Low x physics at strong coupling from AdS/CFT

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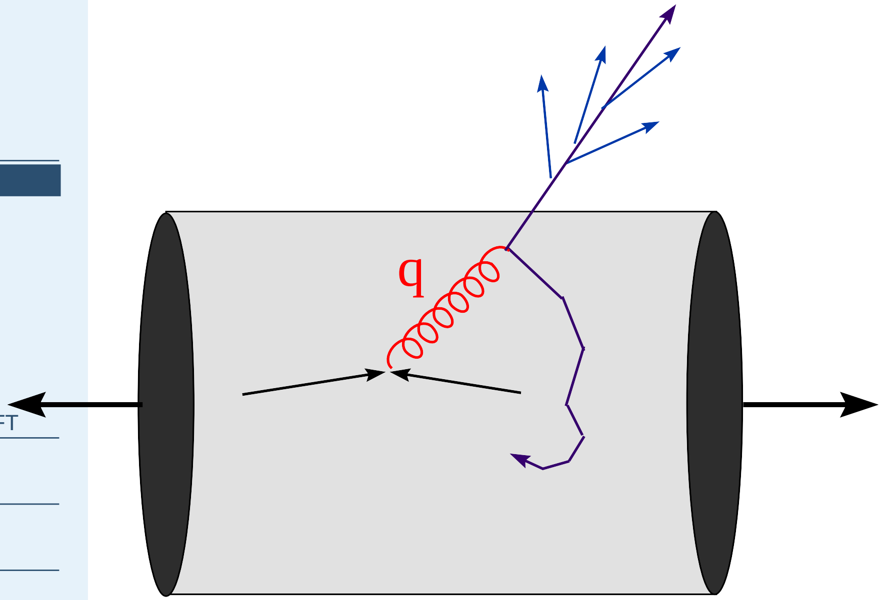
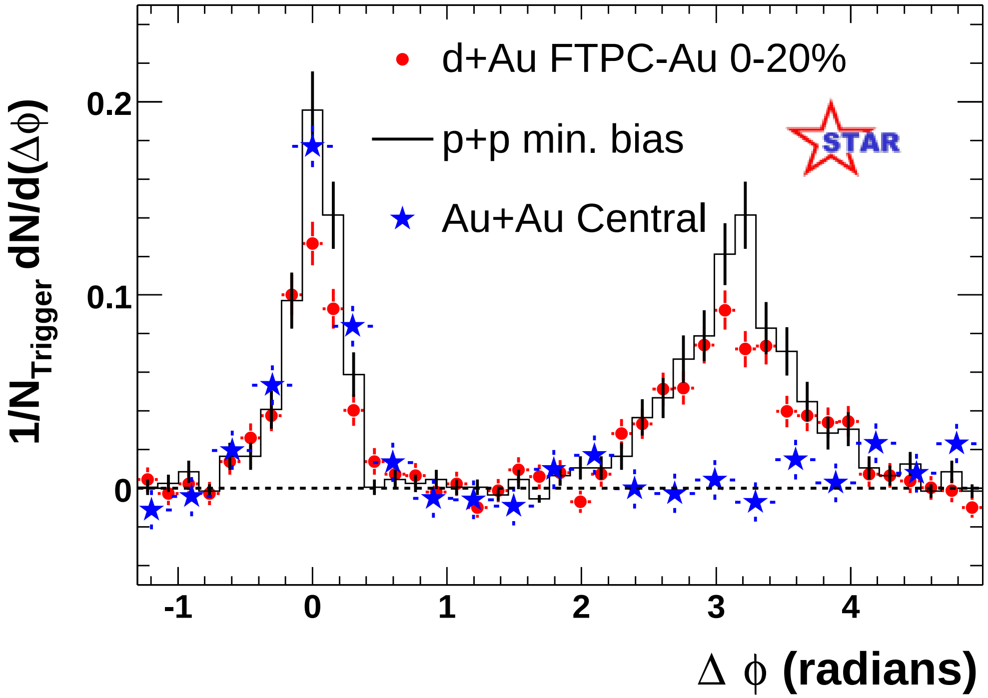
Collaboration with Yoshitaka Hatta and Al Mueller
(arXiv:0710.2148, 0710.5297, and 0803.2481)

Introduction

- Experimental evidence at RHIC & Lattice QCD suggest that the matter produced in a HIC might be strongly interacting
  - "strongly–coupled Quark–Gluon Plasma" (cf. H. Caines)
- Most such evidence refers to long–range ($\gg 1/T$) properties
  - hydrodynamics, thermalization, thermodynamics ...
- ... but some 'hard probes' ($E,\, Q \gg T$) are concerned as well
  - energy loss, momentum broadening, meson dissociation
- AdS/CFT: a framework to study strong coupling (limit)
- Main question: Is this regime of QCD mostly on the 'strong coupling' side, or rather on the 'weak–coupling' one ?
- Compare AdS/CFT results with all that we know, or we believe to know, about QCD

Nucleus–nucleus collision: Jet quenching

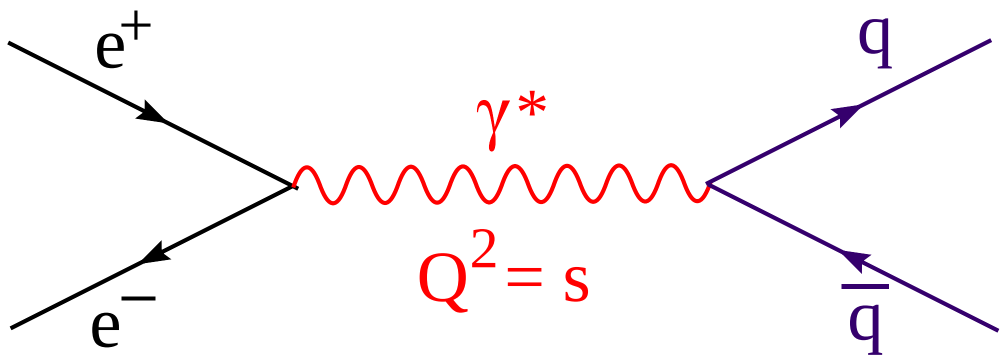




- The "away–side" jet has disappeared !
  absorbtion (or energy loss, or "jet quenching") in the medium
- The matter produced in a heavy ion collision is opaque
  high density, strong interactions, ... or both

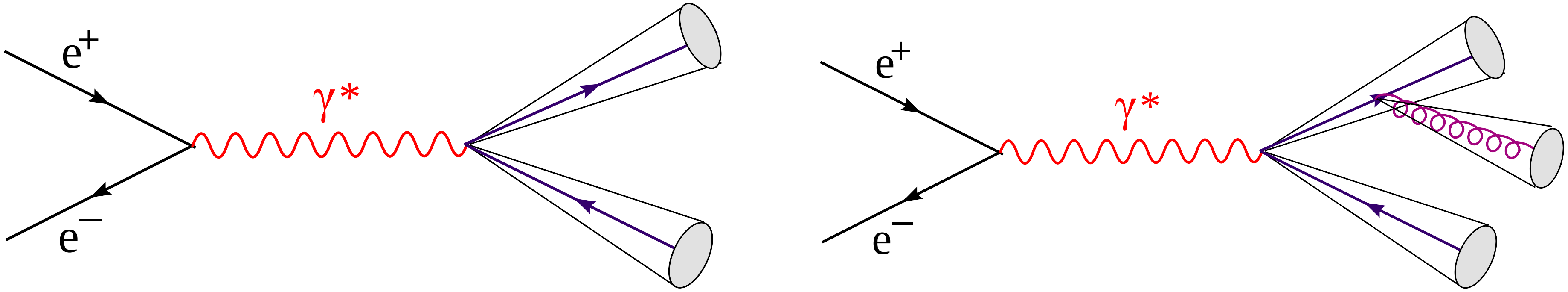
e^+e^- annihilation: Jets in pQCD

- How would a high–energy jet interact in a strongly coupled plasma ?
- How to produce jets in the first place ?
- Weakly–coupled QCD: $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$



- Decay of a time–like photon: $Q^2 \equiv q^\mu q_\mu = s > 0$
- The structure of the final state is determined by
  - parton branching & hadronisation

Parton branching at weak coupling

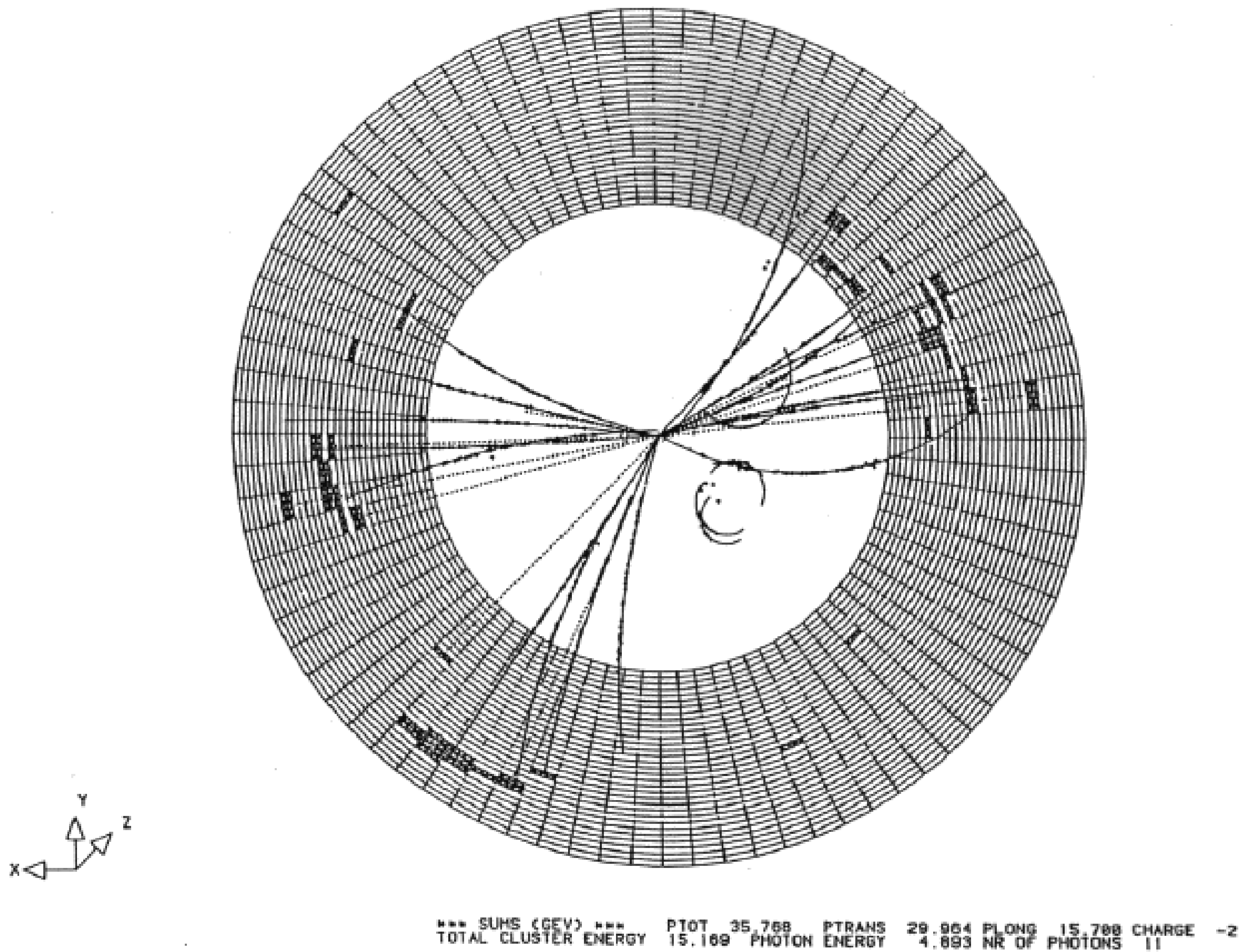


- Bremsstrahlung favors the emission of soft and collinear gluons $\Longrightarrow$ few, well collimated, jets
- e^+e^- cross–section computable in perturbation theory

$$\sigma(s) = \sigma_{\rm QED} \times \left(3\sum_f e_f^2\right)\left(1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right)$$

- Multi–jet ($n \geq 3$) events appear, but are comparatively rare

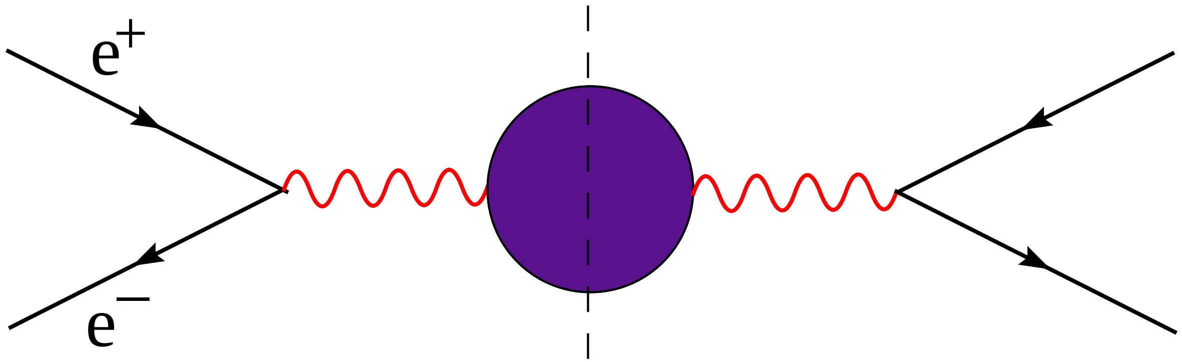
3–jet event at OPAL (CERN)



