Stability and the transverse charge density of the pion and the nucleon

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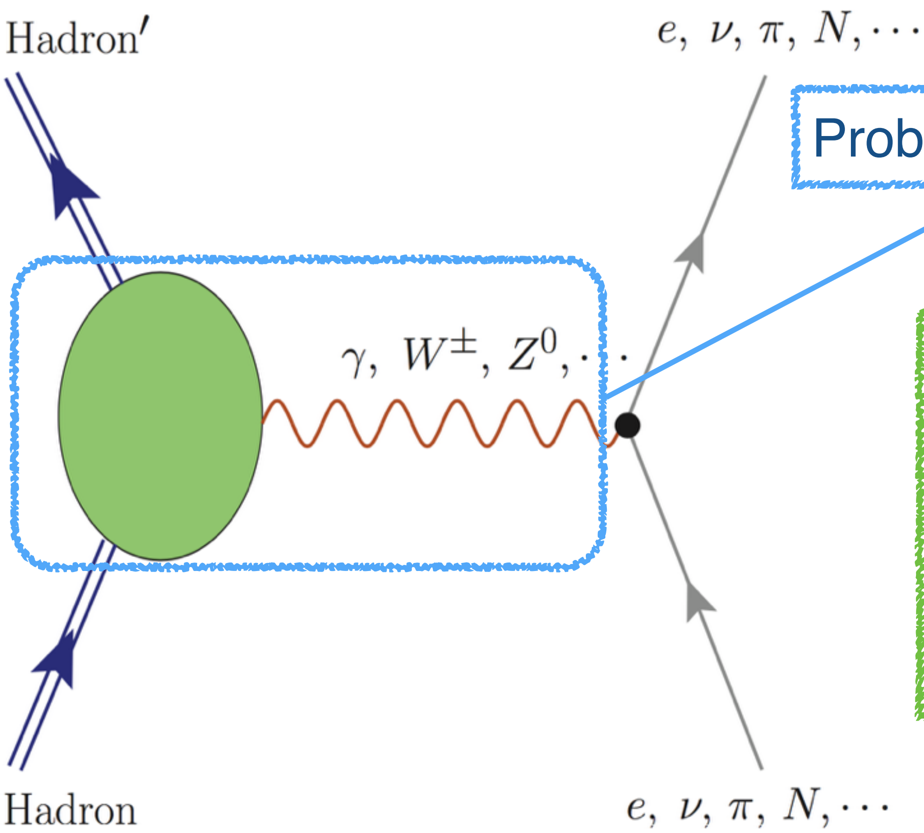
Traditional form factors



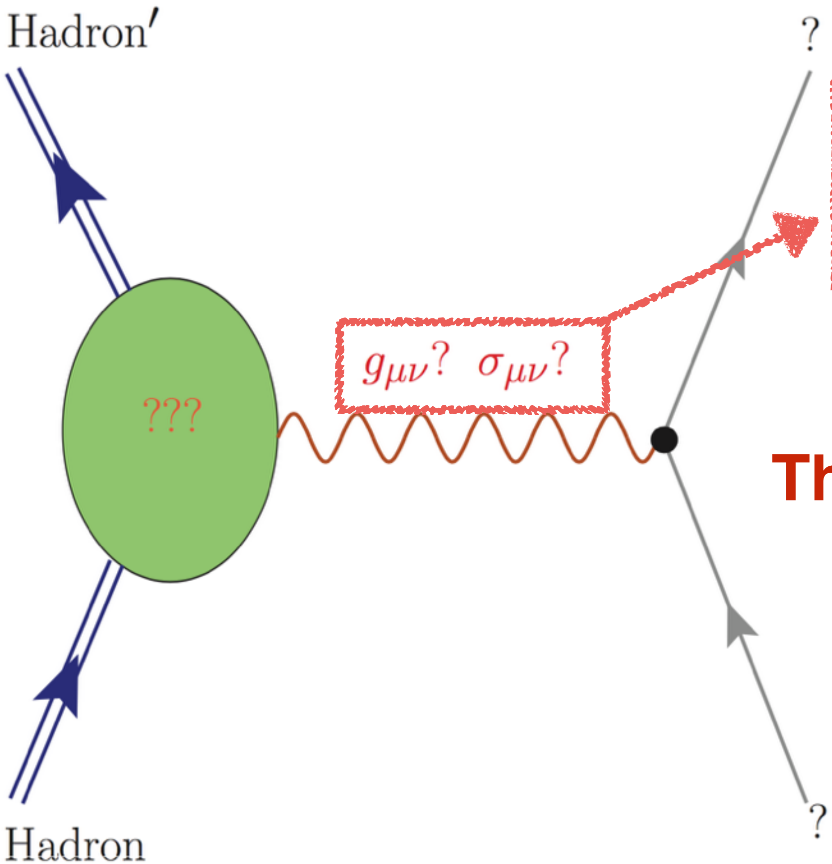
Probes examine the structure of a hadron

- Electromagnetic form factors (Vector form factors)
- Axial-vector form factors
- Scalar form factors
- Pseudoscalar form factors

Energy–Momentum Tensor Form Factors

Energy-Momentum Tensor form factors & Tensor Form factors

These form factors are as equally important as vector and axial-vector form factors (Energy & Momentum distributions & transversity, resp.)!



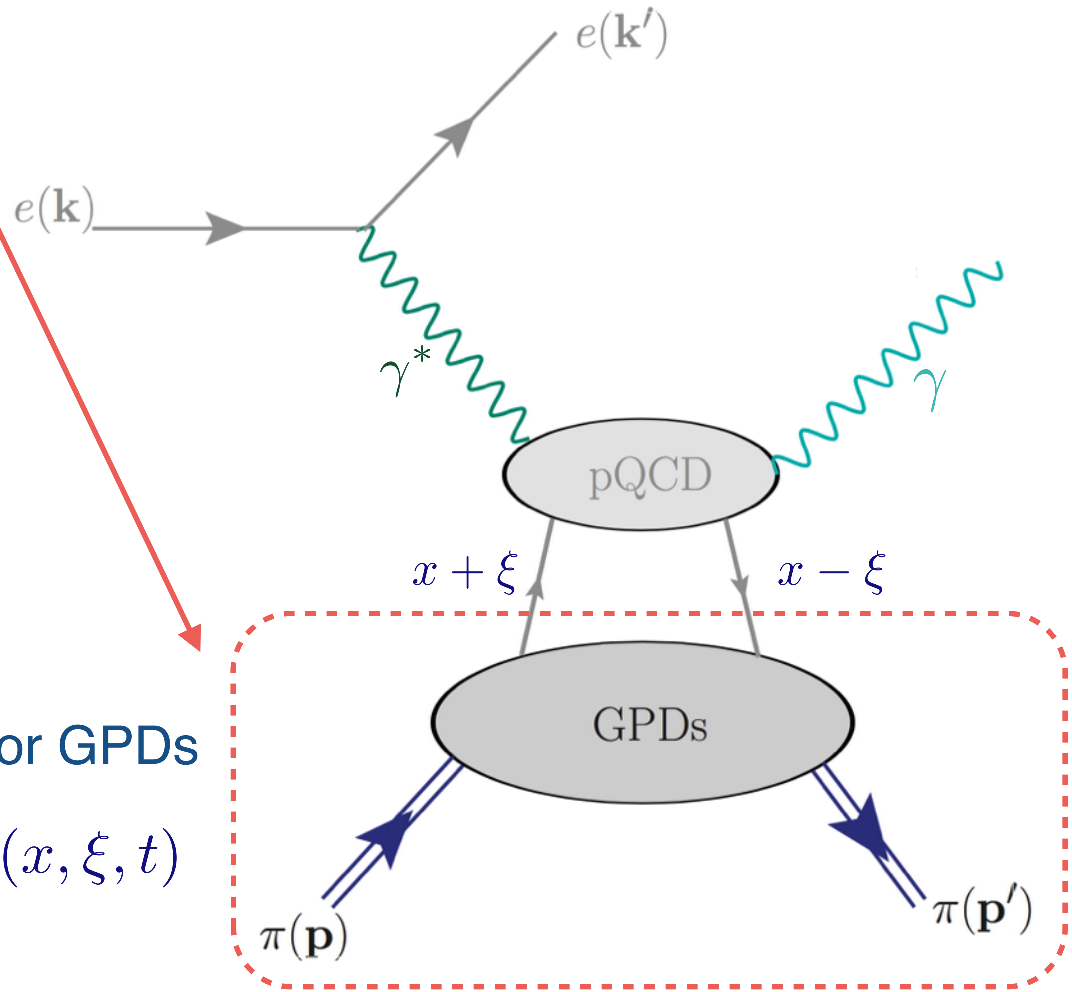
- A probe for the EMTFF: Graviton too weakly coupled to hadrons
- A probe for the tensor FF: Unknown so far

Then, how can we measure them?

Novel view on form factors

Deeply Virtual Compton Scattering

$$2\delta^{ab}\boxed{H_\pi^q(x,\xi,t)} = \frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda(P\cdot n)}\langle\pi^a(p')|\bar{\psi}^q(-\lambda n/2)\not{n}[-\lambda n/2,\lambda n/2]\psi^q(\lambda n/2)|\pi^b(p)\rangle$$



Vector GPDs

$H_\pi^q(x,\xi,t)$

Novel view on form factors

- Form factor as a Mellin moment of the GPD

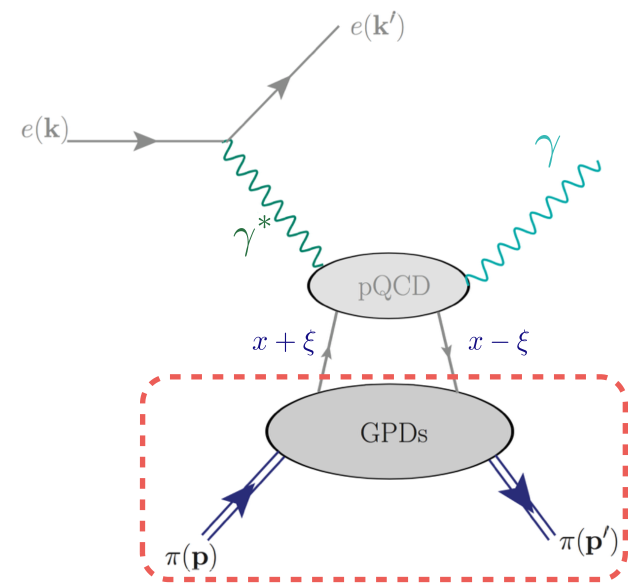
$$\int dx x^{n-1} H_\pi^q(x,\xi,t) = A_{n,0}(t) + \sum_{i=1,\,\mathrm{odd}}^{n} (-2\xi)^{i+1} A_{n,i+1}(t)$$

- Generalized form factors of the pion

 - pion EM form factor as the first Mellin moment:

$$F_\pi(t) = A_{1,0}(t)$$

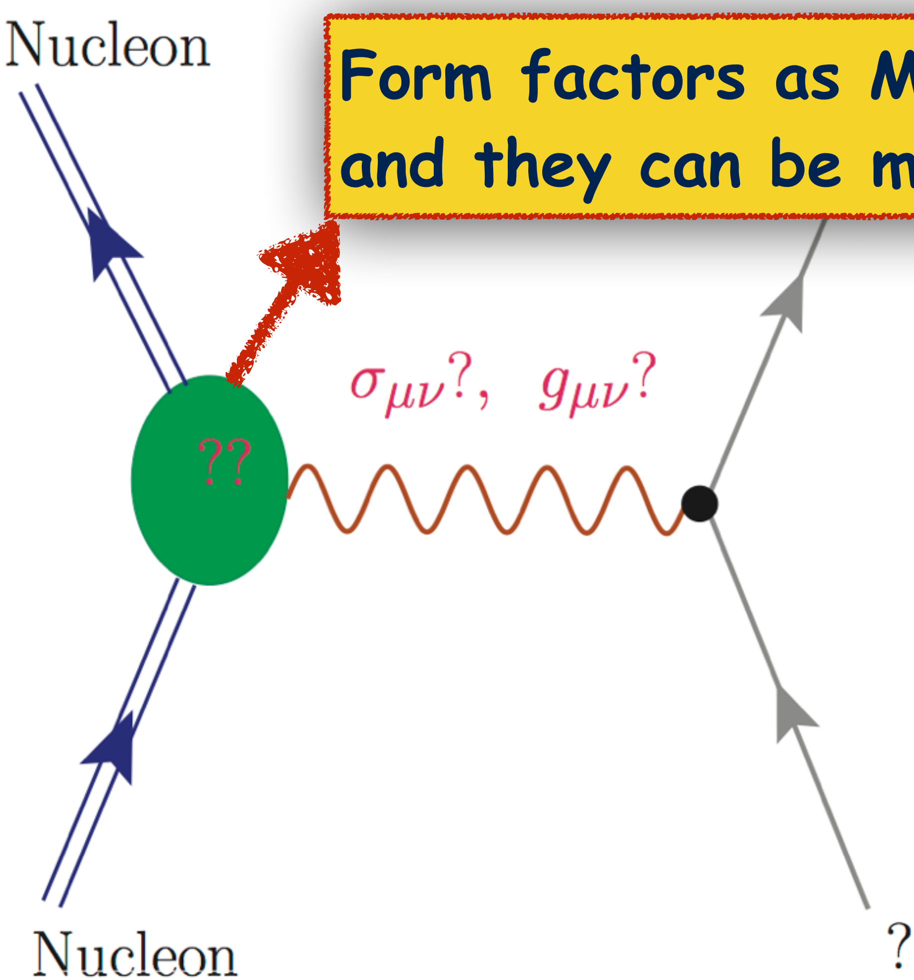
- **EMTFFs as the second Mellin moments, which are the subjects of the present talk.**



Generalised Parton Distributions

Probes are unknown for **Tensor form factors** and the **Energy-Momentum Tensor form factors** but

Form factors as Mellin moments of the GPDs and they can be measured!



