Further study of the NΩ dibaryon within constituent quark models

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Outline

- Introduction
- Quark models and calculation methods
- Results and discussions
- Summary

Introduction

- exotic states

Dibaryons: H, d*, di-Ω, NΩ, ……

Pentaquarks: Θ^+ , Ξ^{--} , ……

Charmonium-like states: X(3872), Y(3940), Z(3900), Z(4430),……

Plausible molecular baryons: $\Sigma_c\bar{D}$, $\Lambda_c\bar{D}$, DN, D_sN, ……

Dibaryons with heavy quarks: $\Lambda_c\Lambda_c$, $\Sigma_c\Sigma_c$, $\Xi_c\Xi_c$,
$N\Lambda_c$, $N\Sigma_c$, $N\Xi_{cc}$, $N\Omega_c$, ……

(1) d*, IJ=03

Predicted as an **"inevitable" dibaryon** state.

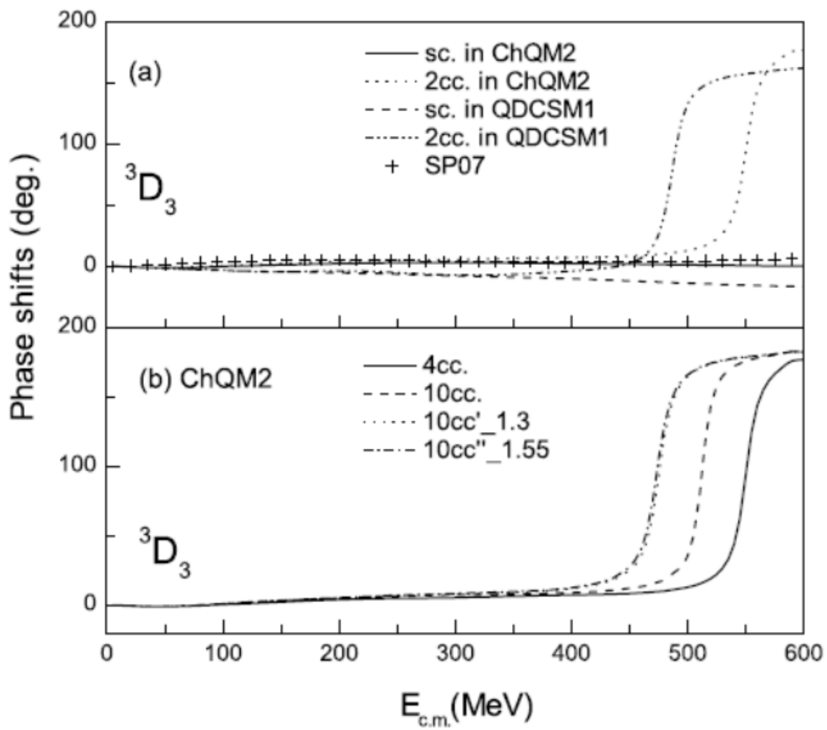
PRC 39,1889 (1989)

A dibaryon **resonance** in NN scattering.

PRC 79, 024001 (2009)

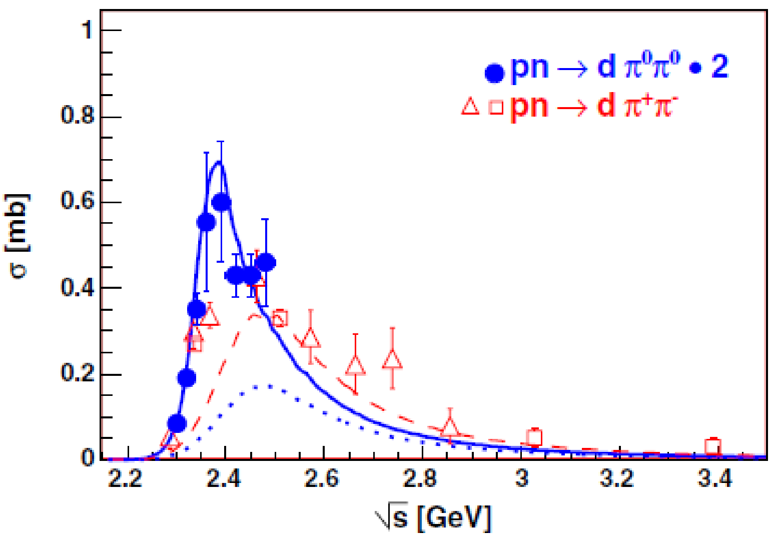
TABLE II. $\Delta\Delta$ or resonance mass M and decay width Γ, in MeV, in five quark models for the $IJ^P = 03^+$ states. The channels included are one channel or 1c (${}^7S_3^{\Delta\Delta}$ only), two coupled channels or 2cc (1c + ${}^3D_3^{NN}$), 4cc (2cc + ${}^{7,3}D_3^{\Delta\Delta}$), and 10cc as described in the text. The pure ${}^7S_3^{\Delta\Delta}$ bound-state mass for ChQM1 is 2456 MeV [2].