Current–current correlator

- Total cross–section given by the optical theorem

$$\sigma(e^+e^-) \;=\; \frac{1}{2s}\,\ell^{\mu\nu}\,\mathrm{Im}\,\Pi_{\mu\nu}(q)$$

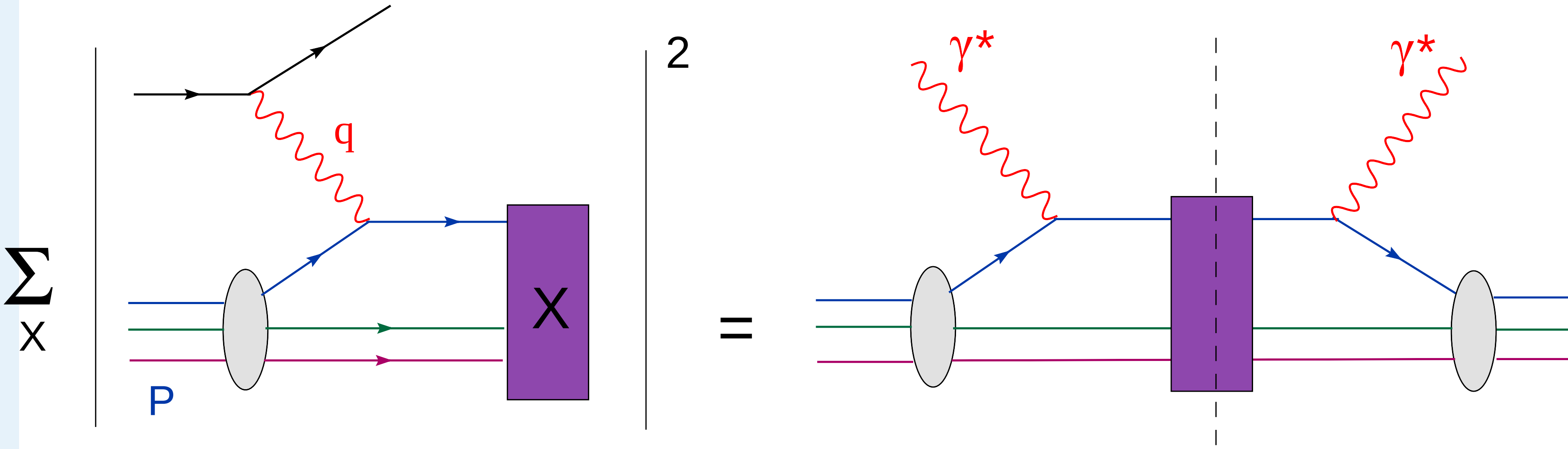


$$\Pi_{\mu\nu}(q) \;\equiv\; i\int \mathrm{d}^4x\,\mathrm{e}^{-iq\cdot x}\,\langle 0\,|\mathrm{T}\,\{J_\mu(x)J_\nu(0)\}\,|\,0\rangle$$

$$J^\mu \;=\; \sum_f e_f\,\bar{q}_f\,\gamma^\mu\,q_f$$: quark electromagnetic current

- Valid to leading order in $\alpha_{\rm em}$ but all orders in α_s

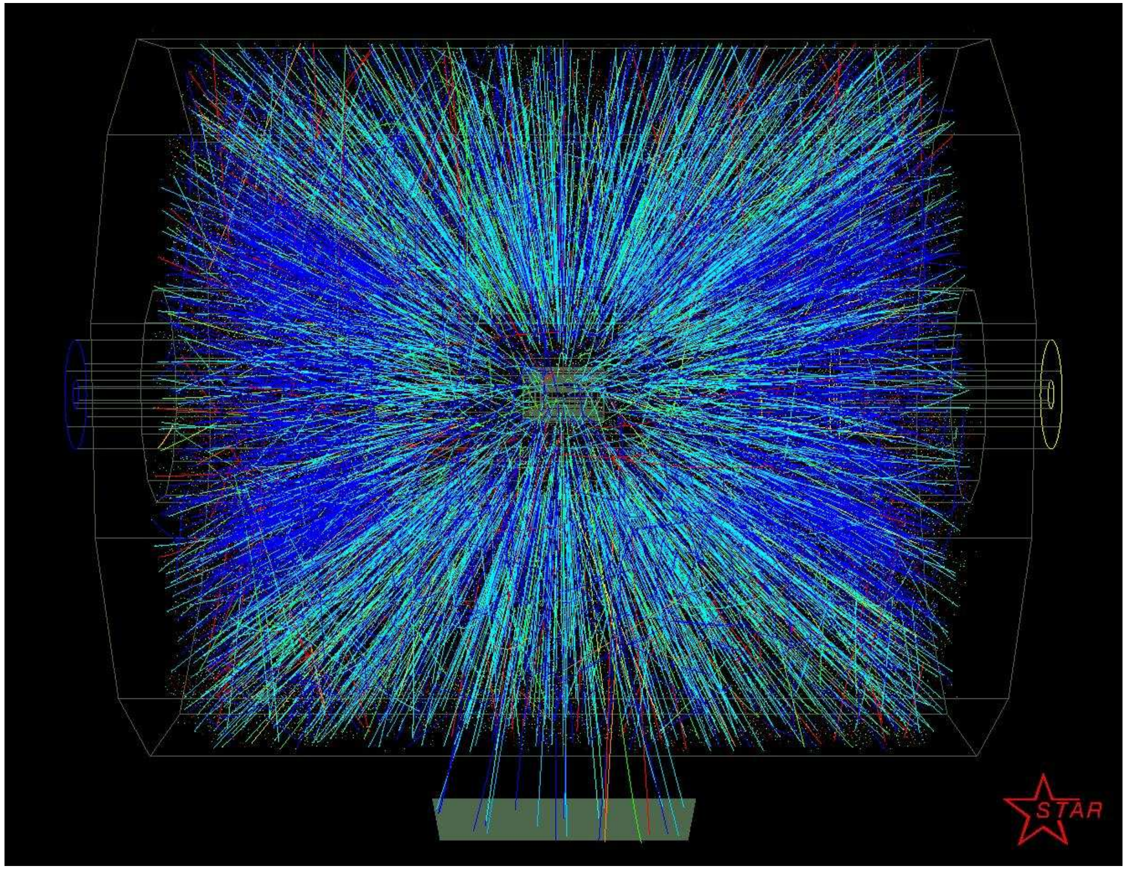
DIS: Parton picture



$$F_{1,2}(x,Q^2)\ \sim\ \mathrm{Im}\int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\, \langle P\,|\mathrm{T}\,\{J_\mu(x)J_\nu(0)\}\,|\,P\rangle$$

- Space–like current: $Q^2\ \equiv\ -q^\mu q_\mu\ \geq\ 0$ and $x\ \equiv\ \frac{Q^2}{2P\cdot q}$
- Physical picture: γ^* absorbed by a quark excitation with
  - transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $p_z = xP$

Partons at RHIC



- Partons are actually ‘seen’ (liberated) in the high energy hadron–hadron collisions
  - central rapidity: small–x partons
  - forward/backward rapidities: large–x partons

Electromagnetic current in a plasma

- Retarded polarization tensor: thermal expectation value

$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\theta(x_0)\, \langle\, [J_\mu(x), J_\nu(0)]\, \rangle_T$$

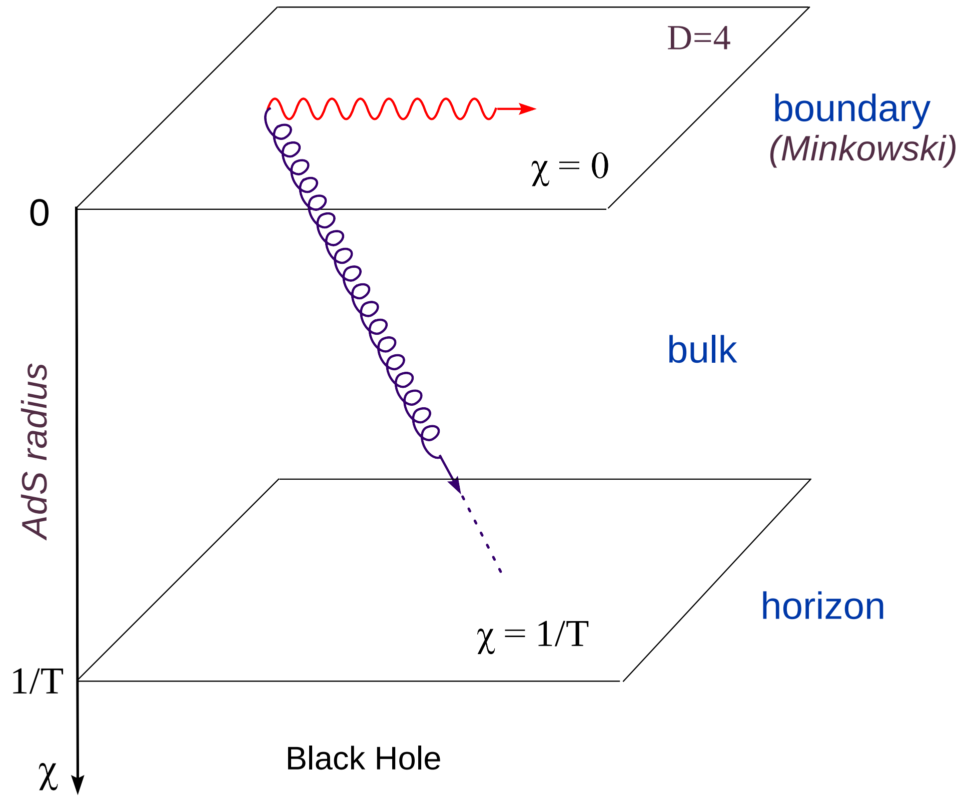
- 'Hard probe' : large virtuality $Q^2 \equiv |q^2| \gg T^2$
- Im $\Pi_{\mu\nu}$: absorption of the current by the plasma
  - time–like current ($q^2 > 0$) : jets
  - space–like current ($q^2 < 0$) : DIS, partons
- Strong 't Hooft coupling (more properly, $N_c \gg 1$) :

  $\lambda \equiv g^2 N_c \gg 1$ with $g^2 \ll 1 \implies$ AdS/CFT correspondence
- $\mathcal{N} = 4$ SYM at finite temperature & $\lambda \to \infty \iff$

  classical gravity in the $AdS_5 \times S^5$–Schwarzschild geometry

DIS off a Black Hole

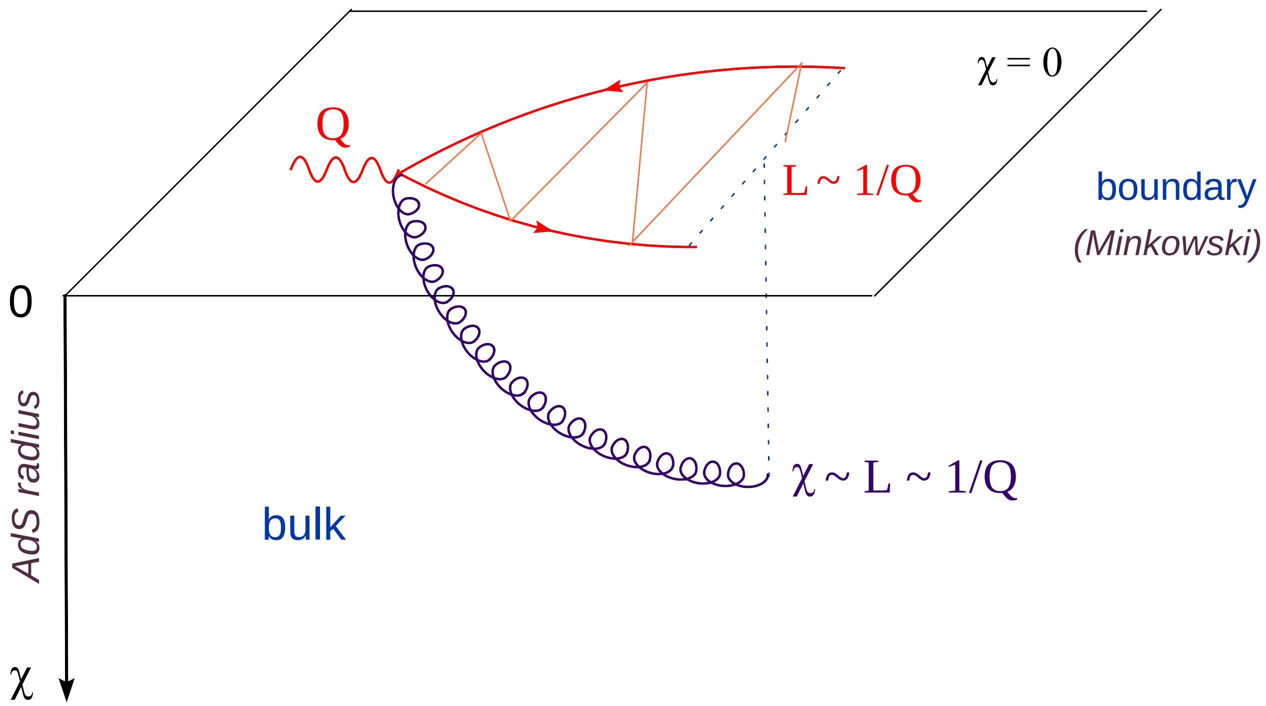
- A Black Hole in the 'radial' dimension (χ) of AdS_5
  - BH horizon at $\chi = 1/T$
  - our Minkowski world : the boundary of AdS_5 at $\chi = 0$



