Parity-violating nucleon-nucleon force in the $1/N_c$ expansion

(D. Phillips, DS and C. Schat)
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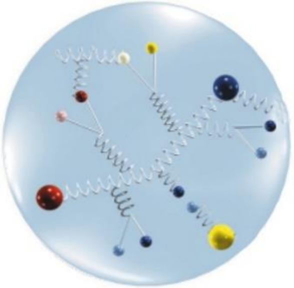
Outline

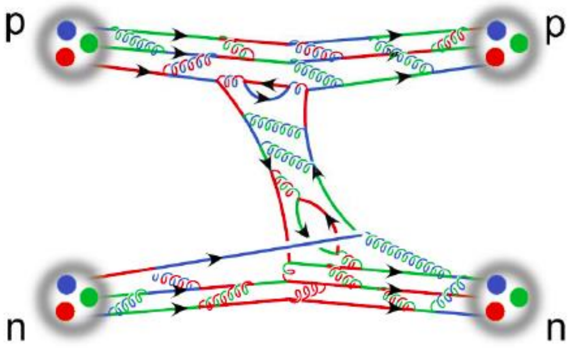
- Introduction
- $1/N_c$ operators
- Parity-violating NN potential in $1/N_c$ expansion
- Matching DDH potential
- N_c scaling of DDH couplings
- Conclusions

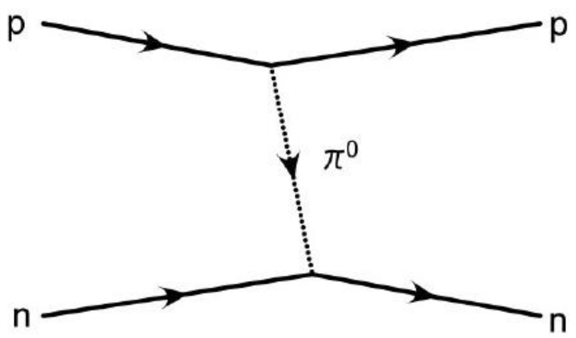
Introduction

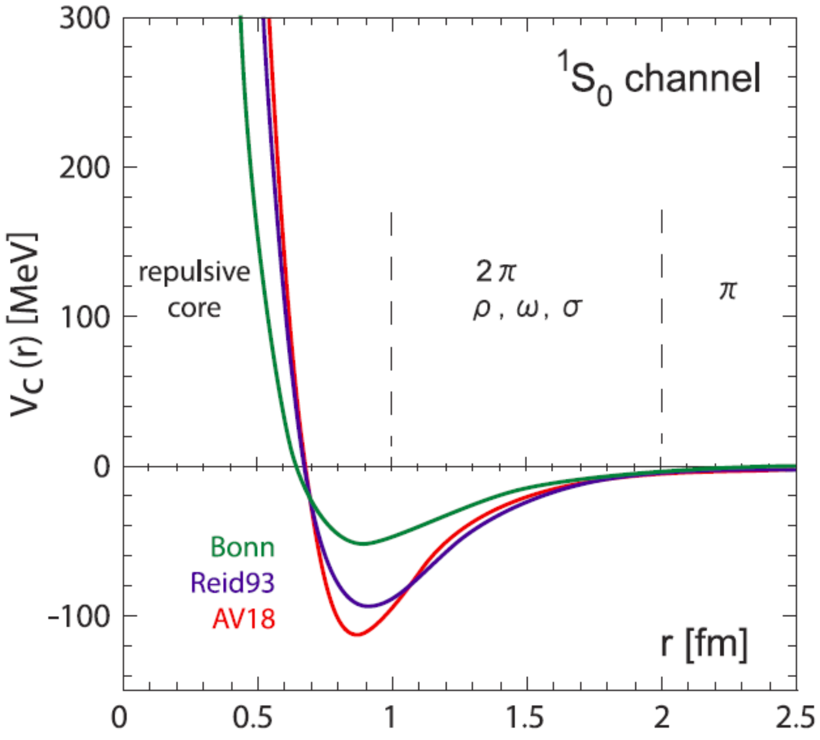
- Nucleon is composite particle which is made of 3 valence quarks.



