QCD, Strings and AdS/CFT, an Introduction

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LOW-x WORKSHOP,
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- Gauge/Gravity Correspondence

  Gauge Fields/String: "Duality" and "Holography"
- AdS/CFT Correspondence for S^4QCD

  Extensions beyond CFT
- A case study: AdS/CFT and QGP Hydrodynamics

  Emergence of a 5d Black Hole Geometry
- AdS/CFT and QGP Dynamics

  Quasi-Perfect Fluidity,Viscosity,Thermalization,...
- Conclusions and Prospects

  Progress and Open Problems of the QCD/String Connection

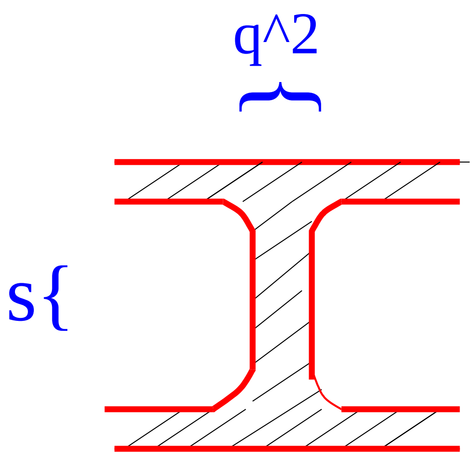
Strong Interactions and Strings

Strong Interactions History

Strings $\not\Leftrightarrow$ QCD $\Leftrightarrow$ Strings

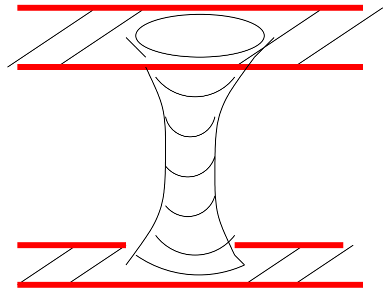
$\boxed{1968} \Rightarrow \boxed{1974} \Rightarrow \boxed{1998} \Rightarrow \boxed{2007} \ldots$

