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Estimation of the Mass of ground state pentaquarks

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Roper resonance

- The roper resonance of radial quantum number N=2 with positive parity has lower mass than three-quark state of radial quantum number N=1 with negative parity.

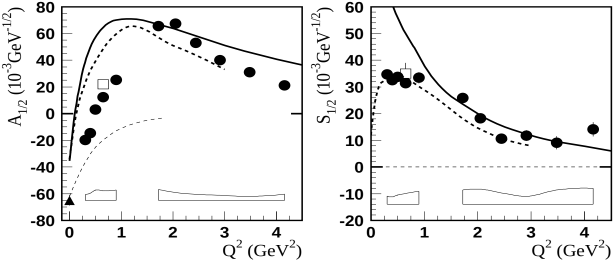
- Roper resonance is usually blamed sitting at a wrong place or intruding the q^3 spectrum.

- It has been studied in any possible picture: normal q^3 first radial excitation, $q^4\bar{q}$ pentaquark, q^3g hybrid, $q^3(q\bar{q})$ resonance...

- Still an open question.

$N_{1/2+}(1440)$

Helicity amplitudes for the $\gamma^* p \to N(1440)$ transition. The thick curves correspond to quark models assuming that $N(1440)$ is a q^3 first radial excitation: dashed (Capstick and Keister, 1995), solid (Aznauryan, 2007). The thin dashed curves are obtained assuming that $N(1440)$ is a $q^3 g$ hybrid state (Li et al., 1992). Figure courtesy to Rev. Mod. Phys. **82**, 1095.



- The sign change in the helicity amplitude as a function of Q^2 suggests a node in the wave function and thus a radially excited state.

$N_{1/2-}(1535)$

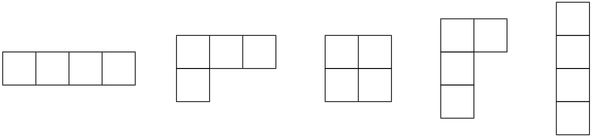
- This resonance is observed at a mass expected in quark models
- Large couplings to the $N\eta$, $N\eta'$, $N\phi$ and $K\Lambda$ but small couplings to the $N\pi$ and $K\Sigma$ are claimed (PDG estimate).
- A large $N\eta$ coupling invites speculation that it might be created dynamically as $N\eta - \Sigma K$ coupled channel effect[Rev. Mod. Phys. **82**, 1095 (2010).
- A large $N\phi$ coupling leads to the proposal that the $N_{1/2-}(1535)$ may have a large component of $uuds\bar{s}$ pentaquark states.

Construction

The construction of pentaquark states is guided by two main rules.

- The pentaquark wave function should be a color singlet;
- The pentaquark wave function should be antisymmetric under any permutation of the four quark configuration.

- The permutation symmetry of the four-quark configuration of pentaquark states is characterized by the S_4 Young tabloids $[4]$, $[31]$, $[22]$, $[211]$ and $[1111]$.



Construction of Pentaquark Wave Functions

- That the pentaquark should be a color singlet demands that the color part of the pentaquark wave function must be a $[222]_1$ singlet.

$$\psi^c_{[222]}(q^4\overline{q}) = \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline & \\ \hline \end{array} \qquad (1)$$

- The color part of the antiquark in pentaquark states is a $[11]_3$ antitriplet

$$\psi^c_{[11]}(\overline{q}) = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \qquad (2)$$

- The color wave function of the four-quark configuration must be a $[211]_3$ triplet

$$\psi^c_{[211]_\lambda}(q^4) = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1} \end{array} \quad \psi^c_{[211]_\rho}(q^4) = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1} \end{array} \quad \psi^c_{[211]_\eta}(q^4) = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1} \end{array} \qquad (3)$$

q^4 Color Wave Functions

- q^4 color wave functions can be derived by applying the λ-, ρ- and η-type projection operators of the S_4 $IR[211]$ in Yamanouchi basis,

$$\left|\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & R \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array}\right\rangle = P_{[211]_\lambda}(RRGB) \Longrightarrow \psi^c_{[211]_\lambda}(R):$$

$$\frac{1}{\sqrt{16}}(2|RRGB\rangle - 2|RRBG\rangle - |GRRB\rangle - |RGRB\rangle - |BRGR\rangle$$
$$-|RBGR\rangle + |BRRG\rangle + |GRBR\rangle + |RBRG\rangle + |RGBR\rangle)$$