- Abelian current J_μ in 4D $\longleftrightarrow$ Maxwell wave A_μ in AdS_5

The UV/IR correspondence

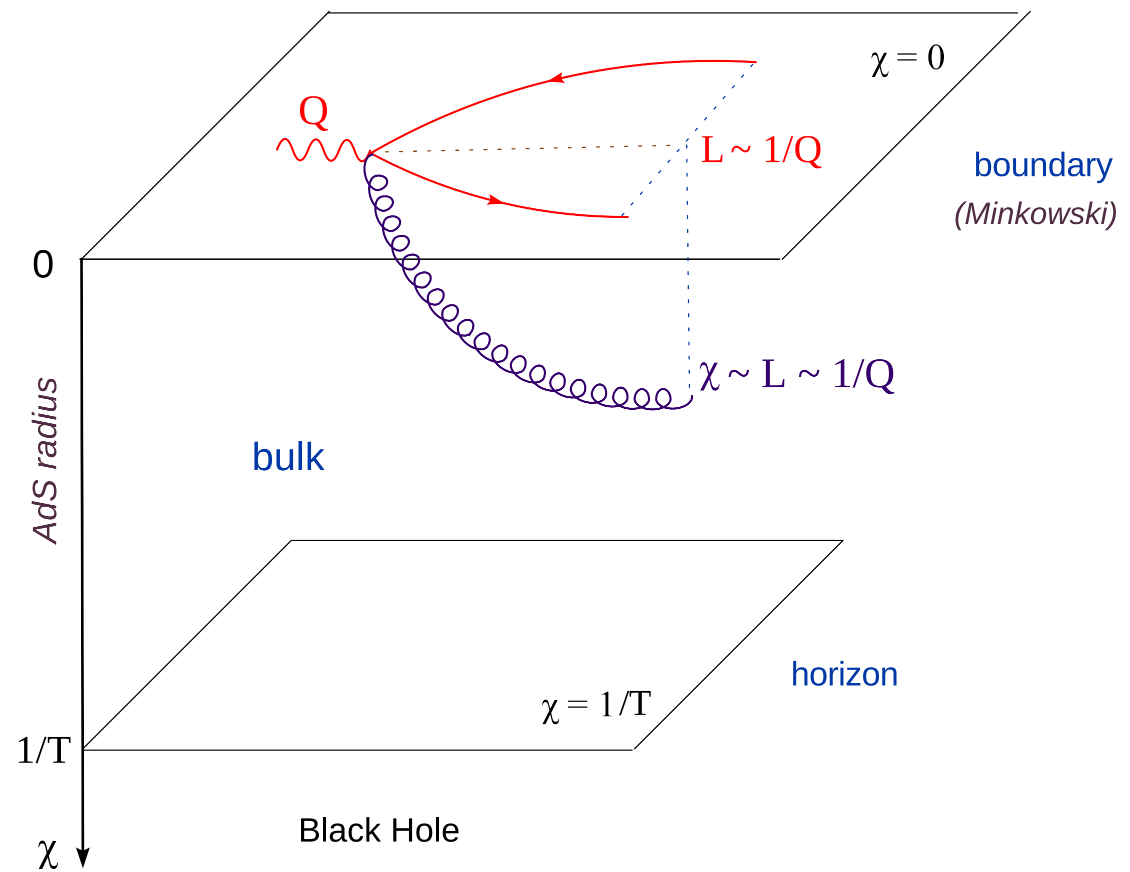
- Radial penetration χ in AdS_5 $\longleftrightarrow$ Transverse size L in $4D$



  - QM : a partonic fluct. grows like $L \sim \sqrt{t}$ up to $L \sim 1/Q$
  - So does the position χ of the wave packet in AdS_5 !
- Important for the physical interpretation of AdS/CFT results

Space–like current

- The wave gets stuck near the boundary: $\chi \lesssim 1/Q \ll 1/T$



- No interaction with the BH (plasma) ...
  unless the energy ω of the wave is high enough !

Saturation momentum

- $J_\mu(x) \propto \mathrm{e}^{-i\omega t + iqz}$: $Q^2 = q^2 - \omega^2 > 0$ & $x = \frac{Q^2}{2\omega T}$
- The criterion for interacting with the plasma

$$\underbrace{Q}_{\text{potential barrier}} \lesssim \underbrace{\frac{\omega T^2}{Q^2}}_{\text{gravitational potential}}$$

Recall: Gravitational interactions are proportional to the energy density

Saturation momentum

- $J_\mu(x) \propto \mathrm{e}^{-i\omega t+iqz}$: $Q^2 = q^2 - \omega^2 > 0$ & $x = \frac{Q^2}{2\omega T}$
- The criterion for interacting with the plasma

$$\underbrace{Q}_{\text{virtuality barrier}} \lesssim \underbrace{\frac{\omega}{Q^2}}_{\text{lifetime}} \times \underbrace{T^2}_{\text{plasma force}}$$

The partonic fluctuation must live long enough to feel the effects of the plasma

Saturation momentum

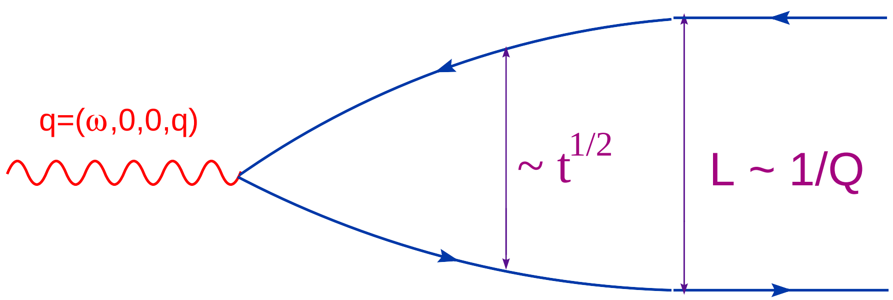
- $J_\mu(x) \propto \mathrm{e}^{-i\omega t+iqz}$: $Q^2 = q^2 - \omega^2 > 0$ & $x = \frac{Q^2}{2\omega T}$
- The criterion for interacting with the plasma

$$\underbrace{Q}_{\text{virtuality barrier}} \lesssim \underbrace{\frac{\omega}{Q^2}}_{\text{lifetime}} \times \underbrace{T^2}_{\text{plasma force}}$$

- High enough energy or, equivalently, low enough Q^2:

$$Q \lesssim Q_s \simeq (\omega T^2)^{1/3} \simeq \frac{T}{x}$$

- $L_s \sim 1/Q_s$: the minimal size for the dipole to feel the plasma



No partons at large x !

$$Q_s(x) \simeq \frac{T}{x} \quad \Longleftrightarrow \quad x_s(Q) \simeq \frac{T}{Q}$$

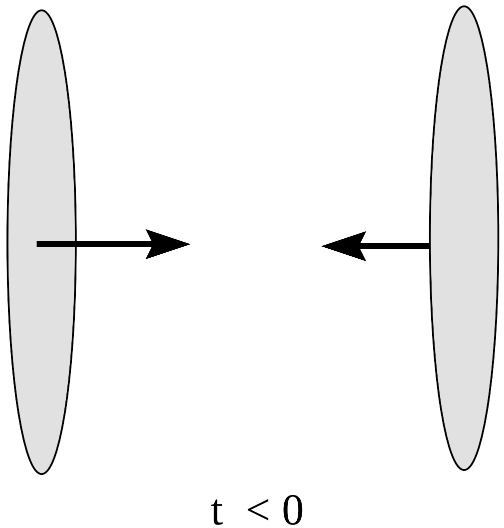
- For $Q > Q_s(x)$ or $x > x_s(Q)$: $F_2(x, Q^2) \approx 0$

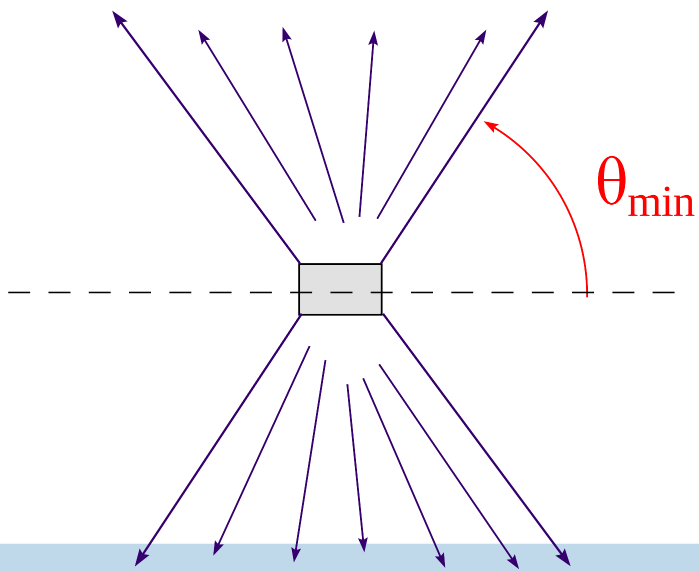
  $\Longrightarrow$ no partons with large momentum fractions $x > x_s$

- 'All partons have branched down to small values of x'

  (Polchinki and Strassler, 02; Hatta, E.I., Mueller, 07; see below !)

- No forward/backward jets in hadron–hadron collisions !





Low x : recovering the partons

- $x \lesssim x_s$: Total absorbtion of the current in the plasma

  $\Longrightarrow$ Unitarity limit ('black disk') for DIS

$$F_2(x, Q^2) \simeq xN_c^2Q^2 \begin{cases} \exp\left[-(x/x_s)^{1/2}\right] & \text{for} \quad x \gg x_s \equiv T/Q \\ 1 & \text{for} \quad x \lesssim x_s \equiv T/Q, \end{cases}$$

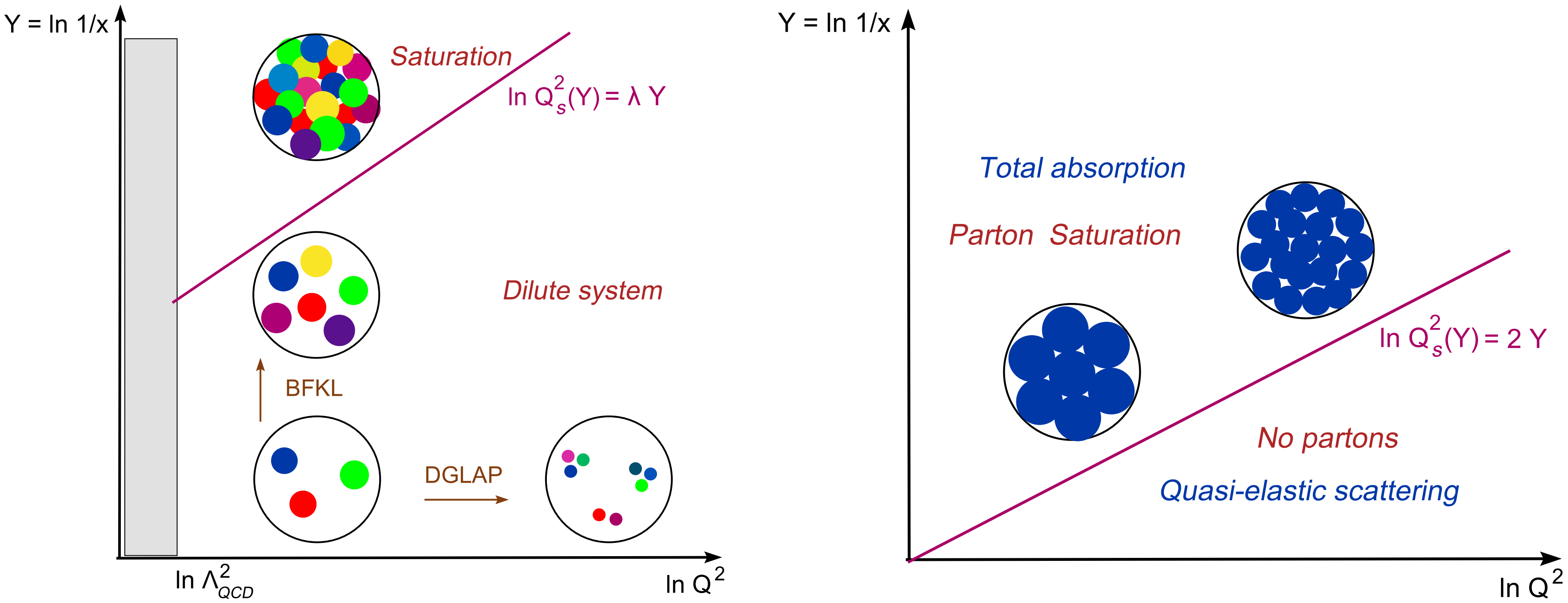
- For $x \lesssim x_s$: occupation numbers of $\mathcal{O}(1) \Longrightarrow$ saturation

$$\frac{1}{x}F_2(x, Q^2) \sim \int^{Q^2} \mathrm{d}^2k_\perp \frac{\mathrm{d}n}{\mathrm{d}^2x_\perp \mathrm{d}^2k_\perp}$$