- Nucleon-nucleon (NN) scattering is used to study the strong-nuclear and electromagnetic forces between the nucleons.







Introduction

- There are parity-violating (PV) NN interactions which manifest the presence of weak interactions between the quarks inside each nucleon.

- The weak force is smaller than the strong force in order ~ 10^{-7}.

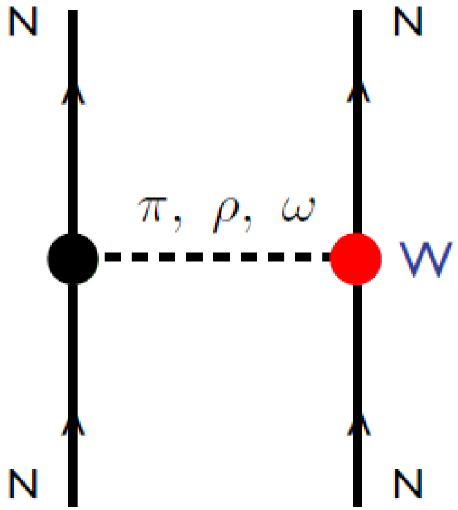
- The analyzing power for the $\vec{p}+p$ scattering experiments from

Bonn	$A_L(\vec{\mathrm{p}}\mathrm{p}; 13.6\ \mathrm{MeV}) = (-0.93 \pm 0.20 \pm 0.05) \times 10^{-7}$
PSI	$A_L(\vec{\mathrm{p}}\mathrm{p}; 45\ \mathrm{MeV}) = (-1.57 \pm 0.23) \times 10^{-7}$
TRIUMF	$A_L(\vec{\mathrm{p}}\mathrm{p}; 221\ \mathrm{MeV} = (+0.84 \pm 0.34) \times 10^{-7}$

demonstrate that PV NN forces exist.

Introduction

- The most popular framework to study the PV NN force is Desplanques, Donoghue, and Holstein (DDH) model which is based on single meson exchange picture.



(Desplanques, Donoghue, and Holstein, 1980)

- 7 meson-nucleon coupling constants: range + isospin structures

$$h^1_\pi\,,\ h^0_\rho\,,\ h^1_\rho\,,\ h^{1\prime}_\rho\,,\ h^2_\rho\,,\ h^0_\omega\,,\ h^1_\omega$$

- PV force: interference between strong (black) and weak (red) vertices

$$\begin{aligned}\mathcal{H}_{\rm wk} &= \frac{h^1_\pi}{\sqrt{2}}\bar{N}(\vec{\tau}\times\vec{\pi})_z N \\ &+ \bar{N}\left(h^0_\rho\vec{\tau}\cdot\vec{\rho}^\mu + h^1_\rho\rho^\mu_z + \frac{h^2_\rho}{2\sqrt{6}}(3\tau_z\rho^\mu_z - \vec{\tau}\cdot\vec{\rho}^\mu)\right)\gamma_\mu\gamma_5 N \\ &+ \bar{N}\left(h^0_\omega\omega^\mu + h^1_\omega\tau_z\omega^\mu\right)\gamma_\mu\gamma_5 N - h^{1\prime}_\rho\bar{N}(\vec{\tau}\times\vec{\rho}^\mu)_z\frac{\sigma_{\mu\nu}k^\nu}{2m_N}\gamma_5 N\end{aligned}$$