N_{ch}	ChQM2		ChQM2a		QDCSM0		QDCSM1		QDCSM3	
	M	Γ	M	Γ	M	Γ	M	Γ	M	Γ
1c	2425	–	2430	–	2413	–	2365	–	2276	–
2cc	2428	17	2433	10	2416	20	2368	20	2278	19
4cc	2413	14	2424	9	2400	14	$\boxed{2357}$	14	2273	17
10cc	$\boxed{2393}$	14			–	–	–	–	–	–
10cc′	2353	17			–	–	–	–	–	–
10cc″	2351	21			–	–	–	–	–	–

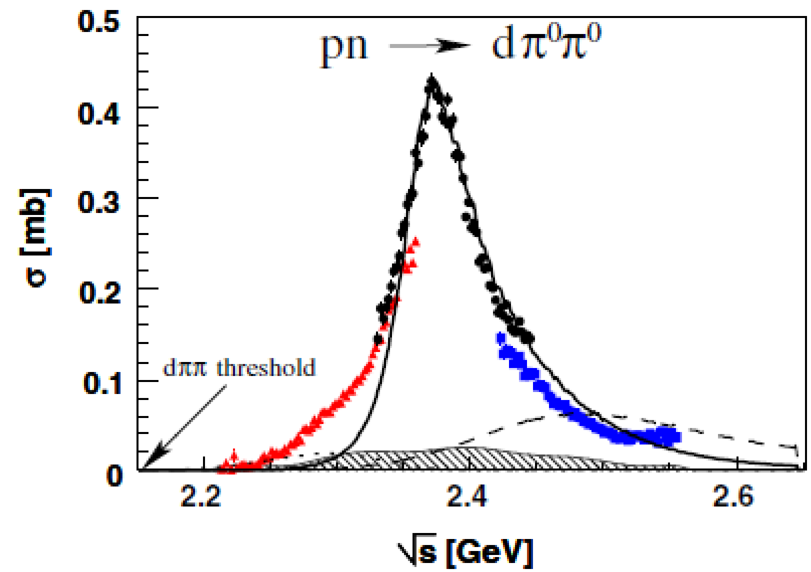


WASA-at-COSY

PRL 102, 052301 (2009)



PRL 106, 242302 (2011)

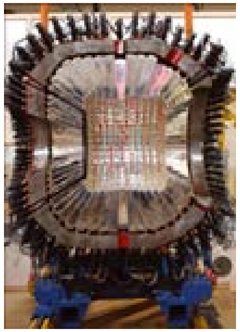


$$m = 2.37\ \mathrm{GeV}, \Gamma \approx 70\ \mathrm{MeV}\ \text{and}\ I(J^P) = 0(3^+)$$

CERN Courier

Aug 26, 2011

COSY finds evidence for an exotic particle…



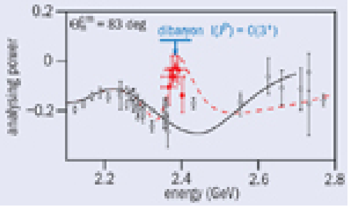
WASA detector

Experiments at the Jülich Cooler Synchrotron, COSY, have found evidence for a new complex state in the two-baryon system, with mass 2.37 GeV and width 70 MeV. The structure, containing six valence quarks, could constitute either an exotic compact particle or a hadronic molecule. The result could cast light on the long-standing question of whether there are eigenstates in the two-baryon system other than the deuteron ground-state. This has awaited an answer since Robert Jaffe first envisaged the possible existence of non-trivial six-quark configurations in QCD in 1977.

The new structure has been observed in high-precision measurements carried out by the WASA-at-COSY collaboration, using the Wide-Angle Shower Apparatus (WASA). The data exhibit a narrow isoscalar resonance-like structure in neutron-proton collisions for events where a deuteron is produced together with a pair of neutral pions. From the differential distributions, the spin-parity of the new system is deduced to

Jul 23, 2014

COSY confirms existence of six-quark states



Energy dependence

Experiments at the Jülich Cooler Synchrotron (COSY) have found compelling evidence for a new state in the two-baryon system, with a mass of 2380 MeV, width of 80 MeV and quantum numbers $I(J^P) = 0(3^+)$. The structure, containing six valence quarks, constitutes a dibaryon, and could be either an exotic compact particle or a hadronic molecule. The result answers the long-standing question of whether there are more eigenstates in the two-baryon system than just the deuteron ground-state. This fundamental question has been awaiting an answer since at least 1964, when first Freeman Dyson and later Robert Jaffe envisaged the possible existence of non-trivial six-quark configurations.

