NNLO QCD corrections to $\gamma\gamma^* \to \eta_c$ transition form factor

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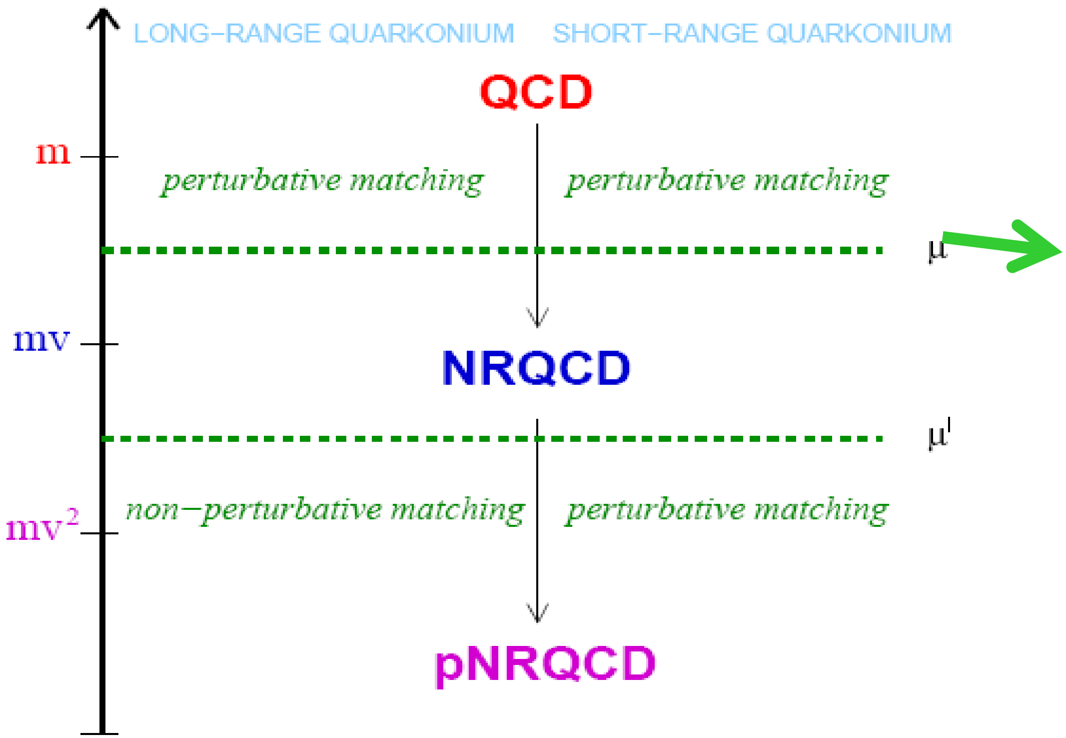
- **Brief introduction to NRQCD factorization**
- **N^3LO QCD correction to** Upsilon -> $e^+ e^-$ as an example
- **NNLO QCD correction to on $\gamma\gamma^* \rightarrow \eta_c$ form factor and confront the BaBar data**

Nonrelativistic QCD (NRQCD): the modern effective field theory to tackle heavy quarkonium

Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1995)



This scale separation is usually referred to as **NRQCD factorization.**

The NRQCD short-dist. coefficients can be computed in perturbation theory, order by order

NRQCD Lagrangian (characterized by velocity expansion)

Velocity-scaling rule

$$\mathcal{L}_{\rm NRQCD} = \mathcal{L}_{\rm light} + \mathcal{L}_{\rm heavy} + \delta\mathcal{L}.$$

$$\mathcal{L}_{\rm light} = -\frac{1}{2}{\rm tr}\, G_{\mu\nu}G^{\mu\nu} + \sum \bar{q}\, i\not{D} q,$$

$$\mathcal{L}_{\rm heavy} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi,$$

$$\begin{aligned}
\delta\mathcal{L}_{\rm bilinear} = & \frac{c_1}{8M^3}\left(\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi\right) \\
& + \frac{c_2}{8M^2}\left(\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\right) \\
& + \frac{c_3}{8M^2}\left(\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right) \\
& + \frac{c_4}{2M}\left(\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi - \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right),
\end{aligned}$$

NRQCD gains popularity in quarkonium physics
(see Brambilla et al. EPJC 2011 for a recent review)

Nowadays, NRQCD becomes standard approach to tackle various quarkonium production and decay processes

charmonium: $v^2/c^2 =$ 0.3

bottomonium: $v^2/c^2 =$ 0.1

For most phenomenologically interesting reactions, at least the NLO QCD corrections are known, K factor is often large.

Exemplified by the e+ e- -> J/Ψ+ ηc at B factories.