DDH potential

$$
\begin{aligned}
V_{DDH}^{\rm PNC}(\vec{r}) &= i\frac{h_\pi^1 \bar{g}_{\pi NN}}{\sqrt{2}}\left(\frac{\vec{\tau}_1\times\vec{\tau}_2}{2}\right)_z(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2m_N}, w_\pi(r)\right] \\
&-g_\rho\left(h_\rho^0\vec{\tau}_1\cdot\vec{\tau}_2 + h_\rho^1\left(\frac{\vec{\tau}_1+\vec{\tau}_2}{2}\right)_z + h_\rho^2\frac{(3\tau_1^z\tau_2^z-\vec{\tau}_1\cdot\vec{\tau}_2)}{2\sqrt{6}}\right) \\
&\quad\times\left((\vec{\sigma}_1-\vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1-\vec{p}_2}{2m_N}, w_\rho(r)\right\} + i\left(1+\frac{\chi_V}{\Lambda}\right)\vec{\sigma}_1\times\vec{\sigma}_2\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2}, w_\rho(r)\right]\right) \\
&-g_\omega\left(h_\omega^0 + h_\omega^1\left(\frac{\vec{\tau}_1+\vec{\tau}_2}{2}\right)_z\right) \\
&\quad\times\left((\vec{\sigma}_1-\vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1-\vec{p}_2}{2m_N}, w_\omega(r)\right\} + i\left(1+\frac{\chi_S}{\Lambda}\right)\vec{\sigma}_1\times\vec{\sigma}_2\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2}, w_\omega(r)\right]\right) \\
&+\left(\frac{\vec{\tau}_1-\vec{\tau}_2}{2}\right)_z(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left(g_\rho h_\rho^1\left\{\frac{\vec{p}_1-\vec{p}_2}{2m_N}, w_\rho(r)\right\} - g_\omega h_\omega^1\left\{\frac{\vec{p}_1-\vec{p}_2}{2m_N}, w_\omega(r)\right\}\right) \\
&-\frac{g_\rho h_\rho^{1'}}{\Lambda}\, i\left(\frac{\vec{\tau}_1\times\vec{\tau}_2}{2}\right)_z(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2}, w_\rho(r)\right].
\end{aligned}
$$