Energy-Momentum Tensor Form Factors

- Given an action $S = \int d^4 \sqrt{-g}\mathcal{L}$

$$T_{\mu\nu} = 2\frac{\delta S}{\delta g^{\mu\nu}} \quad \text{or} \quad T_{\mu\nu} = -\frac{\partial \mathcal{L}}{\partial(\partial^\mu \varphi_a(x))}\partial_\nu \phi_a(x) + g_{\mu\nu}\mathcal{L}$$

$$\partial^\mu T_{\mu\nu} = 0$$

EMT is a conserved quantity.

Scale independent

- Energy-momentum tensor form factors

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}\left[(t g_{\mu\nu} - q_\mu q_\nu)\boxed{\Theta_1(t)} + 2P_\mu P_\nu \boxed{\Theta_2(t)}\right]$$

$$\langle N(p')|T^{Q,G}_{\mu\nu}(0)|N(p)\rangle = \bar{u}(p')\left[\boxed{M_2^{Q,G}(t)}\frac{P_\mu P_\nu}{M_N} + \boxed{J^{Q,G}(t)}\frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\Delta^\rho}{2M_N} + \boxed{d_1^{Q,G}(t)}\frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu}\right]u(p)$$

(also known as the gravitational form factors)

H. Pagels, Phys. Rev. **144**, 1250 (1966)

EMT form factors: Stability of the pion

Chiral quark-(soliton) model

Effective Chiral Action

$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$ by the quark condensate

$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R / \mathrm{SU}(2)_V$: Goldstone bosons $\quad \Sigma \to L\Sigma R^\dagger$

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr}\log\left[i\partial\!\!\!/ + iM\Sigma P_L + iM\Sigma^\dagger P_R + im\mathbf{1}\right]$$

N_c : The number of colors

M : Dynamical quark mass

$\Sigma = \exp(i\pi\cdot\tau/f_\pi)$: Pion field as a pseudo-Goldstone boson

$P_L,\, P_R$: Chiral projection operators

$m = (m_u + m_d)/2$: Current quark mass

A. Manohar, H. Georgi, NPB**234**, 189 (1984) D. Diakonov and V.Y. Petrov, NPB**272**, 457 (1986)

EMT form factors

Energy-momentum Tensor Form factors (Pagels, 1966)

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}\left[(tg_{\mu\nu} - q_\mu q_\nu)\boxed{\Theta_1(t)} + 2P_\mu P_\nu \boxed{\Theta_2(t)}\right]$$

EMTFFs (Gravitational FFs)

$$T_{\mu\nu}(x) = \frac{1}{2}\bar{\psi}(x)\gamma_{\{\mu} i\overleftrightarrow{\partial}_{\nu\}}\psi(x)$$: EMT operator

Isoscalar vector GPDs of the pion

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p')|\bar{\psi}(-\lambda n/2)\not{n}[-\lambda n/2, \lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

The second moment of the GPD

$$\int dx\, x H_\pi^{I=0}(x,\xi,t) = \boxed{A_{20}(t)} + 4\xi^2 \boxed{A_{22}(t)}$$: Generalized form factors of the pion

$$\boxed{\Theta_1 = -4A_{22}^{I=0},\ \Theta_2 = A_{20}^{I=0}}$$

$$\Theta_1(0) - \Theta_2(0) = \mathcal{O}(m_\pi^2)$$

EMT form factors

Time component of the EMT matrix element gives the pion mass.

$$\langle \pi^a(p)|T_{44}(0)|\pi^b(p)\rangle\Big|_{t=0} = -2m_\pi^2 \Theta_2(0)\delta^{ab}$$

The sum of the spatial component of the EMT matrix element gives the pressure of the pion, which should vanish!

$$\langle \pi^a(p)|T_{ii}(0)|\pi^b(p)\rangle\Big|_{t=0} = \frac{3}{2}\delta^{ab} t\,\Theta_1(t)\Big|_{t=0}$$

Zero in the chiral limit

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Stability of the pion

Pressure of the pion beyond the chiral limit

$$\mathcal{P} = \langle \pi^a(p) | T_{ii}(0) | \pi^a(p) \rangle$$
$$= \boxed{\frac{12 N_c m M}{f_\pi^2} \int d\tilde{l} \, \frac{-l^2}{[l^2 + \overline{M}^2]^2}} + \boxed{\frac{12 N_c M^2}{f_\pi^2} \int d\tilde{l} \int_0^1 dx \, \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + \overline{M}^2]^3}}$$

$$i\langle \psi^\dagger \psi \rangle = 8 N_c \int d\tilde{l} \frac{\overline{M}}{[l^2 + \overline{M}^2]}$$

$$f_\pi^2 = 4 N_c \int_0^1 dx \int d\tilde{l} \, \frac{M \overline{M}}{[l^2 + \overline{M}^2 + x(1-x)p^2]^2}$$

Quark condensate

Pion decay constant

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Stability of the pion

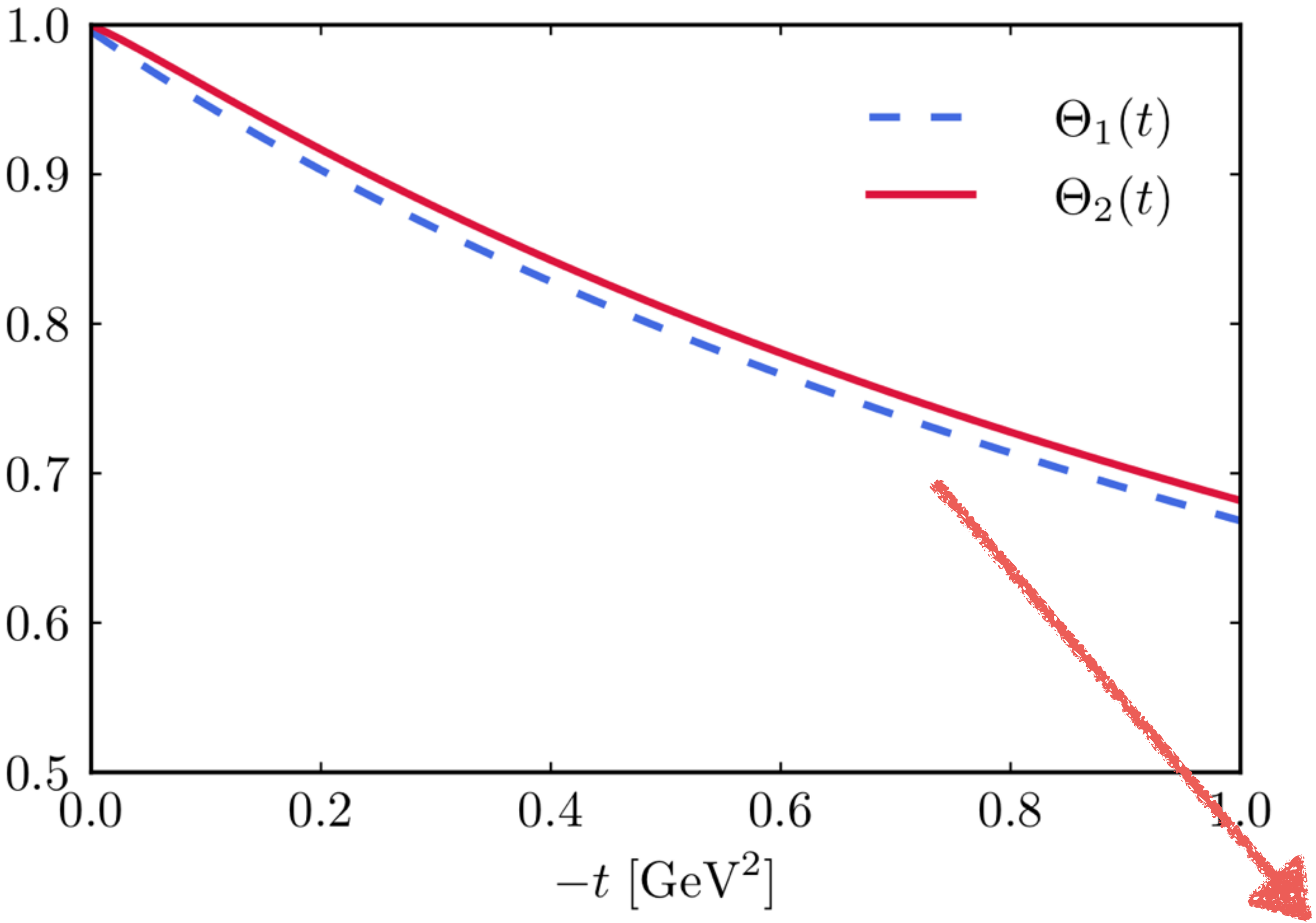
$$\mathcal{P} = \frac{3M}{f_\pi^2 \overline{M}} \left(m \langle \bar{\psi}\psi \rangle + m_\pi^2 f_\pi^2 \right) = 0 \; !$$

by the Gell-Mann-Oakes-Renner relation to linear m order

Physical implications: The stability of the pion should be deeply rooted spontaneous breakdown of chiral symmetry and the pattern of explicit chiral symmetry breaking.

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Energy-momentum Tensor FFs



$$\Theta_1 = \Theta_2$$

in the chiral limit

The difference arises from the explicit chiral symmetry breaking.

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Low-Energy Constants in curved space

Chiral Lagrangian in flat space

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger)$$
$$+ L_1[\mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger)]^2 + L_2 \mathrm{Tr}(D_\mu \Sigma D_\nu U^\dagger)\mathrm{Tr}(D^\mu \Sigma D^\nu \Sigma^\dagger)$$
$$+ L_3 \mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger D_\nu \Sigma D^\nu \Sigma^\dagger) + \cdots$$

Chiral Lagrangian in curved space

$$\mathcal{L} = L_{11} R \mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger) + L_{12} R^{\mu\nu} \mathrm{Tr}(D_\mu \Sigma D_\nu \Sigma^\dagger)$$
$$+ L_{13} R \mathrm{Tr}(\chi \Sigma^\dagger + \Sigma \chi^\dagger) + \cdots$$

The Low-Energy constants can be derived by the Derivative expansion.
(small pion momentum, small pion mass)

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Low-Energy Constants in curved space

$$\Theta_1(q^2) = 1 + \frac{2q^2}{f_\pi^2}(4L_{11} + L_{12}) \boxed{- \frac{16m_\pi^2}{f_\pi^2}(L_{11} - L_{13})} + \ldots$$

$$\Theta_2(q^2) = 1 - \frac{2q^2}{f_\pi^2}L_{12} + \ldots$$

It measures the explicit chiral symmetry breaking.