$\boxed{1968}$



Veneziano Amplitude

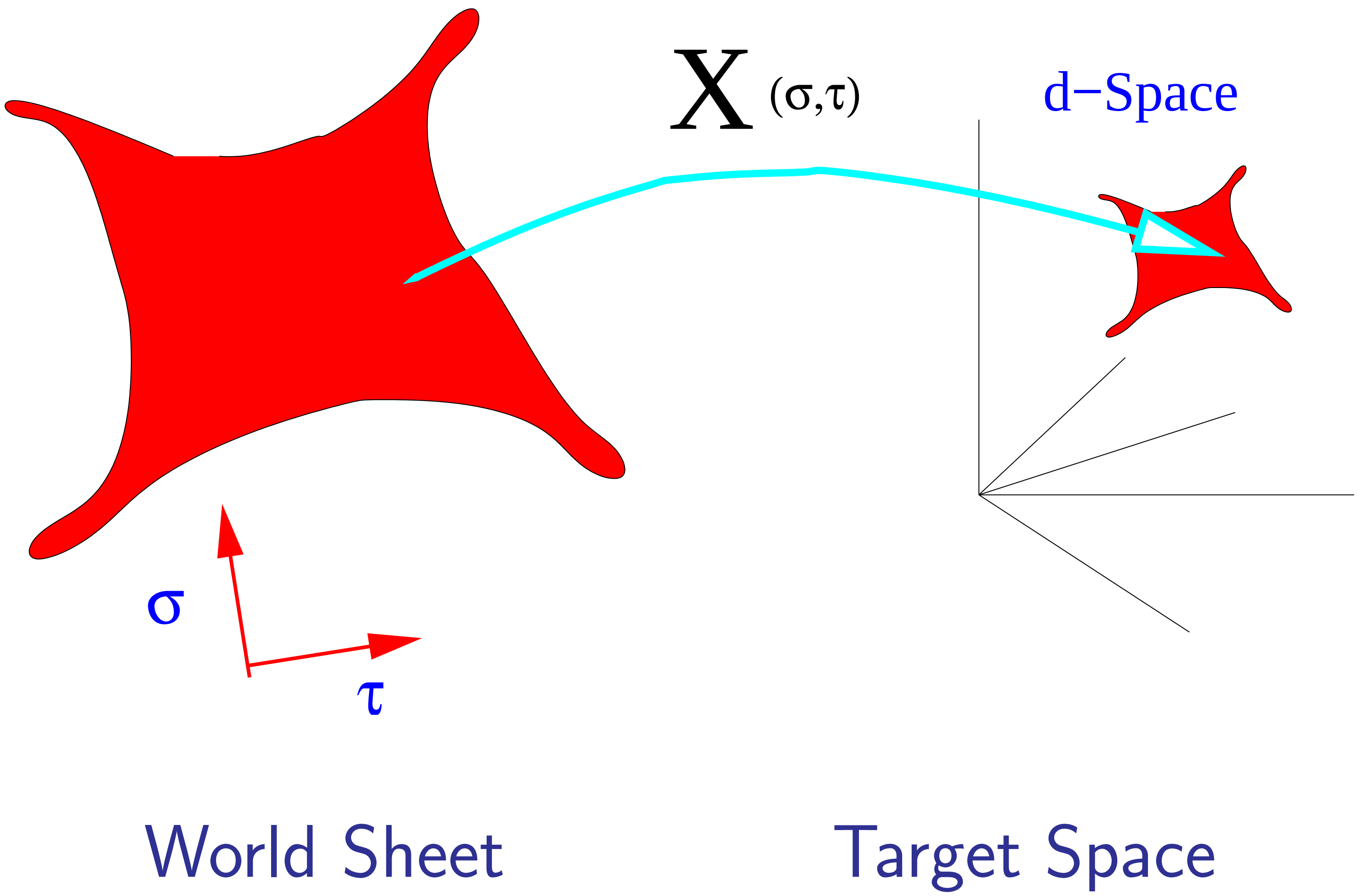
$A_R(s,q\text{^}2)$



Shapiro-Virasoro Amplitude

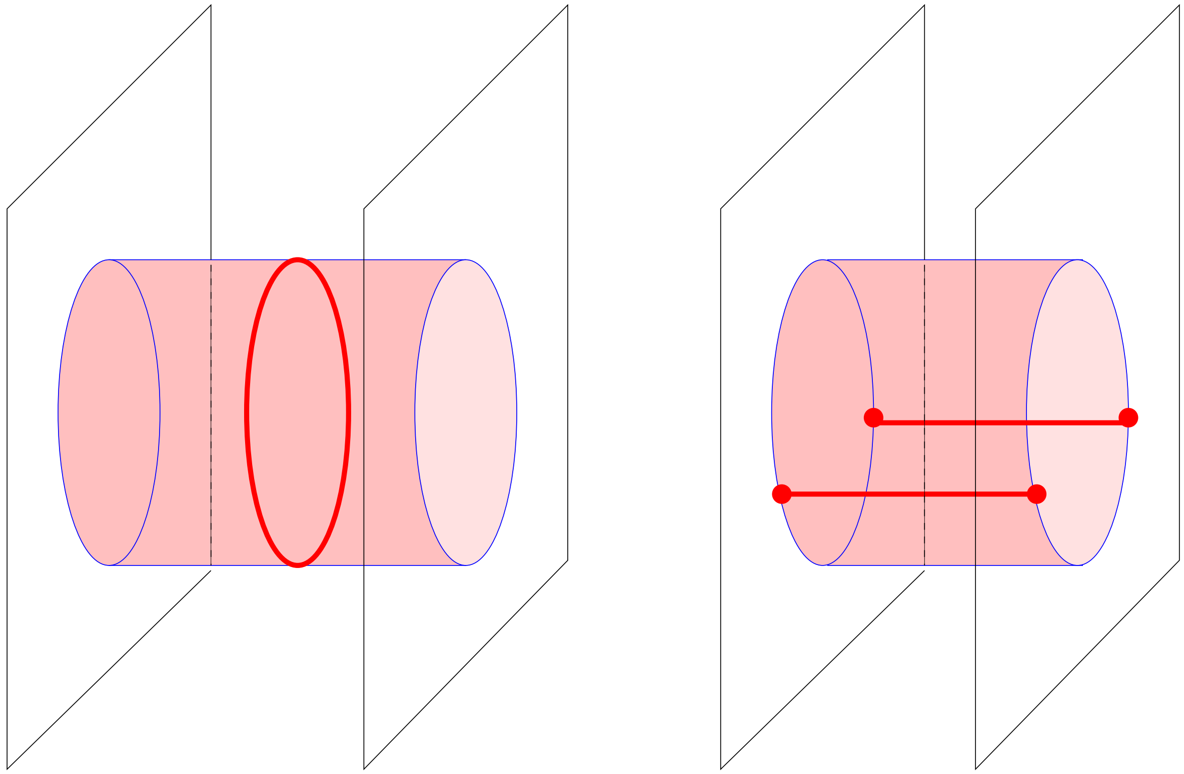
$A_P(s,q\text{^}2)$

String Apparatus



The Gauge-Gravity Correspondence

"Duality": Open String ⇔ Closed String



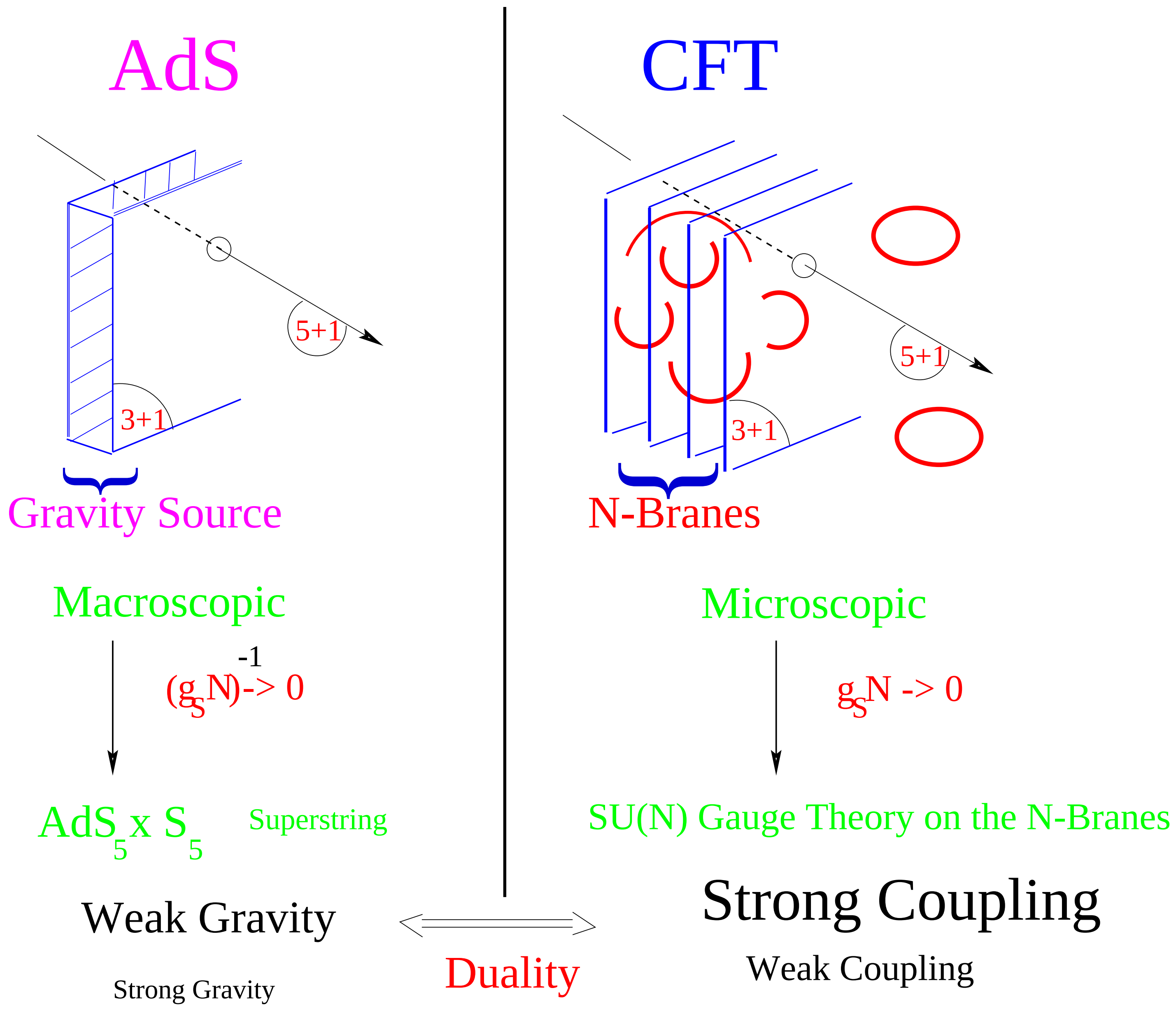
Schomerus, 2006

$$\begin{aligned} Closed\ String &\Leftrightarrow 1-loop\ Open\ String \\ D\!-\!Brane\ \text{``}Universe\text{''} &\Rightarrow Open\ String\ Ending \\ Gravity &\Leftrightarrow Gauge \\ Large/Small\ Distance &\Rightarrow Gravity/Gauge\ Correspondence \end{aligned}$$

AdS/CFT Correspondence

J.Maldacena

1998



DUALITY

- D_3-brane Solution of Supergravity: Horowitz,Strominger, 1991

$$\boxed{ds^2 = f^{-1/2}(-dt^2 + \sum_1^3 dx_i^2) + f^{1/2}(dr^2 + r^2 d\Omega_5)}$$

"Physical" Brane + Extra-Dimensions

$$f = 1 + \frac{R^4}{r^4} \; ; \; R^4 = 4\pi\alpha'^2 g_{YM}^2 N_c$$

- "Maldacena limit":

$$\frac{\alpha'(\to 0)}{r(\to 0)} \to z \; , \; R \; fixed \; \Rightarrow g_{YM}^2 N_c \to \infty$$