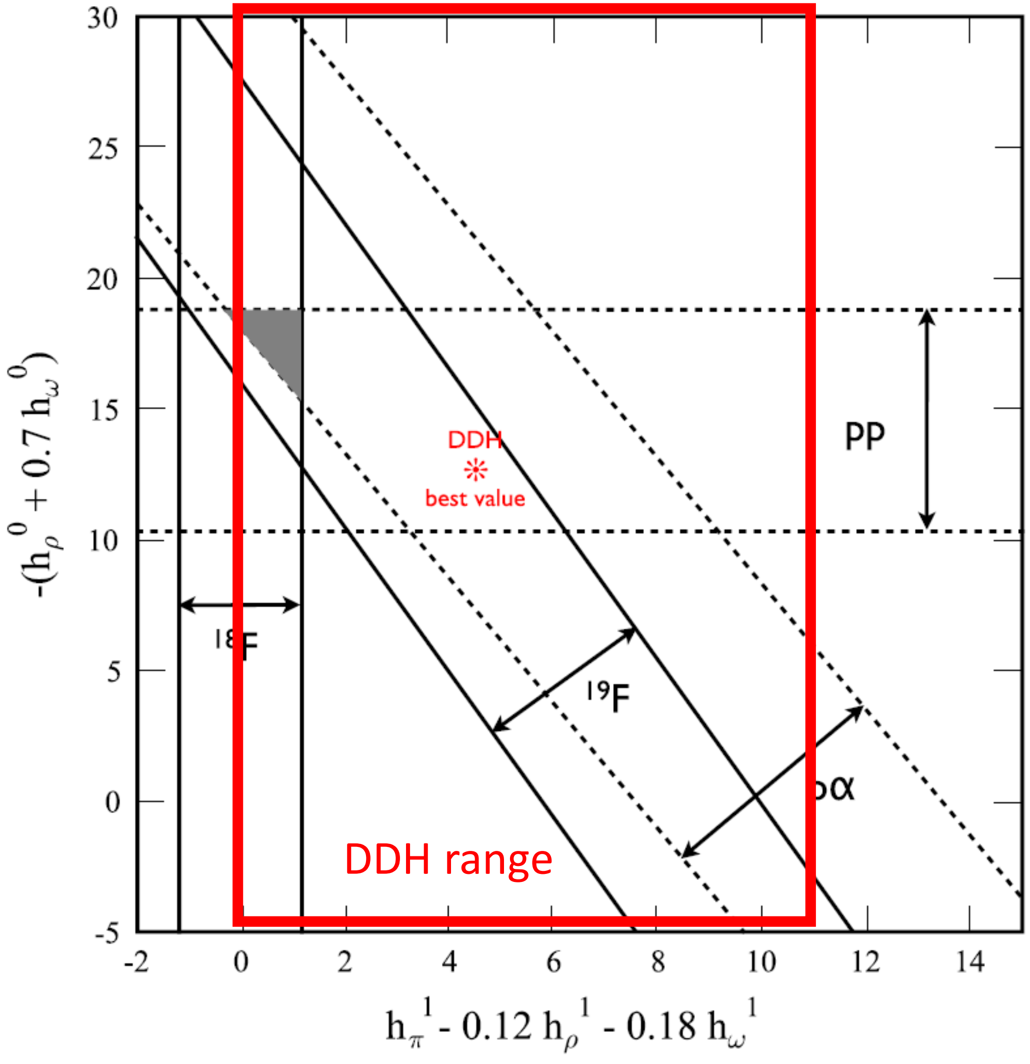
Introduction

Coupling	DDH Reasonable Range	DDH "Best" Value
h_π^1	$0 \to 11.4$	$+4.5$
h_ρ^0	$-30.8 \to 11.4$	-11.4
h_ρ^1	$-0.4 \to 0.1$	-0.2
h_ρ^2	$-11 \to -7.6$	-9.5
$h_\rho^{1\prime}$	–	0
h_ω^0	$-10.3 \to 5.7$	-1.9
h_ω^1	$-1.9 \to 0.76$	-1.1

with unit of 10^{-7}

Coupling values are given by $SU(6)_W$ quark model calculation

(Haxton and Holstein, 2013)



Introduction

This work is to show that Standard Model coupling $\sin^2 \theta_W$ and the $1/N_c$ expansion of QCD predict the operators, and the sizes of the associated DDH coupling constants, which appear in the PV NN potential.

SM effective Lagrangian for PV

$$\mathcal{L}_W^{\rm eff} = \frac{G_F}{\sqrt{2}} \cos^2\theta_C \sum_{i=1}^{2} \left(V^\mu_{(i)} + A^\mu_{(i)} \right)^2 + \frac{G_F}{\sqrt{2}} \left(\cos 2\theta_W \, V^\mu_{(3)} - A^\mu_{(3)} - 2\sin^2\theta_W \, I^\mu \right)^2$$

$$= \frac{G_F}{\sqrt{2}} \left\{ \cos^2\theta_C \sum_{i=1}^{2} V_\mu^{(i)} A^\mu_{(i)} + \cos 2\theta_W \, V^\mu_{(3)} A^\mu_{(3)} - 2\sin^2\theta_W \, I_\mu A^\mu_{(3)} \right\} + \cdots ,$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad V^\mu_{(i)} = \frac{1}{2}\bar{q}\gamma^\mu \tau^{(i)} q, \quad A^\mu_{(i)} = \frac{1}{2}\bar{q}\gamma^\mu\gamma_5\tau^{(i)} q, \qquad I^\mu = \frac{1}{6}\bar{q}\gamma^\mu q$$

Importantly, the factor $\sin^2\theta_W$ ($\approx 1/5$) is the suppression factor only for $\Delta I = 1$ operators.

$$\langle M\, N | \mathcal{H}^{\rm eff}_{4q} | N \rangle \;\; \rightarrow \;\; h^1_\pi \sim h^1_\rho \sim h^{1\prime}_\rho \sim h^1_\omega \sim \sin^2\theta_W$$

$$h^0_\rho \sim h^2_\rho \sim h^0_\omega \sim 1$$

Baryon in large-N_c

- QCD can be expanded number of color in N_c
- bound state of N_c quarks completely anti-symmetric in color
- Ordering of effects in powers of $1/N_c$
- Hadrons in N_c scaling $M_{\text{meson}} \sim N_c^0$ $M_{\text{baryon}} \sim N_c$ (Witten, 1979)

$$f_\pi \sim \sqrt{N_c} \qquad g^A_{n\pi NN} \sim N_c^{1-\frac{n}{2}}$$

- Systematic expanding baryon-matrix elements of QCD quark currents in powers of $1/N_c$
- For baryons: dynamical spin-flavor symmetry in large N_c limit (Luty and March-Russell, 1994; Dashen, Jenkins and Manohar, 1995)