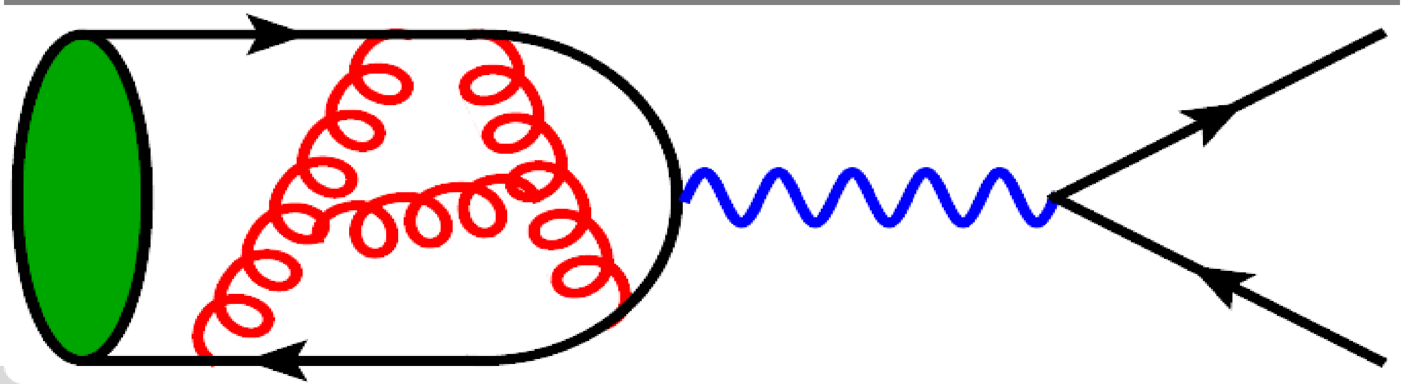
Take Upsilon -> $e^+ e^-$ as example (Stealing the slides of Steinhauser at QWG14)

Karlsruhe Institute of Technology

$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)$ to NNNLO

Matthias Steinhauser | (M. Beneke, Y. Kiyo, P. Marquard, A. Penin, J. Piclum, D. Seidel)

TTP KARLSRUHE QUARKONIUM 2014, CERN, NOVEMBER 10-14, 2014



KIT – University of the State of Baden-Wuerttemberg and National Laboratory of the Helmholtz Association

The state-of-the art knowledge of the NRQCD short-distance coefficient for the electromagnetic current

c_v to 3 loops

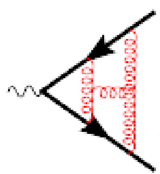
QCD $\longrightarrow$ NRQCD

$$j_v^\mu = \bar{Q}\gamma^\mu Q \quad \longrightarrow \quad \tilde{j}^i = \phi^\dagger \sigma^i \chi$$

$$j_v^i \quad = \quad c_v(\mu)\, \tilde{j}^i + \frac{d_v(\mu)}{6m_Q^2}\phi^\dagger \sigma^i \vec{D}^2 \chi + \ldots$$

$$\boxed{Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \ldots}$$

Γ_v:



$\tilde{\Gamma}_v \equiv 1 \quad \tilde{Z}_2 \equiv 1$

Z_2: [Melnikov,v.Ritbergen'00] [Marquard,Mihaila,Piclum,Steinhauser'07]

$\tilde{Z}_v = 1 + \mathcal{O}(\alpha_s^2)$

[Beneke,Signer,Smirnov'98;
Kniehl,Penin,Steinhauser,Smirnov'03;
Marquard,Piclum,Seidel,Steinhauser'06;
Beneke,Kiyo,Penin'07]

($\overline{\mathrm{MS}}$ scheme)

$$\overline{R_{\eta_c}}(\Lambda) \equiv \sqrt{\frac{2\pi}{N_c}}\, \langle 0|\chi^\dagger \psi(\Lambda)|\eta_c\rangle,$$

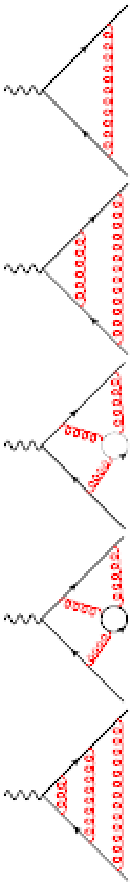
$$\overline{R_{\psi}}(\Lambda)\, \epsilon \equiv \sqrt{\frac{2\pi}{N_c}}\, \langle 0|\chi^\dagger \boldsymbol{\sigma} \psi(\Lambda)|\psi(\epsilon)\rangle\,,$$

Onium-to-vacuum NRQCD matrix element is connected to the wave function at the origin in quark potential model

Now c_V is known to 3-loop order

c_V

$$c_V = 1 + \frac{\alpha_s}{\pi} c_V^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 c_V^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 c_V^{(3)} + \mathcal{O}(\alpha_s^4)$$



$c_V^{(1)}$ [Källen, Sarby'55]

$c_V^{(2)}$ [Czarnecki,Melnikov'97; Beneke,Signer,Smirnov'97]

$c_V^{(3),n_l}$ [Marquard,Piclum,Seidel,Steinhauser'06]

$c_V^{(3),n_h}$ [Marquard,Piclum,Seidel,Steinhauser'08]

$c_V^{(3)}$ [Marquard,Piclum,Seidel,Steinhauser'14]

massive vertices

on-shell quarks: $q_1^2 = q_2^2 = M_Q^2$

$(q_1 + q_2)^2 = 4M_Q^2$

Now c_V is known to 3-loop order

Results for c_v

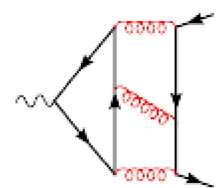
[Marquard,Piclum,Seidel,Steinhauser'14]

$$
\begin{aligned}
c_v(\mu = m_Q) \quad = \quad & 1 - 2.67\frac{\alpha_s}{\pi} + [-44.55 + 0.41 n_l]\left(\frac{\alpha_s}{\pi}\right)^2 \\
& + [-2091(2) + 120.66 n_l - 0.82 n_l^2]\left(\frac{\alpha_s}{\pi}\right)^3 \\
& + \text{singlet terms}
\end{aligned}
$$

