Jet fragmentation at strong coupling

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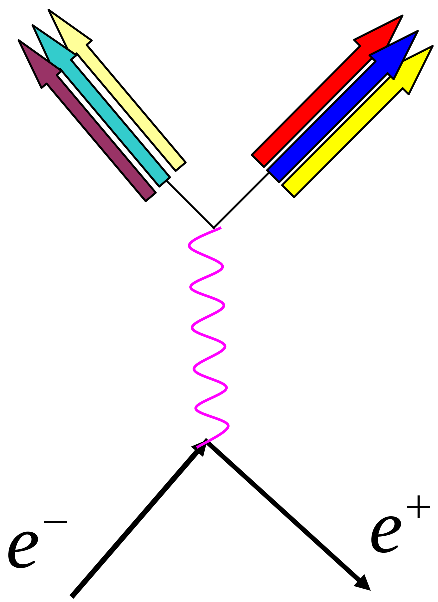
Based on :
Y.H. and T. Matsuo, arxiv:0804.4733 [hep-th],
arXiv:0807.0098 [hep-ph]

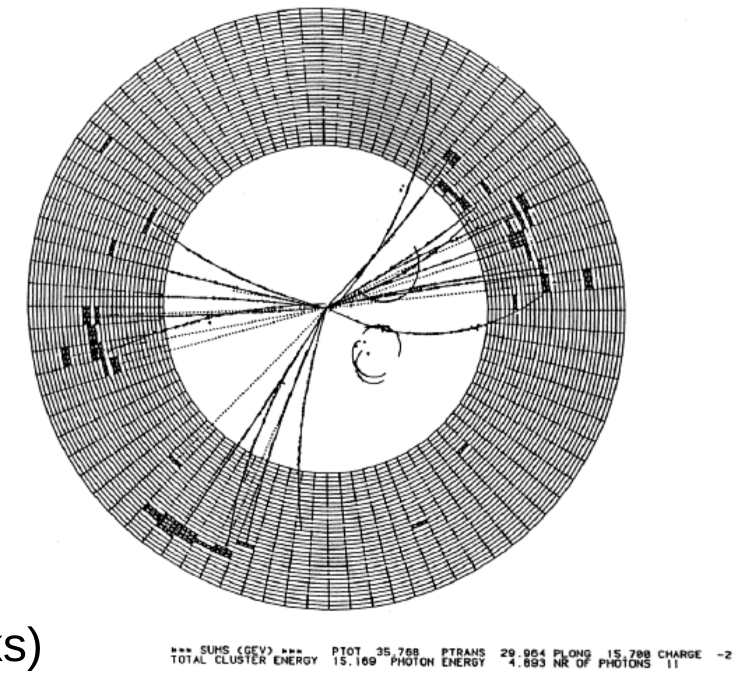
Contents

- e+e- annihilation and jets in QCD
- Jets in N=4 SYM ?
- Thermal hadron spectrum

Jets in QCD

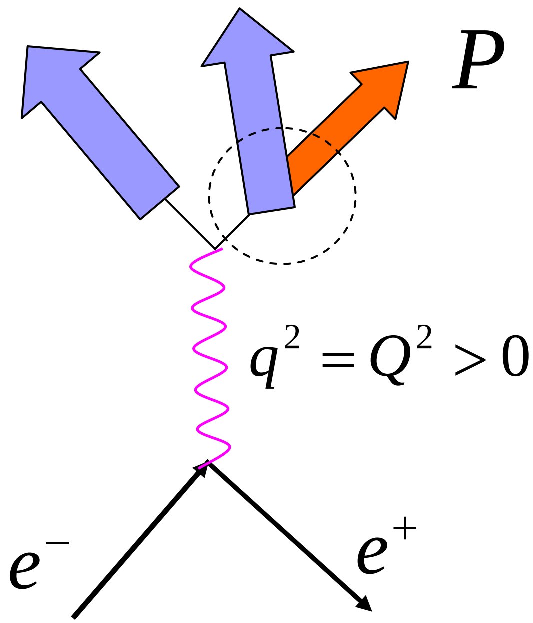


Observation of jets in `75 marks one of the most striking confirmations of QCD and the parton picture



Average angular distribution $1+\cos^2\theta$ reflecting fermionic degrees of freedom (quarks)

Fragmentation function



Count how many hadrons are there inside a quark.