[J.F. Donoghue and H. Leutwyler, Z.Phys.C(1991) 52, 343]

Derivative expansion in curved space

$$L_{11} = \frac{N_c}{192\pi^2} = 1.6 \times 10^{-3}$$

$$L_{12} = -2L_{11} = -3.2 \times 10^{-3}$$

$$L_{13} = -\frac{N_c}{96\pi^2}\frac{M}{B_0}\Gamma\left(0, \frac{M^2}{\Lambda^2}\right) = 0.84 \times 10^{-3}$$

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

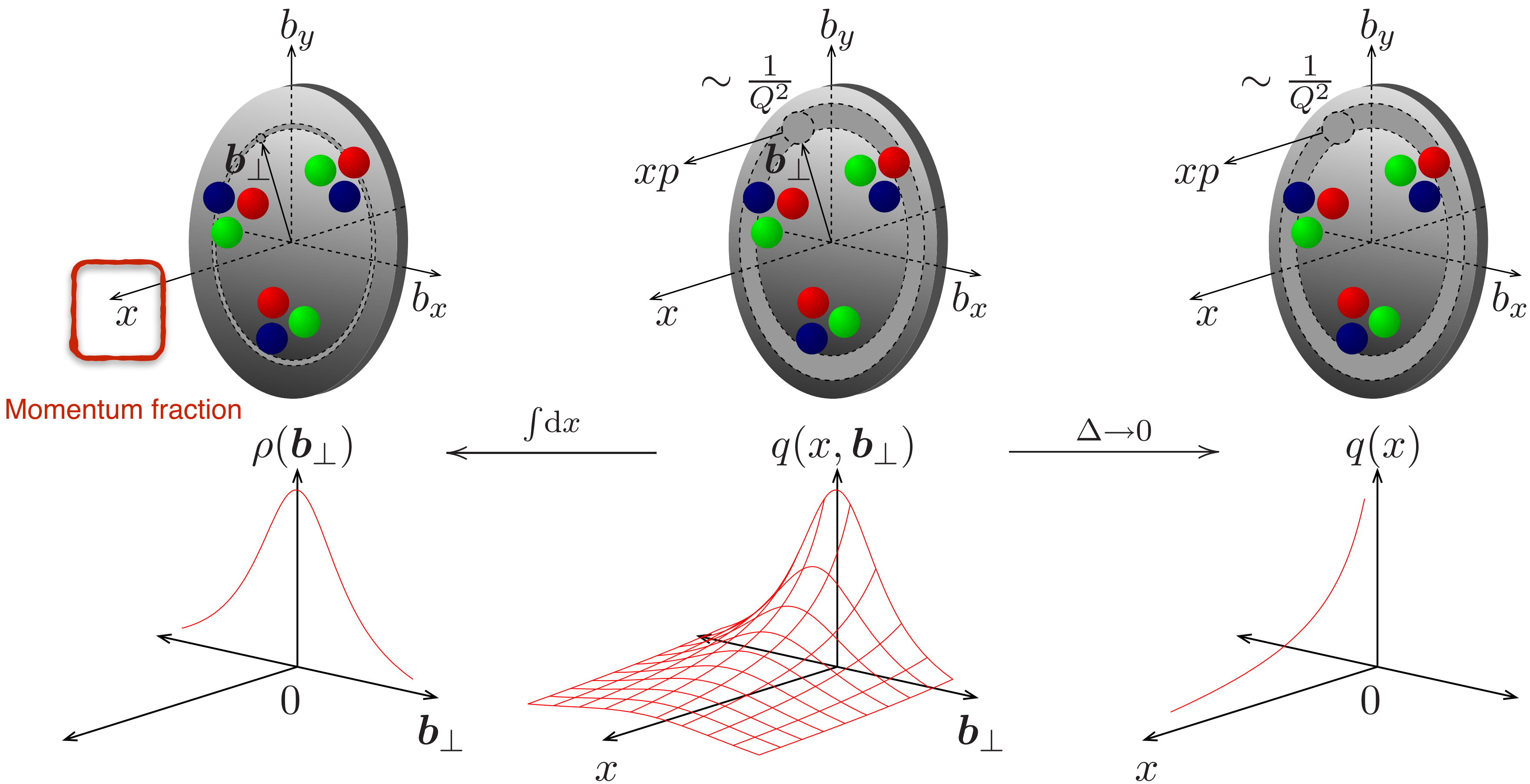
Low-Energy Constants in curved space

	L_{11}	L_{12}	L_{13}
Present Work	**$1.6 * 10^{-3}$**	**$-3.2 * 10^{-3}$**	**$0.84 * 10^{-3}$**
SQM*	$1.6 * 10^{-3}$	$-3.2 * 10^{-3}$	$0.3 * 10^{-3}$
XPT**	$1.4 * 10^{-3}$	$-2.7 * 10^{-3}$	$0.9 * 10^{-3}$

[*Megias *et al*. PRD **70**, 034031 (2004)]
[**J.F. Donoghue and H. Leutwyler, Zeit. PC **52**, 343 (1991)]

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Pion Tomography



Transverse densities of Form factors

GPDs Pion Tomography

Structure functions Parton distributions

D. Brömmel, Dissertation (Regensburg U.)

Transverse charge densities

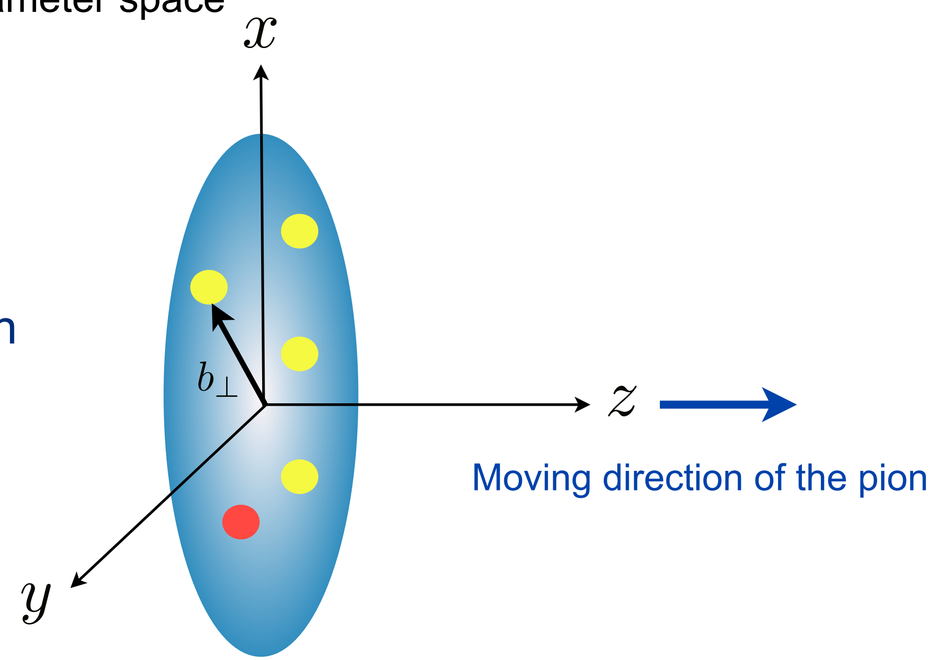
Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

$$q(x,b) = \int \frac{d^2}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_\pi^q(x,0,t)$$

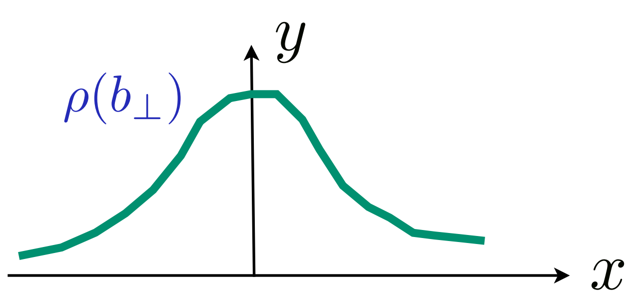
It can be interpreted as the probability distribution of a quark in the transverse plane.

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).

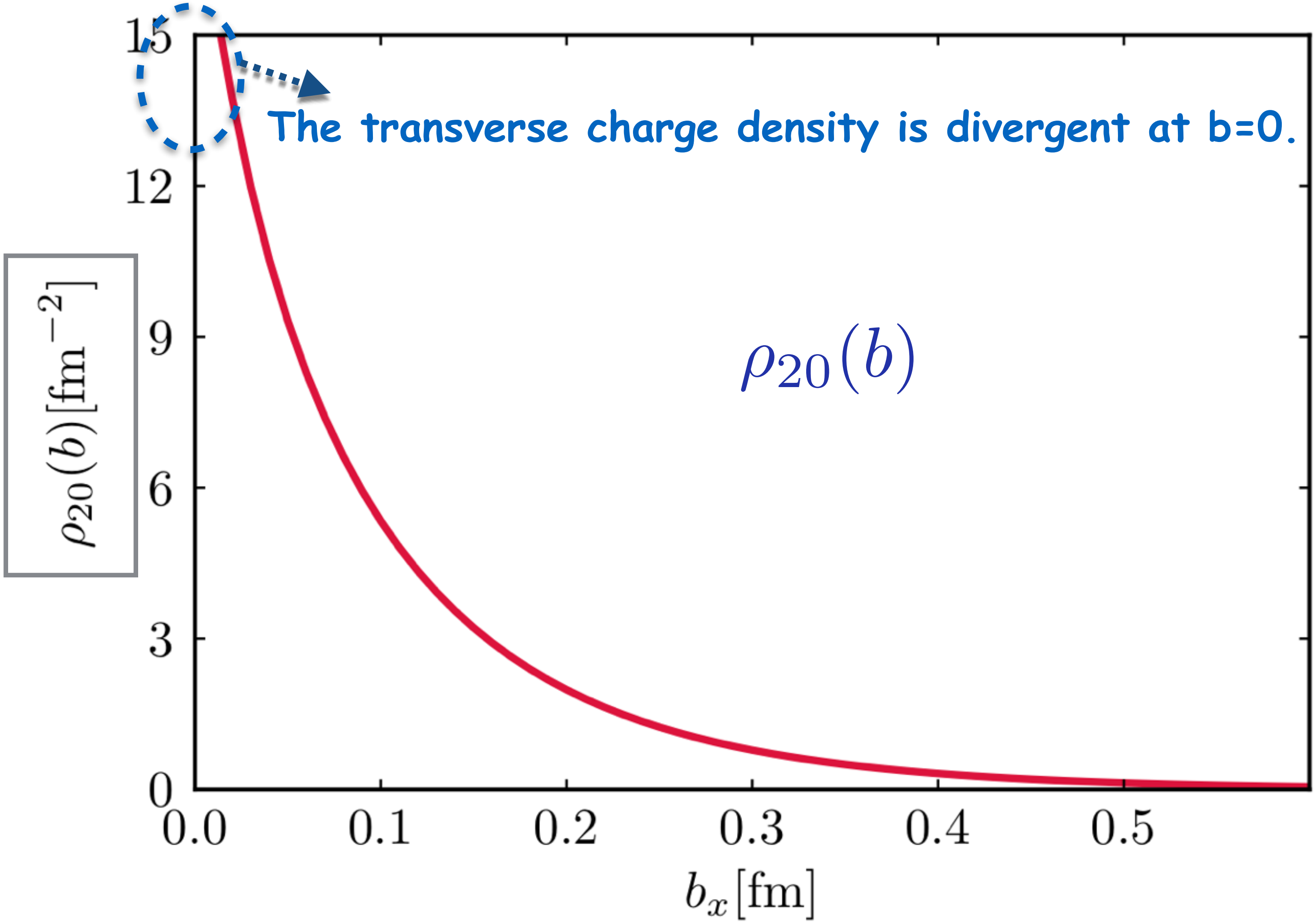


Pion transverse charge densities

$$\rho_{ni}(\mathbf{b}) := \int \frac{d^2q}{(2\pi)^2} A_{ni}(t) e^{i\mathbf{q}\cdot\mathbf{b}}$$