$$\left|\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & R \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array}\right\rangle = P_{[211]_\rho}(RGRB) \Longrightarrow \psi^c_{[211]_\rho}(R):$$

$$\frac{1}{\sqrt{48}}(3|RGRB\rangle - 3|GRRB\rangle + 3|BRRG\rangle - 3|RBRG\rangle + 2|GBRR\rangle$$
$$-2|BGRR\rangle - |BRGR\rangle + |RBGR\rangle + |GRBR\rangle - |RGBR\rangle)$$

q^4 Color Wave Functions

$$\left| \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & R \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\eta}(RGBR) \Longrightarrow \psi^c_{[211]_\eta}(R):$$

$$\frac{1}{\sqrt{6}}(|RGBR\rangle + |GBRR\rangle + |BRGR\rangle - |RBGR\rangle - |GRBR\rangle - |BGRR\rangle)$$

The singlet color wave function $\Psi^c_{[211]_j}$ $(j = \lambda, \rho, \eta)$ of pentaquarks is given by

$$\Psi^c_{[211]_j} = \frac{1}{\sqrt{3}}\left[\psi^c_{[211]_j}(R)\,\bar{R} + \psi^c_{[211]_j}(G)\,\bar{G} + \psi^c_{[211]_j}(B)\,\bar{B}\right]. \quad (4)$$

Matrix representations of S_4

- S4 $[211]$

$$D^{[211]}(12) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad D^{[211]}(23) = \begin{pmatrix} -1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$D^{[211]}(34) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1/3 & 2\sqrt{2}/3 \\ 0 & 2\sqrt{2}/3 & 1/3 \end{pmatrix}$$

- S4 $[31]$

$$D^{[31]}(12) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D^{[31]}(23) = \begin{pmatrix} -1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$D^{[31]}(34) = \begin{pmatrix} 1/3 & 0 & 2\sqrt{2}/3 \\ 0 & 1 & 0 \\ 2\sqrt{2}/3 & 0 & -1/3 \end{pmatrix}$$

q^4 Wave Functions

- To form a fully antisymmetric q^4 wave function, the spatial-spin-flavor part must be $[31]$.
- The total wave function of q^4 systems may be written in the general form,

$$\psi = \sum_{i,j=\lambda,\rho,\eta} a_{ij}\ \psi^{c}_{[211]_i}\psi^{\mathrm{osf}}_{[31]_j} \tag{5}$$

- The coefficients of the antisymmetric ψ can be determined by applying the Yamanouchi-basis representations of the S_4 to the general form. One gets

$$\psi = \frac{1}{\sqrt{3}}\left(\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} - \psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + \psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\right) \tag{6}$$

q^4 Spatial-Flavor-Spin States

- In the same way we can write

$$\Psi^{osf}_{[31]} = \sum_{i,j=S,A,\lambda,\rho,\eta} a_{ij}\Psi^{o}_{[X]_i}\Psi^{sf}_{[Y]_j} \quad (7)$$

$$\Psi^{sf}_{[Z]} = \sum_{i,j=S,A,\lambda,\rho,\eta} a_{ij}\Phi^{f}_{[X]_i}\chi^{s}_{[Y]_j} \quad (8)$$

- The possible spatial-spin-flavor and spin-flavor configurations and explicit forms of the wave functions are determined by applying the S_4 representations in Yamanouchi basis.
- Spatial-spin-flavor configurations:

$[31]_{OSF}$	
$[4]_O$	$[31]_{SF}$
$[1111]_O$	$[211]_{SF}$
$[22]_O$	$[31]_{SF}$, $[211]_{SF}$
$[211]_O$	$[31]_{SF}$, $[211]_{SF}$, $[22]_{SF}$
$[31]_O$	$[4]_{SF}$, $[31]_{SF}$, $[211]_{SF}$, $[22]_{SF}$