$$D_T(x, Q^2)$$

Feynman-x $\quad x \equiv \frac{2P \cdot q}{Q^2} = \frac{2E}{Q}$

First moment

$$\int_0^1 dx \; D_T(x, Q^2) = \langle n \rangle$$

average multiplicity

Second moment

$$\int_0^1 dx \; x D_T(x, Q^2) = 2$$

energy conservation

Evolution equation

The fragmentation functions satisfy a DGLAP-type equation

$$\frac{\partial}{\partial \ln Q^2} D_T(x,Q^2) = \int_x^1 \frac{dz}{z} P_T(z) D_T\left(\frac{x}{z}, Q^2\right)$$

Take a Mellin transform $D_T(j,Q^2) = \int_0^1 dx\ x^{j-1} D_T(x,Q^2)$

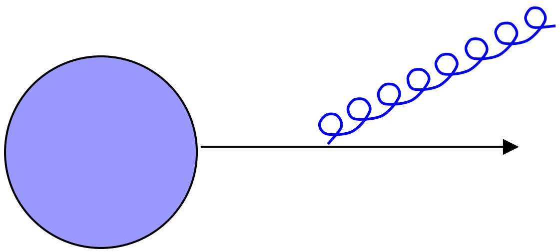
$$\frac{\partial}{\partial \ln Q^2} D_T(j,Q^2) = \gamma_T(j) D_T(j,Q^2)$$

Timelike anomalous dimension

$$\langle n \rangle = D_T(1,Q^2) \propto Q^{2\gamma_T(1)}$$ (assume $\beta = 0$)

Timelike anomalous dimension in QCD

Lowest order perturbation

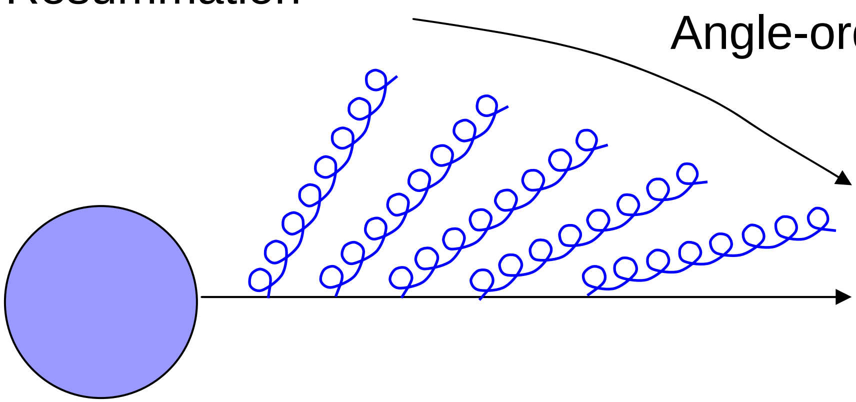


$$\sim \frac{1}{x} \quad \longleftrightarrow \quad \gamma_T(j) \propto \frac{\alpha_s}{j-1}$$