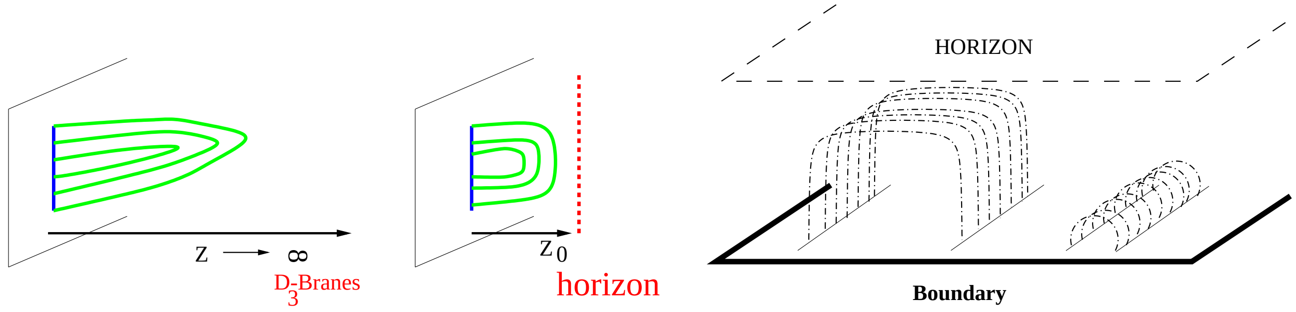
Strong coupling limit

$$\boxed{ds^2 = \frac{1}{R^2 z^2}(-dt^2 + \sum_{1-3} dx_i^2 + dz^2) + R^2 d\Omega_5}$$

Background Structure: $AdS_5 \times S_5$ (same R^2)

HOLOGRAPHY

Exemple of $\langle Wilson\ Lines \rangle$



NON-CONFINING CONFINING GEOMETRY

$$\langle e^{iP\int_C \vec{A}\cdot\vec{dl}} \rangle = \int_\Sigma e^{-\frac{Area(\Sigma)}{\alpha'}} \approx e^{-\frac{Min.Area}{\alpha'}} \times Fluctuations$$

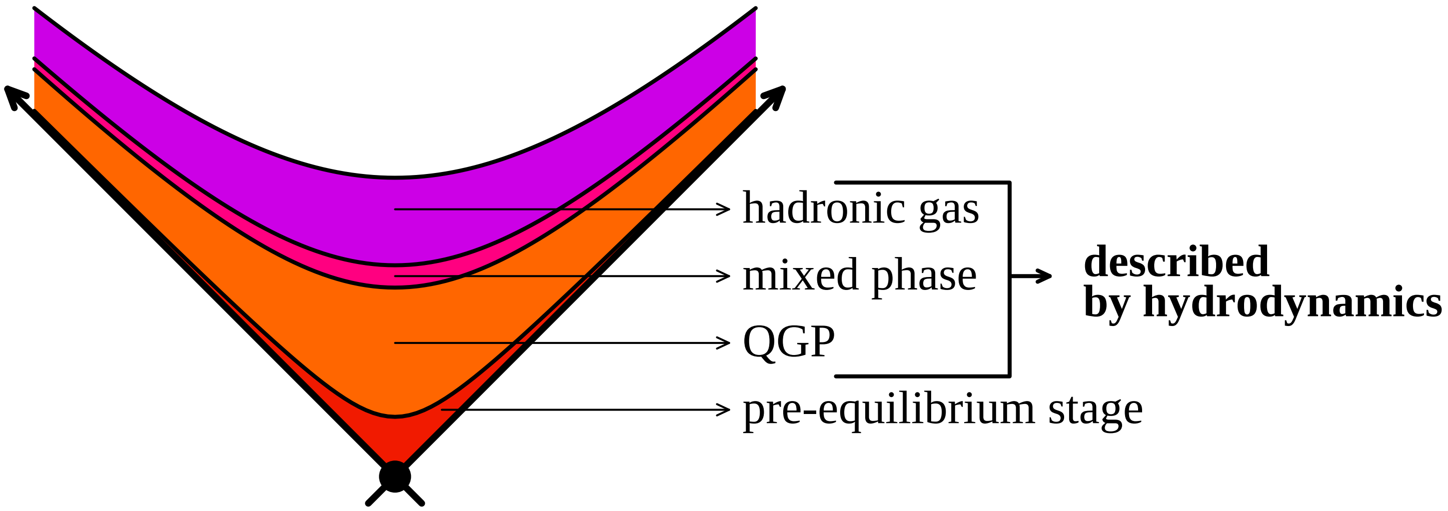
- Conformal case : AdS_5/CFT_4

$$\langle Wilson\ Lines \rangle = e^{T*V(L)} \sim e^{T\times 1/L}$$

- Confining case : ex: Horizon at z_h (Witten 1998)

$$\langle Wilson\ Lines \rangle = e^{T*V(L)} \sim e^{T\times L(\times z_h^2)}$$

2007 A case Study: QGP formation and Relativistic Hydrodynamics



"Abstracted" from Experiments:

- Kinematic Landscape

$$\tau = \sqrt{x_0^2 - x_1^2} \; ; \; \eta = \frac{1}{2} \log \frac{x_0 + x_1}{x_0 - x_1} \; ; \; x_T = \{x_2, x_3\}$$

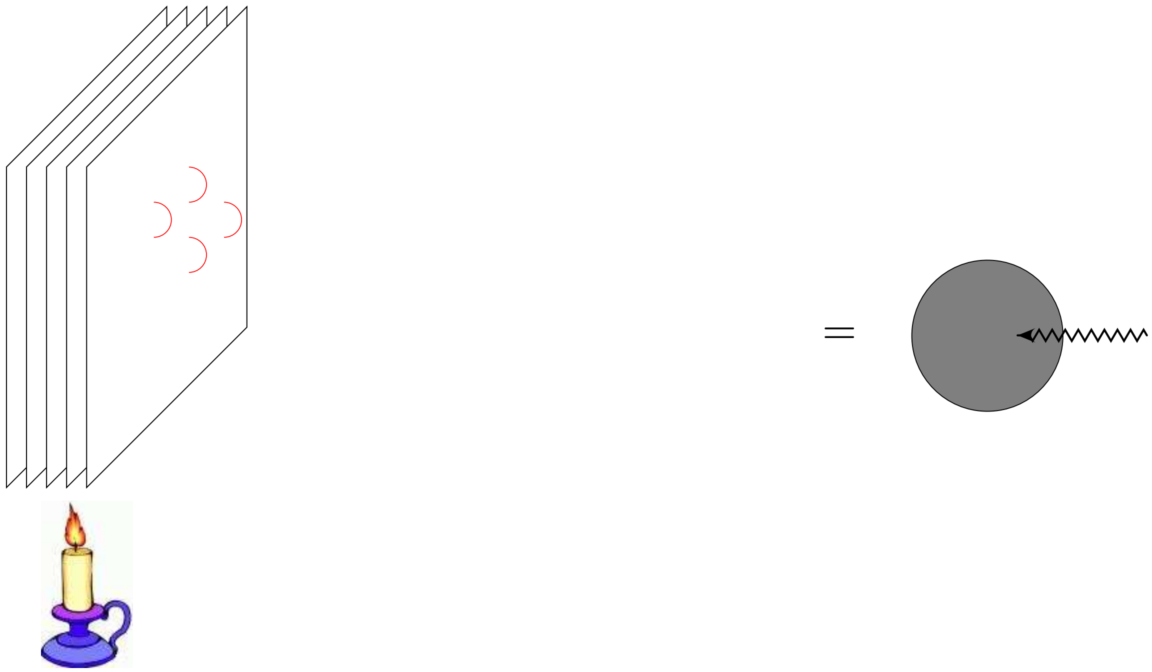
- QGP: (Almost) Perfect fluid behaviour (small Viscosity)

- Fast QGP Formation

Strong Coupling QGP: What Strings can teach us?

Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes
Will be absorbed by the D3 branes.
The process of absorption can be looked at from two different perspectives:



Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

$$\sigma_{abs}(\omega) \propto \int d^4x \, \frac{e^{i\omega t}}{\omega} \, \langle [T_{x_2 x_3}(x), T_{x_2 x_3}(0)] \rangle \Rightarrow \boxed{\frac{\eta}{s} \equiv \frac{\sigma_{abs}(0)/(16\pi \; G)}{A/(4 \; G)} = \frac{1}{4\pi}}$$

Kovtun, Policastro, Son, Starinets (2001-...)

Tool: Holographic Renormalization

K.Skenderis (2002)

- Using Fefferman-Graham Coordinates:

$$ds^2 = \frac{g_{\mu\nu}(z)\ dx^\mu dx^\nu + dz^2}{z^2}$$

- $4d \Leftrightarrow 5d$ metric:

$$g_{\mu\nu}(z) = g^{(0)}_{\mu\nu}(= \eta_{\mu\nu}) + z^2 g^{(2)}_{\mu\nu}(= 0) + z^4 g^{(4)}_{\mu\nu}(\propto \langle T_{\mu\nu}\rangle) + \ldots$$

$+\ldots$: derived from Einstein Eqs.

- $4d$ Constraints

$$T^\mu{}_\mu = 0 \quad ; \quad \mathcal{D}_\nu T^{\mu\nu} = 0$$

EMERGENCE of the 5d BLACK HOLE

Balasubramanian,de Boer,Minic; Myers; Janik,R.P.