- $\mu = m_Q$
- large corrections
- singlet terms: small ($\lesssim 3\%$ of $c^{(2-loop)}$)

  at 2 loops (for axial-vector, scalar, pseudo-scalar current)

  [Kniehl,Onishchenko,Piclum,Steinhauser'06]



The decay rate to 3-loop order

Result

$$\Gamma(\Upsilon(1S) \to \ell^+\ell^-)|_{\rm pole} =$$

$$\frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\Big[1+\alpha_s\left(-2.003+3.979\,L\right)+\alpha_s^2\left(9.05-7.44\,\ln\alpha_s-13.95\,L+10.55\,L^2\right)$$

$$+\alpha_s^3\left(-0.91+4.78_{a_3}+22.07_{b_2\epsilon}+30.22_{c_f}-134.8(1)_{c_g}-14.33\,\ln\alpha_s-17.36\,\ln^2\alpha_s\right.$$

$$\left.+\left(62.08-49.32\,\ln\alpha_s\right)L-55.08\,L^2+23.33\,L^3\right)+\mathcal{O}(\alpha_s^4)\Big]$$

$$\stackrel{\mu=3.5\text{ GeV}}{=}\quad\frac{2^5\alpha^2\alpha_s^3 m_b}{3^5}\big[1+1.166\alpha_s+15.2\alpha_s^2+(66.5+4.8_{a_3}$$

$$+22.1_{b_2\epsilon}+30.2_{c_f}-134.8(1)_{c_g})\,\alpha_s^3+\mathcal{O}(\alpha_s^4)\big]$$

$$=\quad\frac{2^5\alpha^2\alpha_s^3 m_b^{\rm pole}}{3^5}\left[1+0.28+0.88-0.16\right]$$

$$=\quad\left[1.04\pm0.04(\alpha_s)^{+0.02}_{-0.15}(\mu)\right]\text{ keV}$$

$$=\quad\frac{2^5\alpha^2\alpha_s^3 m_b^{\rm PS}}{3^5}\left[1+0.37+0.95-0.04\right]$$

$$=\quad\left[1.08\pm0.05(\alpha_s)^{+0.01}_{-0.20}(\mu)\right]\text{ keV}$$

$\alpha_s = 0.1184 \pm 0.001$

$3 \le \mu \le 10$ GeV

$L = \ln[\mu/(4m_b\alpha_s/3)]$

Now let us switch the gear, consider the simplest exclusive meson production process:

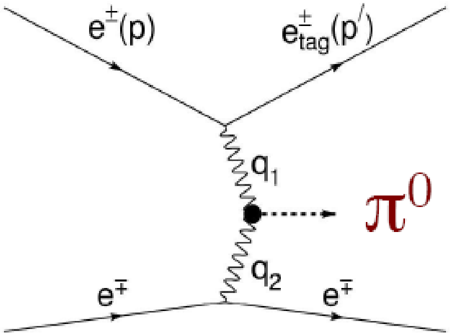
$\gamma\gamma^* \rightarrow$ C-even meson

Surprise brought by BaBar two-photon experiment on $\gamma\gamma^* \rightarrow \pi^0$

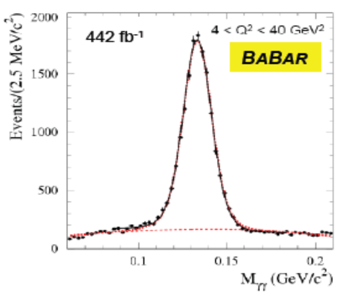
Transition

Form Factor π^0

$e^+e^- \rightarrow e^+e^-\pi^0$



Pion Production: $e^+e^- \rightarrow e^+e^-\pi^0$



Fit function: sum of signal+background

$N \approx 14\,000 \pm 140 \pm 170$

$\sigma = 7.5 \pm 0.1$ MeV/c^2

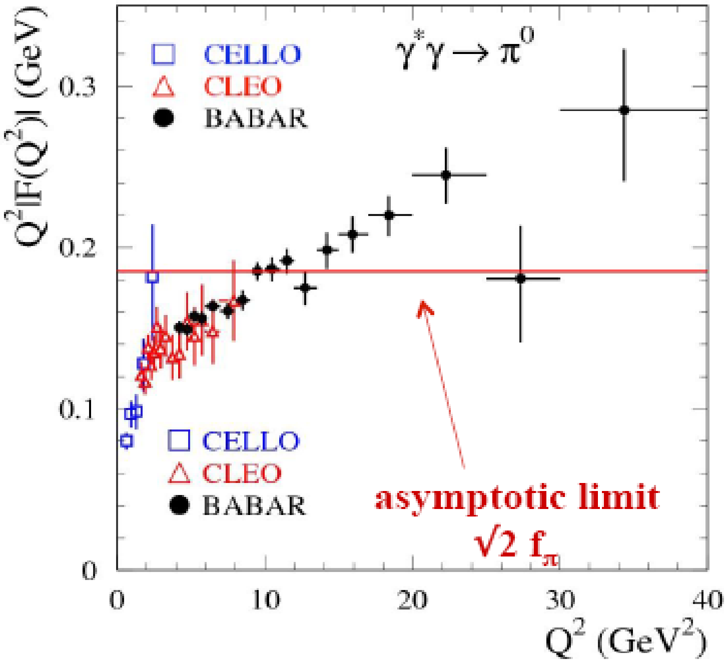
17 intervals in Q^2

BaBar, PRD 80 (2009),052002

Surprise brought by BaBar two-photon experiment on $\gamma\gamma^* \rightarrow \pi^0$