Soft singularity

$$\gamma_T(1) = \infty \quad !!$$

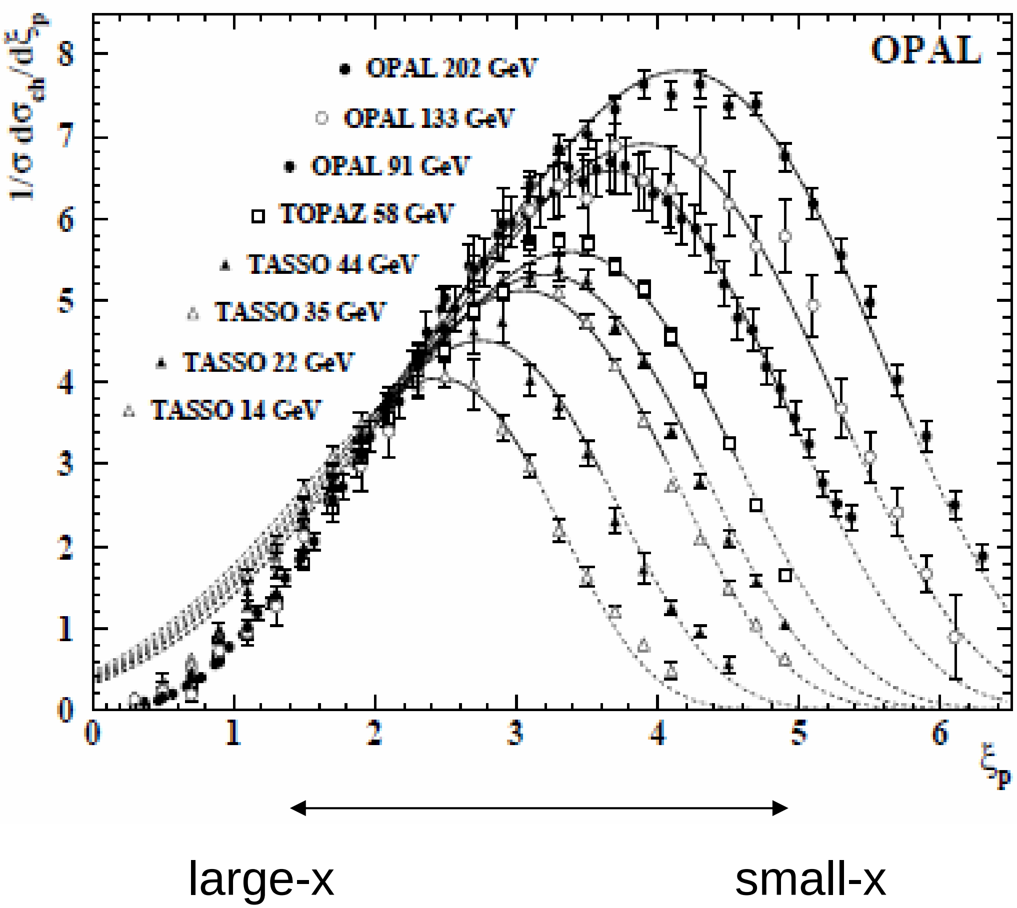
Resummation



$$\gamma_T(j) = \frac{1}{4}\left[\sqrt{(j-1)^2 + \frac{8N\alpha_s}{\pi}} - (j-1)\right]$$

Basseto, Ciafaloni, Marchesini, ‘80
Mueller, `81

Inclusive spectrum



$$\frac{x}{\sigma}\frac{d\sigma}{dx} \propto xD_T(x,Q^2)$$

'Hump-backed' shape consistent with pQCD with coherence.

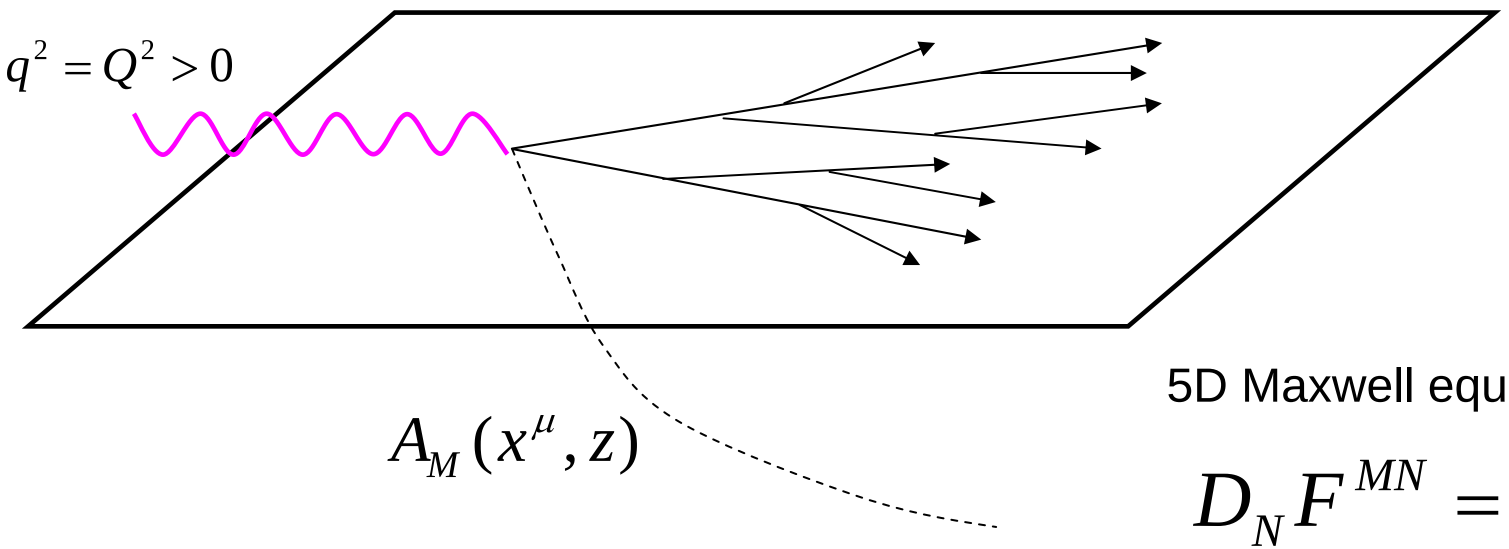
Roughly an inverse Gaussian in $\ln 1/x$ peaked at