Effective operators in 1/Nc expansion

Building blocks

$$I^a = \sum_{\alpha=1}^{N_c} \frac{\tau_\alpha^a}{2} \qquad S^i = \sum_{\alpha=1}^{N_c} \frac{\sigma_\alpha^i}{2} \qquad G^{ia} = \sum_{\alpha=1}^{N_c} \frac{\sigma_\alpha^i \tau_\alpha^a}{4}$$

$$< \mathrm{N}|I|\mathrm{N} > \sim N_c^0\,, \quad < \mathrm{N}|S|\mathrm{N} > \sim N_c^0\,, \quad < \mathrm{N}|G|\mathrm{N} > \sim N_c$$

Only valid for the N → N piece of the Hilbert space

Effective Hamiltonian

$$H = N_c \sum_{s,t,m} v_{stm} \left(\frac{S}{N_c}\right)^s \left(\frac{I}{N_c}\right)^t \left(\frac{G}{N_c}\right)^m$$

(Carone et al.,1994; Luty and March-Russell, 1994; Phillips and Schat, 2013)

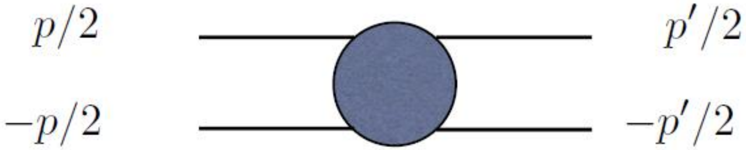
Operators N_c scaling and its transformations

O	Order	$O_{\tau\tau}$	Order	S	T
$\mathbb{1}$	1	$\tau_1 \cdot \tau_2$	$1/N_c^2$	0	$+$
$\sigma_1 \cdot \sigma_2$	$1/N_c^2$	$\sigma_1 \cdot \sigma_2\ \tau_1 \cdot \tau_2$	1	0	$+$
σ_1^i	$1/N_c$	$\sigma_1^i\ \tau_1 \cdot \tau_2$	$1/N_c$	1	$-$
σ_2^i	$1/N_c$	$\sigma_2^i\ \tau_1 \cdot \tau_2$	$1/N_c$	1	$-$
$(\sigma_1 \times \sigma_2)^k$	$1/N_c^2$	$(\sigma_1 \times \sigma_2)^k\ \tau_1 \cdot \tau_2$	1	1	$+$
$[\sigma_1^i \sigma_2^j]_2$	$1/N_c^2$	$[\sigma_1^i \sigma_2^j]_2\ \tau_1 \cdot \tau_2$	1	2	$+$

	T	P	P_{12}	Order-N_c
$\mathbf{p}_+$	$-$	$-$	$-$	$1/N_c$
$\mathbf{p}_-$	$+$	$-$	$-$	1

$$\mathbf{p}_\pm = \mathbf{p}' \pm \mathbf{p}$$

$$\mathbf{p}_+ \cdot \mathbf{p}_- = 0$$



(Phillips and Schat, 2013)

PV potential in $1/N_c$ operator expansion

$$U_{PV}^{N_c} = N_c \Big(U_P^1(\mathbf{p}_-^2)\, [\mathbf{p}_- \cdot (\sigma_1 \times \sigma_2)\, \tau_1 \cdot \tau_2]$$
$$+ U_P^2(\mathbf{p}_-^2)\, [\mathbf{p}_- \cdot (\sigma_1 \times \sigma_2)\, [\tau_1\, \tau_2]_2^{zz}] \Big) ,$$

$$U_{PV}^{N_c^0} = N_c^0 \Big(U_P^3(\mathbf{p}_-^2)\, [\mathbf{p}_+ \cdot (\sigma_1\, \tau_1^z - \sigma_2\, \tau_2^z)] + U_P^4(\mathbf{p}_-^2)\, [\mathbf{p}_- \cdot (\sigma_1 + \sigma_2)\, (\tau_1 \times \tau_2)^z]$$
$$+ U_P^5(\mathbf{p}_-^2)\, [\mathbf{p}_- \cdot (\sigma_1 \times \sigma_2)\, (\tau_1 + \tau_2)^z] + U_D^1(\mathbf{p}_-^2)\, [[(\mathbf{p}_+ \times \mathbf{p}_-)^i\, \mathbf{p}_-^j]_2 \cdot [\sigma_1^i\, \sigma_2^j]_2\, (\tau_1 \times \tau_2)^z] \Big)$$