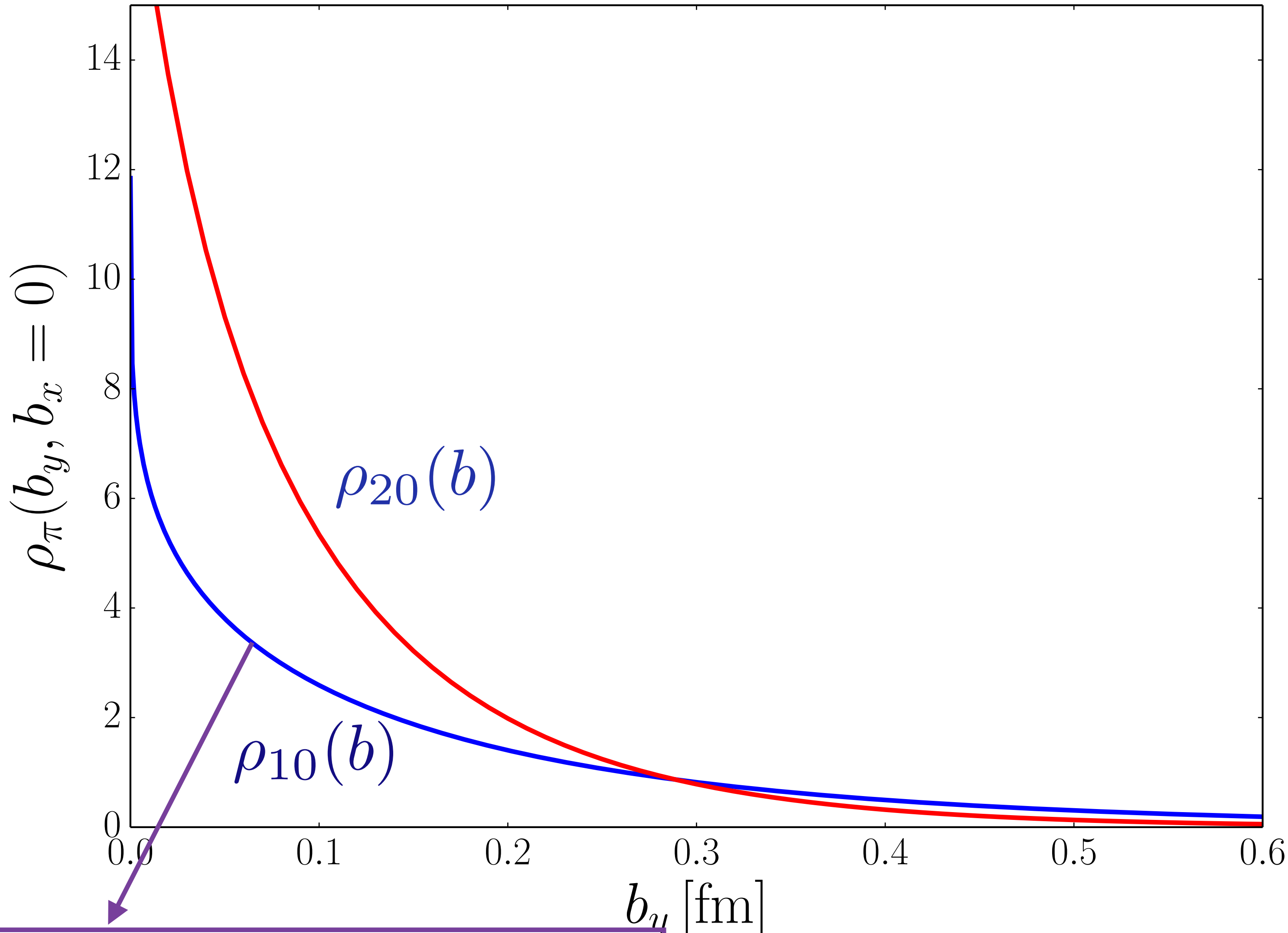


Transverse charge density of the pion



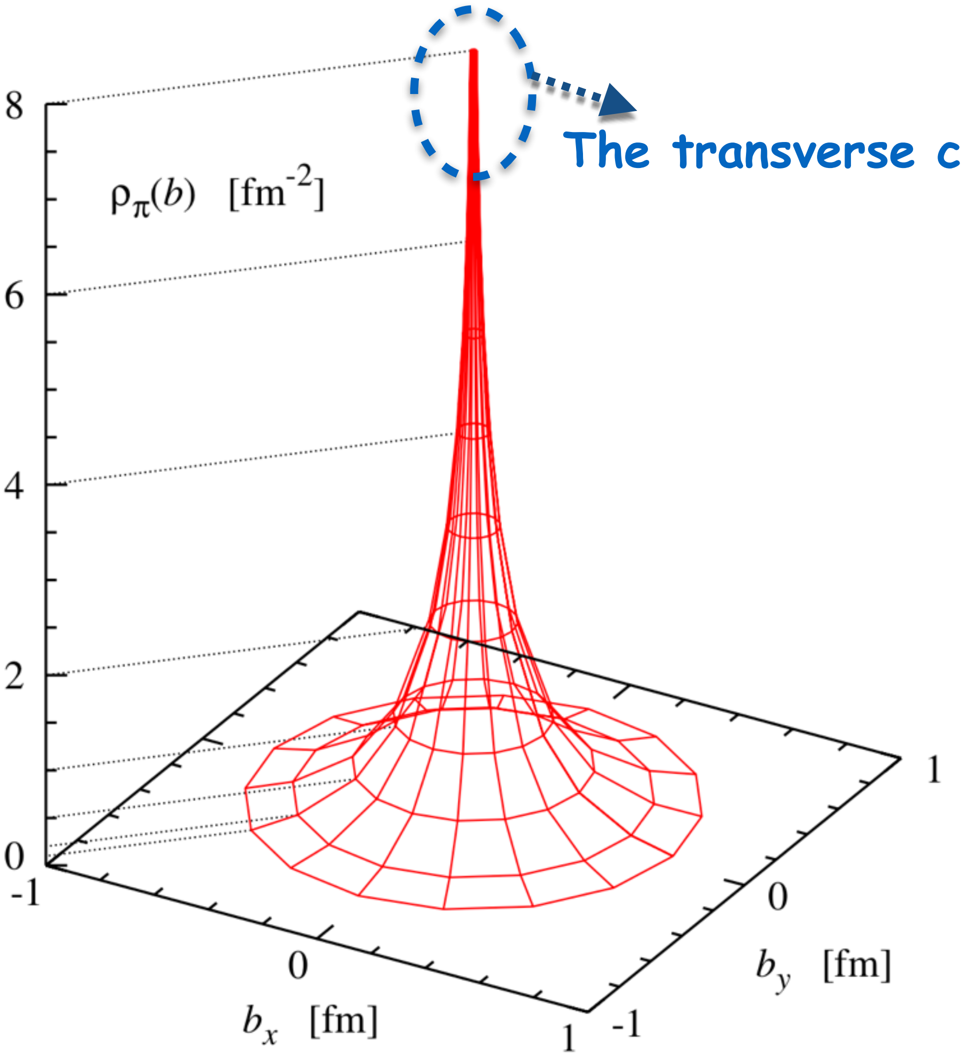
H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Transverse charge density of the pion



Transverse charge density from the EMFF

Transverse charge density of the pion



The transverse charge density is divergent at b=0.

$$\rho_{10}(b)$$

G. Miller, A. Strikman, C. Weiss, PRD83, 013001 (2011)

EMT form factors:

Stability of the nucleon

EMT form factors

Energy-momentum tensor form factors

$$\langle N(p')|T_{\mu\nu}^{Q,G}(0)|N(p)\rangle = \bar{u}(p')\left[M_2^{Q,G}(t)\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}\right]u(p)$$

GPDs

$$\int \frac{dx^-}{4\pi}\langle P', S'|\bar{q}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{0}_\perp)|P,S\rangle$$

$$= \frac{1}{2\bar{p}^+}\bar{u}(p',s')\left(\gamma^+ H_q(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M_N}E_q(x,\xi,t)\right)u(p,s)$$

The EMT form factors as the second moments of the isoscalar vector GPDs

$$\int_{-1}^{1} dx\,x\sum_f H_q(x,\xi,t) = M_2^Q(t) + \frac{4}{5}d_1^Q(t)\xi^2,$$

$$\int_{-1}^{1} dx\,x\sum_f E_q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5}d_1^Q(t)\xi^2,$$

EMT form factors

In the Breit frame,

$$T^{Q}_{\mu\nu}(\mathbf{r},\mathbf{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\exp(i\Delta\cdot\mathbf{r})\langle p', S'|T^{Q}_{\mu\nu}(0)|p, S\rangle$$

$$M_2(t) - \frac{t}{4M_N^2}\left(M_2(t) - 2J(t) + \frac{4}{5}d_1(t)\right) = \frac{1}{M_N}\int d^3r e^{-i\mathbf{r}\cdot\Delta}T_{00}(\mathbf{r},\mathbf{s})$$

Momentum fractions carried by quarks and gluons

$$M_2^Q(0) = \int_0^1 \mathrm{d}x \sum_q x\boxed{(f_1^q + f_1^{\bar{q}})}(x),$$

Unpolarized parton distributions

$$M_2^G(0) = \int_0^1 \mathrm{d}x x f_1^g(x),$$

EMT form factors

$$J^Q(t) + \frac{2t}{3} J^{Q'}(t) = \int \mathrm{d}^3\mathbf{r} e^{-i\mathbf{r}\boldsymbol{\Delta}} \varepsilon^{ijk} s_i r_j T^Q_{0k}(\mathbf{r}, \mathbf{s}),$$

$$d_1^Q(t) + \frac{4t}{3} d_1^{Q'}(t) + \frac{4t^2}{15} d_1^{Q''}(t)$$
$$= -\frac{M_N}{2} \int \mathrm{d}^3\mathbf{r} e^{-i\mathbf{r}\boldsymbol{\Delta}} T^Q_{ij}(\mathbf{r}) \left(r^i r^j - \frac{\mathbf{r}^2}{3} \delta^{ij} \right),$$

Constraints

$$M_2(0) = \frac{1}{M_N} \int \mathrm{d}^3\mathbf{r} T_{00}(\mathbf{r}, \mathbf{s}) = 1,$$ Nucleon Mass

$$J(0) = \int \mathrm{d}^3\mathbf{r} \varepsilon^{ijk} s_i r_j T_{0k}(\mathbf{r}, \mathbf{s}) = \frac{1}{2},$$ Nucleon Spin

$$d_1(0) = -\frac{M_N}{2} \int \mathrm{d}^3\mathbf{r} T_{ij}(\mathbf{r}) \left(r^i r^j - \frac{\mathbf{r}^2}{3} \delta^{ij} \right) \equiv d_1$$ D-term

Stability of the nucleon

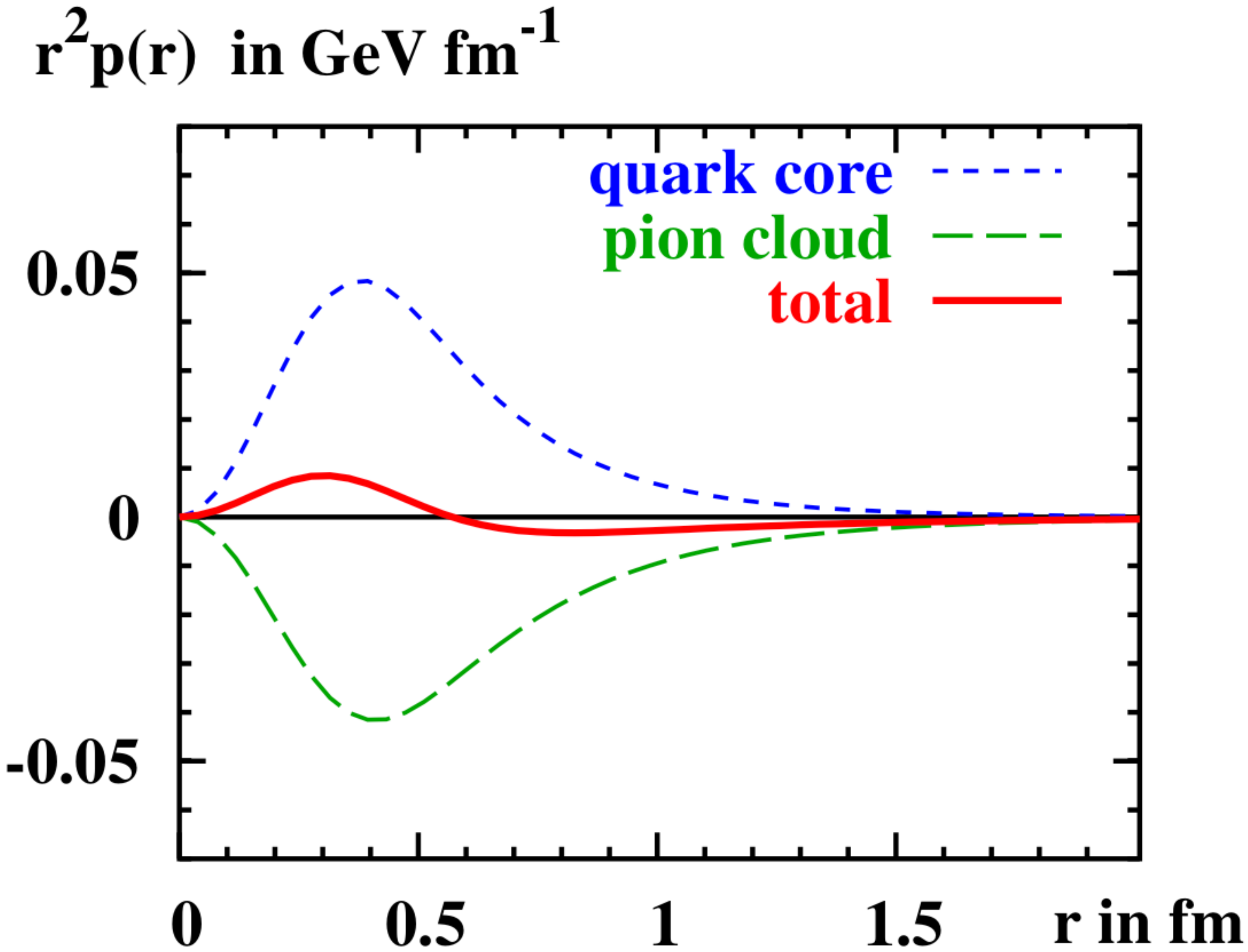
$$T_{ij}(\mathbf{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + \boxed{p(r)}\delta_{ij}$$