- 4d Perfect Fluid "on the brane"

$$\langle T_{\mu\nu}\rangle \propto g^{(4)}_{\mu\nu} = \begin{pmatrix} 3/z_0^4 = \epsilon & 0 & 0 & 0 \\ 0 & 1/z_0^4 = p_1 & 0 & 0 \\ 0 & 0 & 1/z_0^4 = p_2 & 0 \\ 0 & 0 & 0 & 1/z_0^4 = p_3 \end{pmatrix}$$

- Holographic Renormalisation (Resummed)

$$ds^2 = -\frac{(1 - z^4/z_0^4)^2}{(1 + z^4/z_0^4)z^2}dt^2 + (1 + z^4/z_0^4)\frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

- $\Rightarrow 5d$ Black Brane with horizon at $z_0 \sim T_0^{-3}$

$$ds^2 = -\frac{1 - \tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2}dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1 - \tilde{z}^4/\tilde{z}_0^4}\frac{d\tilde{z}^2}{\tilde{z}^2}$$

$$z \to \tilde{z} = \; z\Big/\sqrt{1+\frac{z^4}{z_0^4}}$$

AdS/CFT: From Statics to Dynamics

R.Janik, RP (2005)

- Constraint Equations

$$\mathrm{T}^{\mu}{}_{\mu} \equiv -T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} + 2T_{xx} = 0$$
$$\mathrm{D}_{\nu}T^{\mu\nu} \equiv \tau\frac{d}{d\tau}T_{\tau\tau} + T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} = 0$$

- Boost-invariant Tensor

$$T_{\mu\nu} = \begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3\frac{d}{d\tau}f(\tau) - \tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau) + \frac{1}{2}\tau\frac{d}{d\tau}f(\tau) & 0 \\ 0 & 0 & 0 & \dots \end{pmatrix}$$

- 1-parameter family of proper-time evolution

$f(\tau) \propto \tau^{-s}$: $T_{\mu\nu}t^{\mu}t^{\nu} \geq 0 \Rightarrow 0 < s < 4$

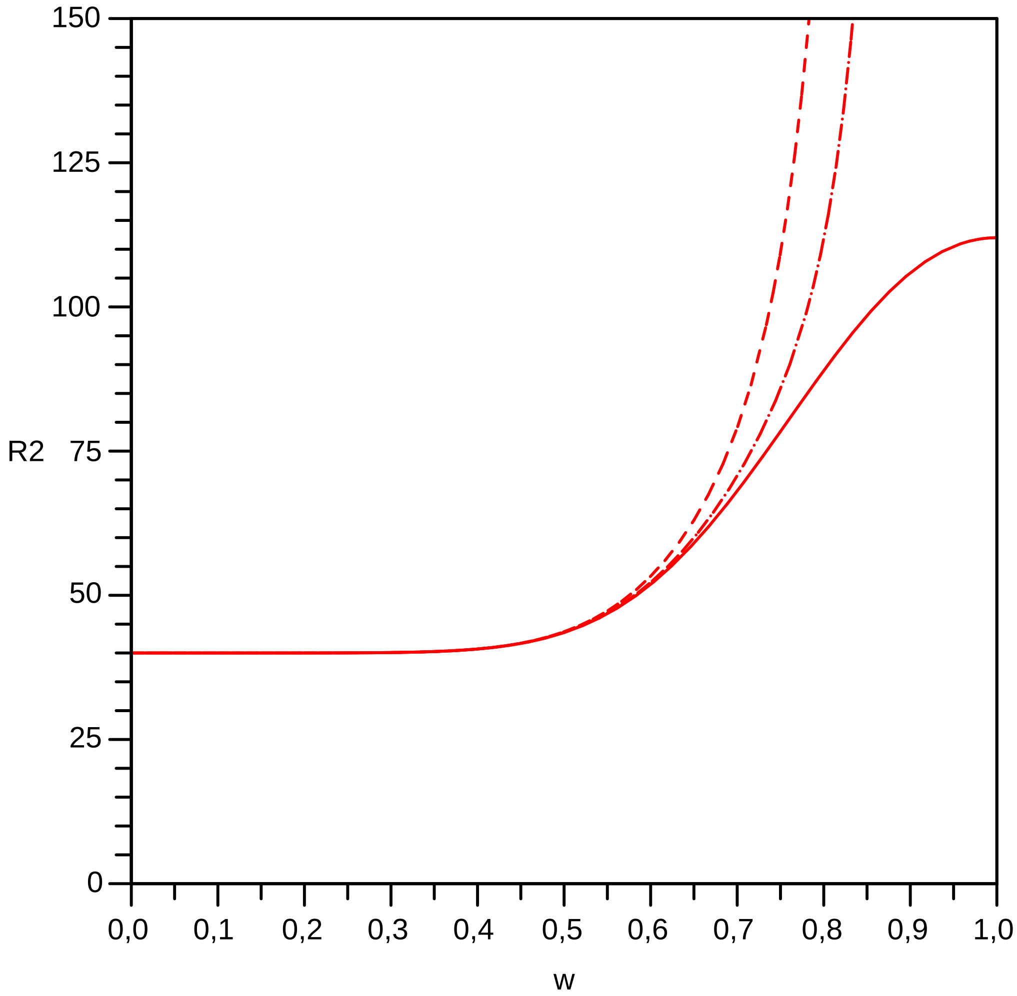
$f(\tau) \propto \tau^{-\frac{4}{3}}$: Perfect Fluid