- http://cerncourier.com/cws/article/cern/

(2) Systematically searching for strange dibaryon states

PRC 83,015202 (2011)

S	I	J	State	M(MeV)	Γ(MeV)	Decay channels
-1	$\frac{1}{2}$	3	$\Delta\Sigma^*$	2440-2540	48-118	$N\Lambda$ or $N\Sigma$
-2	0	2	$N\Xi^*$	2400-2430	10-11	$N\Xi$ or $\Lambda\Lambda$
-2	1	3	$\Delta\Xi^*$	2620-2660	50-90	$N\Xi$
-3	$\frac{1}{2}$	2	$N\Omega$	2528-2547	2-4	$\Lambda\Xi$
-3	$\frac{3}{2}$	3	$\Sigma^*\Xi^*$	2788-2795	50-60	$\Sigma\Xi$

(3) $N\Omega$, IJ=1/2 2

Might be a narrow **resonance**.

A very narrow **resonance**.

PRC 51, 34

A weakly **bound** state.

A narrow **resonance** in $\Lambda\Xi$ scatteri

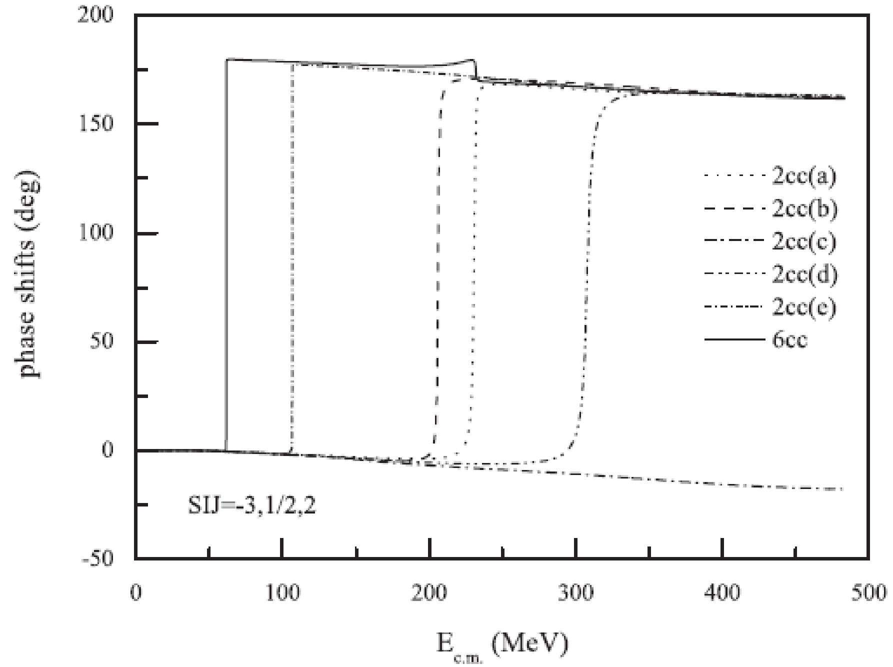


FIG. 10. The D-wave $\Lambda\Xi$ phase shifts with $I, J = 1/2, 2$ in QDCSM. 2cc(a), 2cc(b), 2cc(c), 2cc(d), and 2cc(e) stand for $\Lambda\Xi$ coupling to $\Sigma\Xi^*$, $\Xi\Sigma^*$, $\Lambda\Xi^*$, $\Sigma^*\Xi^*$, and $N\Omega$. The six-channel coupling is denoted by 6cc.

HALQCD Collaboration, arXiv:1403.7284

Being searched by RHIC-STAR Collaboration

Is it possible to observe $\Lambda\Xi$ scattering in the experiment?

$$\Xi \to \Lambda\pi, \qquad \Lambda \to p\pi$$

Neither Λ nor Ξ is stable under strong interactions.
It is **difficult** to observe $\Lambda\Xi$ scattering in the experiment.

NΩ scattering is possible to be observed in the experiment.

$$\Omega \to \Lambda K$$

N is stable under strong interactions.

Further theoretical work:

- NΩ scattering phase shifts;
- The scattering length and the effective range;
- The binding energy.

- Theoretical methods

Lattice QCD

QCD sum rule

Phenomenological models:

On the hadron level:

One-boson-exchange (OBE) model,

One-pion-exchange (OPE) model

......