$$\frac{1}{x} = \sqrt{\frac{Q}{\Lambda}}$$

Mueller,
Dokshitzer, Fadin, Khoze

e+e- annihilation in strongly coupled N=4 SYM

Hofman, Maldacena arXiv:0803.1467 [hep-th]
Y.H., Iancu, Mueller, arXiv:0803.2481 [hep-th]
Y.H., Matsuo arXiv:0804.4733 [hep-th]
Y.H., Matsuo arXiv:0807.0098 [hep-ph]



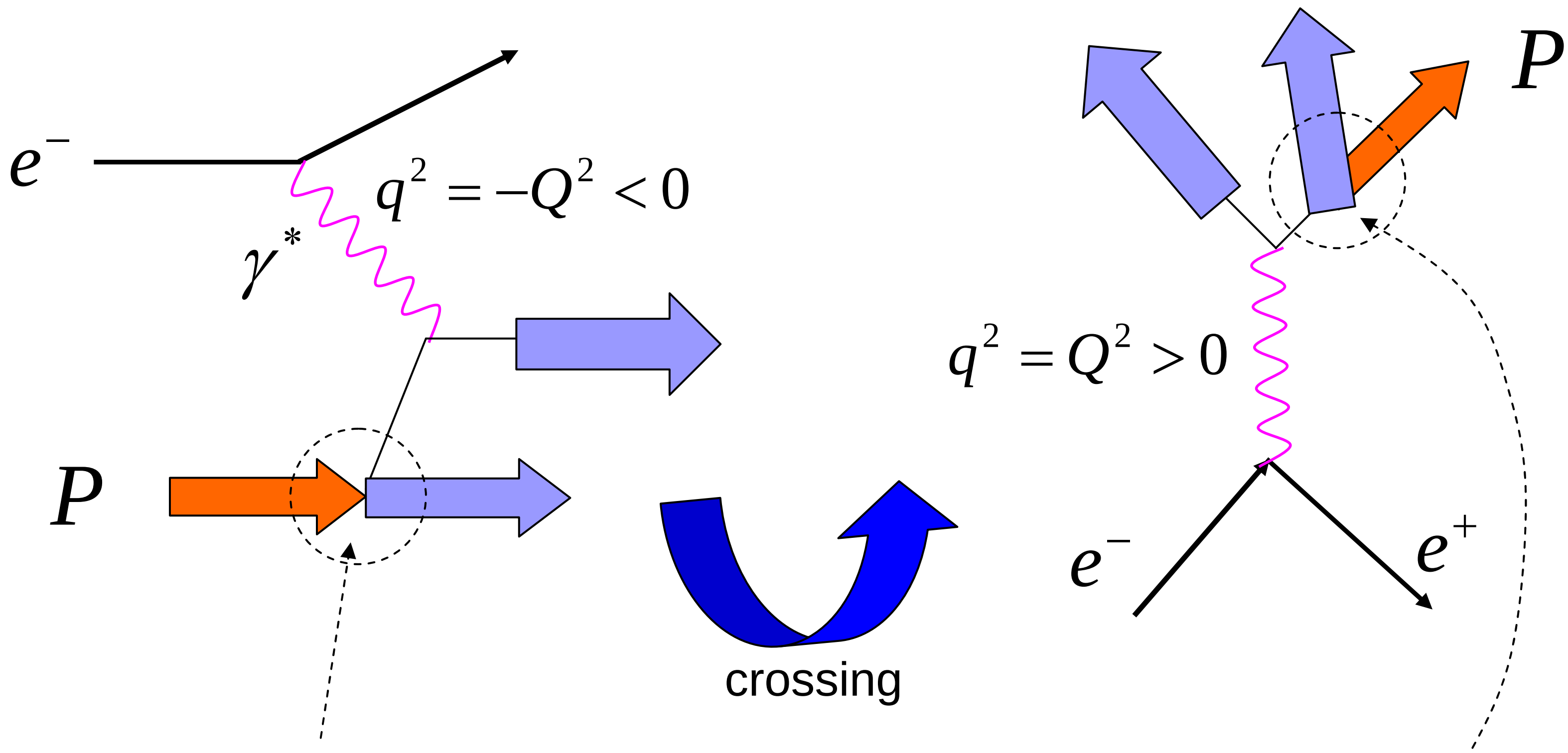
5D Maxwell equation

$$D_N F^{MN} = 0$$

Want to compute $\gamma_T(j)$ at strong coupling

But there is no corresponding operator in gauge theory !

DIS vs. e+e-



Parton distribution function

$$D_S(x, Q^2)$$

Bjorken variable $x \equiv \frac{Q^2}{2P \cdot q}$

Fragmentation function

$$D_T(x, Q^2)$$

Feynman variable $x \equiv \frac{2P \cdot q}{Q^2}$

A reciprocity relation

DGRAP equation

$$\frac{\partial}{\partial \ln Q^2} D_{S/T}(j,Q^2) = \gamma_{S/T}(j) D_{S/T}(j,Q^2)$$

The two anomalous dimensions derive from a single function

$$\gamma_S(j) = f(j - \gamma_S(j))$$