$$U_{PV}^{N_c^{-1}} = N_c^{-1} \Big(U_P^6(\mathbf{p}_-^2)\, [\mathbf{p}_- \cdot (\sigma_1 \times \sigma_2)] + U_P^7(\mathbf{p}_-^2)\, [\mathbf{p}_+^2\, \mathbf{p}_- \cdot (\sigma_1 \times \sigma_2)\, \tau_1 \cdot \tau_2] + U_P^8(\mathbf{p}_-^2)\, [\mathbf{p}_+ \cdot (\sigma_1 - \sigma_2)]$$
$$+ U_P^9(\mathbf{p}_-^2)\, [\mathbf{p}_+ \cdot (\sigma_1 - \sigma_2)\, \tau_1 \cdot \tau_2] + U_P^{10}(\mathbf{p}_-^2)\, [\mathbf{p}_+ \cdot (\sigma_1 - \sigma_2)\, [\tau_1\, \tau_2]_2^{zz}] + U_P^{11}(\mathbf{p}_-^2)\, [\mathbf{p}_+^2\, \mathbf{p}_- \cdot (\sigma_1 \times \sigma_2)\, [\tau_1\, \tau_2]_2^{zz}] \Big)$$

Strong couplings N_c scaling

$$\chi_{V,S} \Rightarrow m_N \, \xi_{V,S} / \Lambda_\chi$$

(Kaplan and Manohar, 1997)

$$g_{IS} \sim N_c^{\frac{1}{2}-|I-S|}$$

$$g_{00} = g_\omega \sim \sqrt{N_c}\,, \qquad g_{01} = g_\omega \, \xi_S \sim \frac{1}{\sqrt{N_c}}\,, \qquad \xi_S \sim \frac{1}{N_c}\,,$$

$$g_{10} = g_\rho \sim \frac{1}{\sqrt{N_c}}\,, \qquad g_{11} = g_\rho \, \xi_V \sim \sqrt{N_c}\,, \qquad \xi_V \sim N_c\,,$$

$$g_{\pi NN} \sim N_c^{3/2} \qquad \Lambda_\chi \sim N_c^0$$

Matching $1/N_c$ PV and DDH

$$\bar{g}_{\pi NN}\, h_\pi^1 \sim \mathcal{O}(\sin^2\theta_W\, N_c)\,, \qquad g_\rho\, h_\rho^0 \sim \mathcal{O}(N_c^0)\,, \qquad g_\rho\, h_\rho^1 \sim \mathcal{O}(\sin^2\theta_W\, N_c^{-1})\,,$$

$$g_\rho\, h_\rho^2 \sim \mathcal{O}(N_c^0)\,, \qquad g_\rho\, \xi_V\, h_\rho^0 \sim \mathcal{O}(N_c)\,, \qquad g_\rho\, \xi_V\, h_\rho^1 \sim \mathcal{O}(\sin^2\theta_W\, N_c^0)\,,$$

$$g_\rho\, \xi_V\, h_\rho^2 \sim \mathcal{O}(N_c)\,, \qquad g_\omega\, h_\omega^0 \sim \mathcal{O}(N_c^0)\,, \qquad g_\omega\, h_\omega^1 \sim \mathcal{O}(\sin^2\theta_W\, N_c)\,,$$

$$g_\omega\, \xi_S\, h_\omega^0 \sim \mathcal{O}(N_c^{-1})\,, \qquad g_\omega\, \xi_S\, h_\omega^1 \sim \mathcal{O}(\sin^2\theta_W\, N_c^0)\,, \qquad g_\rho\, h_\rho^1 \sim \mathcal{O}(\sin^2\theta_W\, N_c^{-1})\,,$$

