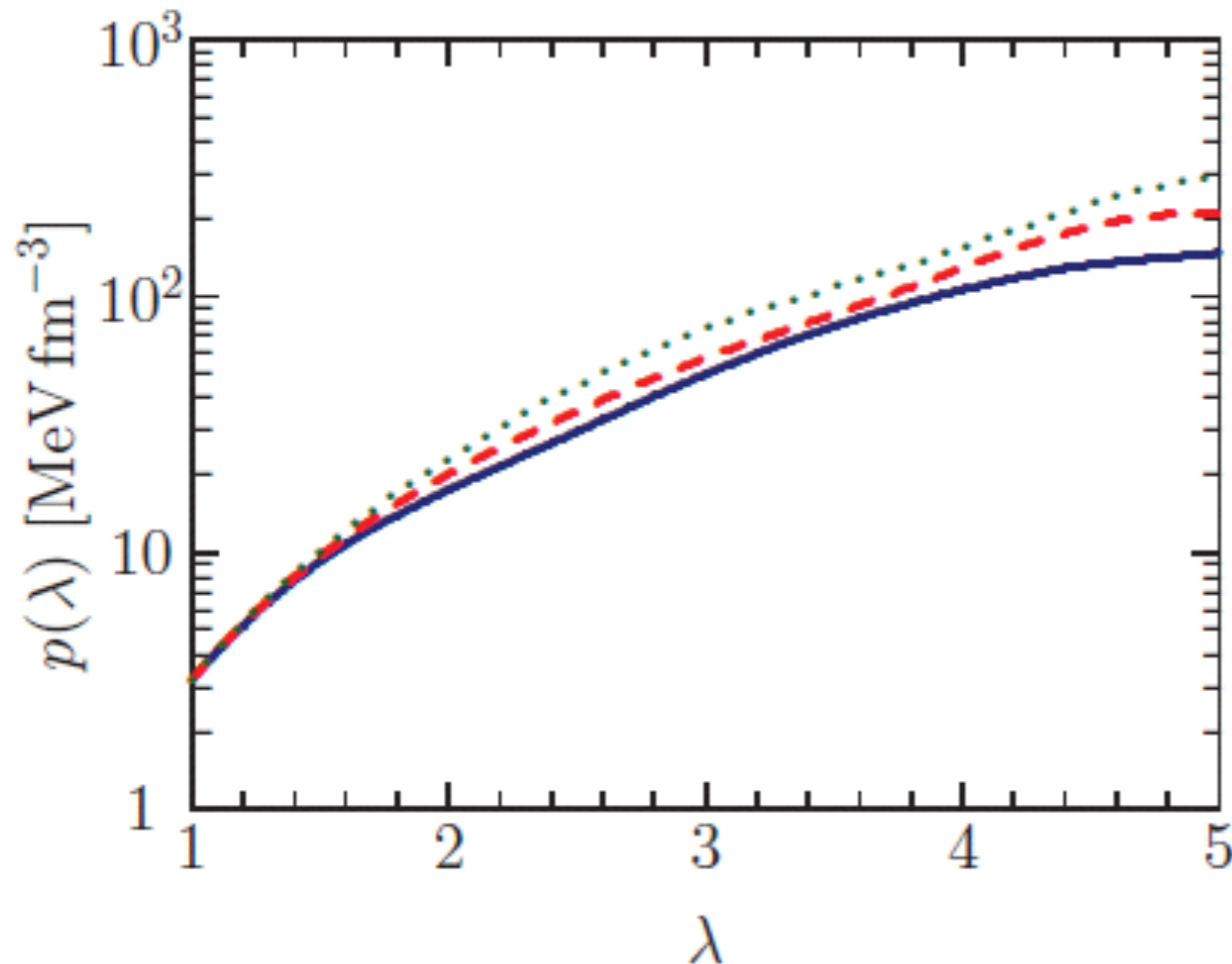
For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions [PRC 58, 1804 (1998)] are given by stars.  
(From arigonna 2 body interactions + 3 body interactions)



