Problems in nucleon structure study

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I.Introduction

- Nucleon is an SU(3) color gauge system, atom is an U(1) em gauge system. To study the internal structure of atom and nucleon, the mass(energy)-momentum, spin, orbital angular momentum distribution among the constituents are fundamental problems.
- Our experience on the atomic, molecular, nuclear internal structure study seems to show that these problems are trivial in principle except the many-body complication.
- However, to take the gauge invariance, canonical quantization, Lorentz covariance full into account, this is not trivial in principle.
- 6 years ago we raised questions about the precise meaning of momentum, spin and orbital angular momentum of quark, gluon in nucleon structure. It seems that up to now there is no consensus in this community.
- Today I discuss problems in the nucleon structure study.

II.Symmetric or asymmetric energy-momentum(E-M) tensor

- The E-M tensor of the interacting quark-gluon system is the starting point to study the momentum, spin and orbital angular momentum distribution among the quark, gluon constituents.
- The Lagrangian of an interacting quark-gluon field system is

$$\mathcal{L}(x) = \frac{1}{2}\bar{\psi}(i\gamma^{\mu}(\partial_{\mu} + igA_{\mu}^{a}\frac{\lambda^{a}}{2}) - m)\psi + h.c. - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu}^{a}$$

- From the interacting quark-gluon Lagrangian, follow the standard Noether recipe one can obtain the E-M tensor of this system,

$$T_{as}^{\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\partial^{\nu}\psi + h.c. - F^{\mu\rho a}\partial^{\nu}A_{\rho}^{a} - g^{\mu\nu}\mathcal{L}(x)$$

- This E-M tensor is neither symmetric nor gauge invariant.
- One can follow the Belinfante recipe to get the symmetric one and add a surface term to make it gauge invariant,

$$T_{sy}^{\mu\nu} = \frac{i}{4}\bar{\psi}(\gamma^{\mu}D^{\nu} + \gamma^{\nu}D^{\mu}) + h.c. - F^{\mu\rho a}F_{\rho}^{\nu a} - g^{\mu\nu}\mathcal{L}(X),$$

$$D^{\mu} = \partial^{\mu} + igA^{\mu a}\frac{\lambda^{a}}{2}.$$

- It is believed that one has the freedom to make choice of the form of the E-M tensor, because to add a surface term will not change the conservation law satisfied by the E-M tensor.
- The symmetric and gauge invariant one is prefered because the Einstain gravitation equation needs symmetric one.
- In fact the E-M tensor density of em-field is measurable and there are already experimental hints to refute the symmetric one .