- Energy sum rule:

$$\mathcal{E} = T^2 \int_0^1 \mathrm{d}x\, F_2(x, Q^2) \sim T^2 \left[xF_2(x, Q^2)\right]_{x=x_s} \sim N_c^2T^4 \checkmark$$

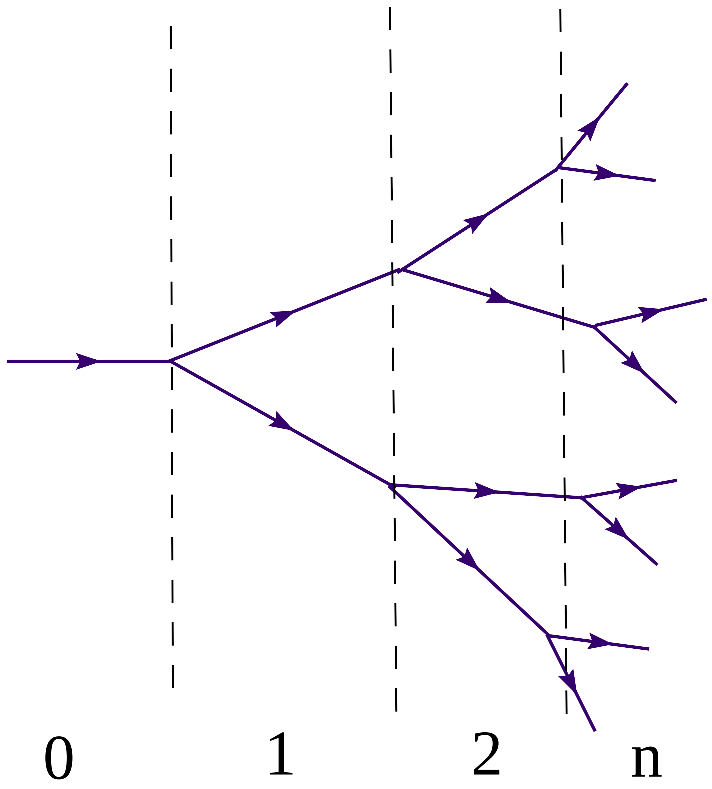
Saturation line: weak vs. strong coupling



- Saturation exponent : $Q_s^2(x) \propto 1/x^{\lambda_s} \equiv e^{\lambda_s Y}$
  - weak coupling (LO BFKL): $\lambda_s \approx 1.23\, g^2 N_c$
  - phenomenology & NLO BFKL: $\lambda_s \approx 0.2 \div 0.3$
  - strong coupling (plasma): $\lambda_s = 2$ (graviton)

Medium induced branching

- Universal energy loss mechanism, active at partonic level
  - no reason why branching should stop at 2 parton level
  - no reason to favour special corners of phase–space



$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

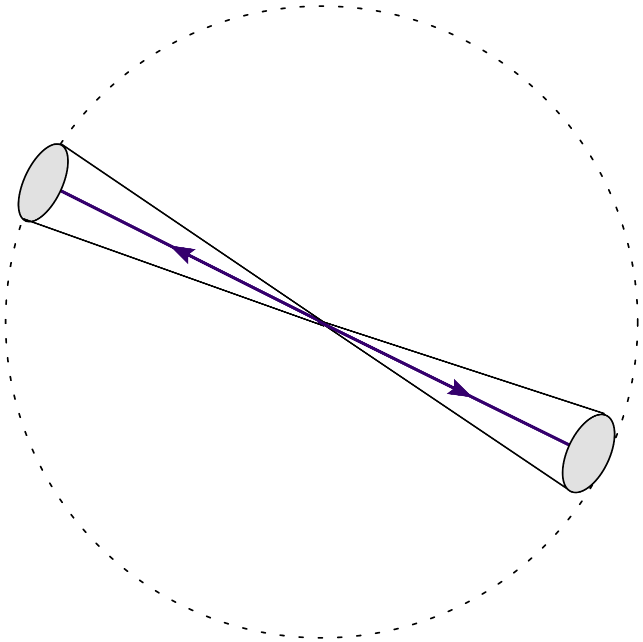
$$Q_n \sim \frac{Q_{n-1}}{2} \quad \text{(vacuum)}$$

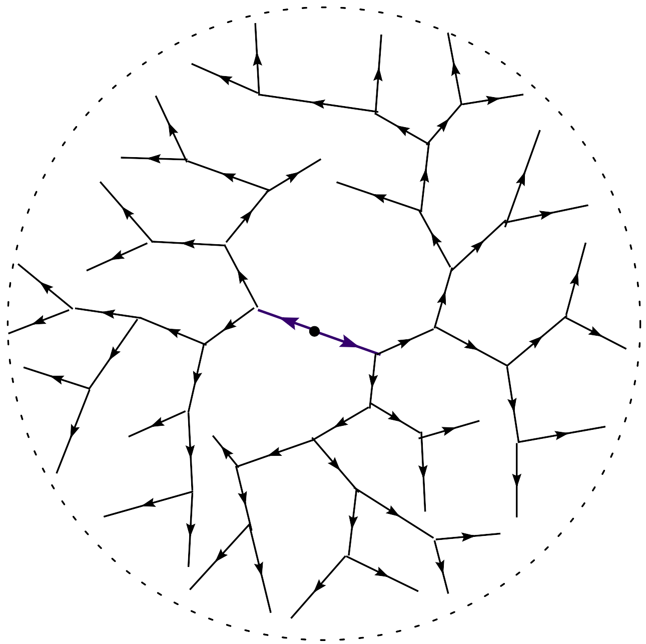
$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

$$\frac{\Delta Q_n}{\Delta t_n} \sim -T^2 \quad \text{(plasma)}$$

- Qualitative agreement with all the results from AdS/CFT

e^+e^- at strong coupling



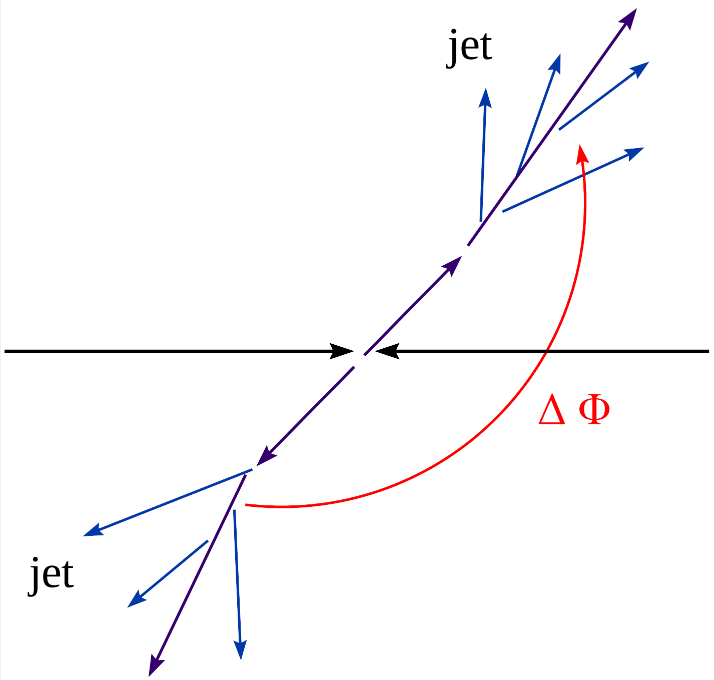


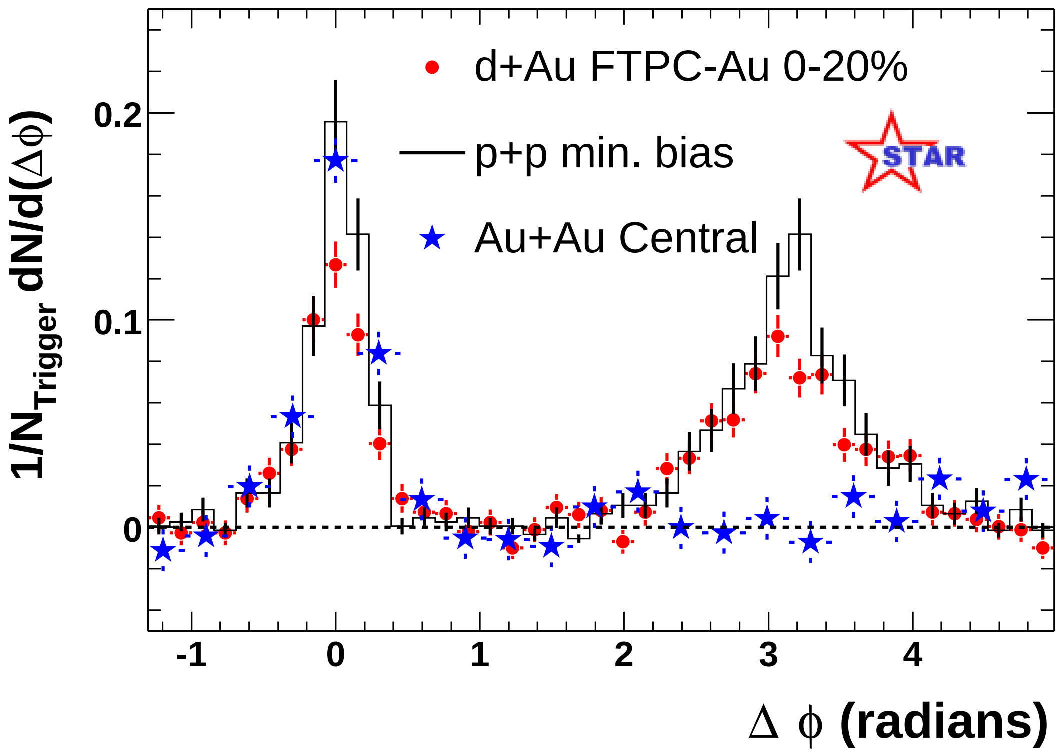
- Partons keep splitting down to $Q \sim \Lambda$ (or $Q \sim T$)
- In the COM frame $\longrightarrow$ spherical distribution
  - ▷ similar conclusion by Hofman and Maldacena, 08
- No jets in e^+e^- annihilation at strong coupling !
  - ▷ see also the talk by Yoshitaka Hatta

Conclusions

- Hard probes & high-energy physics appears to be quite different at strong coupling as compared to QCD
  - no forward/backward particle production in HIC
  - no jets in e^+e^- annihilation
  - different mechanism for jet quenching
    - ▷ see talks by Cyrille Marquet and Al Mueller
- Not so surprising: by asymptotic freedom, hard & high–energy physics in QCD is weakly coupled
- Are AdS/CFT methods useless for HIC ? Not necessarily so !
  - some observables receive contributions from several scales, from soft to hard: use AdS/CFT in the soft sector
    - ▷ see talk by Al Mueller
  - ... and long–range properties: hydro, thermalization, etc
    - ▷ cf. talk by Robi Peschanski

Jets in proton–proton collisions

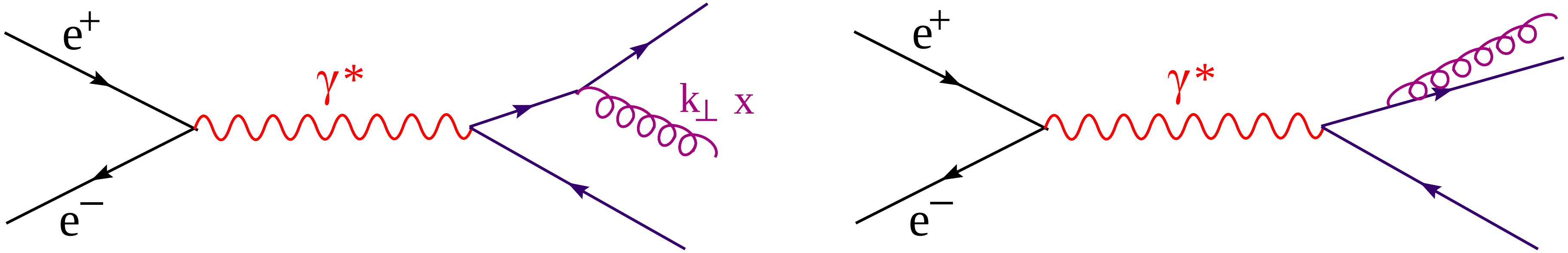




- Azimuthal correlations between the produced jets:

  a peak at $\Delta\Phi = 180°$

Parton branching at weak coupling



- The parton picture is meaningful indeed (time separation)

$$t_{\rm branch} \sim \frac{k_0}{k_\perp^2} \ll t_{\rm hadr} \sim \frac{k_0}{\Lambda^2_{\rm QCD}} \quad \text{if} \quad k_\perp^2 \gg \Lambda^2_{\rm QCD}$$