# Asymmetric matter

## Pressure in neutron matter

[ UY, PRC88 (2013) ]



For comparison: Right figure from  
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).  
(Deduced from experimental flow data and simulations studies)

# Asymmetric matter

## Low density behaviour of symmetry energy

For comparison:  
Trippa-Colo-Vigezzi  
[PRC 77, 061304 (2008)];  
From analysis of GDR  
(208Pb).

$$23.3 < \varepsilon_s(\rho = 0.1\text{fm}^{-3}) < 24.9 \text{ MeV}$$

Consequently one can  
predict in this model:

$$K_\tau = K_s - 6L_s$$

$$K_{0,2} = K_\tau - \frac{Q}{K_0} L_s$$

$\varepsilon_s(\rho_0)$	$L_s$	$K_s$	$K_\tau$	$K_{0,2}$	$\varepsilon_s(0.1\text{fm}^{-3})$
[MeV]	[MeV]	[MeV]	[MeV]	[MeV]	[MeV]
32	40	-181	-301	-257	25.15
32	50	-160	-310	-254	24.15
32	60	-126	-306	-239	23.22
32	70	-80	-290	-211	22.37
32	80	-21	-261	-172	21.57
32	90	50	-220	-119	20.82
32	100	134	-166	-55	20.13

# Neutron stars

## Neutron star properties [UY, arXiv:1506.06481[nucl-th]]

- TOV equations

$$-\frac{dP(r)}{dr} = \frac{G\mathcal{E}(r)\mathcal{M}(r)}{r^2} \left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1} \left(1 + \frac{P(r)}{\mathcal{E}(r)}\right) \left(1 + \frac{4\pi r^3 P(r)'}{\mathcal{M}(r)}\right)$$