q^4 Spin-Flavor States

	$[4]_{FS}$		
$[4]_{FS}[22]_F[22]_S$	$[4]_{FS}[31]_F[31]_S$	$[4]_{FS}[4]_F[4]_S$	
	$[31]_{FS}$		
$[31]_{FS}[31]_F[22]_S$	$[31]_{FS}[31]_F[31]_S$	$[31]_{FS}[31]_F[4]_S$	$[31]_{FS}[211]_F[22]_S$
$[31]_{FS}[211]_F[31]_S$	$[31]_{FS}[22]_F[31]_S$	$[31]_{FS}[4]_F[31]_S$	
	$[22]_{FS}$		
$[22]_{FS}[22]_F[22]_S$	$[22]_{FS}[22]_F[4]_S$	$[22]_{FS}[4]_F[22]_S$	$[22]_{FS}[211]_F[31]_S$
$[22]_{FS}[31]_F[31]_S$			
	$[211]_{FS}$		
$[211]_{FS}[211]_F[22]_S$	$[211]_{FS}[211]_F[31]_S$	$[211]_{FS}[211]_F[4]_S$	$[211]_{FS}[22]_F[31]_S$
$[211]_{FS}[31]_F[22]_S$	$[211]_{FS}[31]_F[31]_S$		

Spin-Flavor Wave Functions

- Taking $[31]_{FS}[22]_F[31]_S$ configuration as an example, the spin-flavor wave functions of various permutation symmetries take a general form,

$$\psi^{\rm sf} = \sum_{i=\lambda,\rho}\ \sum_{j=\lambda,\rho,\eta} a_{ij}\, \phi_{[22]_i}\chi_{[31]_j}$$

- a_{ij} can be determined by acting the permutations of S_4 on the general form. The spin-flavor wave functions for the q^4 cluster are derived as,

$$\begin{aligned}
\psi^{\rm sf}_{[31]_\lambda} &= -\frac{1}{2}\phi_{[22]_\rho}\chi_{[31]_\rho} + \frac{1}{2}\phi_{[22]_\lambda}\chi_{[31]_\lambda} + \frac{1}{\sqrt{2}}\phi_{[22]_\lambda}\chi_{[31]_\eta} \\
\psi^{\rm sf}_{[31]_\rho} &= -\frac{1}{2}\phi_{[22]_\rho}\chi_{[31]_\lambda} - \frac{1}{2}\phi_{[22]_\lambda}\chi_{[31]_\rho} + \frac{1}{\sqrt{2}}\phi_{[22]_\rho}\chi_{[31]_\eta} \\
\psi^{\rm sf}_{[31]_\eta} &= \frac{1}{\sqrt{2}}\phi_{[22]_\rho}\chi_{[31]_\rho} + \frac{1}{\sqrt{2}}\phi_{[22]_\lambda}\chi_{[31]_\lambda}
\end{aligned}$$

Flavor Wave Functions

- Operate $P_{\lambda,\rho}$ on any q^4 flavor state, for example, $(uudd)$, the flavor wave functions are derived,

$$\left|\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array}, \begin{array}{|c|c|}\hline u & u \\ \hline d & d \\ \hline\end{array}\right\rangle = P_\lambda(uudd)$$

$$\Longrightarrow \phi_{[22]_\lambda} = \frac{1}{2\sqrt{3}}(2uudd + 2dduu - udud - uddu - duud - dudu)$$

$$\left|\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array}, \begin{array}{|c|c|}\hline u & u \\ \hline d & d \\ \hline\end{array}\right\rangle = P_\rho(udud)$$

$$\Longrightarrow \phi_{[22]_\rho} = \frac{1}{2}(udud + dudu - duud - uddu) \quad (9)$$

- With $I = I_3 = 0$

$$\Phi_{[22]_\rho} = \phi_{[22]_\rho}\bar{s}, \quad \Phi_{[22]_\lambda} = \phi_{[22]_\lambda}\bar{s}. \quad (10)$$

- The flavor states with other values of isospin (I, I_3) and strangeness S can be derived the same way in (T_3, Y) basis.