$f(\tau) \propto \tau^{-1}$: Free streaming

$f(\tau) \propto \tau^{-0}$: "Full Anisotropy" $\epsilon = p_{\perp} = -p_L$

AdS/CFT $\Rightarrow$ Perfect Fluid at large τ

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$$



$s = \frac{4}{3} \pm .1$

A nonsingular background selects a moving Black Hole geometry corresponding to the perfect fluid at large proper-times

Dual of a Perfect fluid in S^{4}QCD "heavy ion experiment" : a Moving Black Hole

$$\boxed{v = \frac{z}{\tau^{1/3}}}$$

- Asymptotic metric

$$\boxed{ds^2 = \frac{1}{z^2}\left[-\frac{\left(1-\frac{e_0}{3}\ \frac{z^4}{\tau^{4/3}}\right)^2}{1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2+\left(1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)\left(\tau^2 dy^2+dx_\perp^2\right)\right]+\frac{dz^2}{z^2}}$$

- Black Hole off in the 5th dimension

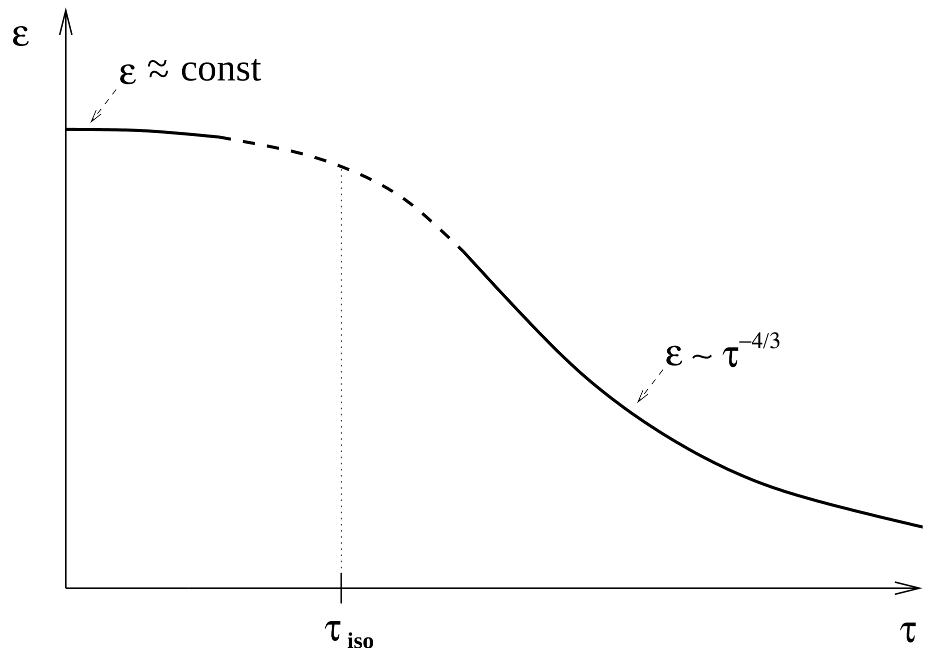
$$Horizon: \ z_h = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \cdot \tau^{\frac{1}{3}} \ .$$

$$Temperature: \ T(\tau) \sim \frac{1}{z_h} \sim \tau^{-\frac{1}{3}}$$

$$Entropy: \ S(\tau) \sim Area \sim \tau \cdot \frac{1}{z_h^3} \sim const$$

AdS/CFT: Anisotropy at small τ

Kovchegov, Taliotis arXiv:0705.1234



Evaluation of τ_{iso}

$$Matching: \; z_h^{late}(\tau) = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \equiv z_h^{early}(\tau) = \tau$$

$$Isotropization: \; \tau_{iso} = \left(\frac{3N_c^2}{2\pi^2 e_0}\right)^{3/8}$$

$$Evaluation: \epsilon(\tau) = \; e_0 \, \tau^{4/3}|_{\tau=.6} \sim 15 \, GeV\, fermi^{-3} \Rightarrow \tau_{iso} \sim .3_{fermi}$$

Conclusions

In progress:

- Gauge-Gravity Correspondence
  A promising way towards QCD at strong couplings
- Results on AdS/CFT $\rightarrow S^4$QCD Hydrodynamics
  Perfect Fluid, Viscosity, Isotropization
- Other studies
  Jet Quenching, Quark Dragging, and many others...