Optical evidence

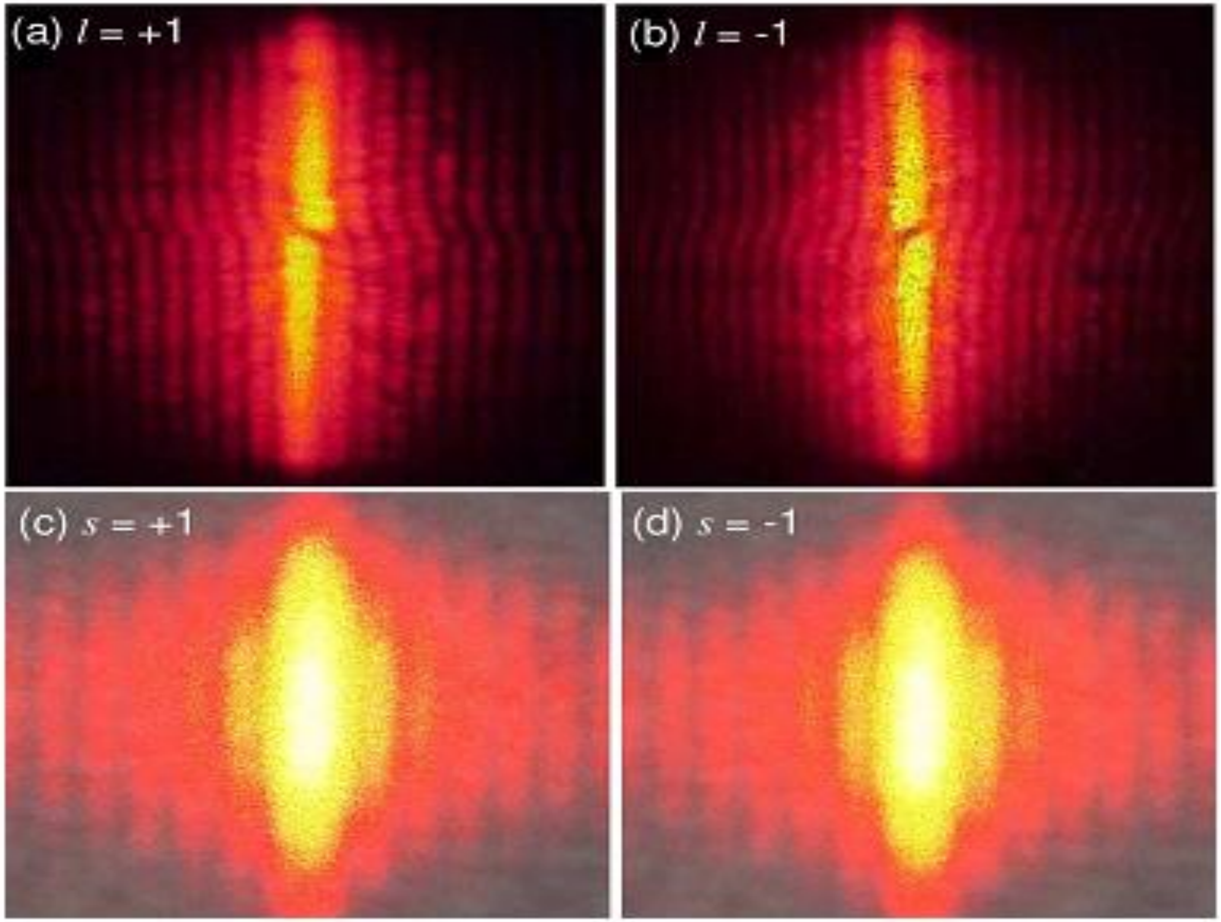

FIG. 1: Single-slit diffraction patterns of light with spin s and orbital angular momentum l. (a) $l = +1$, (b) $l = -1$, (c) $s = +1$, (d) $s = -1$. (a) and (b) are quoted from Fig. 5 of Ref. [5]. Nonzero l leads to distortion of the diffraction fringes, while nonzero s does not. By a careful analysis, this simple but critical difference can tell that the energy-momentum tensor cannot be symmetric for spin-polarized photons.

- For symmetric em E-M tensor, there should be no difference of the diffraction pattern for an orbital or spin polarized light beam if the total

$$J_z = l_z \sim or \sim s_z = \pm 1$$

- For asymmetric em E-M tensor, there should be difference of the diffraction pattern between orbital and spin polarized beams, because only for orbital polarized beam there is momentum density circular flow in the transverse plane.

A detailed analysis had been given in

arXiv:1211.4407[physics.class-ph]

Spin is different from orbital angular momentum

- In 1920's one already knew that the fundamental spin can not be related to the orbital motion. So to use the symmetric E-M tensor to express the angular momentum operator as

$$\begin{aligned} \vec{J} &= \int d^3x \vec{x} \times (\vec{E} \times \vec{B}) \\ &= \int d^3x \vec{x} \times E^i \vec{\nabla} A^i + \int d^3x \vec{E} \times \vec{A} \equiv \vec{L} + \vec{S}. \end{aligned}$$

physically is misleading even though mathematically correct. It also leads to the wrong idea that the Poynting vector is not only the energy flow but also the momentum density flow of em field.

The Belinfante derivation of the symmetric E-M tensor and in turn the derived momentum and angular momentum density operators physically is misleading too.