Spin Wave Functions

- Operate $P_{\lambda,\rho,\eta}$ on any q^4 spin state, for example, the state spin=1

$$\left|\begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \uparrow \\ \hline \downarrow \\ \cline{1-1}\end{array}\right\rangle = P_{[31]_\lambda}(\uparrow\uparrow\downarrow\uparrow)$$

$$\Longrightarrow \chi_{[31]_\lambda}(1,1) = \frac{1}{\sqrt{6}}\ |\ 2\uparrow\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow\ \rangle$$

$$\left|\begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \uparrow \\ \hline \downarrow \\ \cline{1-1}\end{array}\right\rangle = P_{[31]_\rho}(\uparrow\downarrow\uparrow\uparrow)$$

$$\Longrightarrow \chi_{[31]_\rho}(1,1) = \frac{1}{\sqrt{2}}\ |\ \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow\rangle$$

$$\left|\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \uparrow \\ \hline \downarrow \\ \cline{1-1}\end{array}\right\rangle = P_{[31]_\eta}(\uparrow\uparrow\uparrow\downarrow)$$

$$\Longrightarrow \chi_{[31]_\eta}(1,1) = \frac{1}{2\sqrt{3}}\ |\ 3\uparrow\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\uparrow\ \rangle$$

Spin Wave Functions

- Combine the spin wave functions above with the one of the antiquark, the total spin wave function of the pentaquark states is

$$\chi(q^4\bar{s})_{[31]_\alpha}(3/2,1/2) = \sqrt{\frac{2}{3}}\,\chi_{[31]_\alpha}(s_{q^4}=1, m_{q^4}=1)\chi_{\bar{s}}(-1/2) - \sqrt{\frac{1}{3}}\,\chi_{[31]_\alpha}(s_{q^4}=1, m_{q^4}=0)\chi_{\bar{s}}(1/2) \quad (11)$$

- with $\alpha = \rho$, λ, η. The states with other values of the projection m_s can be obtained the same way. The expectation values of same spin but different projectors are the same.

Spatial Wave Functions

A complete basis of certain symmetry may be constructed with pentaquark systems in the harmonic oscillator interaction with at least four quarks are the same mass.

$$H = \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + \frac{p_\eta^2}{2m} + \frac{p_\xi^2}{2m'} + \frac{1}{2}C\left(\lambda^2 + \rho^2 + \eta^2 + \xi^2\right) \quad (12)$$

where

$$\begin{aligned}
\vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}) \\
\vec{\lambda} &= \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3}) \\
\vec{\eta} &= \frac{1}{\sqrt{12}}(\vec{r_1} + \vec{r_2} + \vec{r_3} - 3\vec{r_4}) \\
\vec{\xi} &= \frac{1}{\sqrt{20}}(\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} - 4\vec{r_5}) \quad (13)
\end{aligned}$$

Spatial Wave Functions

Spatial wave functions take the general form,

$$\begin{aligned}\Psi^{\mathrm{o}}_{NLM} \;=\; & \sum_{n_\lambda,n_\rho,n_\eta,n_\xi,l_\lambda,l_\rho,l_\eta,l_\xi} A(n_\lambda,n_\rho,n_\eta,n_\xi,l_\lambda,l_\rho,l_\eta,l_\xi) \\ & \cdot\Psi_{n_\lambda l_\lambda m_\lambda}(\vec{\lambda})\Psi_{n_\rho l_\rho m_\rho}(\vec{\rho})\Psi_{n_\eta l_\eta m_\eta}(\vec{\eta})\Psi_{n_\xi l_\xi m_\xi}(\vec{\xi}) \\ & \cdot C(l_\lambda,l_\rho,m_\lambda,m_\rho,l_{\lambda\rho},m_{\lambda\rho}) \\ & \cdot C(l_\eta,l_\xi,m_\eta,m_\xi,l_{\eta\xi},m_{\eta\xi}) \\ & \cdot C(l_{\lambda\rho},l_{\eta\xi},m_{\lambda\rho},m_{\eta\xi},LM) \qquad (14)\end{aligned}$$

with $N = 2(n_\lambda + n_\rho + n_\eta + n_\xi) + l_\lambda + l_\rho + l_\eta + l_\xi$

Spatial Wave Functions

Spatial wave functions take the general form,

$$\begin{aligned}\Psi^{\text{O}}_{NLM} = & \sum_{n_\lambda,n_\rho,n_\eta,n_\xi,l_\lambda,l_\rho,l_\eta,l_\xi} A(n_\lambda,n_\rho,n_\eta,n_\xi,l_\lambda,l_\rho,l_\eta,l_\xi) \\ & \cdot\Psi_{n_\lambda l_\lambda m_\lambda}(\vec{\lambda})\Psi_{n_\rho l_\rho m_\rho}(\vec{\rho})\Psi_{n_\eta l_\eta m_\eta}(\vec{\eta})\Psi_{n_\xi l_\xi m_\xi}(\vec{\xi}) \\ & \cdot C(l_\lambda,l_\rho,m_\lambda,m_\rho,l_{\lambda\rho},m_{\lambda\rho}) \\ & \cdot C(l_{\lambda\rho},l_\eta,m_{\lambda\rho},m_\eta,l_{\lambda\rho\eta},m_{\lambda\rho\eta}) \\ & \cdot C(l_{\lambda\rho\eta},l_\xi,m_{\lambda\rho\eta},m_\xi,LM)\end{aligned} \quad (15)$$