$$\boxed{\int_0^\infty dr\, r^2 p(r) = 0}$$

: Stability condition of the nucleon

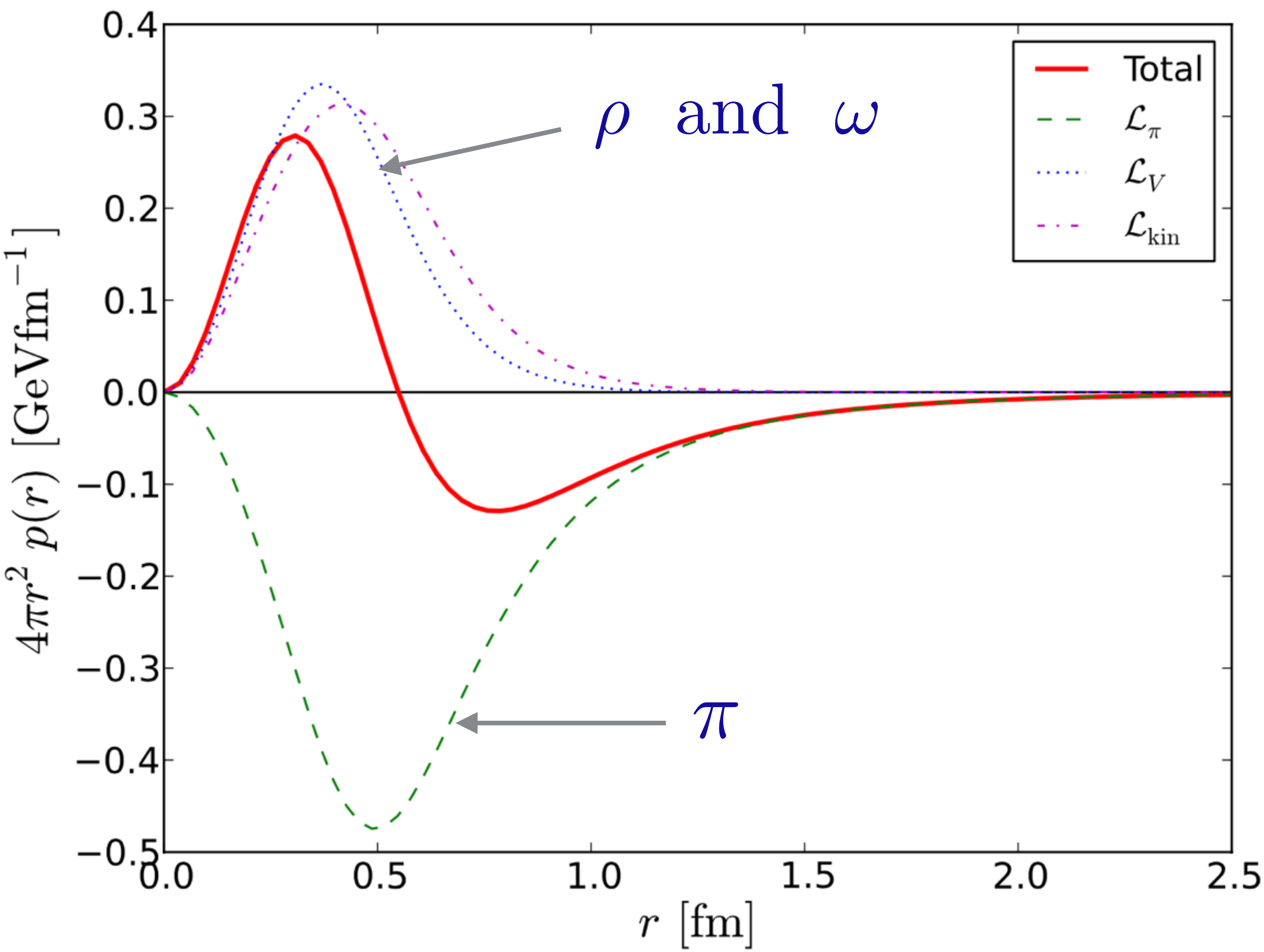
Any model for the nucleon should satisfy this condition!

Stability of the nucleon: XQSM



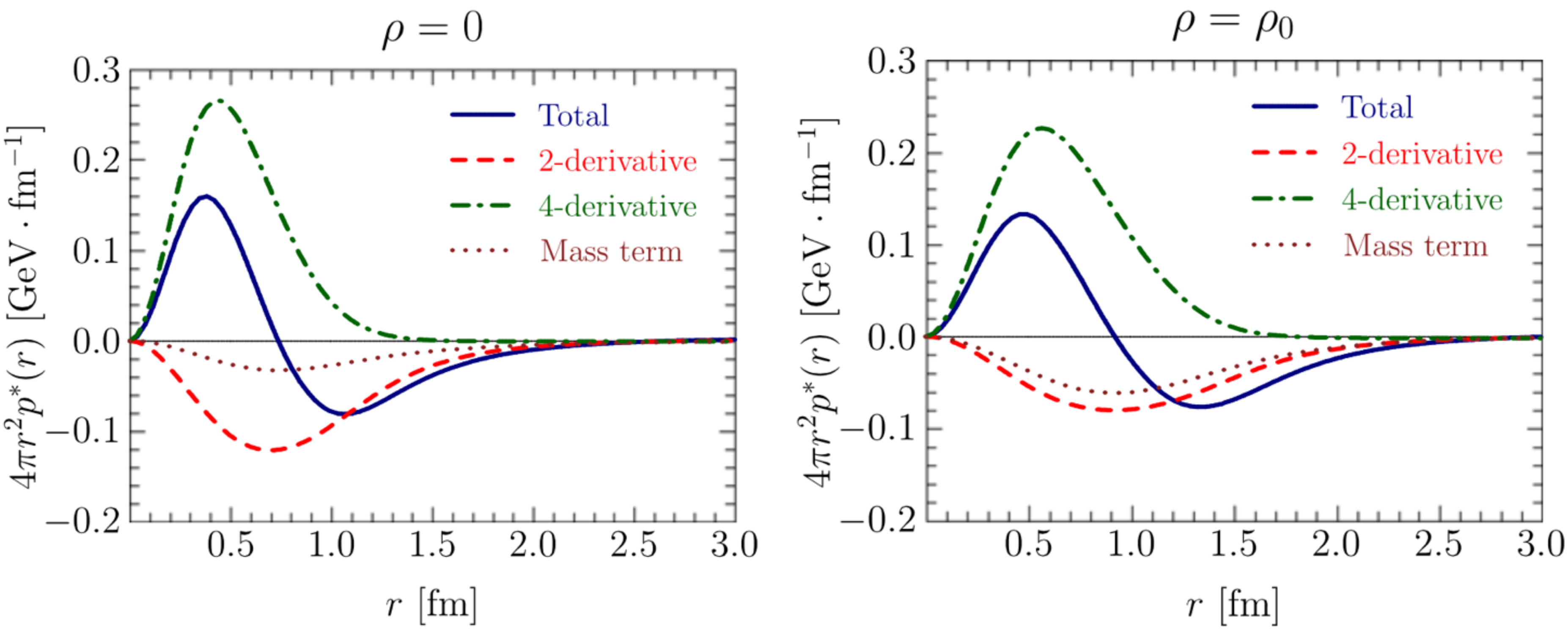
Goeke et al, PRD 75, 094021

Stability of the nucleon: pi-rho-omega model



J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

Stability of the nucleon: Skyrme



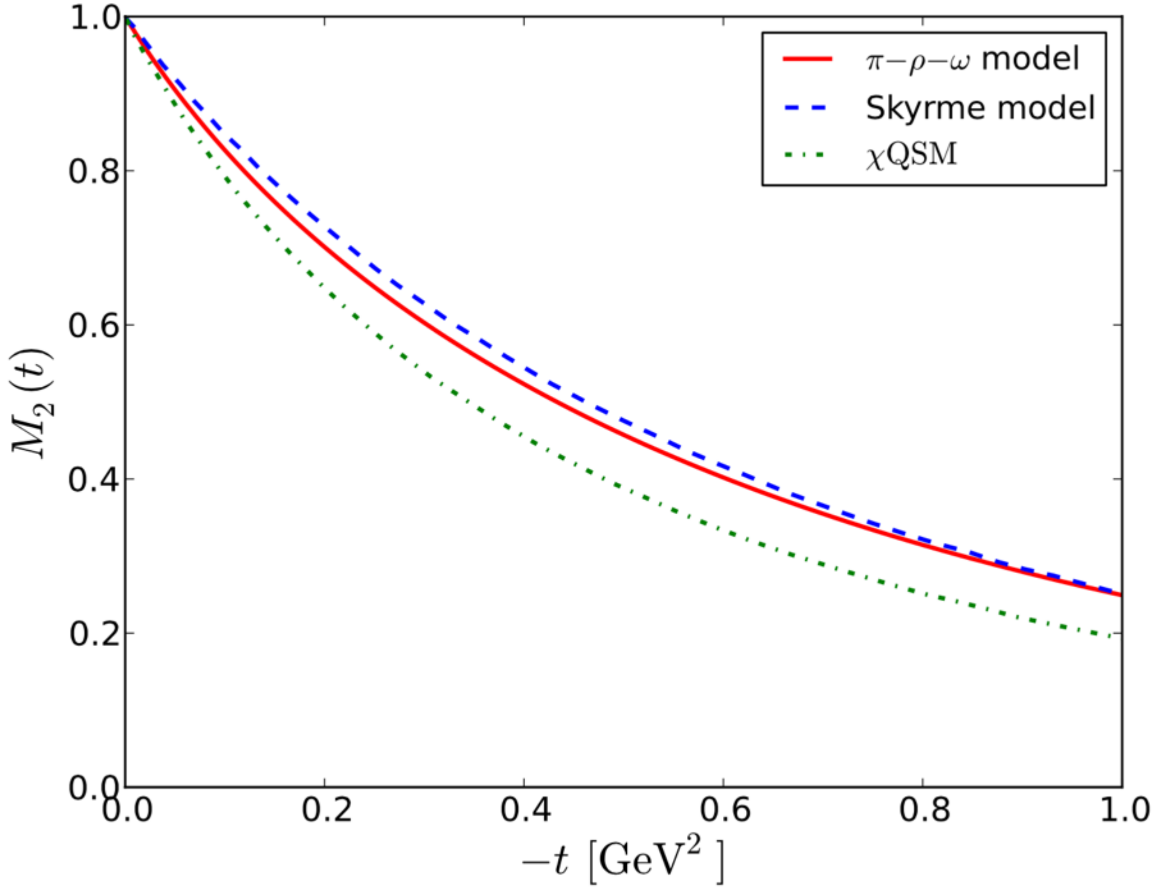
It is quite nontrivial to satisfy the stability of the nucleon!