$$\gamma_T(j) = f(j + \gamma_T(j))$$

Dokshitzer, Marchesini, Salam, ‘06

$f(j)$ respects the Gribov-Lipatov reciprocity

Checked up to three loops (!) in QCD   Mitov, Moch, Vogt, `06

Application to AdS/CFT   Basso, Korchemsky, ‘07

Assume this is valid at strong coupling and see where it leads to.

Average multiplicity at strong coupling

$$\gamma_S(j) = \frac{j}{2} - \frac{1}{2}\sqrt{2\sqrt{\lambda}(j - j_0)} \quad \xleftrightarrow{\text{crossing}} \quad \gamma_T(j) = -\frac{1}{2}\left(j - j_0 - \frac{j^2}{2\sqrt{\lambda}} \right)$$

Kotikov, Lipatov, Onishchenko, Velizhanin `05
Brower, Polchinski, Strassler, Tan, `06

$$n(Q) \propto (Q/\Lambda)^{2\gamma_T(1)} = (Q/\Lambda)^{1-3/2\sqrt{\lambda}}$$

Y.H., Matsuo '08

c.f. in perturbation theory, $n(Q) \propto Q^{\sqrt{\frac{\lambda}{2\pi^2}}}$

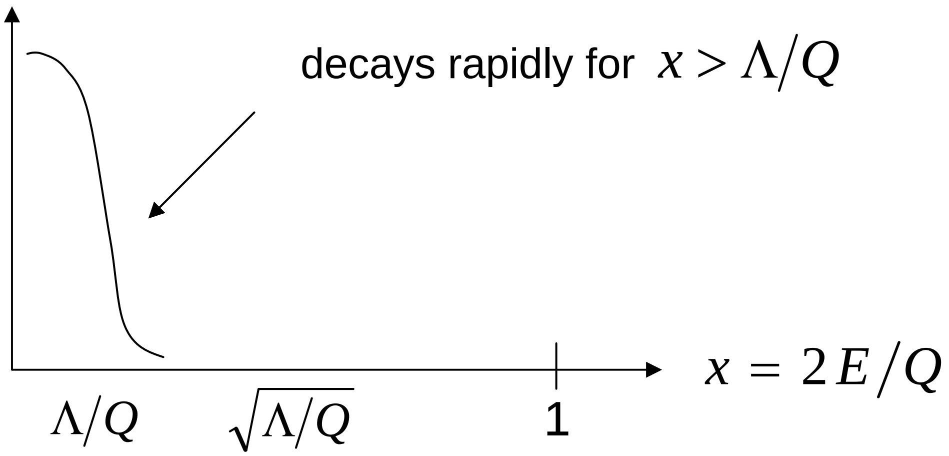
c.f. heuristic argument $n(Q) \propto Q$ Y.H., Iancu, Mueller '08

Jets at strong coupling?