with $N = 2(n_\lambda + n_\rho + n_\eta + n_\xi) + l_\lambda + l_\rho + l_\eta + l_\xi$

- The coefficients A are determined by applying the Yamanouchi basis representations of the S_4. Various types of spatial wave functions with the $[4], [31], [22], [211]$ and $[1111]$ symmetries are worked out.

Constituent Quark Models

- Hamiltonian for a N-quark system:

$$H = \sum_{k=1}^{N} \frac{p_k^2}{2m_k} + C \sum_{i<j}^{N} ([\vec{r}_i - \vec{r}_j]^2 + V_0) + \sum_{n=1}^{N} m_n + H_{hyp} \qquad (16)$$

- One-gluon-exchange contribution ,m_u, m_s stand for $m_n + V_0$, m_i, m_j are dressed masses of quark $m_u^B = m_u + \sqrt{3}w_0$, $m_s^B = m_s + \sqrt{\frac{3m_u}{m_s}}w_0$

$$H_{hyp}^{OGE} = -C_G \sum_{i<j} \frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \qquad (17)$$

- Supposing at least four quark mass is the same, we change the variables to be Jacobi coordinates, the non-interacting part of N-quark system can be rewritten as,

$$H_N = \sum_{i=1}^{N-1} \frac{\vec{\eta}_i^2}{2m} + \frac{\vec{P}^2}{2M} + NC \sum_{i=1}^{N-1} \vec{\xi}_i^2 + N(m + V_0) \qquad (18)$$

Our baryon mass

- In our picture baryons consist of the q^3 component as well as the $q^4\bar{q}$ pentaquark component, the wave function of baryons takes the form,

$$|N\rangle = A_i|q^3\rangle + B_j|q^4\bar{q}\rangle \tag{19}$$

- The baryon mass is estimated,

$$M = A_i^2 M(q^3) + B_j^2 M(q^4\bar{q}), \quad A_i^2 + B_j^2 = 1 \tag{20}$$

- $M(q^3)$ and $M(q^4\bar{q})$ are the masses of the q^3 and the $q^4\bar{q}$ pentaquark system, respectively.

$$m_{q^3} = \sum_{j=1}^{3}(m_j + V_0) + \sum_{i=1}^{2}(n_i + \frac{3}{2})w_i + \Delta m^{OGE}(q^3),$$

$$m_{q^4\bar{q}} = \sum_{j=1}^{5}(m_j + V_0) + \sum_{i=1}^{4}(n_i + \frac{3}{2})w_i + \Delta m^{OGE}(q^4\bar{q}) \tag{21}$$

Fixing of model parameters

- The parameters are determined by fitting the theoretical results to PDG of 8 isospin baryon states.

$$\Delta m = \sum_{i=1}^{8} (m^i_{theory} - m^i_0)^2 \tag{22}$$

- $\Delta m = 1498.87 \mathrm{MeV}^2$

$$m_u \approx 122.9 \text{ MeV}, \quad m_s \approx 399.1 \text{ MeV}$$
$$C_g \approx 18.97 \text{ MeV}, \quad w_0 \approx 138.1 \text{ MeV}$$
$$\sqrt{3}w_0 = 239.2 \text{ MeV}, \quad \sqrt{5}w_0 = 308.8 \text{ MeV}$$

- The constitute quark mass m_i^B, m_j^B and $k_B = \frac{m_s^B}{m_u^B}$

$$m_u^B = m_u + \sqrt{3}w_0 = 362.1 \text{ MeV}, \quad m_s^B = m_s + \sqrt{\frac{3m_u}{m_s}}w_0 = 531.9 \text{ MeV},$$

$$k_B = \frac{m_s^B}{m_u^B} = 1.469$$

Estimations of ground state pentaquark($uuuu\bar{u}$) masses with OGE

$$m_u \approx 122.9 \text{ MeV}, \quad m_s \approx 399.1 \text{ MeV}, \quad C_g \approx 18.97 \text{ MeV},$$
$$w_0 \approx 138.1 \text{ MeV}, \quad k_B = 1.469$$