HChK et al, PLB 718, 625 (2012)

EMT form factors: Results

Mass form factors

$$M_2(t) = \frac{1}{M_{\rm sol}} \int \mathrm{d}^3 r\, T_{00}(r)\, j_0(r\sqrt{-t}) - \frac{t}{5M_{\rm sol}^2} d_1(t)$$

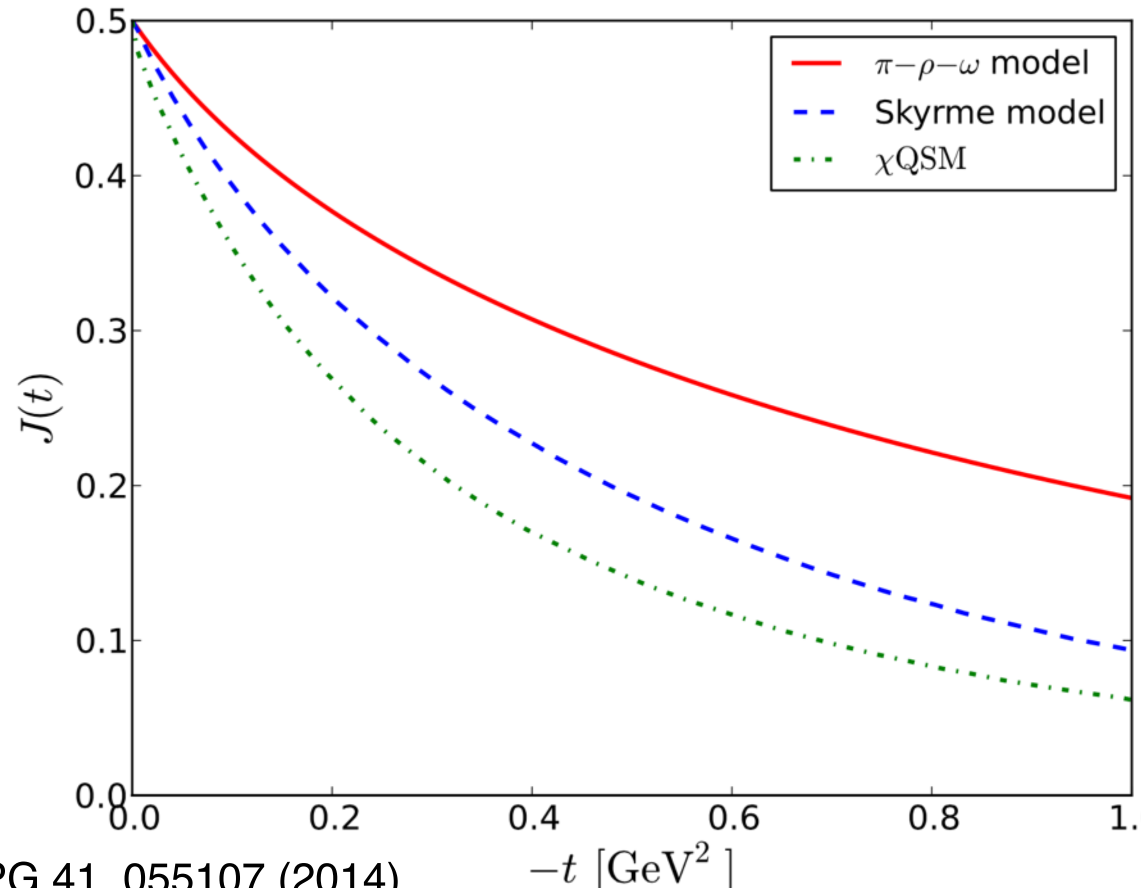


J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

EMT form factors: Results

Spin form factors

$$J(t) = 3\int \mathrm{d}^3r\, \rho_J(r)\, \frac{j_1(r\sqrt{-t})}{r\sqrt{-t}} \qquad T^{0i}(\vec{r},\vec{s}) = \frac{\mathrm{e}^{ilm} r^l s^m}{(\vec{s}\times\vec{r})^2}\,\rho_J(r)$$



J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

EMT form factors: Results

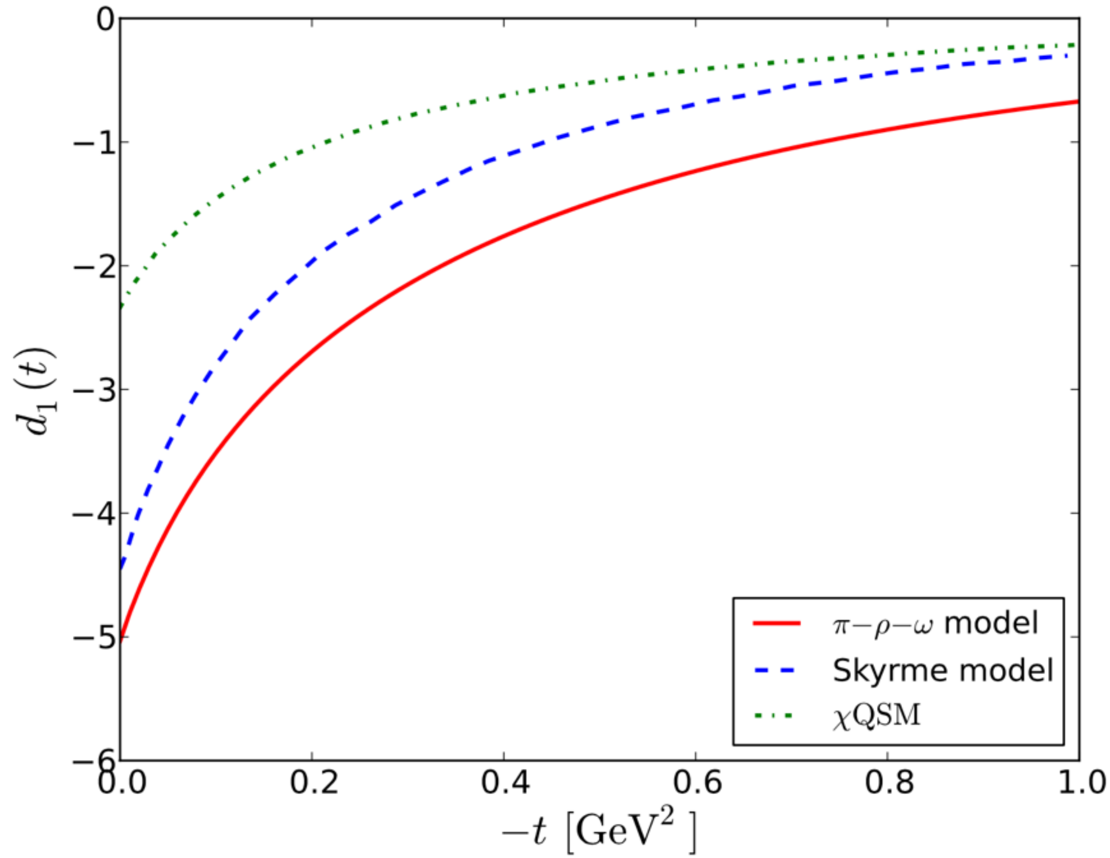
d1 form factors

$$d_1(t) = \frac{15M_{\rm sol}}{2}\int \mathrm{d}^3r\; p(r)\;\frac{j_0(r\sqrt{-t})}{t}$$

$$T^{ij}(r) = s(r)\left(\frac{r^ir^j}{r^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\delta^{ij}$$

$$\boxed{d_1 < 0}$$

To secure the stability of a particle

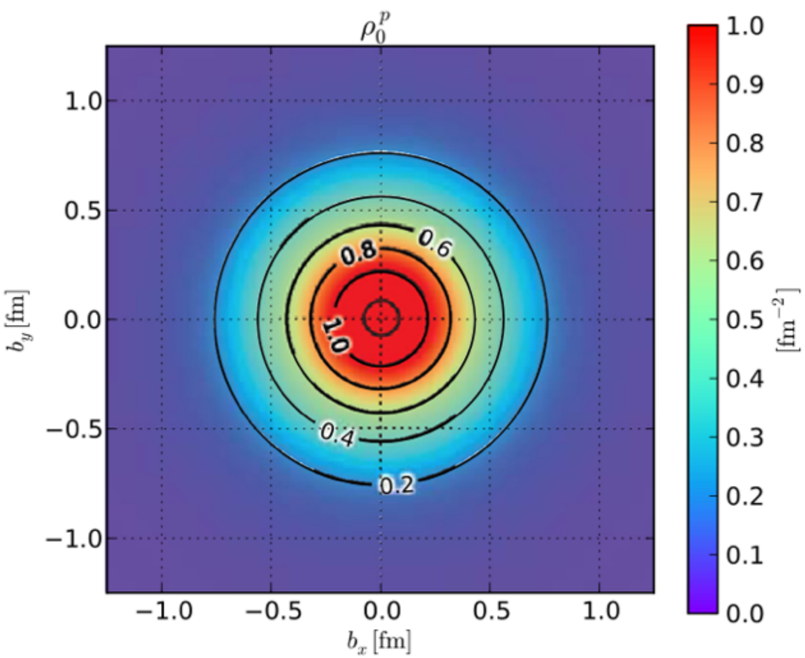


J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

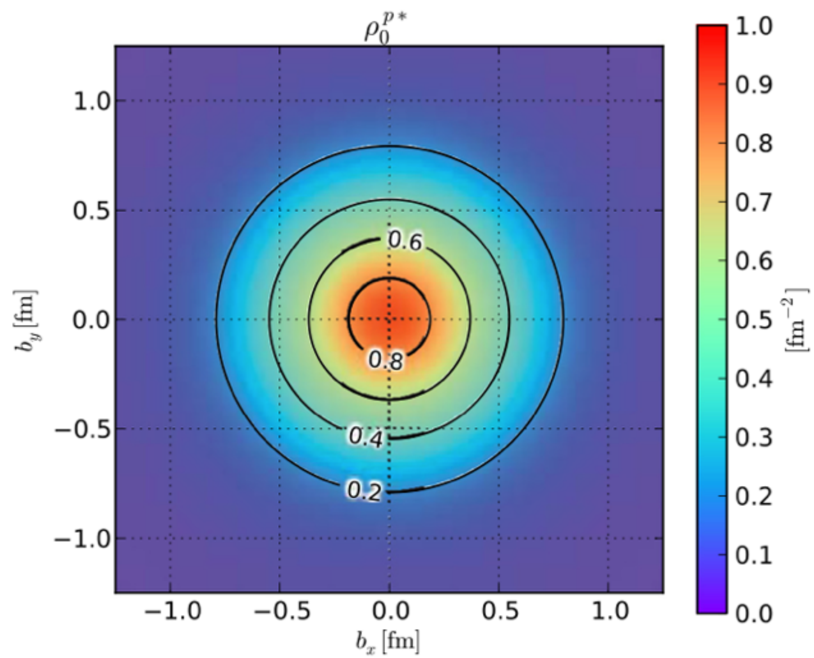
Results

Transverse charge densities inside an **unpolarized proton in nuclear matter**

Free-space Skyrme Model



Medium-Modified Skyrme Model



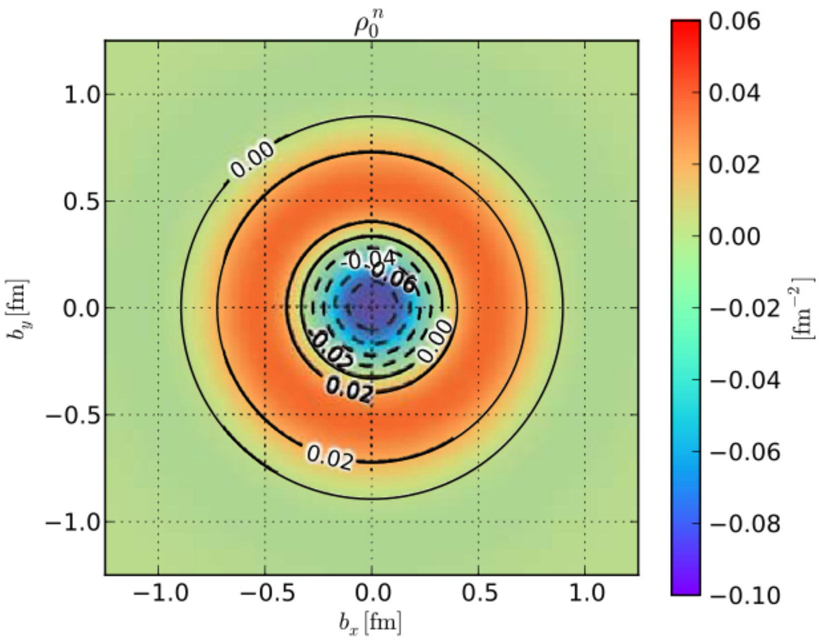
The proton bulges out in nuclear matter!