The π^0 Transition Form Factor

Comparison of the result of experiment to the QCD limit



- Experiment:
  In Q^2 range 4-9 GeV^2 CLEO results are consistent with more precise BaBar data

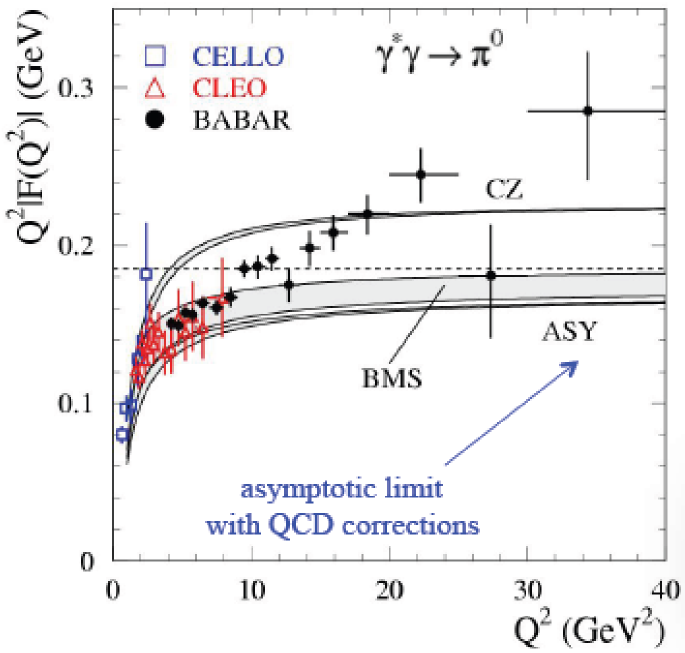
- QCD prediction (Brodsky-Lepage '79):
  at high Q^2 data should reach asymptotic limit (either from below or from above)

  $Q^2 F(Q^2) = \sqrt{2} f_\pi = 0.185$ GeV
  assuming the asymptotic DA

BaBar measurement poses acute challenge to theoretical explanation

The π^0 Transition Form Factor

Comparison of the result of experiment to the QCD limit

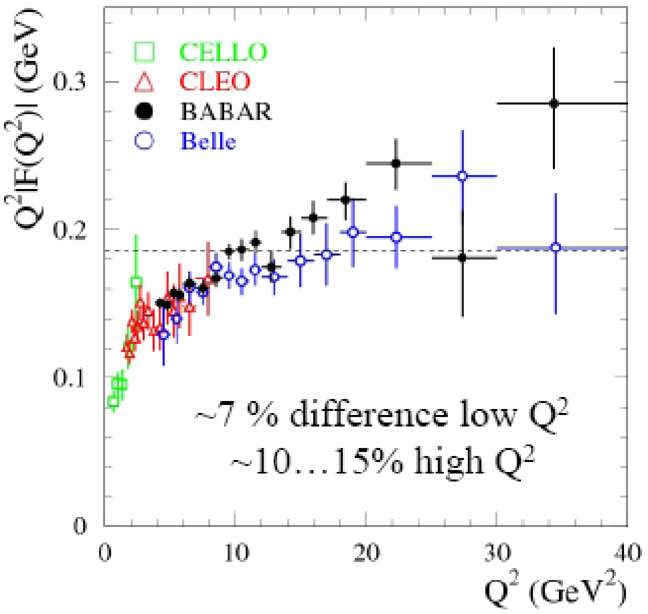


QCD radiative corrections:
To compare DAs with exptl. data, QCD radiative corrections need to be taken into account:
Use Bakulev-Mikhailov-Stefanis light-cone sum rule theory at NLO pQCD + twist-4 power corrections

Model dependence of rad. correct. at low Q^2 might be large! How low?

Belle did not confirm BaBar measurement on $\gamma\gamma^* \rightarrow \pi^0$! Situation needs clarification

Comparison with BELLE, arXiv:1205.3249

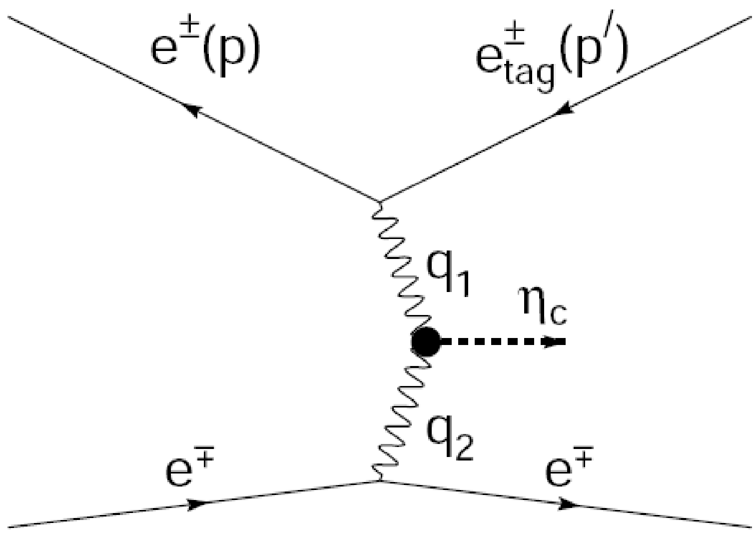


- Difference BABAR – BELLE ~$2\sigma_{syst}$
- BELLE has lower detection efficiency (~factor 2)
- BELLE has higher systematic uncertainties