Configurations	Spin (or J)	$\Delta m(q^4\overline{q})_{total}(\text{MeV})$	$M(q^4\overline{q})$ (MeV)
$\Psi^{sf}_{[31]_{FS}[4]_F[31]_S}$	$\frac{1}{2}, \frac{3}{2}$	556, 253	3024, 2720
$\Psi^{sf}_{[31]_{FS}[31]_F[4]_S}$	$\frac{3}{2}, \frac{5}{2}$	0, 253	2467, 2720
$\Psi^{sf}_{[31]_{FS}[31]_F[31]_S}$	$\frac{1}{2}, \frac{3}{2}$	101, 25	2568, 2493
$\Psi^{sf}_{[31]_{FS}[31]_F[22]_S}$	$\frac{1}{2}$	0	2467
$\Psi^{sf}_{[31]_{FS}[22]_F[31]_S}$	$\frac{1}{2}, \frac{3}{2}$	-354, 25	2113, 2493

Estimations of ground state pentaquark($uuus\bar{u}$) masses with OGE

Configurations	Spin (or J)	$\Delta m(q^4\overline{q})_{total}$(MeV)	$M(q^4\overline{q})$ (MeV)
$\Psi^{sf}_{[31]_{FS}[4]_F[31]_S}$	$\frac{1}{2}, \frac{3}{2}$	483, 204	3073, 2794
$\Psi^{sf}_{[31]_{FS}[31]_F[4]_S}$	$\frac{3}{2}, \frac{5}{2}$	34, 221	2624, 2811
$\Psi^{sf}_{[31]_{FS}[31]_F[31]_S}$	$\frac{1}{2}, \frac{3}{2}$	89, 19	2679, 2609
$\Psi^{sf}_{[31]_{FS}[31]_F[22]_S}$	$\frac{1}{2}$	0	2590
$\Psi^{sf}_{[31]_{FS}[211]_F[31]_S}$	$\frac{1}{2}, \frac{3}{2}$	-444, -96	2146, 2494
$\Psi^{sf}_{[31]_{FS}[211]_F[22]_S}$	$\frac{1}{2}$	-255	2335
$\Psi^{sf}_{[31]_{FS}[22]_F[31]_S}$	$\frac{1}{2}, \frac{3}{2}$	-318, 37	2272, 2627

Summary

- Yamanouchi basis approach is the very tool for constructing multiquark states.
- The ground state pentaquark mass is higher than $2000\mathrm{MeV}$ as we expected.
- The constitute quark model with OGE explain the mass well in the ground state baryons.
- The excited q^3 baryon and pentaquark mass may need including spin tensor, spin-orbit interaction and coupling to reproduce the baryon mass spectrum.

Thank you for your attention!

Error of the theoretical results

Baryon	PDG Data (MeV)	Calculated mass (MeV)	Error (MeV)
N	938	934.7	-3.3
Δ	1232	1238.2	6.2
Λ	1115	1131.6	16.6
Σ	1193	1163.7	-29.1
Σ^*	1385	1386.6	1.6
Θ	1315	1331.8	16.8
Θ^*	1530	1527.4	-2.6
Ω	1672	1666.0	-6.0

- The quark mass in one-gluon-exchange contribution should be the dressed mass which include bare quark mass and harmonic oscillator energy.
- The goldstone-boson-exchange contribution can be ignored.

Error of the theoretical results

$$m_u^3 \approx 126.3 \text{ MeV}, \quad m_s^3 \approx 399.4 \text{ MeV}, \quad m_u^2 \approx 192.7 \text{ MeV}, \quad m_s^2 \approx 382.0 \text{ MeV},$$
$$C_m \approx 18.96 \text{ MeV}, \quad w_0 \approx 136.2 \text{ MeV}, \qquad (23)$$