On the quark level:

Isgur quark model , Chiral quark model,

Quark delocalization color screening model

......

Quark models and calculation methods

- **Quark Models**

1. Quark delocalization color screening model (QDCSM)

- QDCSM was developed by Nanjing-Los Alamos collaboration in1990s aimed to multi-quark study. PRL 69, 2901 (1992)

- Two new ingredients (based on quark cluster model configuration):

  quark delocalization (orbital excitation)
  color screening (color structure)

  Provide the intermediate-range attraction.

  Take into account the effect of hidden-color channel-coupling.

  PRC 84, 064001 (2011)

- Apply to baryon-baryon interaction and dibaryons

2. Chiral quark model (ChQM)

Provide the intermediate-range attraction by σ meson-exchange.

Rep. Prog. Phys. 68, 965 (2005)

SU(2) ChQM: only σ meson-exchange;

$$V_{ij}^{\sigma} = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2} m_\sigma \left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \right]$$

SU(3) ChQM: full scalar octet meson-exchange.

PRC 75, 034002 (2007)

$$V_{ij}^{\sigma_a} = V_{a_0}(r_{ij}) \sum_{a=1}^{3} \lambda_i^a \cdot \lambda_j^a + V_\kappa(r_{ij}) \sum_{a=4}^{7} \lambda_i^a \cdot \lambda_j^a + V_{f_0}(r_{ij}) \lambda_i^8 \cdot \lambda_j^8 + V_\sigma(r_{ij}) \lambda_i^0 \cdot \lambda_j^0$$

$$H = \sum_{i=1}^{6}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} + \sum_{j>i=1}^{6}\left(V_{ij}^{C} + V_{ij}^{G} + V_{ij}^{\chi} + V_{ij}^{\sigma}\right)$$

$$V_{ij}^{G} = \frac{1}{4}\alpha_s \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left[\frac{1}{r_{ij}} - \frac{\pi}{2}\delta(\boldsymbol{r}_{ij})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3m_i m_j}\right) - \frac{3}{4m_i m_j r_{ij}^3} S_{ij}\right]$$

$$V_{ij}^{\chi} = V_\pi(\boldsymbol{r}_{ij})\sum_{a=1}^{3}\lambda_i^a \cdot \lambda_j^a + V_K(\boldsymbol{r}_{ij})\sum_{a=4}^{7}\lambda_i^a \cdot \lambda_j^a + V_\eta(\boldsymbol{r}_{ij})\left[(\lambda_i^8 \cdot \lambda_j^8)\cos\theta_P - (\lambda_i^0 \cdot \lambda_j^0)\sin\theta_P\right]$$

$$V_{ij}^{C} = -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c ({r_{ij}}^2 + v_0)$$

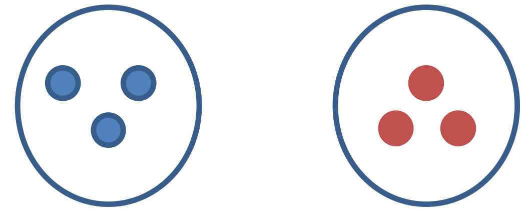
$$\Downarrow$$

$$V_{ij}^{C} = \begin{cases} -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \ (r_{ij}^2 + v_0) & \text{if } i,j \text{ in the same} \\ & \text{baryon orbit} \\ -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \ \left(\frac{1-e^{-\mu_{ij}\mathbf{r}_{ij}^2}}{\mu_{ij}} + v_0\right) & \text{otherwise} \end{cases}$$

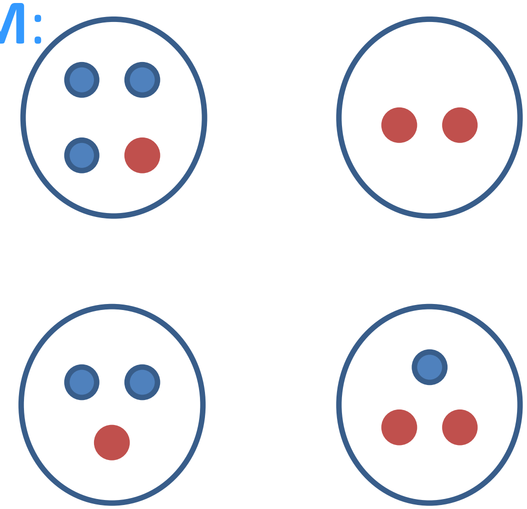
The single-particle orbital wave functions in **ChQM**:

$$\phi_\alpha(\vec{S}_i) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\vec{r}_\alpha - \vec{S}_i/2)^2},$$

$$\phi_\beta(-\vec{S}_i) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\vec{r}_\beta + \vec{S}_i/2)^2},$$