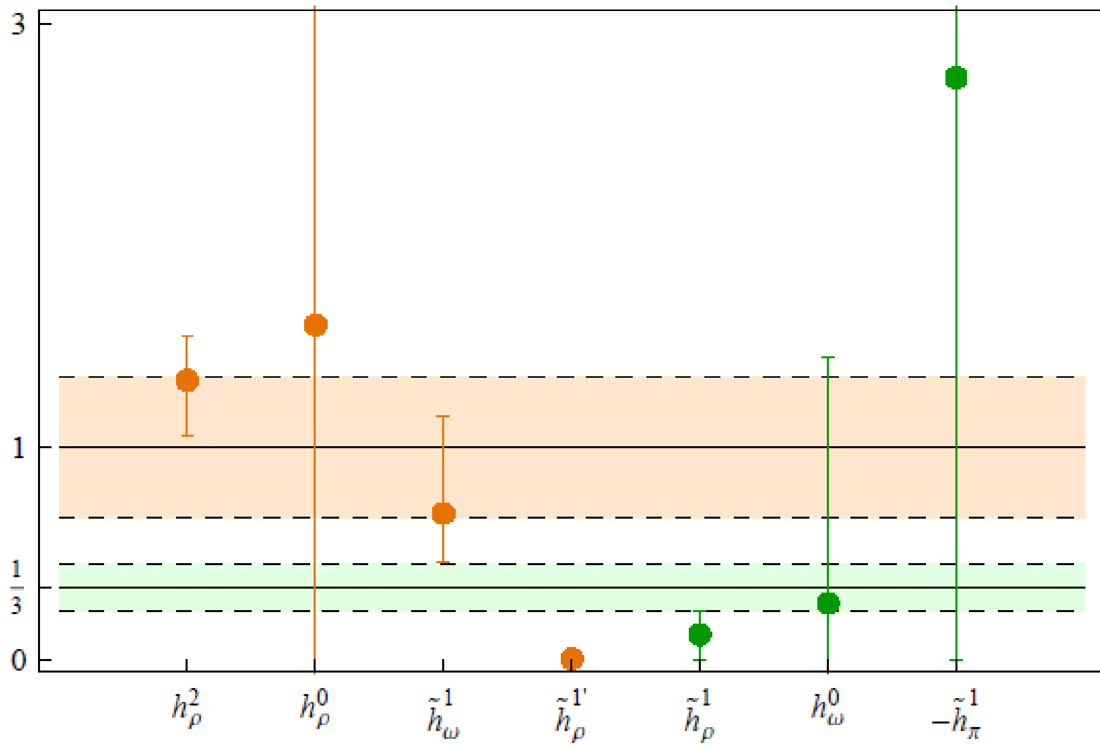
$$g_\omega\, h_\omega^1 \sim \mathcal{O}(\sin^2\theta_W\, N_c) \qquad g_\rho\, h_\rho^{1'} \sim \mathcal{O}(\sin^2\theta_W\, N_c^0)\,,$$