- Gluon emission rate to lowest order in pQCD :

$$d\mathcal{P}_{\rm Brem} \sim \alpha_s(k_\perp^2)\,\frac{d^2k_\perp}{k_\perp^2}\,\frac{dx}{x}$$

- Hard gluon ($k_\perp \sim \sqrt{s}$ & $x \sim \mathcal{O}(1)$) : small probability $\sim \alpha_s(s)$
- Collinear and/or soft gluons ($k_\perp \ll \sqrt{s}$ & $x \ll 1$) : jets

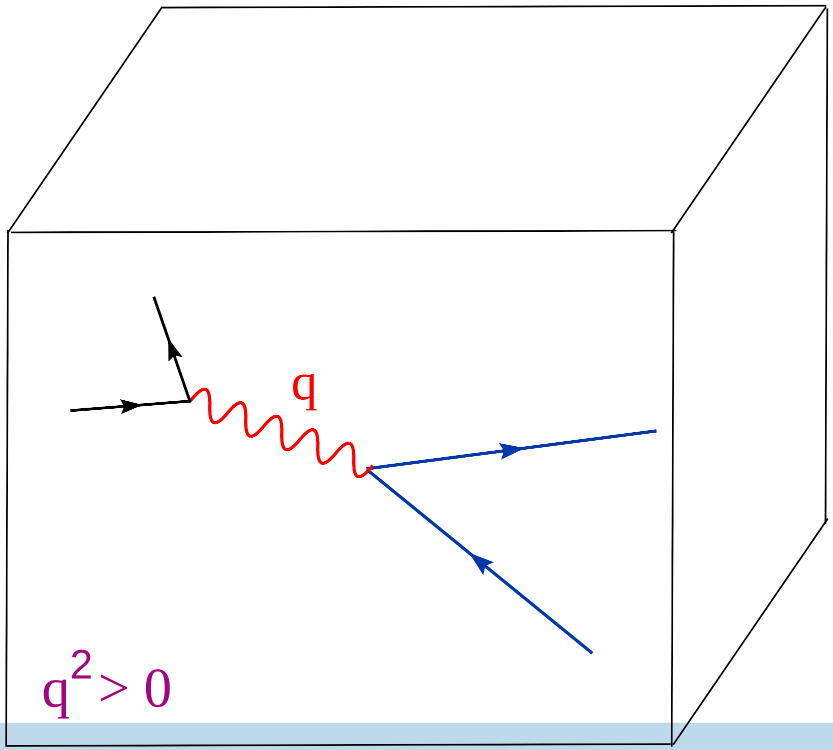
Electromagnetic current in a plasma

- Retarded polarization tensor: thermal expectation value

$$\Pi_{\mu\nu}(q) \equiv \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\theta(x_0)\, \langle\, [J_\mu(x), J_\nu(0)]\,\rangle_T$$

- Time–like current ($q^2 > 0$) :

  jet production & their subsequent interactions in the plasma



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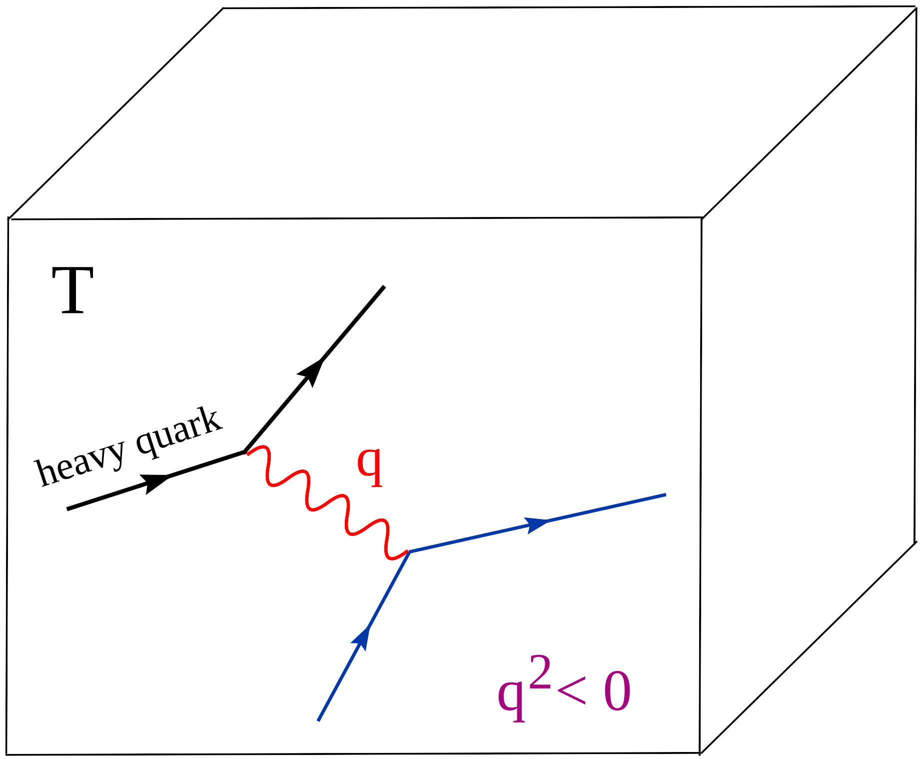
- Space–like current ($q^2 < 0$) : DIS & parton structure



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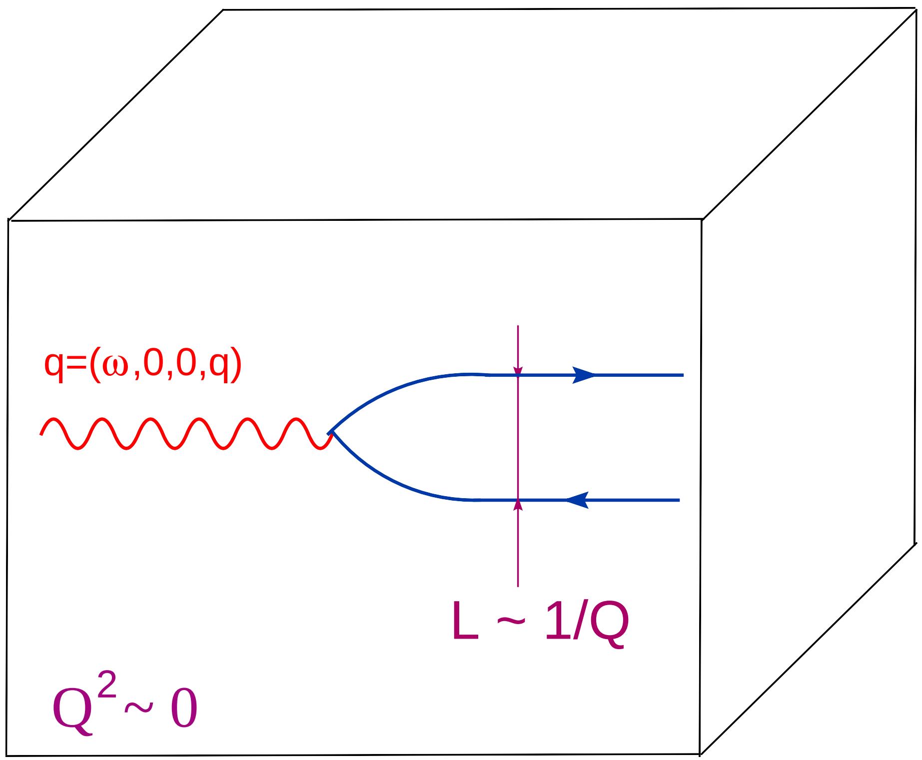
- High–energy current ($Q^2 \approx 0$) : ‘meson’ in the plasma



Electromagnetic current in a plasma

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- 'Hard probe' : large virtuality $Q^2 \equiv |q^2| \gg T^2$
- Strong coupling : AdS/CFT correspondence

  'Gauge theory at strong coupling' (more properly, large N_c)

  $\Longleftrightarrow$ 'String theory in $D = 10$ at weak coupling'
- Not exactly QCD ! $\mathcal{N} = 4$ SYM (conformal & SUSY)
  - no confinement, no asymptotic freedom, no asymptotic states, no intrinsic mass scale, no fundamental fermions
- ... but presumably less important for the QGP phase of QCD

AdS/CFT ... in a nutshell

- A gauge–string, 'weak–strong', duality:

  gauge theory in $D = 4$ at strong coupling ($\lambda \gg 1$) $\Longleftrightarrow$

  string theory in $D = 10$ at weak coupling and low curvature

- Not exactly QCD ! $\mathcal{N} = 4$ SYM (conformal & SUSY)
  - no confinement, no asymptotic freedom, no asymptotic states, no intrinsic mass scale, no fundamental fermions
- ... but presumably less important for the QGP phase of QCD
  - deconfined, small 'trace anomaly', no asymptotic states, similar thermodynamics for QCD and pure Yang–Mills th.
- Strong–coupling limit $\lambda \to \infty$:

  $\mathcal{N} = 4$ SYM at finite temperature & $\lambda \to \infty$ $\Longleftrightarrow$

  classical gravity in the $AdS_5 \times S^5$–Schwarzschild geometry

AdS/CFT at finite temperature

- $\mathcal{N}=4$ SYM at finite temperature $\Longleftrightarrow$ type IIB string theory in $AdS_5 \times S^5$–Schwarzschild geometry

$$\mathrm{d}s^2 = \frac{r^2}{R^2}(-f(r)\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2) + \frac{R^2}{r^2 f(r)}\mathrm{d}r^2 + R^2 \mathrm{d}\Omega_5^2$$

where $f(r) = 1 - (r_0^4/r^4)$ and r_0 = the BH horizon

- 'A Black Hole in the radial dimension of AdS_5' (a Black D–Brane: homogeneous in the physical 4D)

- The correspondence (for parameters) :

$$4\pi g_s = g^2\,, \qquad (R/l_s)^4 = g^2 N_c \equiv \lambda\,, \qquad r_0/R^2 = \pi T$$

- 'Strong coupling limit': $\lambda \to \infty$ for fixed $g^2 \ll 1$ ($N_c \to \infty$)

$\longrightarrow$ 'supergravity approximation' (classical gravity)

Parton saturation at strong coupling

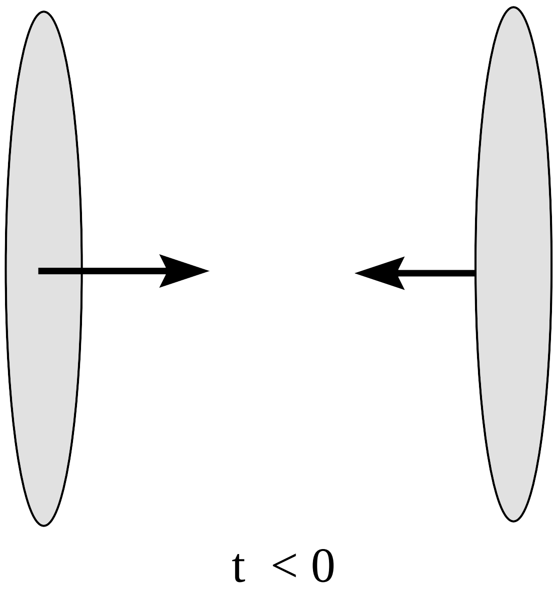
- For $Q > Q_s(x) = T/x$ or $x > x_s(Q) = T/Q$:
  the DIS cross section ($F_2(x, Q^2)$) is essentially zero
  $\Longrightarrow$ no partons with large momentum fractions $x > x_s$
- For $Q \lesssim Q_s(x)$ or $x \lesssim x_s(Q)$: total absorbtion
  $\Longrightarrow$ parton saturation (similar to pQCD)
- 'All partons have branched down to small values of x'
- Similar conclusions for a single 'hadron' ('dilaton') target
  (Polchinki and Strassler, 02; Hatta, E.I., Mueller, 07)

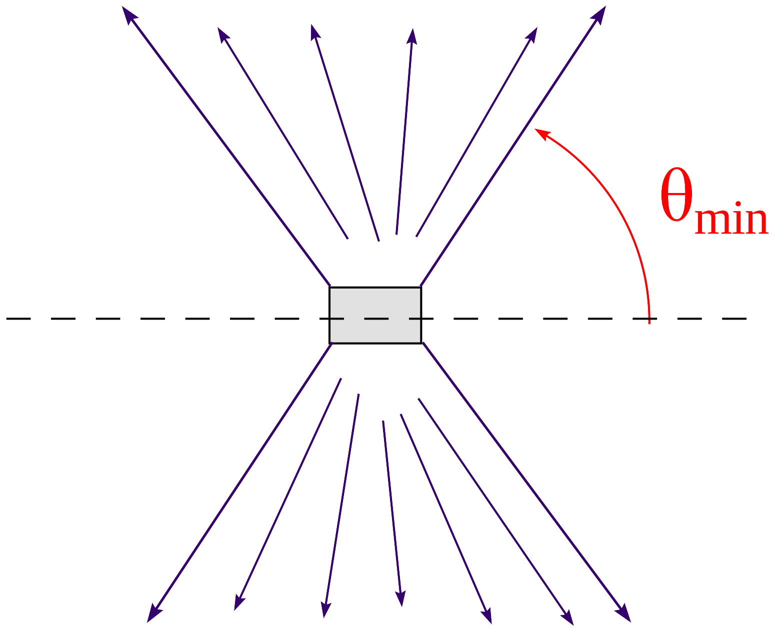
$Q_s^2(x) \sim \dfrac{1}{x}$ (dilaton) as opposed to $Q_s^2(x) \sim \dfrac{1}{x^{0.3}}$ (pQCD)

$\Longrightarrow$ faster increase with $1/x$ (spin $j = 2$ graviton exchange)

No forward jets !