- Energy-pressure relation

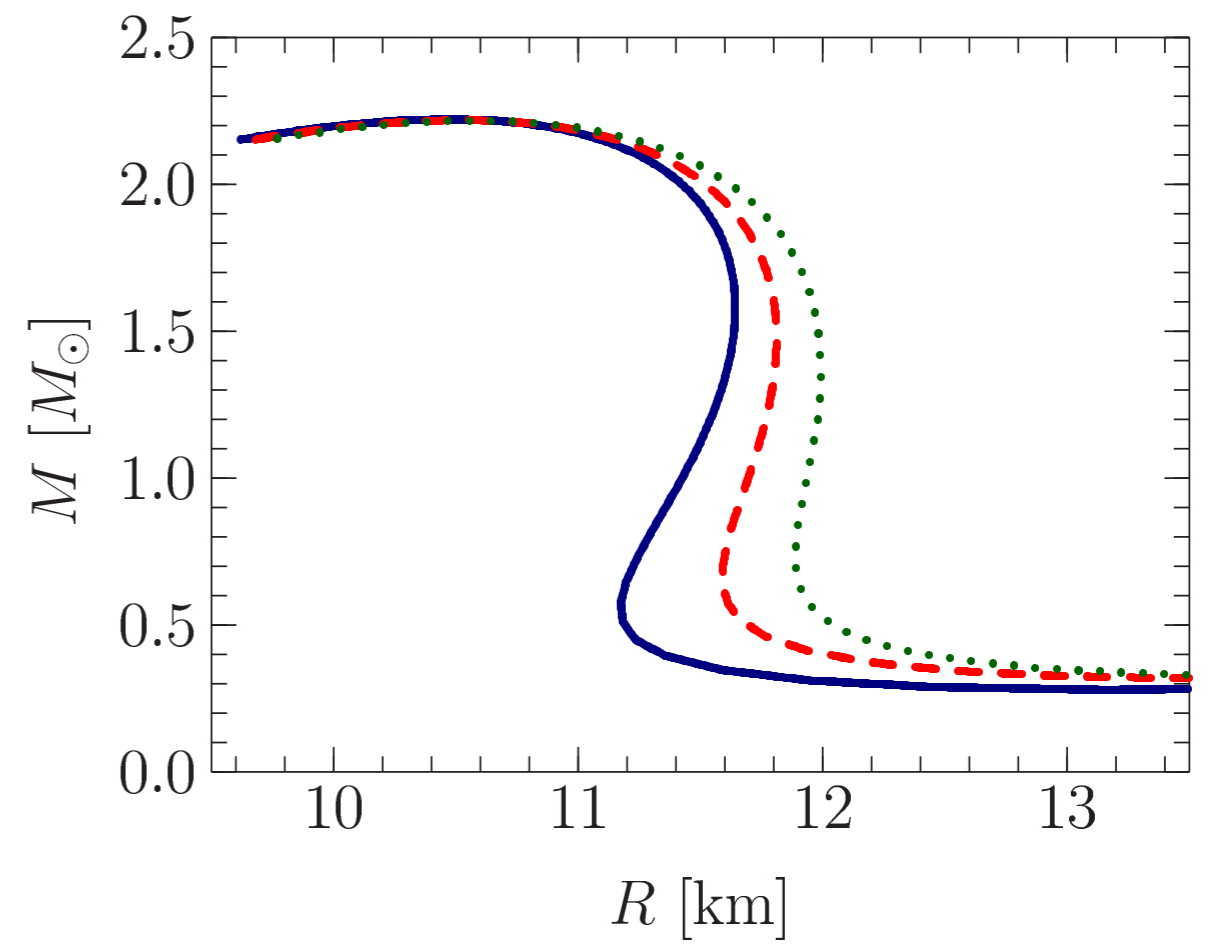
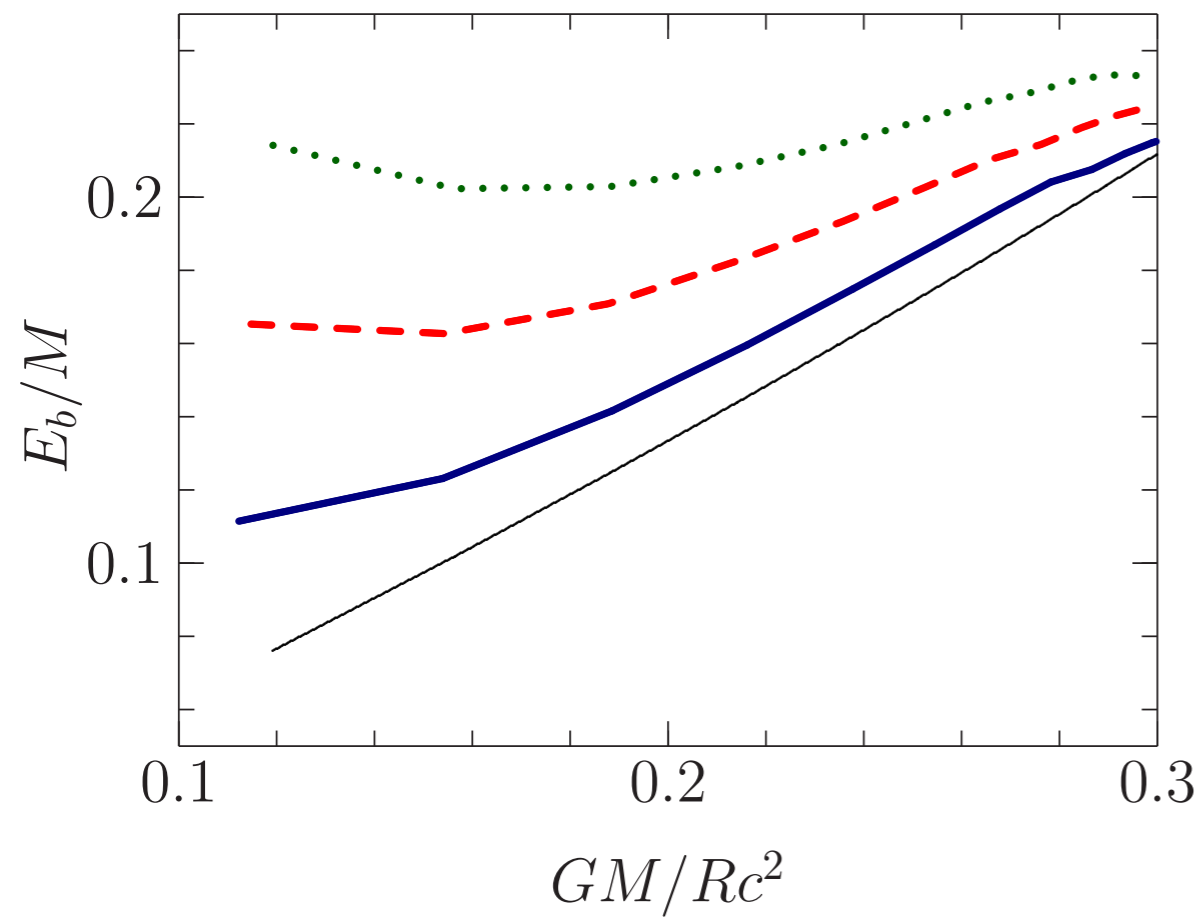
$$P = P(\mathcal{E}) \quad \begin{aligned} P(\lambda) &= \rho_0 \lambda^2 \frac{\partial \varepsilon(\lambda, 1)}{\partial \lambda}, \\ \mathcal{E}(\lambda) &= [\varepsilon(\lambda, 1) + m_N] \lambda \rho_0. \end{aligned}$$