Delocalized orbital wave functions in **QDCSM**:



$$\psi_\alpha(\mathbf{S}_i, \epsilon) = (\phi_\alpha(\mathbf{S}_i) + \epsilon\phi_\alpha(-\mathbf{S}_i))/N(\epsilon),$$

$$\psi_\beta(-\mathbf{S}_i, \epsilon) = (\phi_\beta(-\mathbf{S}_i) + \epsilon\phi_\beta(\mathbf{S}_i))/N(\epsilon),$$

$$N(\epsilon) = \sqrt{1 + \epsilon^2 + 2\epsilon e^{-S_i^2/4b^2}},$$

$$\phi_\alpha(\mathbf{S}_i) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\mathbf{r}_\alpha - \mathbf{S}_i/2)^2},$$

$$\phi_\beta(-\mathbf{S}_i) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\mathbf{r}_\beta + \mathbf{S}_i/2)^2}.$$

➢ Calculation methods

(1) Resonating group method (RGM)

In RGM, the multi-quark wave function is approximated by the cluster wave function,

$$\psi(\xi_1, \xi_2, \boldsymbol{R}) = \mathcal{A}[\phi(\xi_1)\phi(\xi_2)\chi(\boldsymbol{R})]$$

The internal motions of clusters are frozen and the relative motion wave function satisfies the following RGM equation

$$\int H(R'', R')\chi(R')dR' = E\int N(R'', R')\chi(R')dR'$$

$$\left\{\begin{array}{c} H(R'', R') \\ N(R'', R') \end{array}\right\} = \left\langle A[\phi_1\phi_2\delta(R - R'')] \middle| \left\{\begin{array}{c} H \\ 1 \end{array}\right\} \middle| A[\phi_1\phi_2\delta(R - R')] \right\rangle$$

RGM equation

$$\int L(R'', R')\chi(R')dR' = 0$$

where

$$L(R'', R') = H(R'', R') - EN(R'', R')$$

$$= [-\frac{\nabla^2_{R'}}{2\mu} + V^D_{rel}(R') - E_{rel}]\delta(R'' - R') + H^{EX}(R'', R') - EN^{EX}(R'', R').$$

(2) Generating coordinates method (GCM)

Extending the relative motion wave function to the Gaussian function:

$$\chi(\vec{R}) = \sum_i C_i \chi_i(\vec{R}) = (\frac{3}{2\pi b^2})^{3/4} \sum_i C_i e^{-\frac{3}{4}(\vec{R}-\vec{S}_i)^2/b^2}$$

$$\phi_C(\vec{R_C}) = (\frac{6}{\pi b^2})^{\frac{3}{4}} e^{-\frac{3}{b^2}(\vec{R_c})^2}$$

$$\Psi_{6q} = A \sum_i C_i \prod_{\alpha=1}^{3} \phi_\alpha(\vec{S_i}) \prod_{\beta=4}^{6} \phi_\beta(-\vec{S_i}) \\ [\eta_{I_1 S_1}(B_1)\eta_{I_2 S_2}(B_2)]^{IS} [\chi_c(B_1)\chi_c(B_2)]^{[\sigma]}$$

$$\Psi_{6q} \;=\; A\sum_{k}\sum_{i,L_k} C_{k,i,L_k}\int\frac{d\Omega_{S_i}}{\sqrt{4\pi}}\prod_{\alpha=1}^{3}\psi_\alpha(\vec{S}_i,\epsilon)\prod_{\beta=4}^{6}\psi_\beta(-\vec{S}_i,\epsilon)$$
$$\left[[\eta_{I_{1k}S_{1k}}(B_{1k})\eta_{I_{2k}S_{2k}}(B_{2k})]^{IS_k}Y^{L_k}(\hat{\vec{S}}_i)\right]^{J}[\chi_C(B_1)\chi_c(B_2)]^{[\sigma]}$$

$$\int H(\vec{R},\vec{R}')\chi(\vec{R}')d\vec{R}' = E\int N(\vec{R},\vec{R}')\chi(\vec{R}')d\vec{R}'$$

$$\Downarrow$$

$$\sum_{j,k,L_k} C_{j,k,L_k} H_{i,j}^{k',L'_k,k,L_k} = E\sum_{j,k,L_k} C_{j,k,L_k} N_{i,j}^{k',L'_k,k,L_k}\delta_{L'_k,L_k}$$