$\gamma_T(j) \approx 1 - \frac{j}{2}$ in the supergravity limit $\lambda \to \infty$

The inclusive distribution is peaked at the kinematic lower limit

$$D_T(x, Q^2) = Q^2 F\left(\frac{Qx}{\Lambda}\right)$$

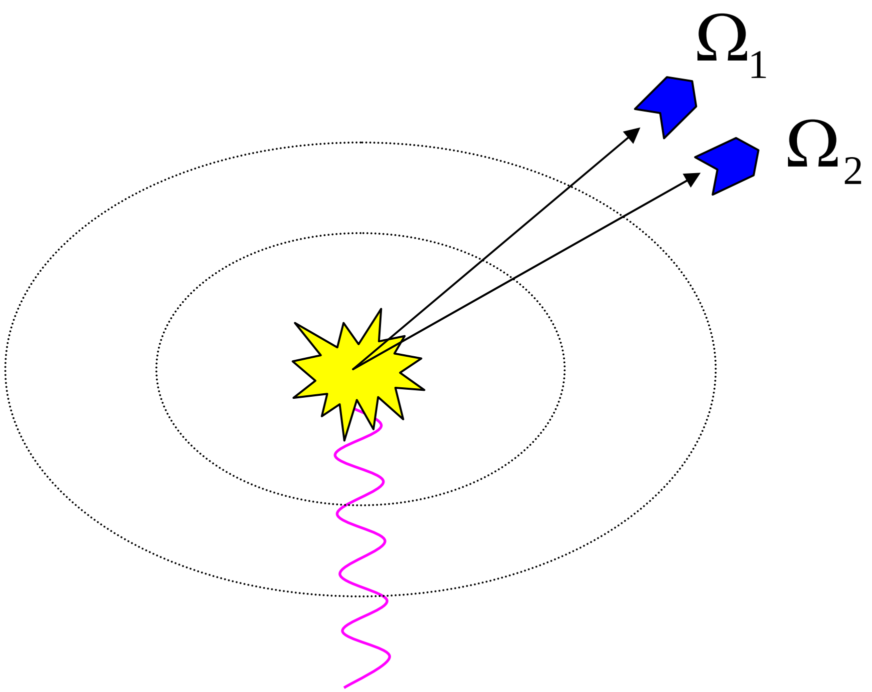


At strong coupling, branching is so fast and efficient. There are no partons at large-x !

Energy correlation function

Hofman, Maldacena `08

Energy distribution is spherical for any λ
Correlations disappear as $\lambda \to \infty$



$$\langle \mathcal{E}(\Omega_1)\mathcal{E}(\Omega_2)\rangle \sim \frac{1}{|\theta_{12}|^{2+2\gamma_S(3)}}$$

$$\gamma_S(3) = O(\lambda) << 1 \quad \text{weak coupling}$$
$$= -\lambda^{1/4}/\sqrt{2} \quad \text{strong coupling}$$

All the Q/Λ particles have the minimal four momentum $\sim \Lambda$ and are spherically emitted. There are no jets at strong coupling !