Investigation on $\gamma\gamma^* \rightarrow \eta_c$

Motivation

In collaboration with **Feng, Sang** (arxiv:**1505.02665**)



$$q_2^2 \approx 0$$

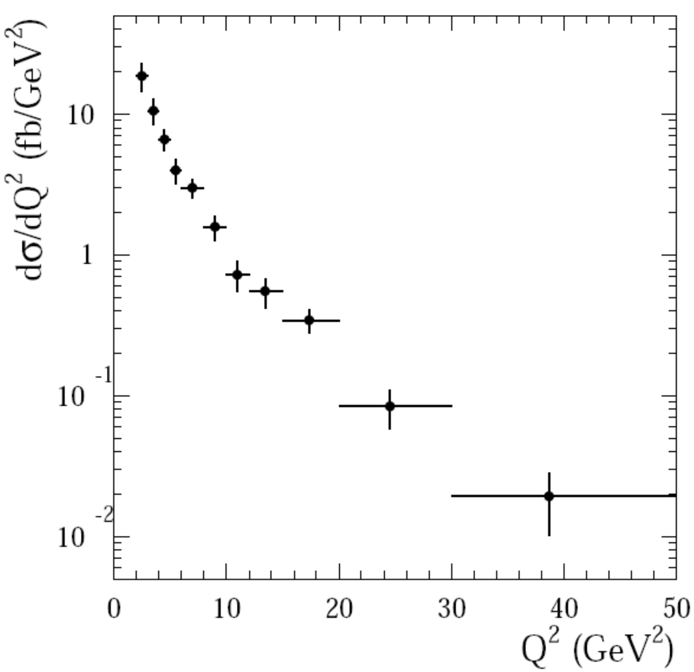
$$q_1^2 = -Q^2 = (p' - p)^2$$

Babar measures the $\gamma\gamma^* \rightarrow \eta_c$ transition form factor in the momentum transfer range from **2 to 50 GeV^2**.

Investigation on $\gamma\gamma^* \to \eta_c$

Motivation

BaBar Collaboration: **Phys.Rev. D81 (2010) 052010**



Q^2 interval (GeV^2)	$\overline{Q^2}$ (GeV^2)	$d\sigma/dQ^2(\overline{Q^2})$ (fb/GeV^2)	$\lvert F(\overline{Q^2})/F(0)\rvert$
2–3	2.49	$18.7 \pm 4.2 \pm 0.8$	0.740 ± 0.085
3–4	3.49	$10.6 \pm 2.1 \pm 0.8$	0.680 ± 0.073
4–5	4.49	$6.62 \pm 1.18 \pm 0.19$	0.629 ± 0.057
5–6	5.49	$4.00 \pm 0.80 \pm 0.10$	0.555 ± 0.056
6–8	6.96	$3.00 \pm 0.43 \pm 0.17$	0.563 ± 0.043
8–10	8.97	$1.58 \pm 0.30 \pm 0.08$	0.490 ± 0.049
10–12	10.97	$0.72 \pm 0.17 \pm 0.05$	0.385 ± 0.048
12–15	13.44	$0.55 \pm 0.13 \pm 0.03$	0.395 ± 0.047
15–20	17.35	$0.34 \pm 0.07 \pm 0.01$	0.385 ± 0.038
20–30	24.53	$0.084 \pm 0.026 \pm 0.004$	0.261 ± 0.041
30–50	38.68	$0.019 \pm 0.009 \pm 0.001$	0.204 ± 0.049

$$\frac{d\sigma(e^+e^- \to \eta_c e^+e^-)}{dQ^2} \times \mathcal{B}(\eta_c \to K\bar{K}\pi)$$

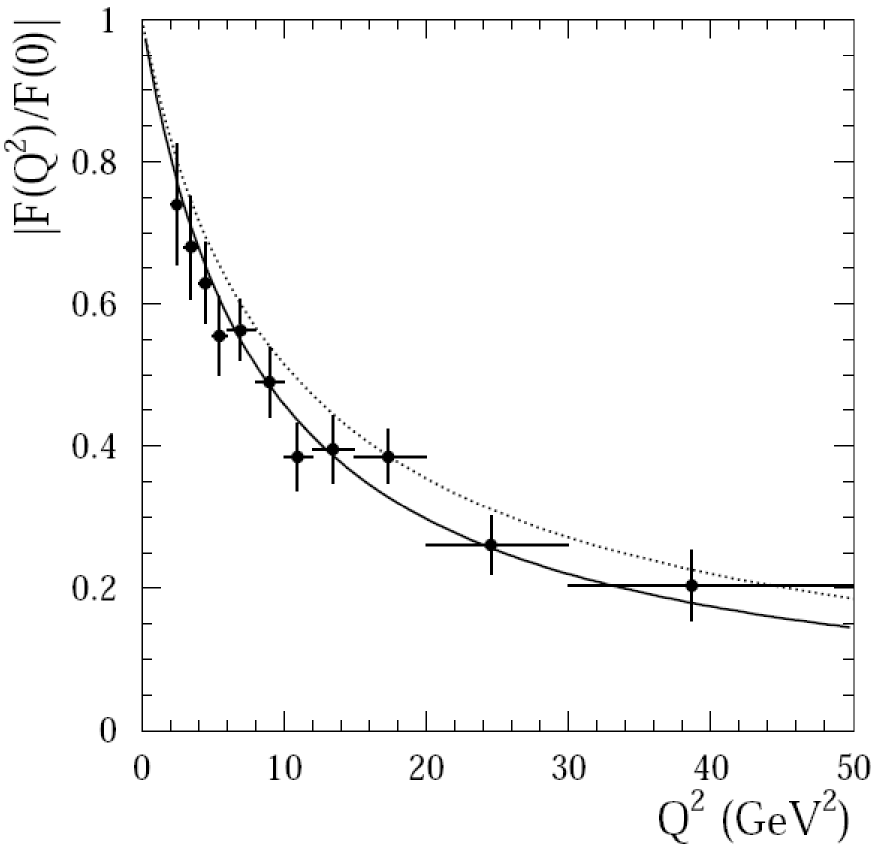
$F(Q^2)$: $\gamma^*\gamma \to \eta_c$ form factor