(3) Kohn-Hulthen-Kato(KHK) variational method

M.Kamimura, Supp.Prog.Theo.Phys.62,236 (1977)

$$u_t(R) = \sum_{i=0}^{n} c_i u_i(R)$$

$$u_i(R) = \begin{cases} \alpha_i u_i^{(in)}(R), & R < R_c, \\ (h_L^{(-)}(k,R) + s_i h_L^{(+)}(k,R))R, & R > R_c \end{cases}$$

$$\frac{u_i^{(in)}(R)}{R} = \sqrt{4\pi}\left(\frac{3}{2\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{3}{4b^2}(R^2+r_i^2)} i^L j_L\left(-i\frac{3}{2b^2}Rr_i\right)$$

$$\sum_{i=0}^{n} c_i = 1, \qquad \sum_{i=0}^{n} c_i s_i = S_t.$$

$$\Downarrow$$

$$c_0 = 1 - \sum_{i=1}^{n} c_i \quad \Rightarrow \quad u_t(R) = u_0(R) + \sum_{i=1}^{n} c_i (u_i(R) - u_0(R))$$

$$\langle \delta\Psi' | H - E | \Psi \rangle = 0$$

$$\sum_{j=1}^{n} \mathcal{L}_{ij} c_j = \mathcal{M}_i, (i = 1 \sim n)$$

$$\mathcal{L}_{ij} = \mathcal{K}_{ij} - \mathcal{K}_{i0} - \mathcal{K}_{0j} + \mathcal{K}_{00},$$

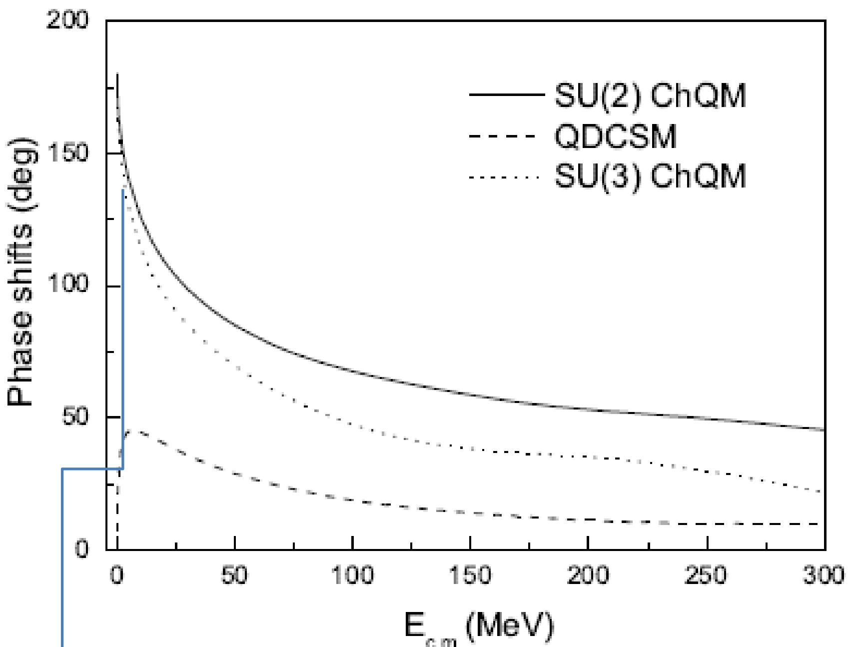
$$\mathcal{M}_i = \mathcal{K}_{00} - \mathcal{K}_{i0}$$

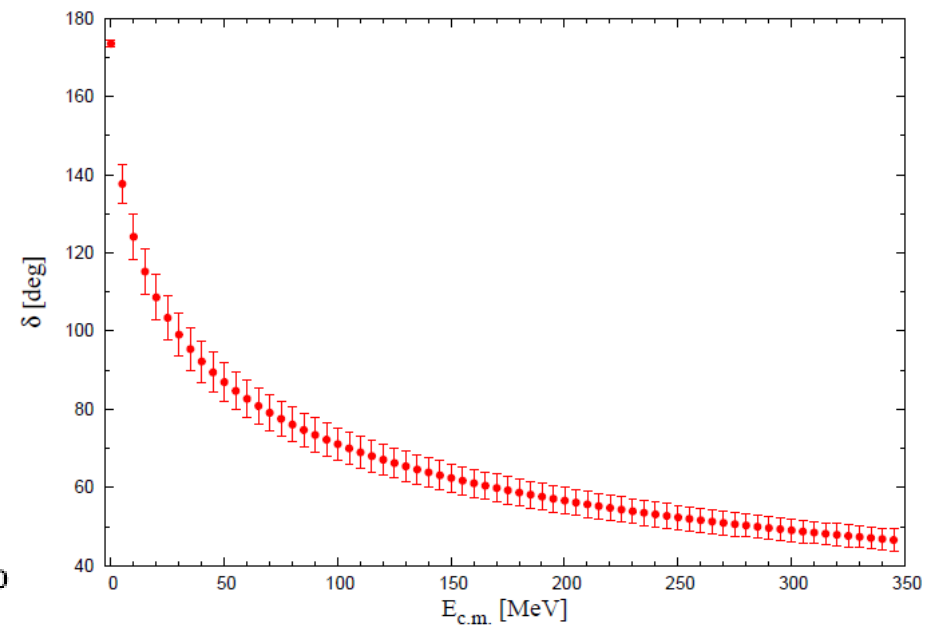
$$\mathcal{K}_{ij} = \langle \phi_A(\xi_A)\phi_B(\xi_B)u_i(R)/R \cdot Y_{LM}(\hat{R}) | H - E | \mathcal{A}[\phi_A(\xi_A)\phi_B(\xi_B)u_j(R)/R \cdot Y_{LM}(\hat{R})] \rangle$$