U. Yakhshiev and HChK, PLB 726, 375 (2013)

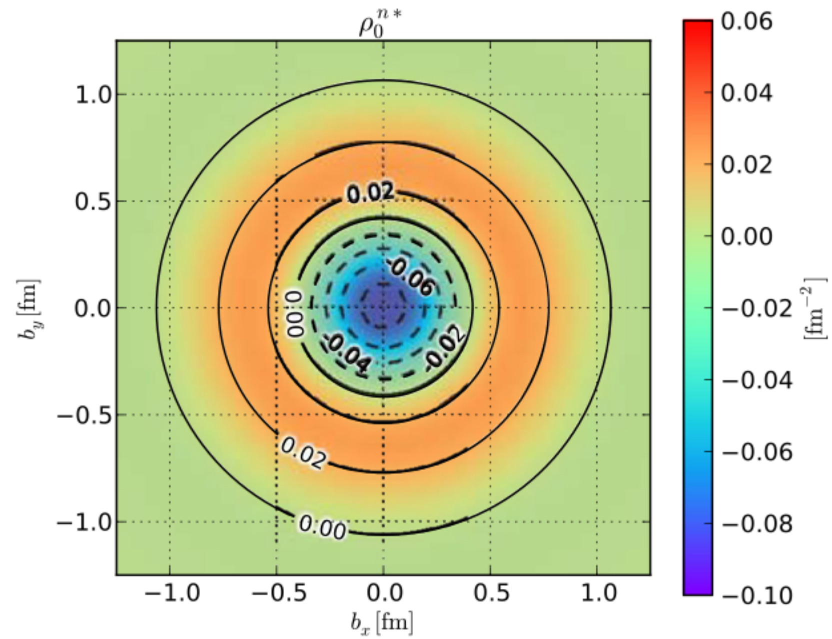
Results

Transverse charge densities inside an **unpolarized neutron in nuclear matter**

Free-space Skyrme Model



Medium-Modified Skyrme Model



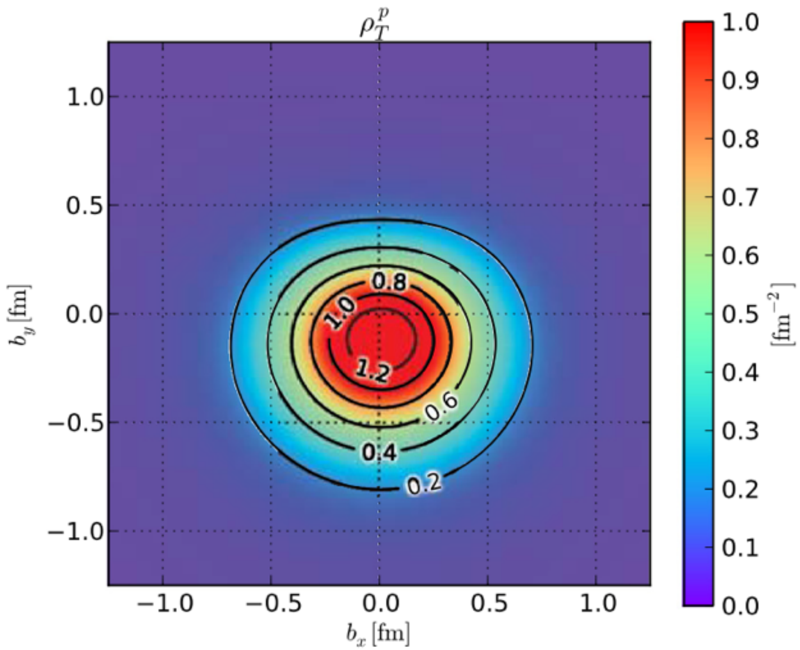
The neutron also bulges out in nuclear matter!

U. Yakhshiev and HChK, PLB 726, 375 (2013)

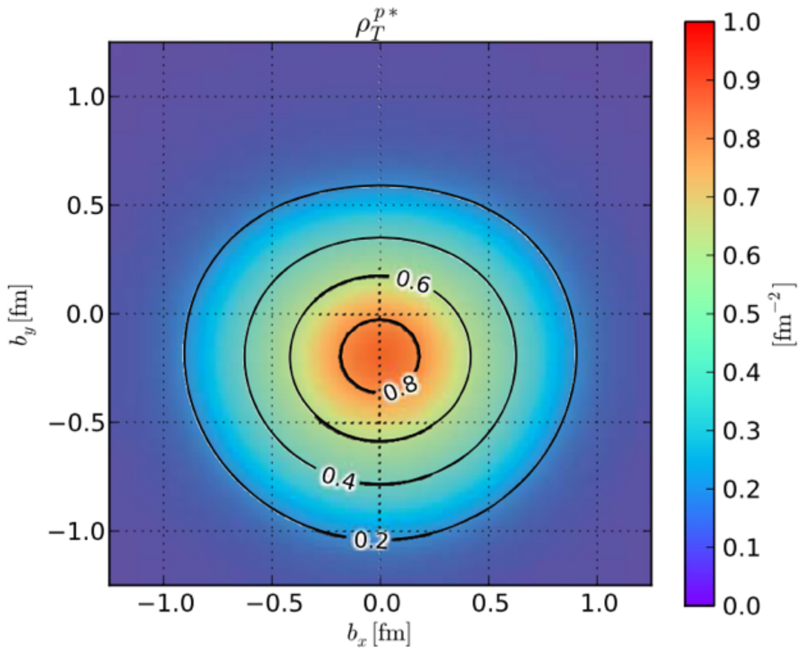
Results

Transverse charge densities inside an **polarized proton in nuclear matter**

Free-space Skyrme Model



Medium-Modified Skyrme Model



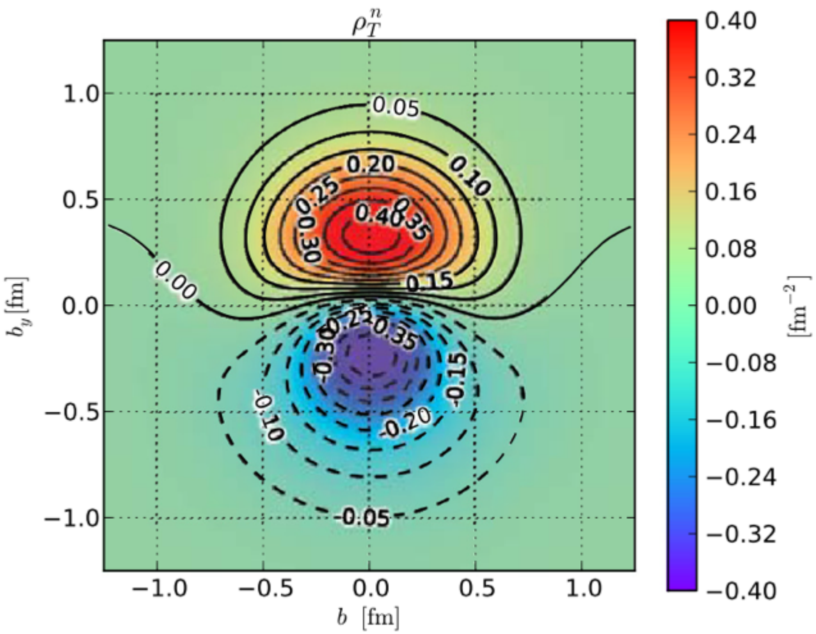
A similar feature also in nuclear matter

U. Yakhshiev and HChK, PLB 726, 375 (2013)

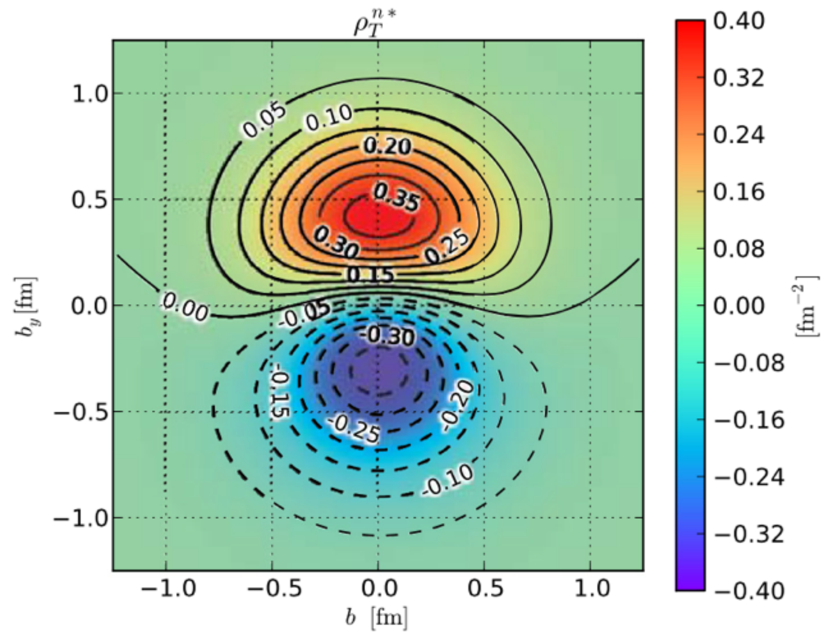
Results

Transverse charge densities inside an **polarized neutron in nuclear matter**

Free-space Skyrme Model



Medium-Modified Skyrme Model



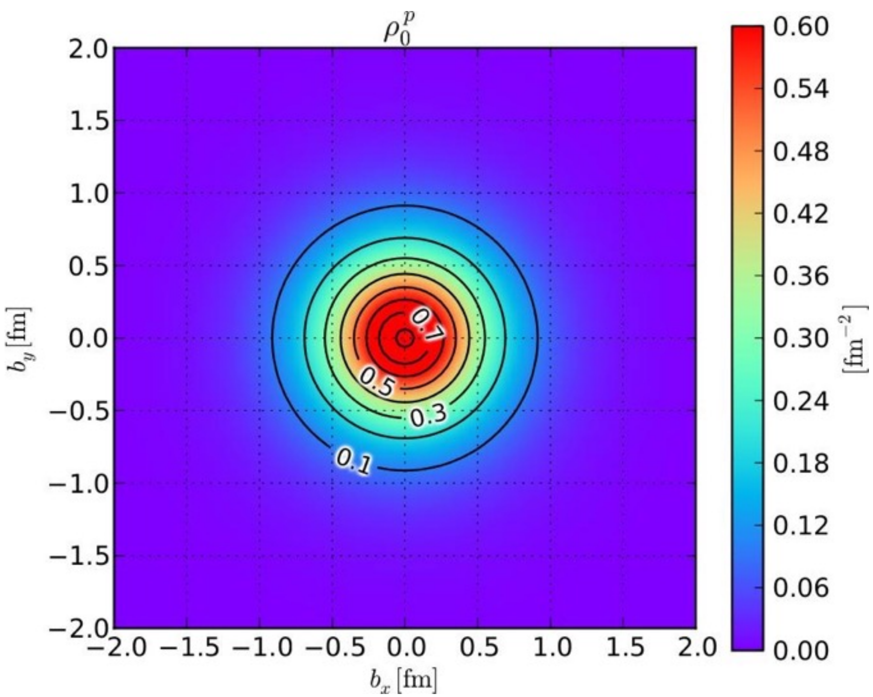
A similar feature also in nuclear matter

U. Yakhshiev and HChK, PLB 726, 375 (2013)

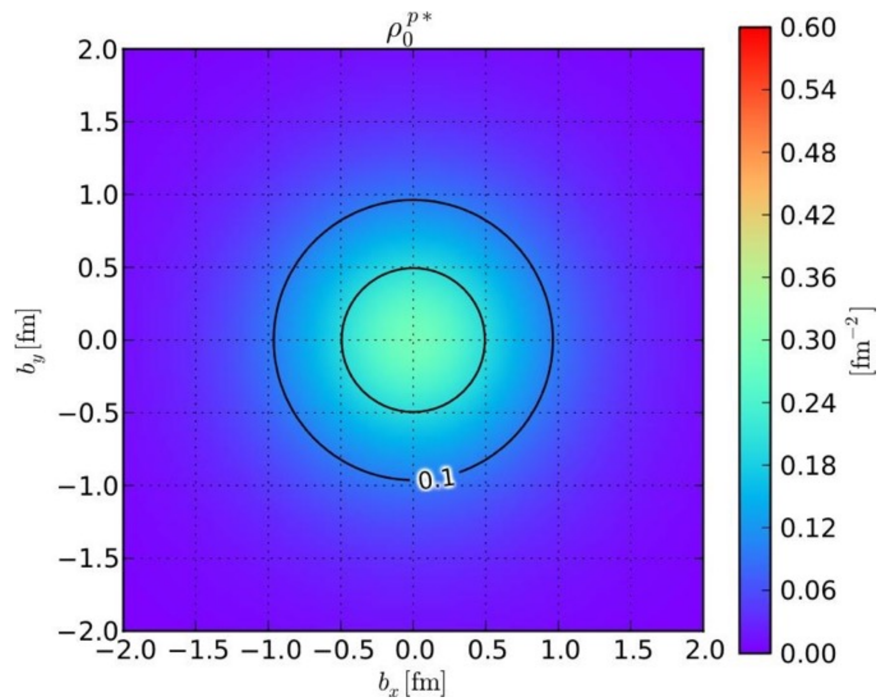
Results

Transverse charge densities inside an **unpolarized proton in nuclear matter**

Free-space pi-rho-omega Model



Medium-Modified pi-rho-omega Model

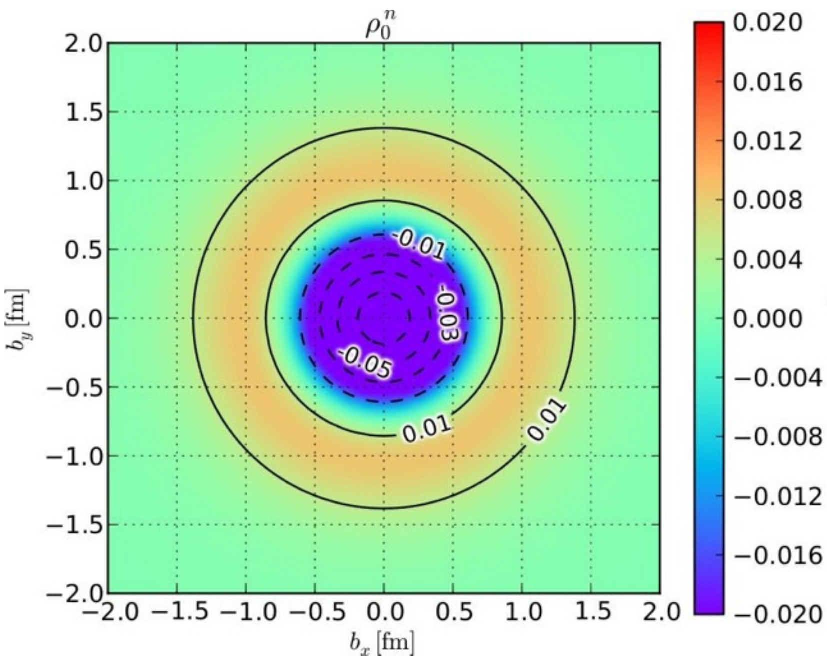


J.H. Jung, U. Yakhshiev and HChK, PRD 89, 114021 (2014)