$F(0)$: $\eta_c \to \gamma\gamma$ form factor

Investigation on $\gamma\gamma^* \rightarrow \eta_c$

Motivation

BaBar Collaboration: **Phys.Rev. D81 (2010) 052010**



The solid curve is from a simple monopole fit:

$$|F(Q^2)/F(0)| = \frac{1}{1+Q^2/\Lambda}$$

with $\Lambda = 8.5 \pm 0.6 \pm 0.7$ GeV^2

The dotted curve is from pQCD prediction

Feldmann and Kroll, Phys. Lett. B 413, 410 (1997)

Previous investigation on $\gamma\gamma^* \rightarrow \eta_c$

- $k_\perp$ factorization: Feldmann *et.al.*, Cao *and Huang*
- Lattice QCD: Dudek *et.al.*,
- **J/ψ** -pole-dominance: Lees *et.al.*,
- QCD sum rules: Lucha *et.al.*,
- light-front quark model: Geng *et.al.*,

All yield predictions compatible with the data, at least in the small Q^2 range.

So far, so good. Unlike $\gamma\gamma^* \rightarrow \pi^0$, there is no open puzzle here

So, why should we bother to investigate this process in NRQCD framework?

1. The momentum transferred is not large enough to warrant light-cone approach

2. Model-independent method is always welcome

3. In the normalized form factor, nonperturbative NRQCD matrix element cancels. Therefore, our predictions are free from any freely adjustable parameters!

4. This simple exclusive reaction serves stringent test against NRQCD

Investigation on $\gamma\gamma^* \rightarrow \eta_c$

Theoretical descriptions in NRQCD

Definition for form factor:

$$\langle \eta_c(p) | J^\mu | \gamma(k,\varepsilon) \rangle = i e^2 \epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu q_\rho k_\sigma F(Q^2)$$

NRQCD factorization defines:

Factorization scale

$$F(Q^2) = C(Q, m, \mu_R, \mu_\Lambda) \frac{\langle \eta_c | \psi^\dagger \chi(\mu_\Lambda) | 0 \rangle}{\sqrt{m}} + \mathcal{O}(v^2)$$

Short-distance coefficient
We are going to compute it!

$$\overline{R_{\eta_c}}(\Lambda) \equiv \sqrt{\frac{2\pi}{N_c}} \langle 0 | \chi^\dagger \psi(\Lambda) | \eta_c \rangle ,$$

$$\overline{R_\psi}(\Lambda)\, \epsilon \equiv \sqrt{\frac{2\pi}{N_c}} \langle 0 | \chi^\dagger \boldsymbol{\sigma} \psi(\Lambda) | \psi(\epsilon) \rangle \ ,$$

Investigation on $\gamma\gamma^* \to \eta_c$

Theoretical descriptions

Upon general consideration, the SDC can be written as

$$C(Q,m,\mu_R,\mu_\Lambda) = C^{(0)}(Q,m)\Bigg\{1 + C_F\frac{\alpha_s(\mu_R)}{\pi}f^{(1)}(\tau) + \frac{\alpha_s^2}{\pi^2}\left[\frac{\beta_0}{4}\ln\frac{\mu_R^2}{Q^2+m^2}C_F f^{(1)}(\tau) - \pi^2 C_F\left(C_F+\frac{C_A}{2}\right)\times\ln\frac{\mu_\Lambda}{m} + f^{(2)}(\tau)\right] + \mathcal{O}(\alpha_s^3)\Bigg\},$$

RGE invariance

IR pole matches anomalous dimension of NRQCD pseudo-scalar density

NLO predictions

$$C^{(0)}(Q,m) = \frac{4e_c^2}{Q^2+4m^2},$$

Tree-level SDC

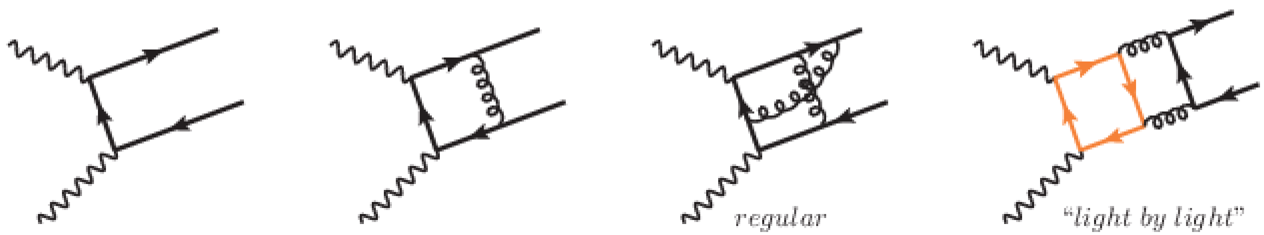
$$f^{(1)}(\tau) = \frac{\pi^2(3-\tau)}{6(4+\tau)} - \frac{20+9\tau}{4(2+\tau)} - \frac{\tau(8+3\tau)}{4(2+\tau)^2}\ln\frac{4+\tau}{2}$$
$$+3\sqrt{\frac{\tau}{4+\tau}}\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}} + \frac{2-\tau}{4+\tau}\left(\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}}\right)^2$$
$$-\frac{\tau}{2(4+\tau)}\mathrm{Li}_2\left(-\frac{2+\tau}{2}\right), \qquad (6)$$