- No large–x partons $\Longrightarrow$ no forward/backward jets in a hadron–hadron collision at strong coupling



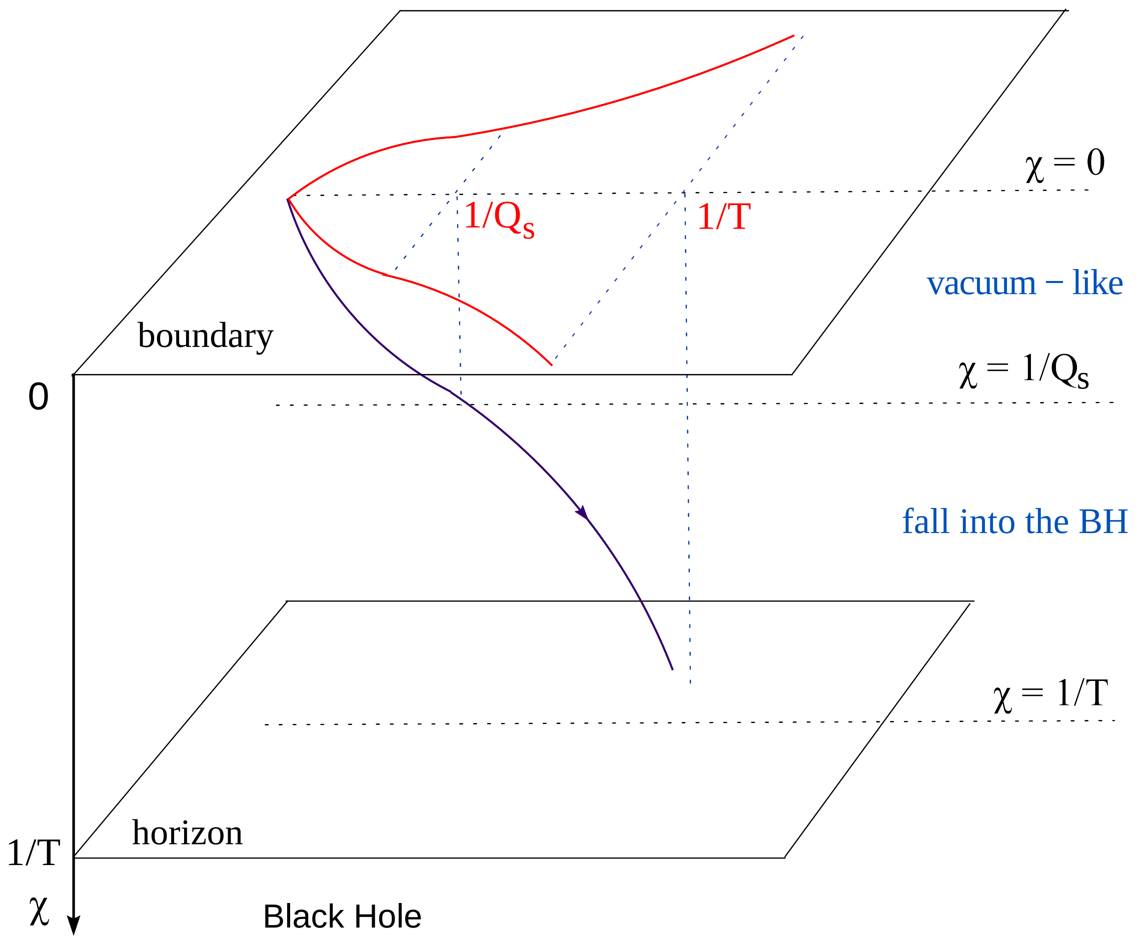


- ‘The Nightmare of CMS’

$$|\eta| \lesssim \eta_{\max}(Q) = \ln \frac{\sqrt{s}}{Q} - \ln \frac{1}{x_s(Q)}\,, \qquad x_s(Q) \sim \frac{\Lambda^2}{Q^2 N_c^2} \ll 1$$

Energy loss at strong coupling

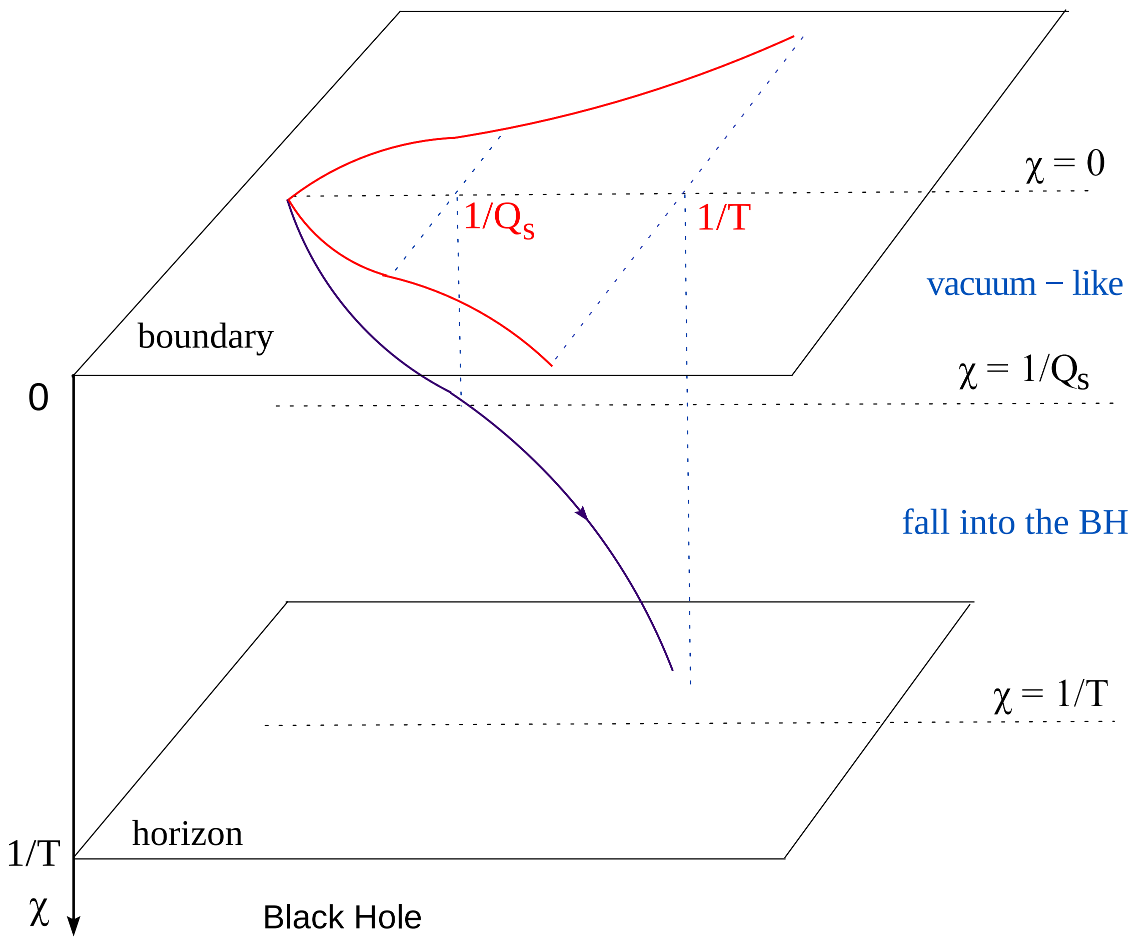
- $Q \lesssim Q_s(x)$ (high energy) : the energy falls into the BH along a massless geodesics (the 'trailing string')



- Same as for a heavy quark *(Herzog et al; Gubser, 2006)*

Energy loss at high energy

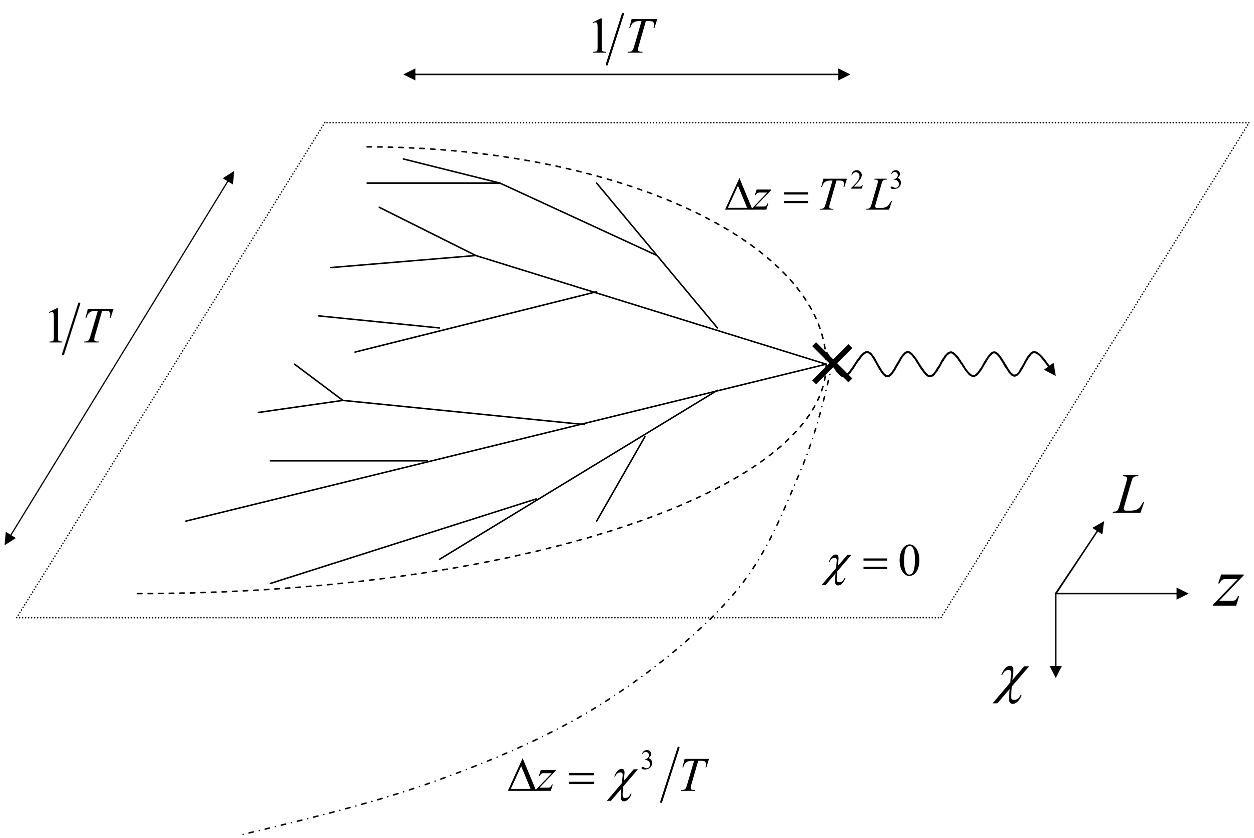
Lifetime of the current: $\Delta t \sim \frac{\omega}{Q_s^2} \sim \frac{1}{T}\left(\frac{\omega}{T}\right)^{1/3}$