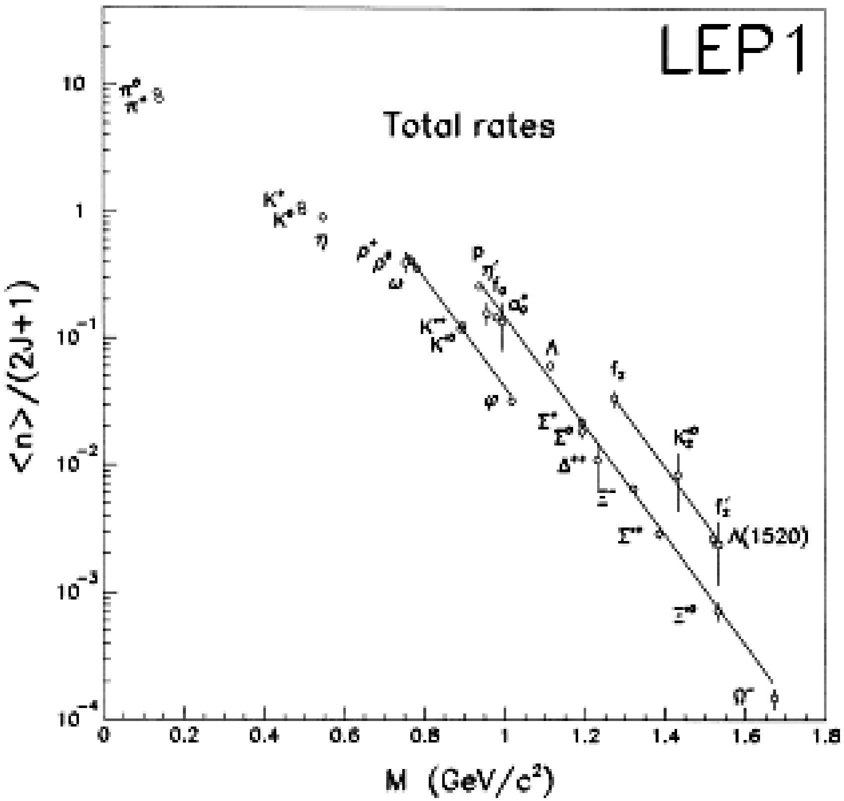
Thermal hadron spectrum

Identified particle yields are well described by a thermal model

$$\frac{N^*}{N} \propto \exp\left(-\frac{M^* - M}{T} \right)$$

The model works in e+e- annihilation, hadron collisions, and heavy-ion collisions

Becattini,
Chliapnikov,
Braun-Munzinger et al.



The origin of the exponential law is not understood,
though there are many speculations (QGP? Hawking radiation?) around…

A statistical model

Bjorken and Brodsky, '70

Total cross section $\sigma_{tot} = \frac{e^4 N_c^2}{32\pi Q^2} = \sum_n \sigma_n$ given by the one-loop result !

Cross section to produce exactly n 'pions' $n \approx Q/\Lambda$

$$\sigma_n = \frac{e^4}{2Q^6}(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k') \prod_{i=1}^{n} \int \frac{d^3 p_i}{2E_i (2\pi)^3}$$
$$\times \langle 0 | j^\mu(0) | p_1, ...p_n \rangle \langle p_1, ...p_n | j^\nu(0) | 0 \rangle (2\pi)^4 \delta^{(4)}(q - \sum_i p_i)$$

Assume
$$\langle 0 | j^\mu(0) | p_1, ...p_n \rangle \langle p_1, ...p_n | j^\nu(0) | 0 \rangle$$
$$\rightarrow a_n (q^\mu q^\nu - g^{\mu\nu} q^2)\, e^{-\beta Q}$$

Then
$$2E \frac{dN}{d^3 p} \sim \frac{n\beta}{\pi\Lambda} e^{-\beta E} \sim \frac{Q\beta}{\pi\Lambda^2} e^{-\beta E}$$

Strikingly similar to the AdS/CFT predictions !

Computing the matrix element

$$\langle 0|\epsilon \cdot j(0)|p_1, \ldots p_n\rangle \sim \frac{g_c^{n+1}}{\alpha' g_c^2}\int dz\, d\Omega_5 \sqrt{-G}\, F(\alpha'\partial^2)(\Phi)^n A_\mu$$

string amplitude

AdS metric

5D scalars

5D photon

$$A_\mu(z) \propto N_c H_1^{(1)}(Qz) \sim \exp(iQz)$$

$$\Phi_i(x,z) = e^{ip_i x}\frac{\sqrt{2}\Lambda z^2}{2\pi^{3/2}R^4} J_{\Delta_+ - 2}(mz)$$

Since $n \propto Q$ is large, do the saddle point approximation in the z-integral

The saddle point $z = z^*$ has an imaginary part.

$$|\langle 0|\epsilon \cdot j|p_1, \ldots p_n\rangle|^2 \sim \frac{N_c^2}{N_c^{2n-2}} \frac{nQ^2}{\Lambda^{2n-4}} e^{-2cQ/\Lambda}$$

Probably absent once the decay width is
Included in the propagators.

Einhorn, '77

$$\gamma^* \to V \to n\pi$$
$$O(N_c^2) \quad O(1)$$

This leads to

$$\frac{dN}{d^3p} \propto e^{-2cE/\Lambda}$$

$c \approx 0.2$ for 'mesons' $\Delta = 2$

$c \approx 0.5$ for `partons' $\Delta = 1$

Summary

- There are not jets in N=4 SYM at strong coupling. (unlike in QCD…)
- The multiplicity rises almost linearly with the energy. (unlike in QCD…)
- The multiplicity ratio exhibits the ‘‘thermal’’ behavior.
  (like in QCD?)