Baryon	PDG Data (MeV)	Calculated mass (MeV)	Error (MeV)
Proton	938	935.0	-3.0
Δ	1232	1238.3	6.3
Λ	1115	1131.4	16.4
Σ	1193	1163.6	-29.4
Σ^*	1385	1386.3	1.3
Θ	1315	1331.7	16.7
Θ^*	1530	1527.3	-2.7
Ω	1672	1666.4	-5.6
ρ	770	775.6	5.6
ω	783	775.6	-7.4
k^*	892	895.7	3.7
ϕ	1020	1018.2	-1.8

Pentaquark Spatial Wave Function, $NLM = 322$

- Symmetric:

$$\begin{aligned}\Psi^S = \frac{1}{3}\,[&-\Psi_{021}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{011}(\xi) + \sqrt{2}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{010}(\xi) \\ &-\Psi_{000}(\lambda)\Psi_{021}(\rho)\Psi_{000}(\eta)\Psi_{011}(\xi) + \sqrt{2}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{010}(\xi) \\ &-\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{021}(\eta)\Psi_{011}(\xi) + \sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{010}(\xi)\,]\end{aligned}$$

- Antisymmetric: Non
- λ and ρ types of [22]: Non
- λ, ρ and η types of [211]: Non

Pentaquark Spatial Wave Function, $NLM = 322$

First Set of λ, ρ and η types of [31]:

$$\Psi^{\lambda[31]} = \frac{1}{\sqrt{6}}[\sqrt{2}\Psi_{010}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \Psi_{011}(\lambda)\Psi_{021}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$$
$$+\sqrt{2}\Psi_{010}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi) - \Psi_{011}(\lambda)\Psi_{000}(\rho)\Psi_{021}(\eta)\Psi_{000}(\xi)]$$

$$\Psi^{\rho[31]} = \frac{1}{\sqrt{6}}[\sqrt{2}\Psi_{022}(\lambda)\Psi_{010}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \Psi_{021}(\lambda)\Psi_{011}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$$
$$+\sqrt{2}\Psi_{000}(\lambda)\Psi_{010}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi) - \Psi_{000}(\lambda)\Psi_{011}(\rho)\Psi_{021}(\eta)\Psi_{000}(\xi)]$$

$$\Psi^{\eta[31]} = \frac{1}{\sqrt{6}}[\sqrt{2}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{010}(\eta)\Psi_{000}(\xi) - \Psi_{021}(\lambda)\Psi_{000}(\rho)\Psi_{011}(\eta)\Psi_{000}(\xi)$$
$$+\sqrt{2}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{010}(\eta)\Psi_{000}(\Xi) - \Psi_{000}(\lambda)\Psi_{021}(\rho)\Psi_{011}(\eta)\Psi_{000}(\Xi)]$$

Pentaquark Spatial Wave Function, $NLM = 322$

Second Set of λ, ρ and η types of [31]:

$$\Psi^{\lambda[31]} = \frac{1}{\sqrt{3}}[\sqrt{2}\Psi_{010}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{022}(\xi) - \Psi_{011}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{021}(\xi)]$$

$$\Psi^{\rho[31]} = \frac{1}{\sqrt{3}}[\sqrt{2}\Psi_{000}(\lambda)\Psi_{010}(\rho)\Psi_{000}(\eta)\Psi_{022}(\xi) - \Psi_{000}(\lambda)\Psi_{011}(\rho)\Psi_{000}(\eta)\Psi_{021}(\xi)]$$

$$\Psi^{\eta[31]} = \frac{1}{\sqrt{3}}[\sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{010}(\eta)\Psi_{022}(\xi) - \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{011}(\eta)\Psi_{021}(\xi)]$$

Pentaquark Spatial Wave Function, Symmetric $NLM = 444$

$$
\begin{aligned}
\Psi^{S_1} &= \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{044}(\xi) \\
\Psi^{S_2} &= \frac{1}{\sqrt{3}}[\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{022}(\xi) + \Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{022}(\xi) \\
&\quad + \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{022}(\xi)] \\
\Psi^{S_3} &= \sqrt{\frac{1}{17}}[\Psi_{033}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{011}(\xi) - \sqrt{7}\Psi_{011}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{011}(\xi) \\
&\quad + \sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{033}(\eta)\Psi_{011}(\xi) - \sqrt{\frac{7}{2}}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{011}(\eta)\Psi_{011}(\xi) \\
&\quad - \sqrt{\frac{7}{2}}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{011}(\eta)\Psi_{011}(\xi)] \\
\Psi^{S_4} &= \sqrt{\frac{5}{57}}[\Psi_{044}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{000}(\lambda)\Psi_{044}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) \\
&\quad + \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{044}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{14}{5}}\Psi_{022}(\lambda)\Psi_{022}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) \\
&\quad + \sqrt{\frac{14}{5}}\Psi_{022}(\lambda)\Psi_{000}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{14}{5}}\Psi_{000}(\lambda)\Psi_{022}(\rho)\Psi_{022}(\eta)\Psi_{000}(\xi)]
\end{aligned}
$$