- Neutron star's mass

$$\mathcal{M}(r) = 4\pi \int_0^r dr r^2 \mathcal{E}(r).$$

# Neutron stars

Neutron star properties [UY, arXiv:1506.06481[nucl-th]]



# Neutron stars

## Neutron star properties [UY, arXiv:1506.06481[nucl-th]]

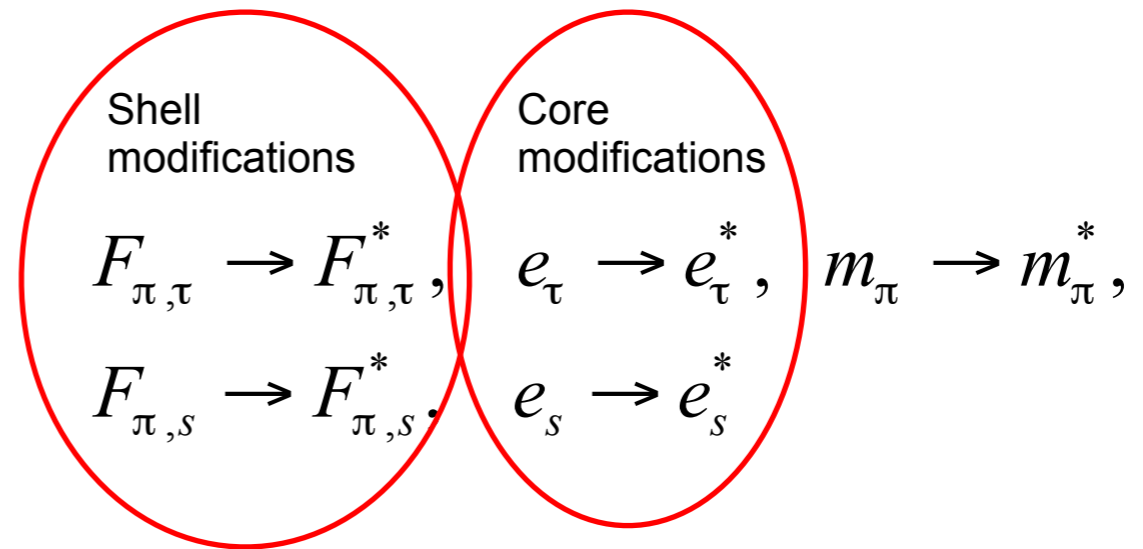
TABLE III: Properties of the neutron stars from the different sets of parameters (see Tables I and II for the values of parameters):  $n_c$  is central number density,  $\rho_c$  is central energy-mass density,  $R$  is radius of the neutron star,  $M_{\max}$  is possible maximal mass,  $A$  is number of baryons in the star,  $E_b$  is binding energy of the star. In the left panel we represent the neutron star properties corresponding to the maximal mass  $M_{\max}$  and in right panel approximately 1.4 solar mass neutron star properties. The last two lines are results from the Ref. [21].