- Same as the 'gluon lifetime' *(Gubser, Gulotta, Pufu, and Rocha,08)*

Trailing string revisited

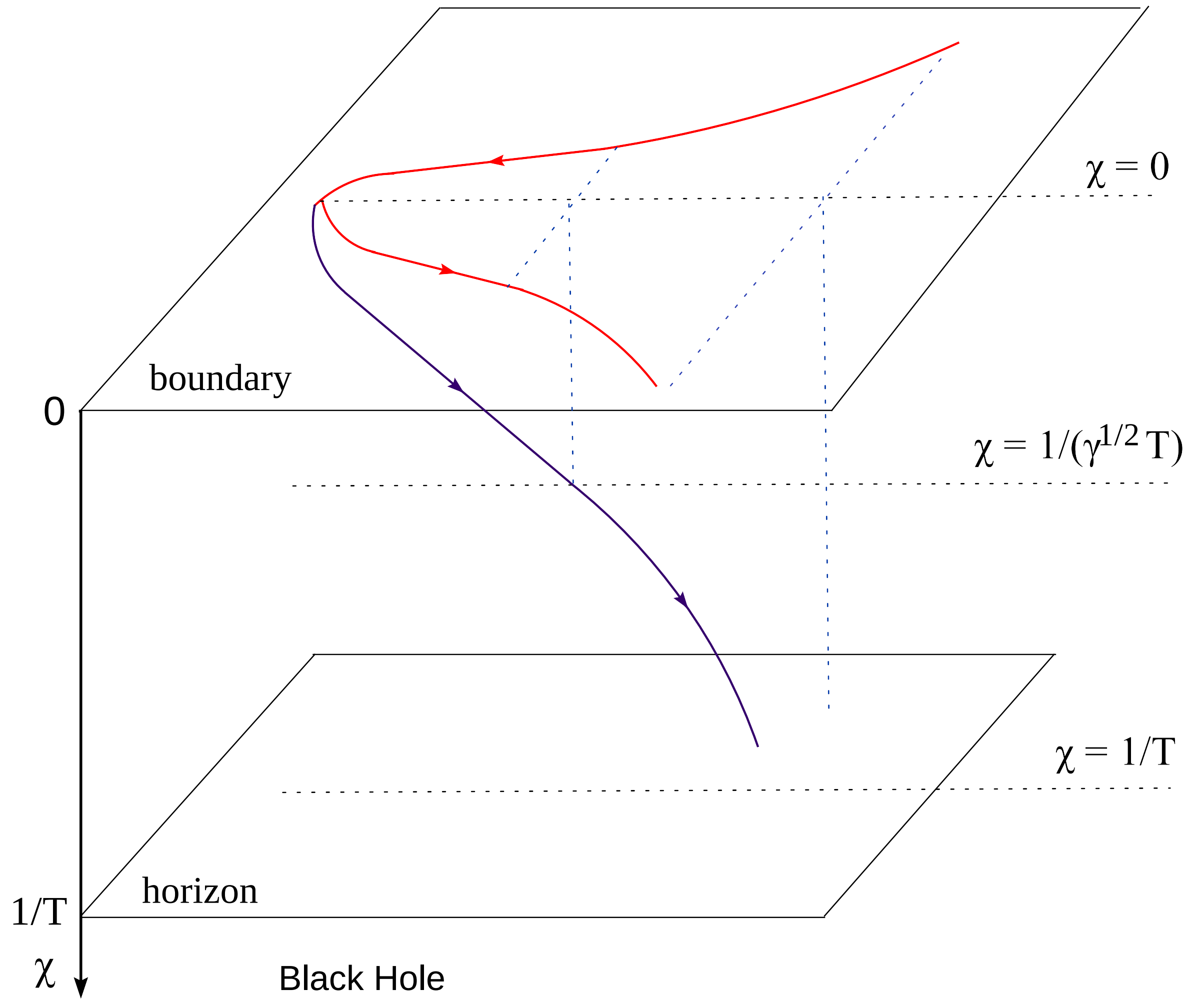
- The enveloping curve of the resulting partonic system coincides with the 'trailing string' *(Herzog et al; Gubser, 2006)*



- ... as it should by virtue of the UV/IR correspondence

Time–like current at strong coupling

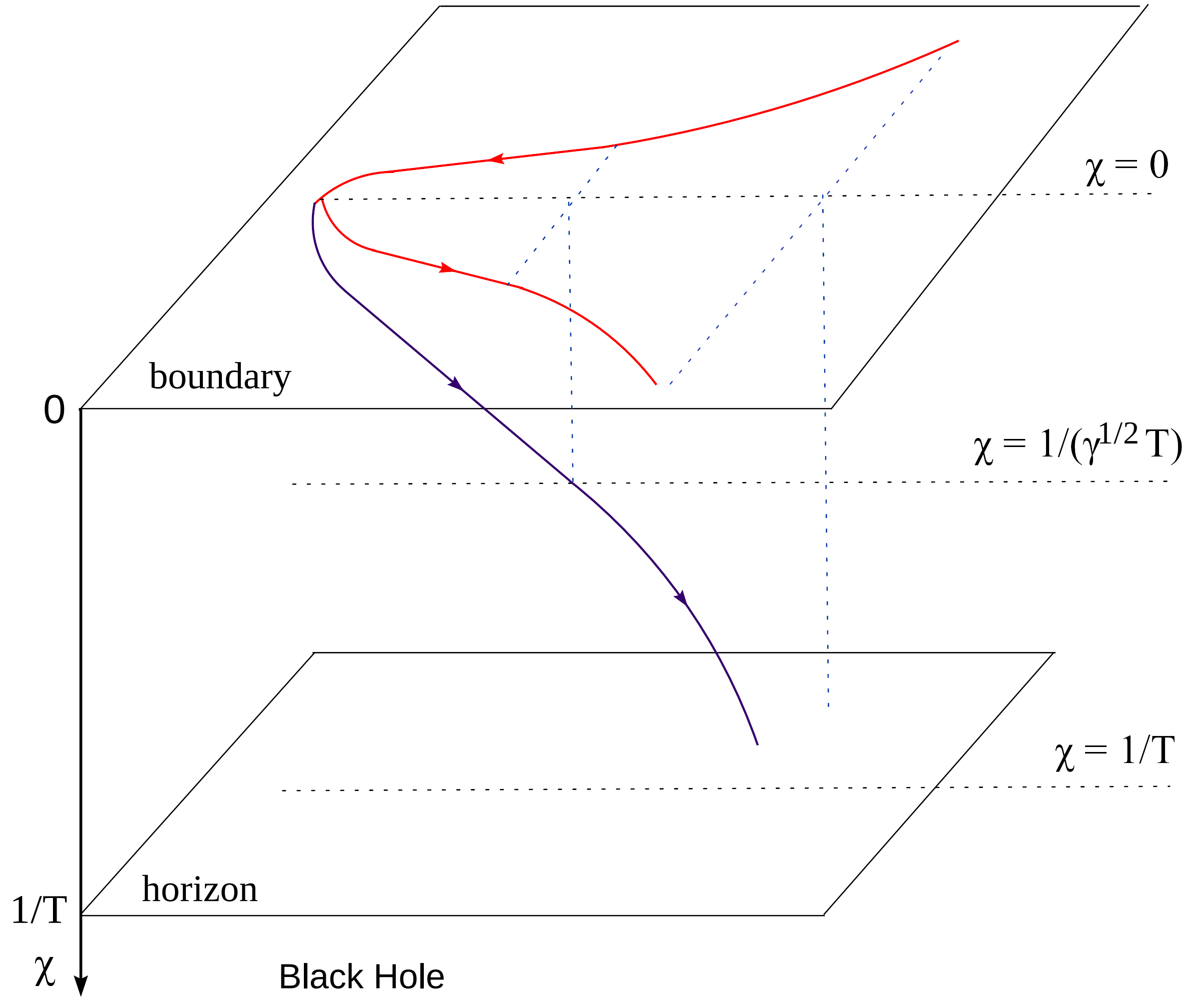
- A time–like current can decay (into partons of $\mathcal{N} = 4$ SYM) already in the vacuum



Early times: free streaming

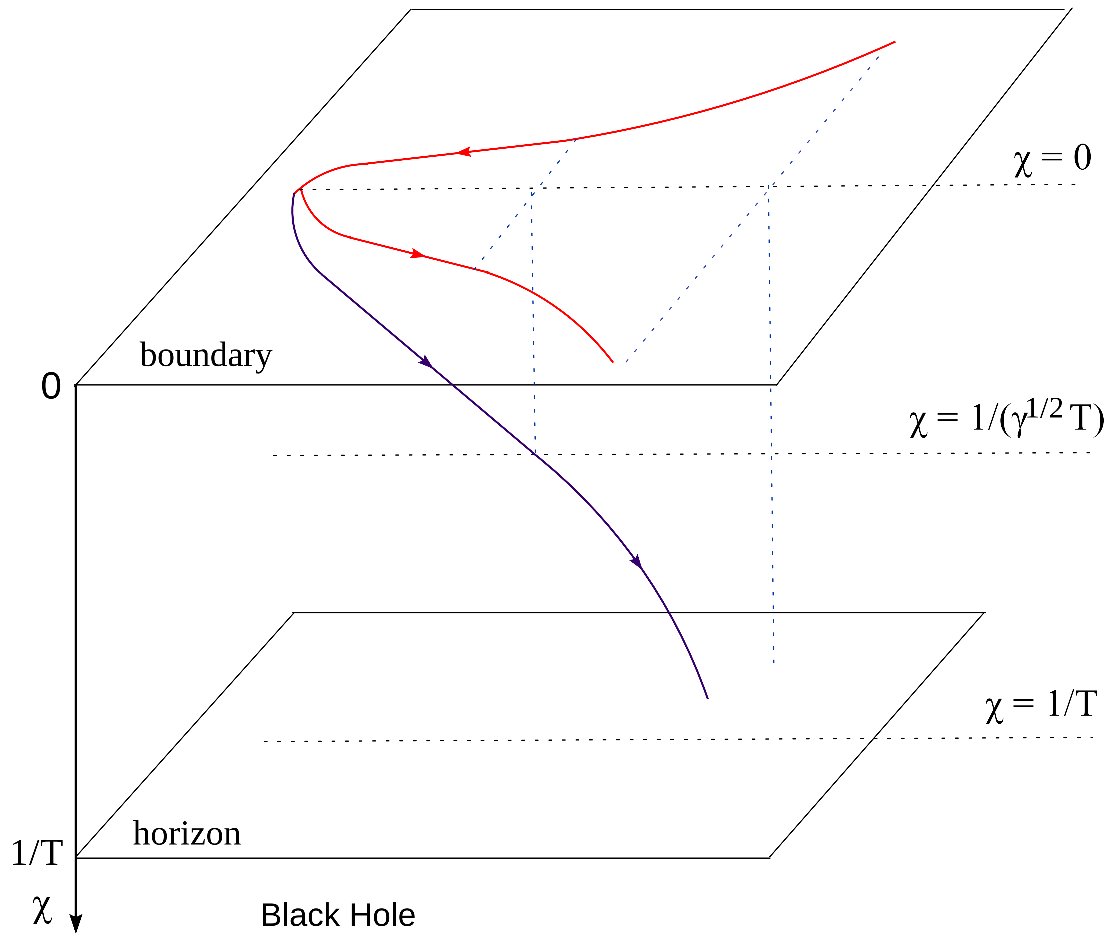
- Early times/small size L :

  free expansion up to the critical size $L_s \sim 1/\sqrt{\gamma}\,T$



Later times : falling into the black hole

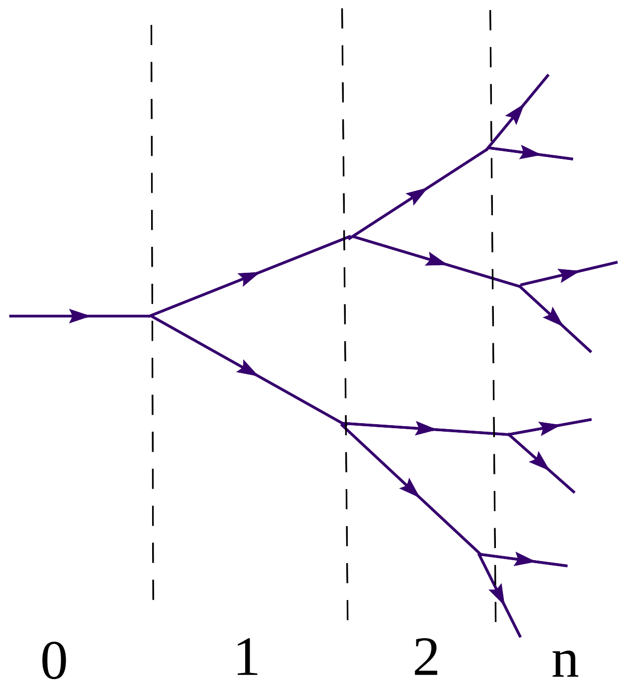
- The partons fall along the 'trailing string', once again



Lifetime of the current/penetration length: $\Delta t \sim \Delta z \sim \dfrac{\sqrt{\gamma}}{T}$

Parton branching at strong coupling

- No reason why branching should stop at 2 parton level !
- No reason to favour special corners of phase–space !