PV couplings N_c scalings

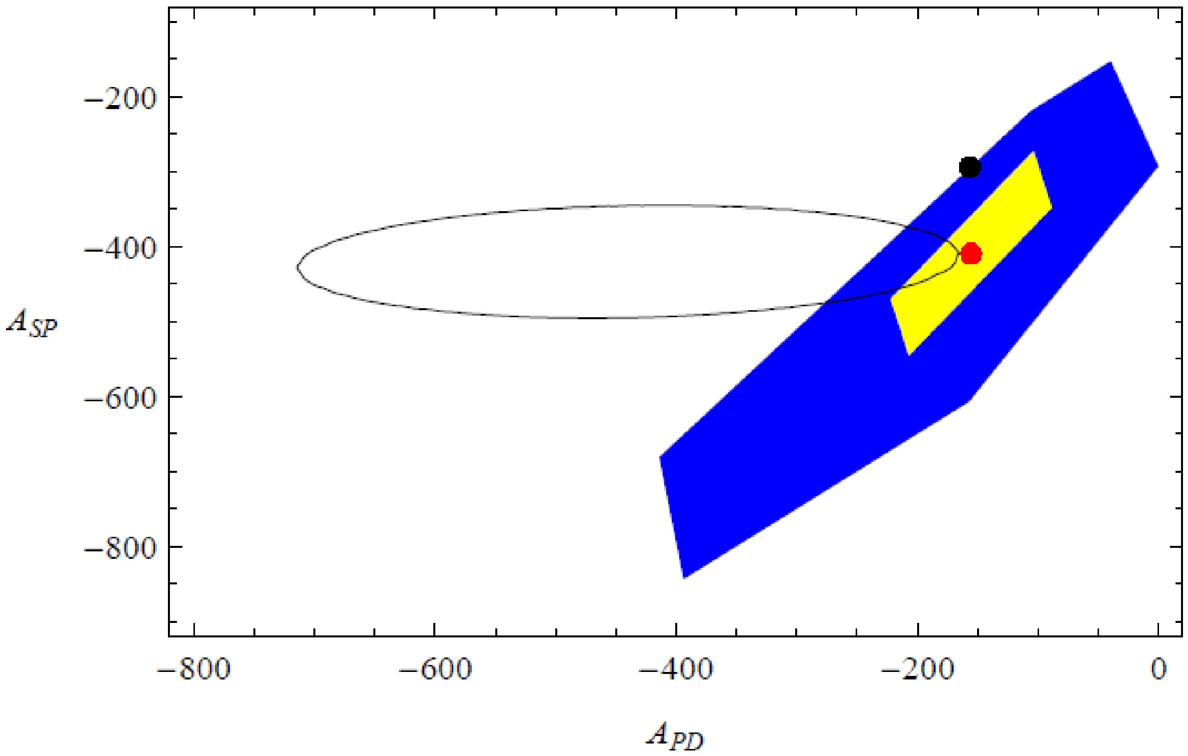
$$h_\rho^0 \sim \sqrt{N_c}\,, \quad \tilde{h}_\rho^1 = \frac{h_\rho^1}{\sin^2\theta_W} \sim \frac{1}{\sqrt{N_c}}\,, \quad h_\rho^2 \sim \sqrt{N_c}$$

$$\tilde{h}_\rho^{1'} = \frac{h_\rho^{1'}}{\sin^2\theta_W} \sim \sqrt{N_c}\,, \quad \tilde{h}_\pi = \frac{h_\pi^1}{\sin^2\theta_W} \sim \frac{1}{\sqrt{N_c}}\,,$$

$$h_\omega^0 \sim \frac{1}{\sqrt{N_c}}\,, \quad \tilde{h}_\omega^1 = \frac{h_\omega^1}{\sin^2\theta_W} \sim \sqrt{N_c}\,.$$



The $\vec{p}+p$ scattering: the plane-wave Born approximation



The couplings in A_{SP} and A_{PS} are occurred in PWBA by including ISI and FSI corrections (Carlson et al., 2002)

$$A_{SP} \equiv g_\rho h_\rho^{pp}(2+\chi_V) + g_\omega h_\omega^{pp}(2+\chi_S),$$
$$A_{PD} \equiv g_\rho h_\rho^{pp}\chi_V + g_\omega h_\omega^{pp}\chi_S,$$

$$\{h_\rho^0, h_\rho^1, h_\rho^2, h_\omega^0, h_\omega^1\} \rightarrow \{-1.0\,, -0.077\,, -1.0\,, -0.33\,, -0.23\} \times 10^{-6}$$

$$\{g_\omega, g_\rho, \xi_S, \xi_V\} \rightarrow \{12.\,, 4.0\,, -0.33\,, 3.0\}$$

$$G_F f_\pi \Lambda_\chi \sim 1.0 \times 10^{-6}$$

$$g_\omega \approx 4\pi$$

(Frunstahl, Serot and Tang, 1997)

Summary

- We apply the $1/N_c$ expansion of QCD to the PV NN potential
- We find 2 operators at $O(N_c)$, 4 operators at $O(N_c^0 \sin^2\theta_W)$ and 6 at $O(1/N_c)$
- The PV observed in $\vec{p}+p$ scattering data is compatible with natural values for the strong and weak coupling constants
- The large-N_c hierarchy of other PV NN force mechanisms is close to estimates of the couplings in DDH models for Delta I =0,2.
- Pion exchange in the PV NN force is suppressed by $\sin^2\theta_W$ and $1/N_c$ ($\approx 1/15$) i.e. DDH best value $\approx 4.5 \times 10^{-7}$ VS $1/N_c \approx 0.77 \times 10^{-7}$. This will be tested by NPDGamma experiment.

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