Set	$n_c$ [fm <sup>-3</sup> ]	$\rho_c$ [10 <sup>15</sup> gr/cm <sup>3</sup> ]	$R$ [km]	$M_{\max}$ [ $M_{\odot}$ ]	$A$ [10 <sup>57</sup> ]	$E_b$ [10 <sup>53</sup> erg]	$n_c$ [fm <sup>-3</sup> ]	$\rho_c$ [10 <sup>15</sup> gr/cm <sup>3</sup> ]	$R$ [km]	$M$ [ $M_{\odot}$ ]	$A$ [10 <sup>57</sup> ]	$E_b$ [10 <sup>53</sup> erg]
III-a	1.046	2.445	10.498	2.226	3.227	8.721	0.479	0.861	11.587	1.402	1.898	3.503
III-b	1.045	2.444	10.547	2.223	3.216	8.557	0.471	0.861	11.772	1.402	1.895	3.453
III-c	1.037	2.424	10.616	2.221	3.200	8.397	0.460	0.832	11.953	1.402	1.887	3.339
III-d	1.047	2.452	10.494	2.221	3.213	8.598	0.481	0.867	11.619	1.402	1.893	3.422
III-e	1.044	2.440	10.554	2.218	3.203	8.495	0.473	0.858	11.809	1.403	1.890	3.384
III-f	1.040	2.433	10.609	2.216	3.189	8.311	0.464	0.842	11.992	1.403	1.887	3.334
SLy230a [21]	1.15	2.69	10.25	2.10	2.99	7.07	0.508	0.925	11.8	1.4	1.85	2.60
SLy230b [21]	1.21	2.85	9.99	2.05	2.91	6.79	0.538	0.985	11.7	1.4	1.85	2.61

# Consistency (difference) with (from) other models

One can find density functionals from the reparametrization scheme  
 [ UY, PRC88 (2013) ]

- Five density dependent parameters



- Rearrangment (technical simplification)

$$1 + C_1 \frac{\rho}{\rho_0} = f_1\left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2\left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$1 + C_3 \frac{\rho}{\rho_0} = f_3\left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right) \frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

# Consistency (difference) with (from) other models

Low energy constants in nuclear at normal nuclear matter density  $\rho_0$

	Present model	ChPT [1]	QCD sum rules [2]
$F_{\pi,t}^* / F_\pi$	0.37	0.74	0.79
$F_{\pi,s}^* / F_\pi$	0.72	< 0	0.78

[1] U. Meissner, J. Oller, A. Wirzba, Annals Phys. 297 (2002) 27.