NLO QCD correction

$$\tau \equiv Q^2/m^2.$$

Investigation on $\gamma\gamma^* \rightarrow \eta_c$

Feynman diagrams



LO NLO NNLO

Numer of diagrams

LO	NLO	NNLO (regular)	NNLO ("light by light")
2	**8**	**108**	**12**

NNLO corrections

$$f^{(2)}(\tau) = f^{(2)}_{\rm reg}(\tau) + f^{(2)}_{\rm lbl}(\tau).$$

regular

Light-by-light
UV/IR finite

Reproduce known NNLO corr. to ηc->γγ

Czarnecki et al. 2001

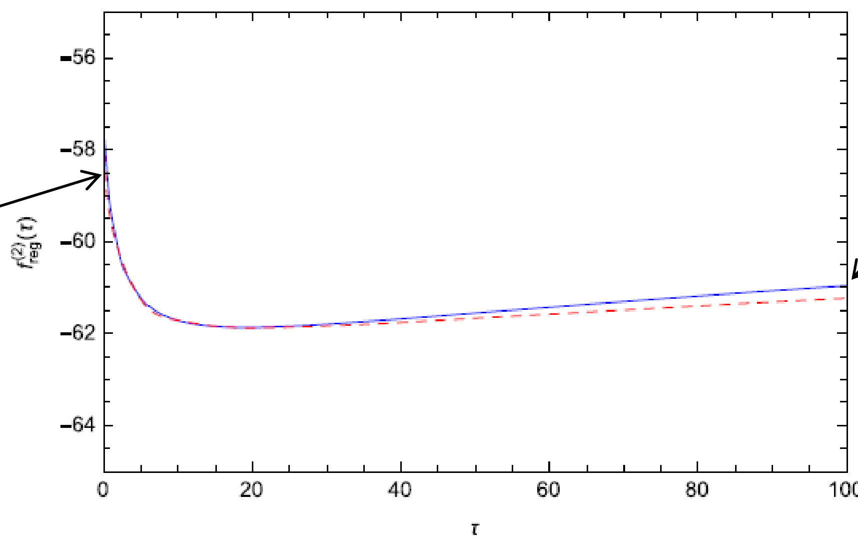


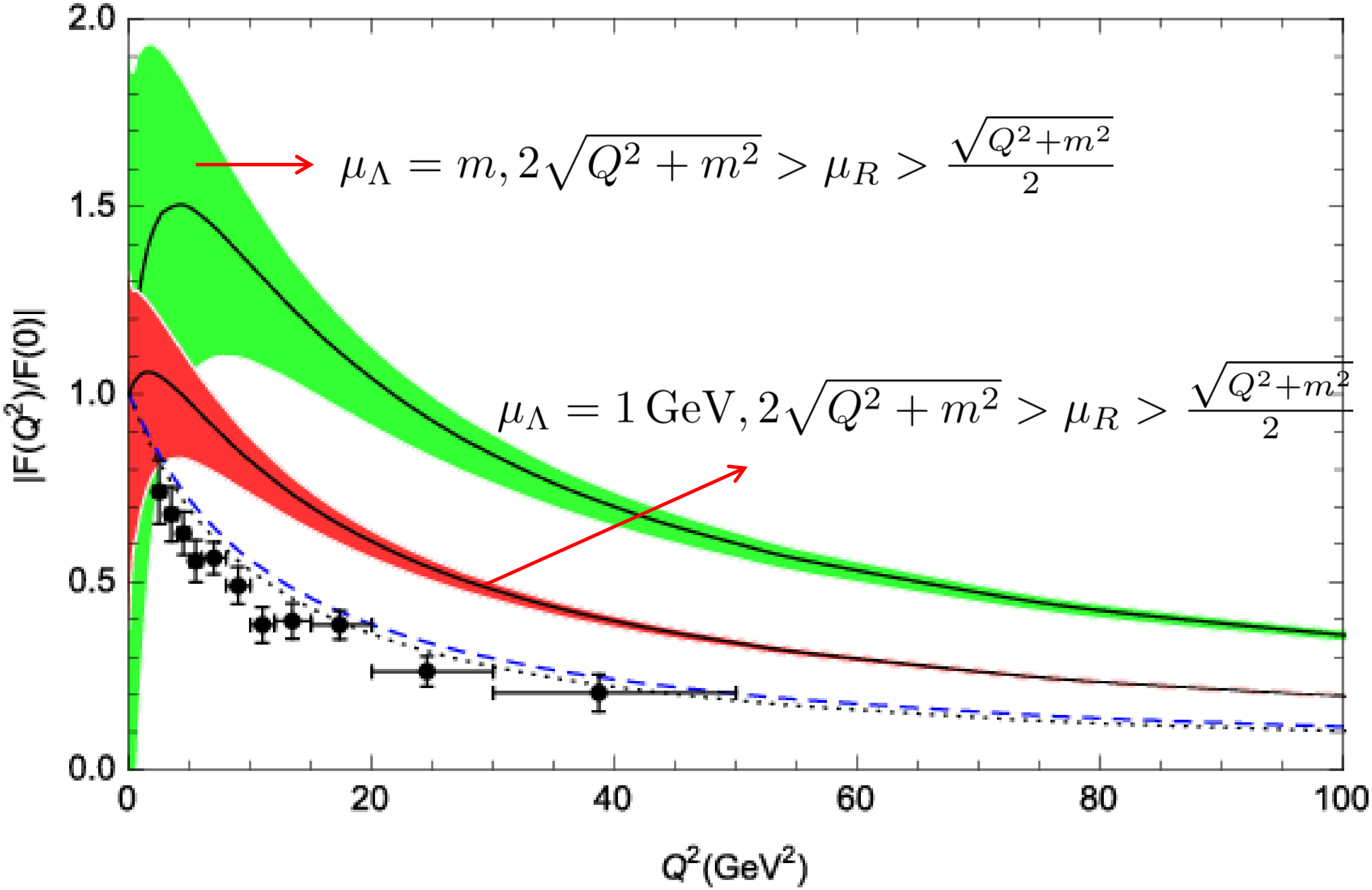
FIG. 2: Profile of $f^{(2)}_{\rm reg}(\tau)$. The solid curve for η_c ($n_L = 3$), and the dashed one for η_b ($n_L = 4$).