Spin half electron field needs asymmetric energy-momentum tensor

- For a spin half electron field symmetry E-M tensor will lead to contradiction between angular momentum and magnetic momentum measurement. Suppose we have a spin polarized electron beam moving along the z direction with momentum density flow $\overrightarrow{K}$,

$$\int (\vec{x} \times \overrightarrow{K})_z dV = \int J_z dV = N \frac{1}{2}$$

$$\frac{K}{E} = \frac{j}{e} = n, \quad \int (\vec{x} \times \overrightarrow{K})_z dV = \frac{E}{e} \int (\vec{x} \times \vec{j})_z dV = \frac{E}{e} \int 2\mu_z dV = N \cdot 1$$

- For symmetric E-M tensor, the momentum density flow is the same as the energy density flow. For spin s=1/2 electron this will lead to a contradiction. The energy flow and momentum density flow should be different.

A detaild analysis had been given in arXiv:1211.2360[gr-qc].

To meet the requirement of a gauge invariant but asymmetric E-M tensor we derived the following one based on the decomposition of gauge potential. Hope experimental colleagues to check further which one is the correct one.

$$T_{cw}^{\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^{\mu}D_{pure}^{\nu}\psi + h.c. - F^{\mu\rho}\partial^{\nu}A_{\rho}^{phys} - g^{\mu\nu}\mathcal{L}(x)$$

III.Gauge invariant decomposition

- Gauge invariant requirement of the individual terms of the momentum and angular momentum operator promoted this wrong development further. For a long time the Ji-decomposition seems to be the unique gauge invariant possibility, which is derived from the symmetric E-M tensor and so the obtained momentum and orbital angular momentum operators are not physical.
- It appears to be simple but if the gauge invariance, canonical quantization rule, Lorentz covariance all take into account, it is not simple.
- The next few pages show the different decomposition of the angular momentum operator of the QED case. QCD case has the same form.

$$J_{QED} = S_e + L_e + S_\gamma + L_\gamma$$

$$\begin{aligned}
\vec{S}_e &= \int d^3x\, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi \\
\vec{L}_e &= \int d^3x\, \psi^\dagger \vec{x} \times \frac{1}{i} \vec{\nabla} \psi \\
\vec{S}_\gamma &= \int d^3x\, \vec{E} \times \vec{A} \\
\vec{L}_\gamma &= \int d^3x\, E^i \vec{x} \times \vec{\nabla} A^i
\end{aligned}$$

$$J_{QED} = S_e + L'_e + J'_\gamma$$

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$$\vec{S}_{\mathrm{e}} = \int d^3x\ \psi^\dagger \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}'_{\mathrm{e}} = \int d^3x\ \psi^\dagger \vec{x} \times \frac{1}{i} \vec{D} \psi$$

$$\vec{J}'_{\gamma} = \int d^3x\ \vec{x} \times (\vec{E} \times \vec{B})$$

Wakamatsu improved this by decomposing the total gluon angular momentum J' into gauge invariant spin and orbital part.

$$J_{QED} = S_e + L_e'' + S_\gamma'' + L_\gamma''$$

$$\begin{aligned}
\vec{S}_e &= \int d^3x\ \psi^\dagger \frac{\vec{\Sigma}}{2} \psi \\
\vec{L}_e'' &= \int d^3x\ \psi^\dagger \vec{x} \times \frac{1}{i} \vec{D}_{phys} \psi \\
\vec{S}_\gamma'' &= \int d^3x\ \vec{E} \times \vec{A}_{phys} \\
\vec{L}_\gamma'' &= \int d^3x\ E^i \vec{x} \times \vec{\nabla} A^i_{phys}
\end{aligned}$$

$$\vec{D}_{phys} \equiv \vec{\nabla} - ie\vec{A}_{pure}, \quad \vec{A}_{pure} + \vec{A}_{phys} \equiv \vec{A}$$

$$\begin{aligned}
\vec{\nabla} \cdot \vec{A}_{phys} &= 0 \\
\vec{\nabla} \times \vec{A}_{pure} &= 0
\end{aligned}$$

Consistent separation of nucleon momentum and angular momentum

Sq Lq Sg Lg

$$\vec{J}_{\text{total}} = \int d^3x \psi^\dagger \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \vec{x} \times \psi^\dagger \frac{1}{i}\vec{D}_{\text{pure}}\psi + \int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}}$$

$$\vec{P}_{\text{total}} = \int d^3x \psi^\dagger \frac{1}{i}\vec{D}_{\text{pure}}\psi + \int d^3x E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}}$$