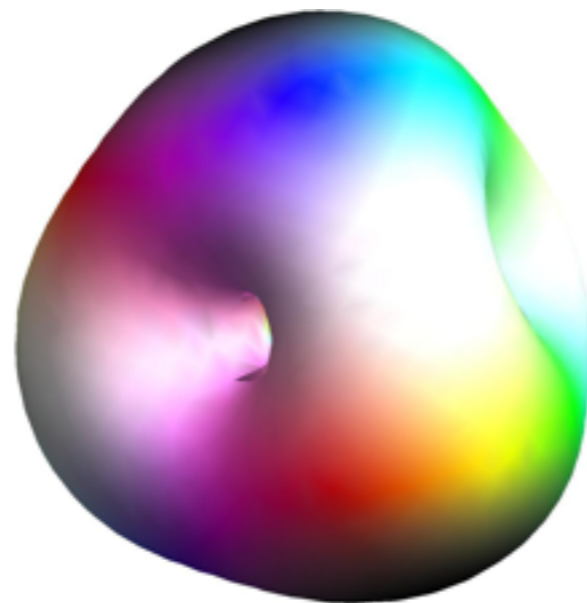
[2] H. Kim, M. Oka, NPA720 (2003) 368.

# Consistency (difference) with (from) other models

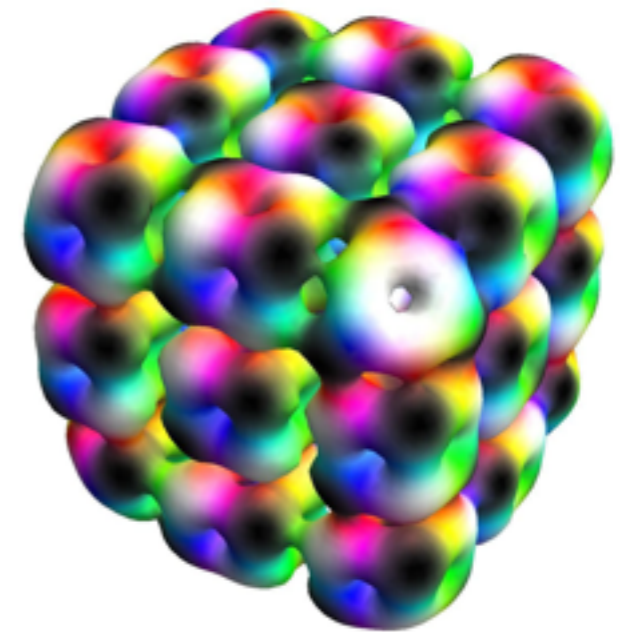
Surface of constant baryon density skyrmions [Feist, D.T.J. *et al.* Phys.Rev. D87 (2013)]



$$B = 1$$



$$B = 3$$



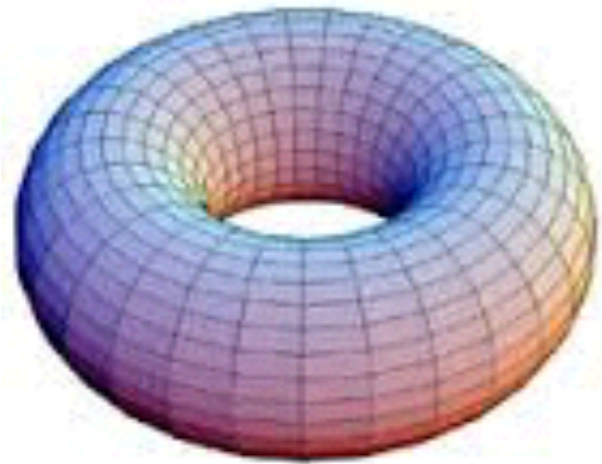
$$B = 104$$

$$\mathcal{L} = \frac{F_\pi^2}{16} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) - \frac{1}{16e^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

Changes from a nucleus to a nucleus (“calibration”)

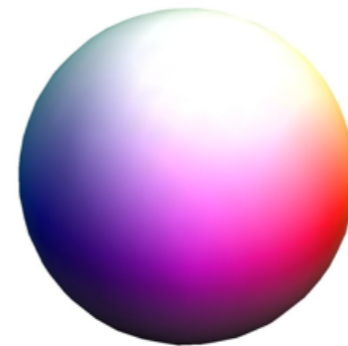
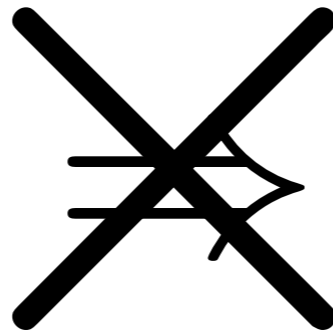
# Consistency (difference) with (from) other models

Topological non-triviality



$B = 2$

?