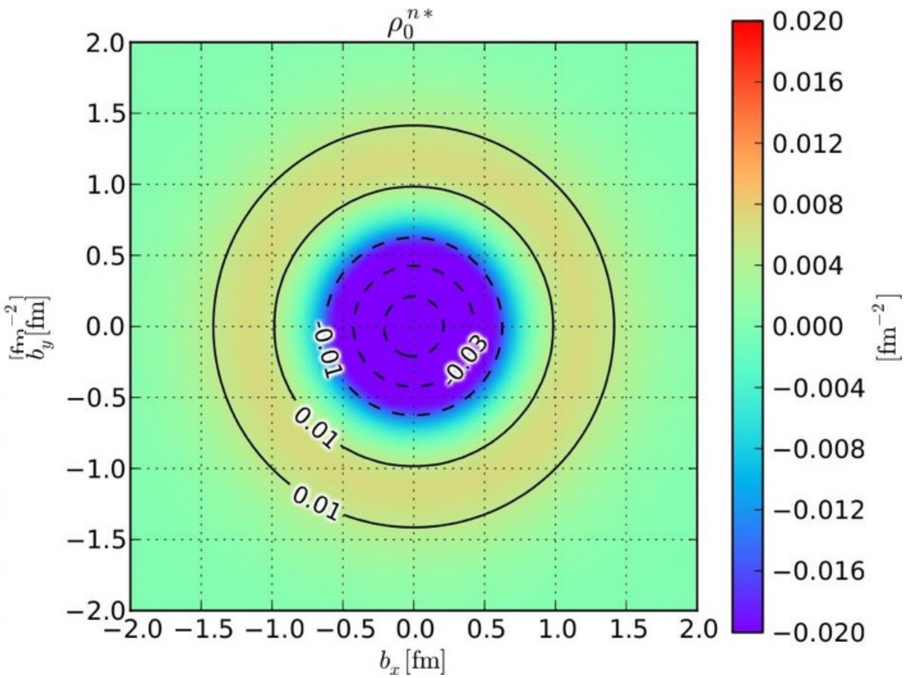
Results

Transverse charge densities inside an **unpolarized neutron in nuclear matter**

Free-space pi-rho-omega Model



Medium-Modified pi-rho-omega Model



J.H. Jung, U. Yakhshiev and HChK, PRD 89, 114021 (2014)

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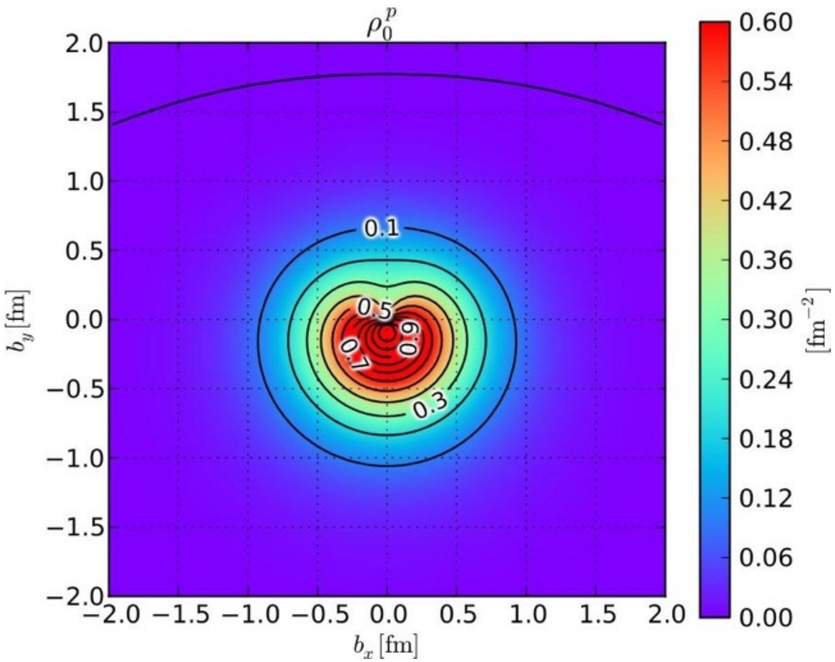

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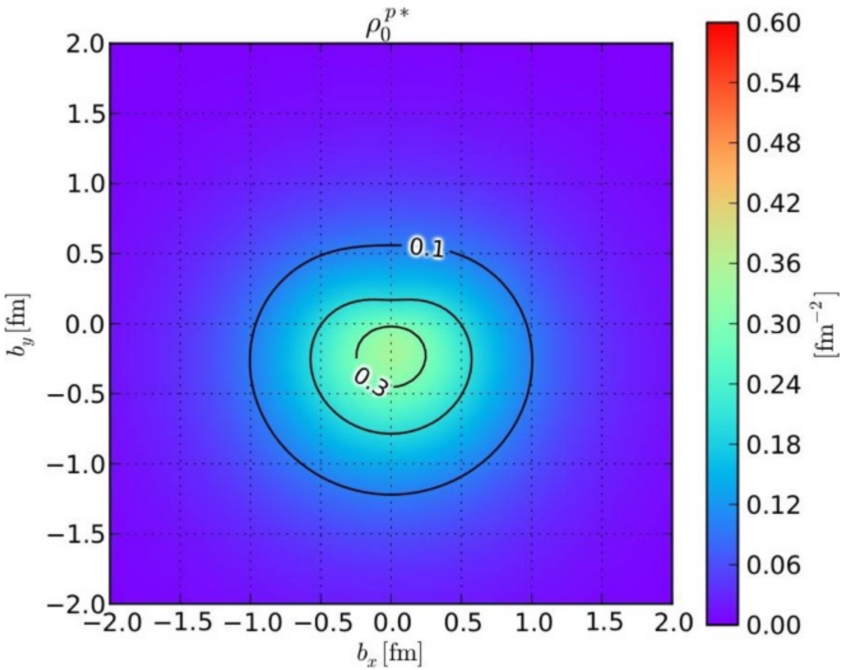

J.H. Jung, U. Yakhshiev and HChK, PRD 89, 114021 (2014)

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Transverse charge densities inside an **polarized neutron in nuclear matter**

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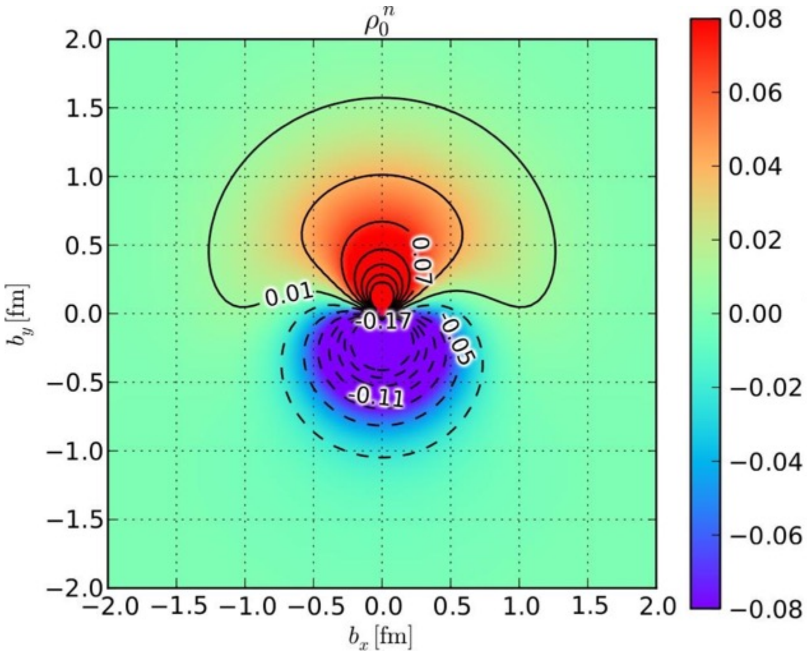
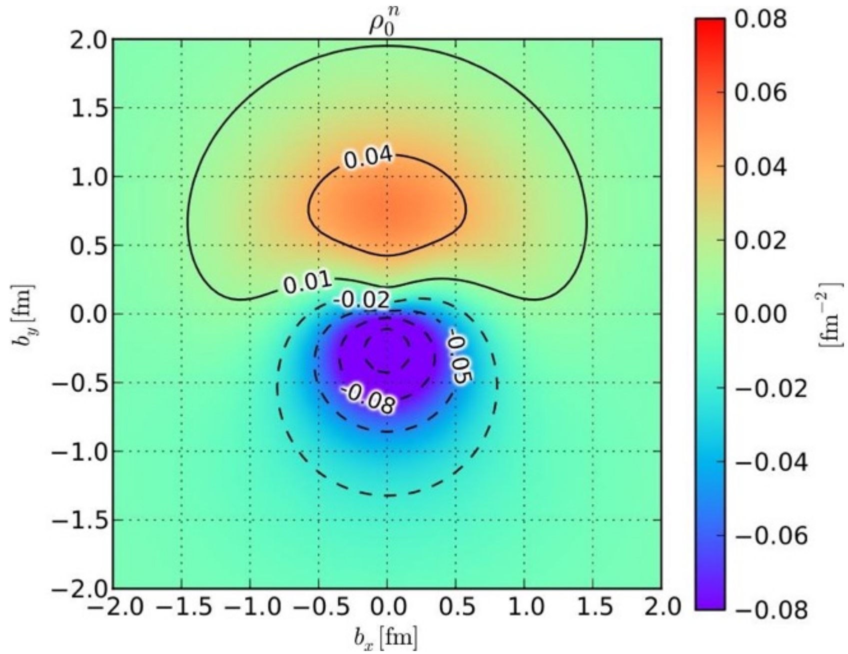


Medium-Modified pi-rho-omega Model



J.H. Jung, U. Yakhshiev and HChK, PRD 89, 114021 (2014)

Summary

- We presented a series of recent studies on the energy-momentum tensor form factors of the pion and the nucleon. The stability of the pion beyond the chiral limit was shown to be secured by the Gell-Mann-Oakes-Renner relation, which implies that the stability of the pion is deeply related to the spontaneous breakdown of chiral symmetry and the pattern of chiral symmetry breaking.
- We also discussed the low-energy constants for the effective chiral Lagrangian in curved space, and the transverse charge densities of the pion in the transverse plane.

Summary

- We also discussed the stability of the nucleon within several chiral soliton models. The stability of the nucleon was acquired from the exact cancellation between the attractive and repulsive contributions. We discussed the transverse charge densities.
- We finally showed that the transverse charge densities are rather informative about the feature of the nucleon in nuclear matter.
- We can extend the present studies to more general form factors and to the GPDs.

Hyeon-Dong Son's Talk on Thursday

Thanks to collaborators!

- Ulubek Yakhshiev, Inha Univ.
- Hyeon-Dong Son, Inha Univ.
- Ju-Hyun Jung, Inha Univ. & Univ. of Graz
- Peter Schweitzer, Univ. Connecticut

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!