Pq Pg

Standard construction of orbital angular momentum $\boxed{\vec{L} = \int d^3x \; \vec{\mathrm{x}} \times \vec{P}}$

criticism to
the new gauge invariant decomposition

- Non-locality: Non-local operators are popular in gauge field theory. All of the parton distribution operators are non-local, only in light-cone gauge becomes local. The new one is local in Coulomb gauge
- Canonical commutation relation:

  Renormalization ruins the canonical commutation relation. However there is no way to avoid the canonical quantization rule.

  hep-ph:0807.3083,0812.4336,0911.0248,1205.6983

Lorentz covariance

- The Lorentz transformation (LT) property of 4-coordinate x^{μ}, momentum p^{μ}, and field tensor $F^{\mu\nu}$ are fixed by measurement. They are transformed by the usual homogeneous LT.
- The LT property of 4-vector potential A^{μ} is gauge dependent, because there are gauge degree of freedom:

Lorentz gauge, usually they are chosen to be transformed by homogeneous LT, but it can also be chosen to be transformed by inhomogeneous LT, i.e.,

$$A^{\mu'}(x') = \Lambda^{\mu}_{\nu} A^{\nu}(x) + \partial^{\mu'}\omega(x')$$

Because there are residual gauge degree of freedom.

- Coulomb gauge, vector potential must be transformed by the inhomogeneous LT to make the transformed potential still satisfy the Coulomb gauge condition, i.e,.

$$A^{\mu'}(x') = \Lambda^{\mu}_{\nu} A^{\nu}(x) + \partial^{\mu'} \omega(x'), \sim\sim \nabla' \cdot A'(x') = 0$$

because homogeneous LT mixed the unphysical components to the vector potential and one must do additional gauge transformation to eliminate the unphysical components.

- Light-cone gauge vector potential must also be transformed by inhomogeneous LT to make the

$$\boldsymbol{A}^{+'} = \mathbf{0}$$

in the new Lorentz frame. Only for limited boosting along the infinite momentum direction light-cone gauge fixing can be preserved automatically!

- The Lorentz frame independence of a theory must be independent of the gauge choice, no matter the Lorentz gauge, Coulomb gauge or light-cone gauge is used.
- The Lorentz transformation form of the gauge potentials are gauge dependent accordingly!

A systematic analysis had been given by C. Lorce in arXiv:1205.6483[hep-ph],PRD87(2013)034031.

Uniqueness of the new decomposition

Gauge invariance is the necessary condition for the measurabilily but not sufficient one.

QED case this physical condition is unique,

$$\vec{\nabla} \cdot \vec{A}_{phys} = 0$$

$$\vec{\nabla} \times \vec{A}_{pure} = 0$$

QCD case the generalization of this condition is under hot debate. We believe the physical one is unique and our physical condition is a generalization of Coulomb gauge , i.e., only two helicity gluon components are physical.

Light-cone gauge is not physical

- Light-cone gauge

$$A^+ = A^0 + A^3 = 0 \rightarrow A^0 = -A^3$$

- Gaussian law gives rise to

$$\nabla^2 A^3 - \nabla \cdot \partial_t \vec{A} = \rho$$

- The gauge has not been fixed completely yet,
- Unphysical components A^0 and A^3 both are

still included in the gauge potential.

- The light-cone momentum in the light-cone gauge still includes unphysical gauge potential contribution.

- E. Leader suggested to use the gauge variant canonical momentum and angular momentum operators as the physical one and tried to prove that the matrix elements of physical states of gauge dependent operator are gauge invariant.
- His argument is based on F. Strocchi and A.S. Wightman’s theory and this theory is limited to the extended Lorentz gauge and so at most only true for very limited gauge transformations.
- Our gauge invariant momentum and angular momentum operator reduce to the canonical one in physical gauge, i.e., they are generators for physical field.

arXiv:1203.1288[hep-ph]

Measurability:

- X.D. Ji argued kinematical orbital angular momentum can be measured through the GPD. B.Q. Ma proved recently it is not true.
- Kinematical momentum and orbital angular momentum are still a mixing of quark and gluon contributions.
- They do not satisfy the momentum and angular momentum canonical commutation relations, and can never be reduced to canonical momentum.
- There is no solution of the eigen-value equation of the kinematical momentum operator.
- Physical momentum and orbital angular momentum satisfy the canonical commutation relations, reduced to the canonical momentum in Coulomb gauge, and are measurable in principle.
- C. Lorce discussed the measurability of the physical momentum and angular momentum in

arXiv:1306.0456[hep-ph]

Four momentum operators

only $\vec{p}_{phys}$ is physical one

$$\vec{p}_{cano} = m\vec{v} + q\vec{A} = \frac{1}{i}\vec{\nabla}$$

$$\vec{p}_{kine} = \vec{p} - q\vec{A} = m\vec{v} = \frac{1}{i}\vec{D}$$

$$p_{lc} = p^{+} - qA^{+}$$

$$\vec{p}_{phys} = m\vec{v} + q\vec{A} - q\vec{A}_{pure} = m\vec{v} + q\vec{A}_{phys} = \frac{\vec{\nabla}}{i} - q\vec{A}_{pure}$$

Quark orbital angular momentum

The kinematical orbital angular momentum

$$\vec{L}_q = \int d^3x\, \vec{x} \times \psi_q^+ (\vec{p} - g\vec{A})\psi_q$$

Is not the canonical orbital angular momentum used in Quantum mechanics. It does not satisfy the angular Momentum algebra

$$\vec{L} \times \vec{L} = i\vec{L}$$

And the gluon contribution is *entangling in it.*

IV.Physical meaning of parton distribution and its evolution

- Jaffe and Bashinsky (arXiv:hep-ph/9804397, NPB536(1999)303) studied the physical meaning of the parton distribution. They conclude that the parton momemtum distribution is a distribution on the eigen-value of the light-cone momentum.

$$p^{+}=-i(\partial^{+}-igA^{+})$$

- Only those observables, the corresponding operators commute with the light-cone momentum, have physical meaningful parton distributions.
- If we insist on this requirement, the 3-D parton momentum distribution and the light-cone quark orbital angular momentum distribution only can be defined in light-cone gauge

$$A^{+}=0$$

- Liang's analysis shows that in the naïve parton model the parton distribution f(x) is a distribution of parton canonical momentum

$$x = p^+ / P^+$$

distribution. Taking into account of the gluon interaction, under the collinear approximation, the parton distribution is the light-cone momentum

$$p^+ - gA^+$$

distribution which is the same as that of Jaffe-Bashinsky.

- Ji et al., also proved that there is only A^μ_{pure} including in the A^+ in the infinite momentum frame.

The kinematic quark momentum

$$\vec{p} - g\vec{A}$$

suggested by X.D. Ji and M. Wakamatsu, derived from the symmetric E-M tensor is neither the right quantum mechanical momentum operator, which can not be reduced to the canonical momentum in any gauge, nor the measured quark momentum!

The DIS measured one is the light-cone momentum

$$p^+ - gA^+$$

which is the light-cone version of $\vec{P}_{phys}$

Evolution of the parton distribution

Most of the evolutions are based on the free parton picture and perturbative QCD.

The measured parton distribution is always a mixing of non-perturbative and perturbative one.

The gluon contribution might be solely due to evolution.

V.What is the key issue of the nucleon structure study?

- Are all of those problems popular in this community really important and will lead to new understanding of nucleon structure, especially will lead to new physics.

For example, how do these parton picture

relate to the QCD confinement,

to the hadron spectroscopy?

VI. Summary

- The E-M tensor density of CED are measurable, the preliminary measurements prefer asymmetric one.
- The decomposition of the momentum and angular momentum of a gauge system into the Fermion and gauge boson parts are not simple if the gauge invariance, canonical quantization and Lorentz covariance are taken into account altogether.
- A decomposition to meet the above requirements has been proposed based on the separation of the gauge potential into A_{phys} and A_{pure}
- There is no consensus after 6 years debates and there are problems on what are the right momentum, orbital angular momentum and spin of Fermion and gauge boson and what are the real impotent problem in nucleon structure study.

Thanks