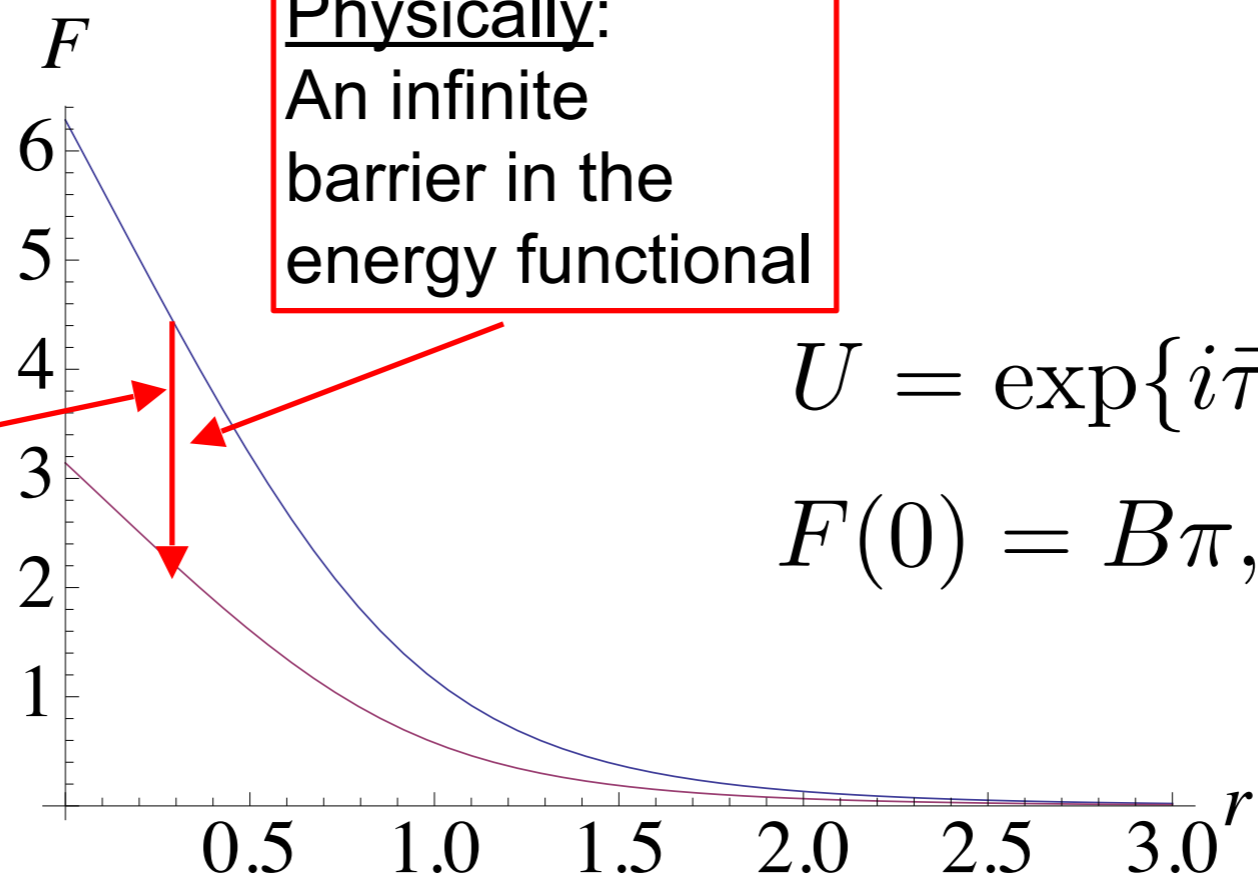
$B = 1$



$B = 1$

Physically:  
An infinite barrier in the energy functional

Mathematically:  
There is no smooth transition between topologically distinct mappings