Asymptotic behavior $\ln^2\tau$
Compatible with Jia, Yang, NPB 2009

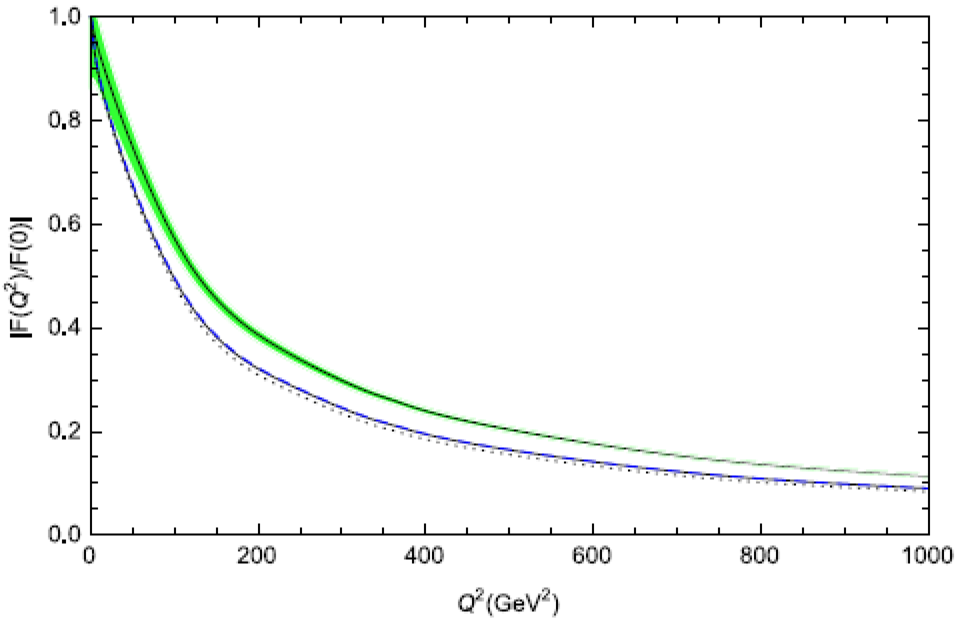
NNLO corrections

τ	1	5	10	25	50
$f_{\mathrm{reg}}^{(2)}$	-59.420(6)	-61.242(6)	-61.721(7)	-61.843(8)	-61.553(8)
$f_{\mathrm{lbl}}^{(2)}$	$0.49(4)$ $-0.65(3)i$	$-0.48(5)$ $-0.72(4)i$	$-1.09(5)$ $-0.71(4)i$	$-2.12(6)$ $-0.69(4)i$	$-3.10(6)$ $-0.68(4)i$
$f_{\mathrm{reg}}^{(2)}$	-59.636(6)	-61.278(6)	-61.716(7)	-61.864(8)	-61.668(8)
$f_{\mathrm{lbl}}^{(2)}$	$0.20(3)$ $-3.11(2)i$	$-1.39(6)$ $-3.38(5)i$	$-2.35(6)$ $-3.46(6)i$	$-3.83(6)$ $-3.51(6)i$	$-5.07(6)$ $-3.52(6)i$

$\gamma\gamma^* \rightarrow \eta_c$: NNLO predictions seriously fails to describe data!



Prediction to $\gamma\gamma^* \rightarrow \eta_b$



Convergence of perturbation series is rather good.
Wait for CEPC/ILC to test our prediction?

Conclusion: $\gamma\gamma^* \to \eta_c$ represents a puzzle in QCD

Can we understand the Babar data in the confine of NRQCD?

- Strong coupling at charm scale is still kind of large, to ensure the convergence of perturbative expansion. $\alpha_s(m_c) = 0.32$
- 3-loop corrections? Not feasible to current technology
- Relativistic corrections? (appears to be small)
- Resum large logarithm of $\ln \frac{\mu_\Lambda}{m}$? (the effect is modest)
- Belle measurement is important to clarify the situation

Thanks for your attention!

Enjoy the beach and sunshine!