$$c_i \xrightarrow{\sum_{i=0}^{n} c_i s_i = S_t} S_t \longrightarrow S_L \xrightarrow{S_L = |S_L| e^{2i\delta_L}} \delta_L$$

Results and discussions

1. The NΩ scattering phase shifts





implies the existence of a bound state

HALQCD Collaboration
arXiv:1403.7284

2. The scattering length, the effective range and the binding energy

$$k cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)$$

$$B = \frac{\hbar^2 \alpha^2}{2\mu} \qquad r_0 = \frac{2}{\alpha}(1 - \frac{1}{\alpha a_0})$$

TABLE II: The scattering length a_0, effective range r_0, and binding energy B of the $N\Omega$ dibaryon.

	a_0 (fm)	r_0 (fm)	B (MeV)	B' (MeV)
SU(2) ChQM	1.87	0.87	23.2	19.6
QDCSM	2.58	0.90	8.1	7.3
SU(3) ChQM	-4.23	2.10	ub	ub

$$\begin{aligned} B_{N\Omega} &= 18.9(5.0)(^{+12.1}_{-1.8}) \text{ MeV}, \\ a_{N\Omega} &= -1.28(0.13)(^{+0.14}_{-0.15}) \text{ fm}, \\ (r_e)_{N\Omega} &= 0.499(0.026)(^{+0.029}_{-0.048}) \text{ fm}. \end{aligned}$$

HALQCD Collaboration
arXiv:1403.7284

3. The effective potential from various terms of NΩ interaction

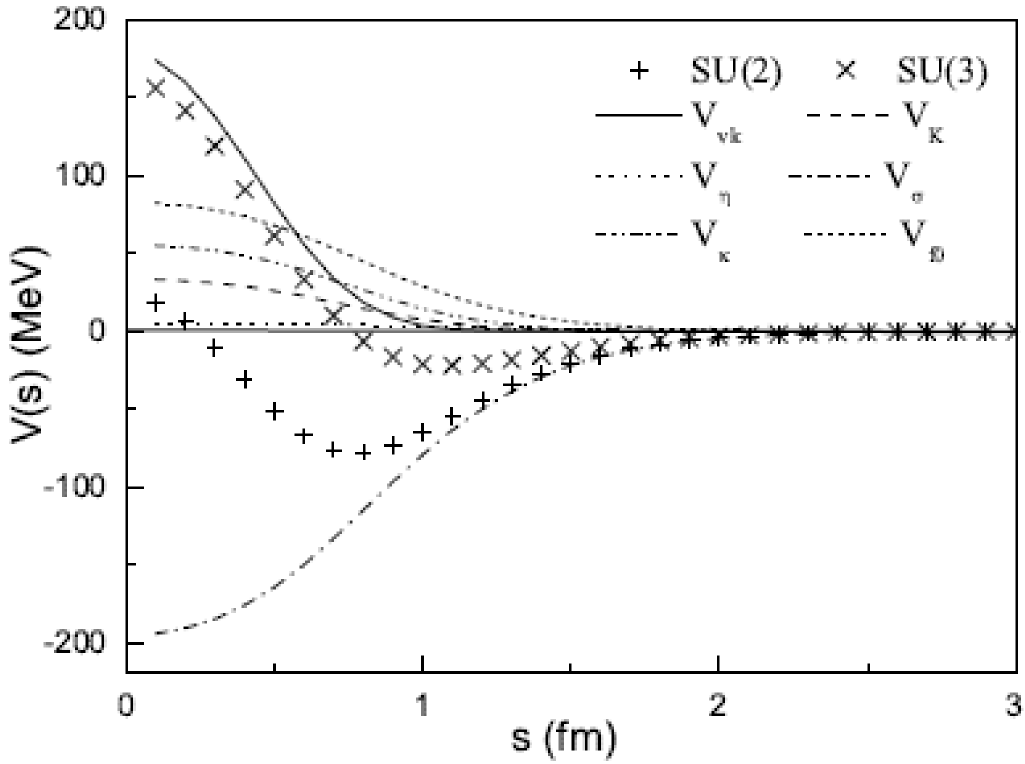


FIG. 2: The contributions to the effective potential from various terms of interactions in ChQM.