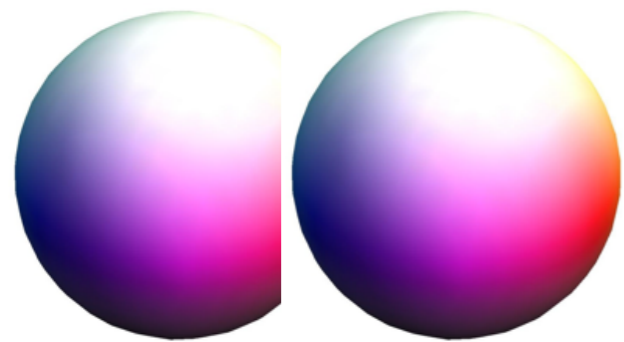


$$U = \exp\{i\vec{\tau}\vec{n}F(r)\}$$

$$F(0) = B\pi, \quad F(\infty) = 0$$

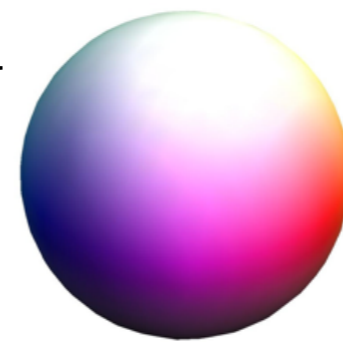
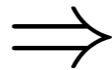
# Consistency (difference) with (from) other models

Physically consistent picture (ansatz product)



$$B = 2$$

Well defined



$$B = 1$$



$$B = 1$$

Overlapping at small distances

Well separated at large distances

One can reproduce SS potential and project it into NN potential.

$$U_{\text{system}} = U(\vec{r}_1)U(\vec{r}_2)$$

$$U = \exp\{i\vec{\tau}\vec{n}F(r)\}$$

$$F(0) = \pi, \quad F(\infty) = 0$$

# Consistency (difference) with (from) other models

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## Other approaches

- Classical crystalline structures
  - Cubic structure
    - [M. Kutschera *et al.* Phys. Rev. Lett. **53** (1984)]
    - [I. R. Klebanov, Nucl. Phys. B **262** (1985)]
  - Phase structure analysis using FCC crystal
    - [H.-J. Lee *et al.* Nucl. Phys. A **723** (2003)]
- Skyrmions in hypersphere
  - System properties from the single skyrmion in hypersphere
    - [N. S. Manton and P. J. Ruback, Phys. Lett. B **181** (1986)]
  -

# Summary and Outlook

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- Within the applicability range, the model describes at same footing
  - the single hadrons properties
    - in separate state
    - in the community of their partners (EM and EMT form factors)
  - as well as the properties of that whole community
    - infinite nuclear matter properties (volume and symmetry energy properties)
    - matter under extreme conditions (neutron stars)
    - few and many nucleon systems (mirror nuclei, rare isotopes, halo nuclei,...)
    - nucleon knock-out reactions
    - effective NN interactions

# Summary and Outlook

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## **Applicability and extensions of the approach**

- Nucleon tomography
  - [H.Ch. Kim, P. Schweitzer, UY, PLB718 (2012)]
  - [H.Ch. Kim, UY, PLB726 (2013)]
  - [J.H.Jung, UY, H.Ch.Kim, Jour. Phys. G41 (2014)]
  - [J.H.Jung, UY, H.Ch.Kim, P. Schweitzer. PRD89 (2014)]
- Nuclear matter [UY, PRC88 (2013)]
- Neutron stars [UY, arXiv:1506.06481[nucl-th]]
- Vector mesons in nuclear matter
  - [J.H.Jung, UY, H.Ch.Kim, PLB 723 (2013)]

**Thank you very much for your attention!**