$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

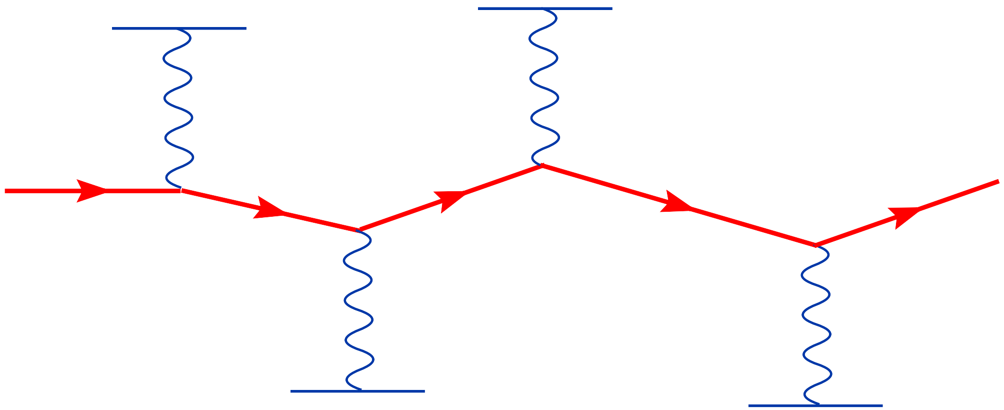
(uncertainty principle)

- Vacuum: 'democratic' branching

$$Q_n \sim \frac{Q_{n-1}}{2} \sim \frac{Q}{2^n} \quad \Longrightarrow \quad L(t) \sim \frac{t}{\gamma} = \sqrt{1 - v_z^2}\, t \ ✓$$

Transverse momentum broadening

- A parton (say, heavy quark) undergoes multiple scattering (random kicks) off the plasma constituents



$$\frac{\mathrm{d}\langle p_\perp^2\rangle}{\mathrm{d}t} \equiv \hat{q} \simeq \alpha_s N_c \frac{xg(x,Q^2)}{N_c^2-1}$$

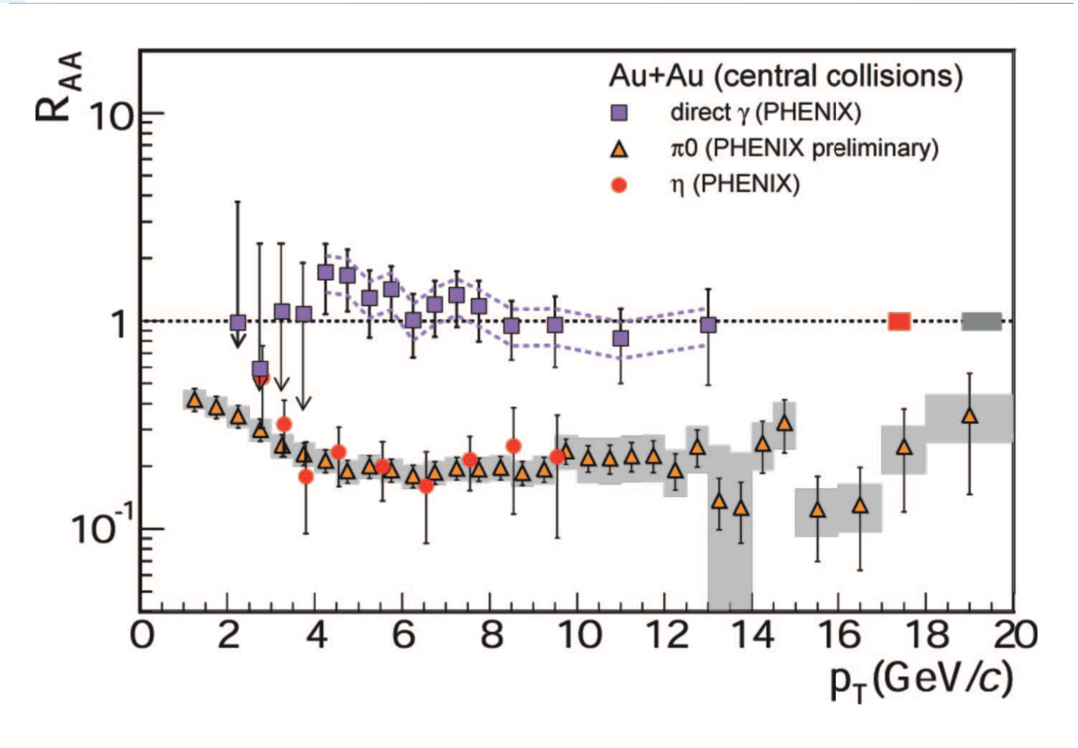
- $xg(x, Q^2)$: gluon distribution per unit volume in the medium
- Weakly–coupled QGP : incoherent sum of the gluon distributions produced by thermal quarks and gluons

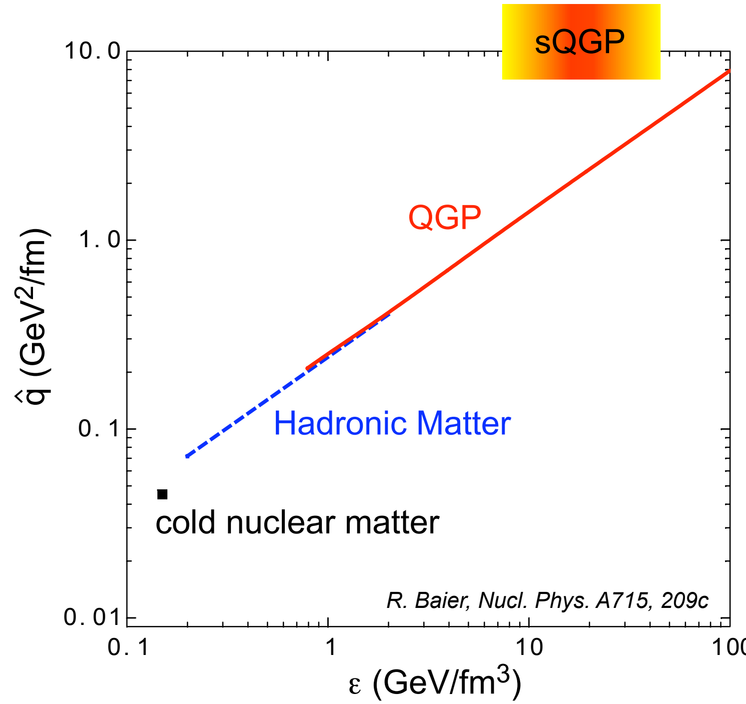
$$xg(x,Q^2) \simeq n_q(T)\, xG_q + n_g(T)\, xG_g\,, \quad \text{with} \quad n_{q,g}(T) \propto T^3$$

Nuclear modification factor

- How to measure $\hat{q}$? Compare AA collisions at RHIC to pp

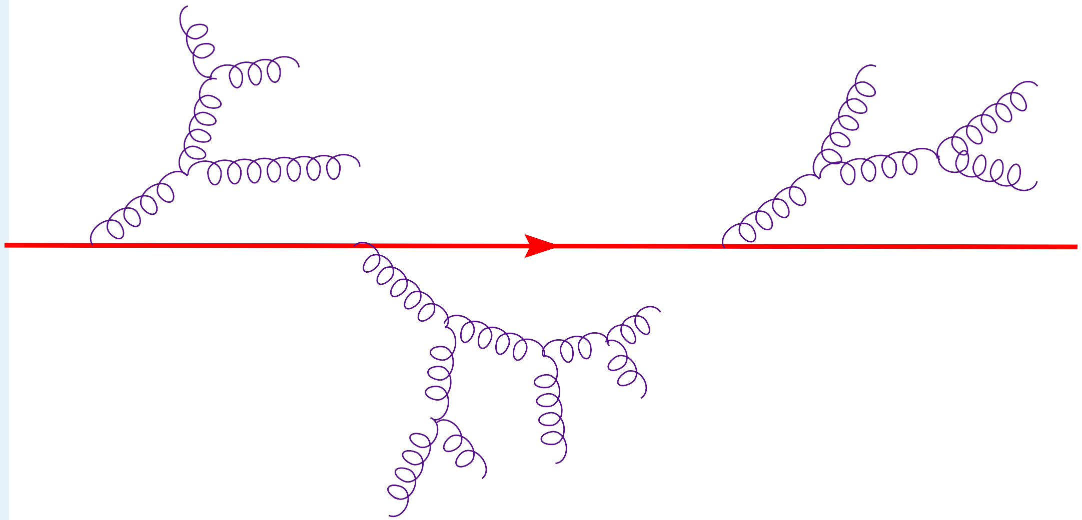
$$R_{AA}(p_\perp) \equiv \frac{Yield(A+A)}{Yield(p+p)\,\times\,A^2}$$





- RHIC data seem to prefer $\hat{q} \simeq 10$ GeV^2/fm, which is too large to be accounted for by weakly–coupled QGP (??)

Heavy Quark: Energy loss

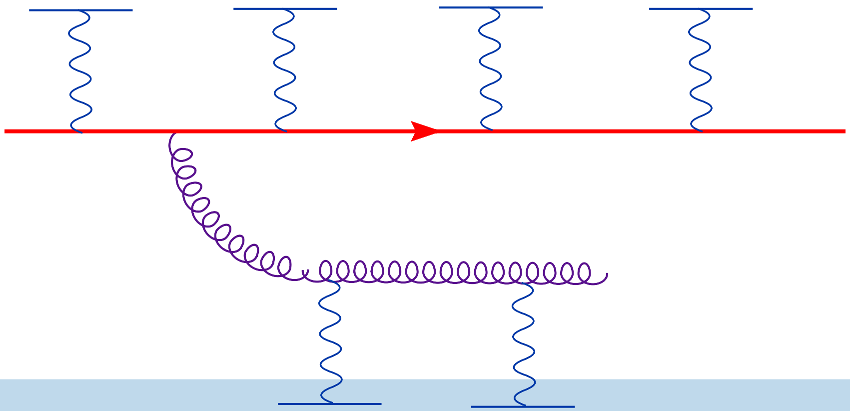


$$Q_s^2 \sim \left(t_{\rm br} T^2\right)^2 \sim \gamma\, T^2$$

$$-\frac{\mathrm{d}E}{\mathrm{d}t} \simeq \sqrt{\lambda}\,\frac{\omega}{(\omega/Q_s^2)} \simeq \sqrt{\lambda}\, Q_s^2$$

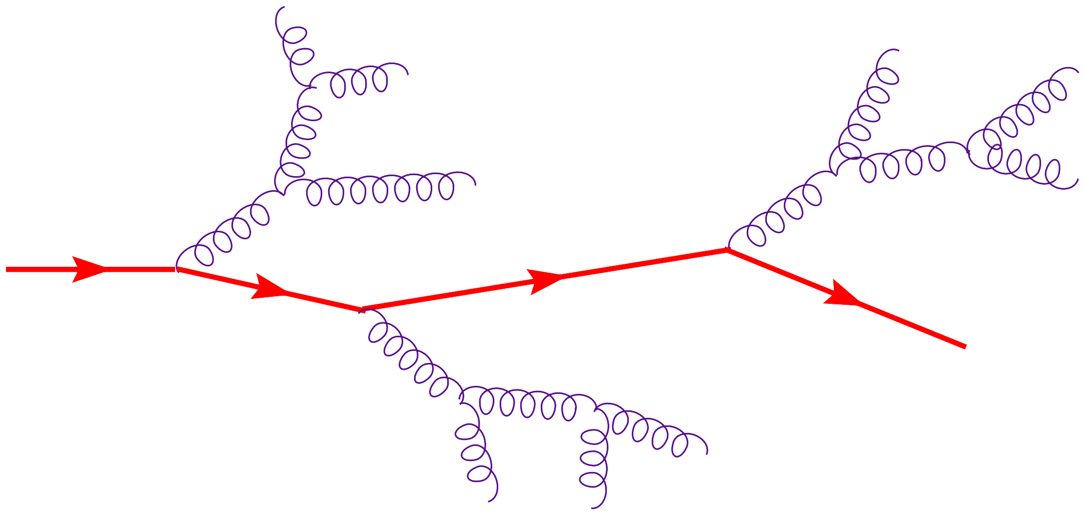
(Herzog et al; Gubser, 2006)

- pQCD : same formula, but with $\sqrt{\lambda} \to g^2 N_c$ and different Q_s



$Q_s^2 \sim t_{\rm br}\, \hat{q}$ (quenching param.)

Transverse momentum broadening

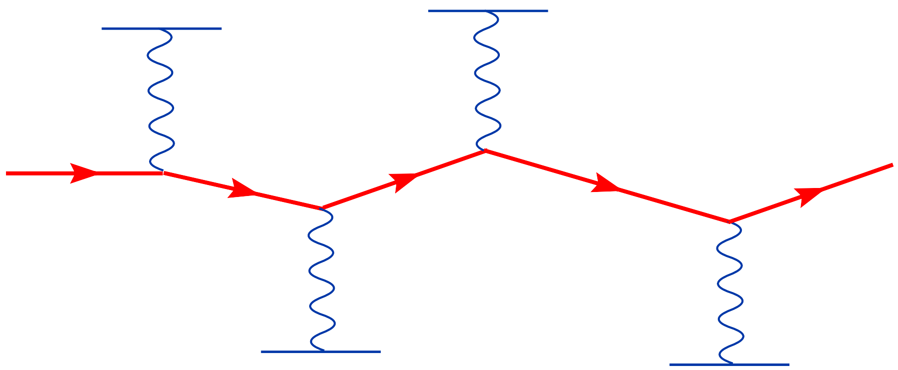


$$\frac{\mathrm{d}p_\perp^2}{\mathrm{d}t} \sim \sqrt{\lambda}\, T^2 Q_s$$

Casalderrey-Solana, Teaney, 2006; Gubser, 2006; Dominguez et al, 2008

see talks by Al Mueller and Cyrille Marquet

- pQCD : different physics ! thermal rescattering



$$\frac{\mathrm{d}p_\perp^2}{\mathrm{d}t} \sim \hat{q}$$