Model parameters

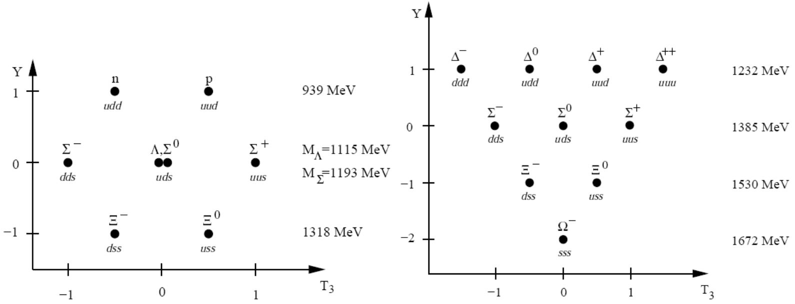
- The harmonic oscillators have different modes for baryon and pentaquark with different potential, here is the mode for constant potential case with m_u, m_s are bare masses.

$$
\begin{aligned}
w_3^{\rho}(uu) &= \sqrt{\frac{6C}{m_u}} = \sqrt{3}w_0, \quad w_3^{\rho}(ss) = \sqrt{\frac{6C}{m_s}} = \sqrt{\frac{3}{k0}}w_0 \\
w_3^{\lambda}(uuu) &= \sqrt{\frac{6C}{m_u}} = \sqrt{3}w_0, \quad w_3^{\lambda}(uus) = \sqrt{\frac{2C(2m_u+m_s)}{m_s}} = \sqrt{\frac{2+k0}{k0}}w_0, \\
w_3^{\lambda}(uss) &= \sqrt{\frac{2C(2m_s+m_u)}{m_s}} = \sqrt{\frac{2k0+1}{k0}}w_0, \quad w_3^{\lambda}(sss) = \sqrt{\frac{6C}{m_s}} = \sqrt{\frac{3}{k0}}w_0.
\end{aligned}
$$

- One-gluon-exchange contribution Δm^{OGE} with m_i, m_j are dressed masses of quark $m_u^B = m_u + \sqrt{3}w_0$, $m_s^B = m_s + \sqrt{\frac{3m_u}{m_s}}w_0$:

$$
\Delta m_{hyp}^{OGE} = \langle\psi|(-C_g \sum_{i<j} \frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j)|\psi\rangle
$$

Baryon octet and decuplet mass



- The parameters are determined by fitting the theoretical results of q^3 baryons to the above 8 masses.

$$\Delta m = \sum_{i=1}^{8} (m^i_{theory} - m^i_0)^2 \tag{25}$$

q^4 Color Wave Functions

q^4 color wave functions can be derived by applying the λ-, ρ- and η-type projection operators of the S_4 $IR[211]$ in Yamanouchi basis,

$$P_{[211]_\lambda}(RRGB) \Longrightarrow \psi^c_{[211]_\lambda}(R)$$
$$P_{[211]_\rho}(RGRB) \Longrightarrow \psi^c_{[211]_\rho}(R)$$
$$P_{[211]_\eta}(RGBR) \Longrightarrow \psi^c_{[211]_\eta}(R)$$

$$\psi^c_{[211]_\eta}(R) = \frac{1}{\sqrt{6}}(|RGBR\rangle + |GBRR\rangle + |BRGR\rangle - |RBGR\rangle - |GRBR\rangle - |BGRR\rangle)$$

The singlet color wave function $\Psi^c_{[211]_j}$ $(j = \lambda, \rho, \eta)$ of pentaquarks is given by

$$\Psi^c_{[211]_j} = \frac{1}{\sqrt{3}}\left[\psi^c_{[211]_j}(R)\,\bar{R} + \psi^c_{[211]_j}(G)\,\bar{G} + \psi^c_{[211]_j}(B)\,\bar{B}\right]. \quad (26)$$