- No quark exchange between N and Ω, no contributions of CON, OGE, OPE.
- the attraction mainly comes from the σ meson-exchange interaction.
- other scalar meson-exchanges provide the repulsive potentials.

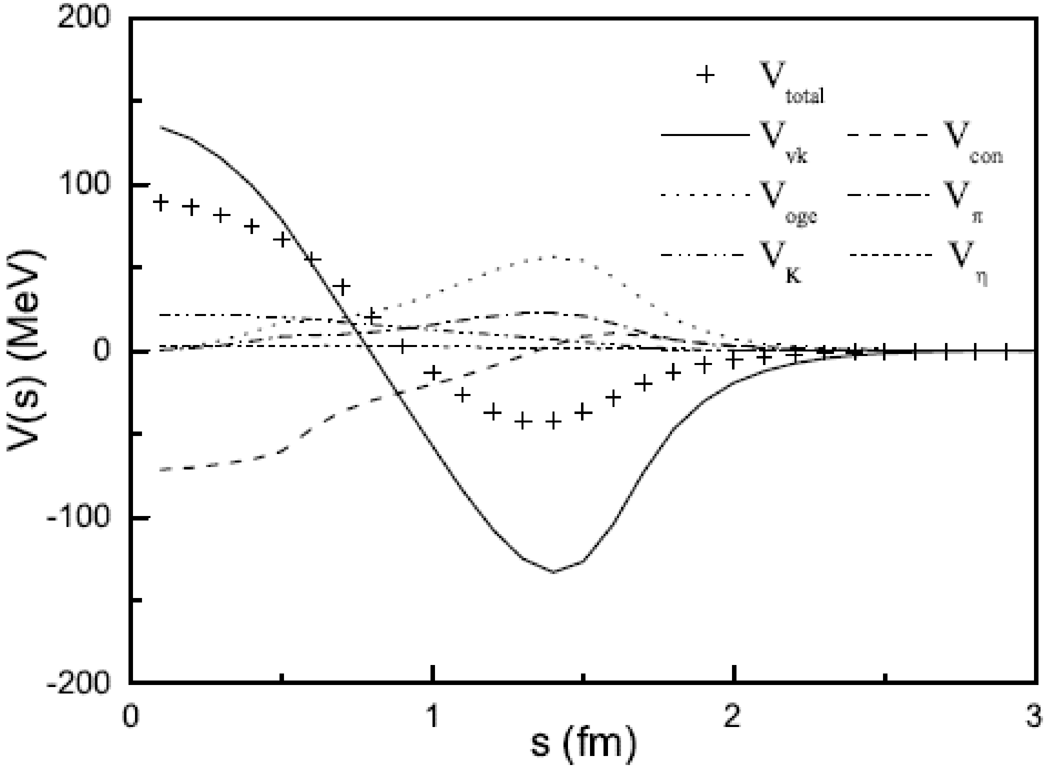


FIG. 3: The contributions to the effective potential from various terms of interactions in QDCSM.

- the attraction mainly comes from the kinetic energy term.
- quark delocalization contributes to the deep attraction of the kinetic energy term.

4. The effect of the channel-coupling

Color-singlet channel: $[111] \otimes [111] \rightarrow [222]$

Hidden-color channel: $[21] \otimes [21] \rightarrow [222]$

TABLE III: The binding energy with channel-coupling.

	B_{sc} (MeV)	B_{5cc} (MeV)	B_{16cc} (MeV)
SU(2) ChQM	19.6	48.8	119.5
QDCSM	7.3	52.1	–
SU(3) ChQM	ub	ub	23.4

- Hidden-color channel-coupling is crucial in binding NΩ.
- QDCSM includes the effect of hidden-color channel-coupling.

Summary

1. NΩ is a narrow resonance in $\Lambda\Xi$ D-wave scattering.

2. NΩ low-energy scattering phase shifts imply the existence of NΩ, which can be observed in RHIC-STAR.

3. Both QDCSM and ChQM, with different intermediate-range attraction mechanisms, can obtain the NΩ dibaryon.

4. Hidden-color channel-coupling is crucial in binding NΩ. QDCSM includes the effect of hidden-color channel-coupling.

Thank you!