In outlook:

- Can we go beyond Boost Invariance?
  From Bjorken to Landau to “real” Hydrodynamics?
- Can we follow the flow from Ions to Hadrons?
  Initial and Final conditions for Hydrodynamics
- From S^4QCD to S^0QCD Hydrodynamics ?
  Can we construct the “Dual” of the QGP?

Some references...

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EXTRA SLIDES

Initial Conditions: Shock Waves (1)

- One Initial Shock Wave

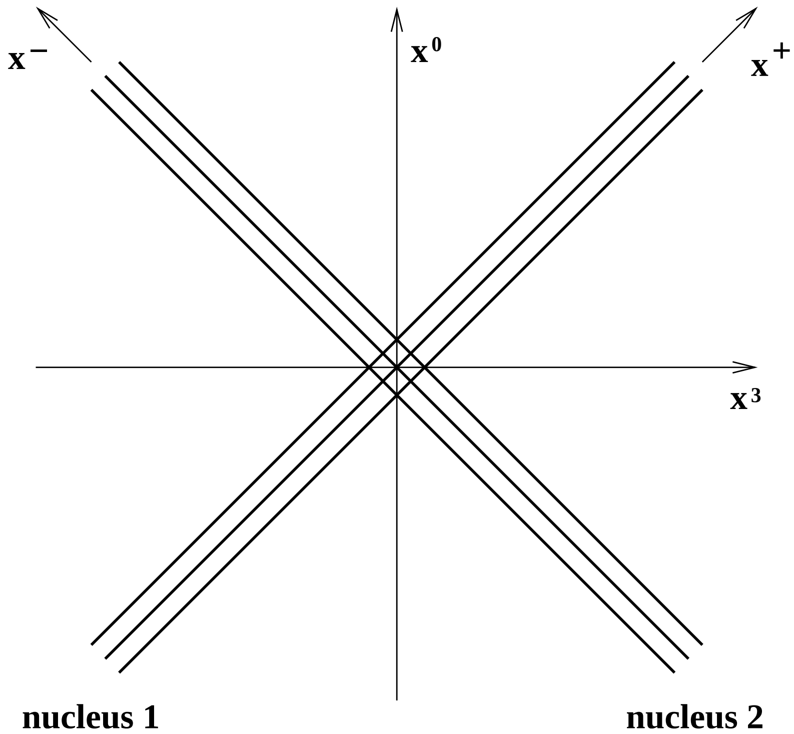
Janik,R.P.(2005)

$$ds^2 = \frac{-2dx^+dx^- + \mu_1 z^4 \delta(x^-) dx^{-2} + d\mathbf{x}_\perp{}^2 + dz^2}{z^2}$$

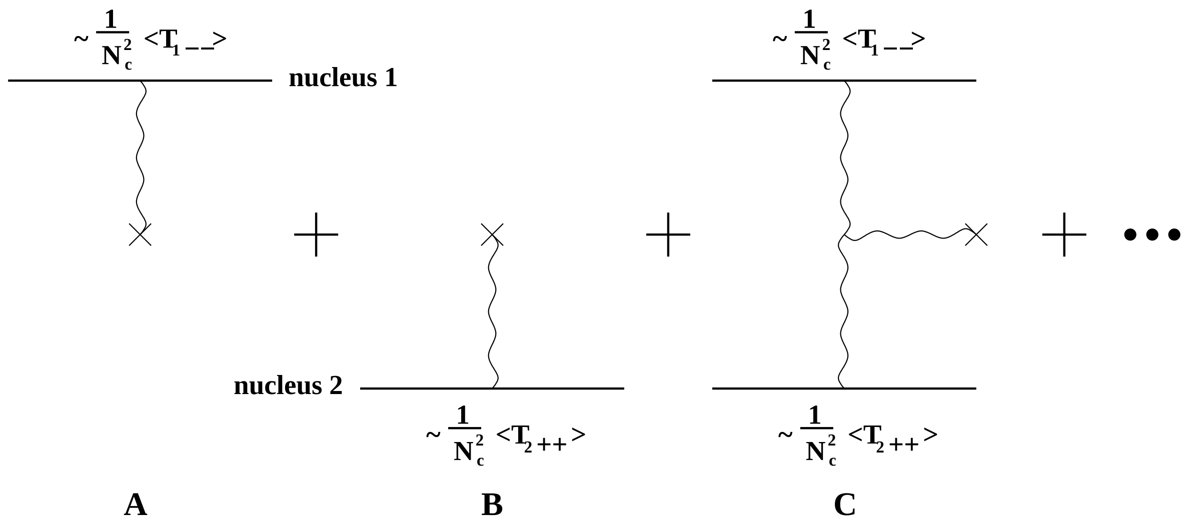
- Shock-wave collisions

Grumiller,Romatschke (2008)

Albacete,Kovchegov,Taliotis (2008)



Initial Conditions: Shock Waves (2)

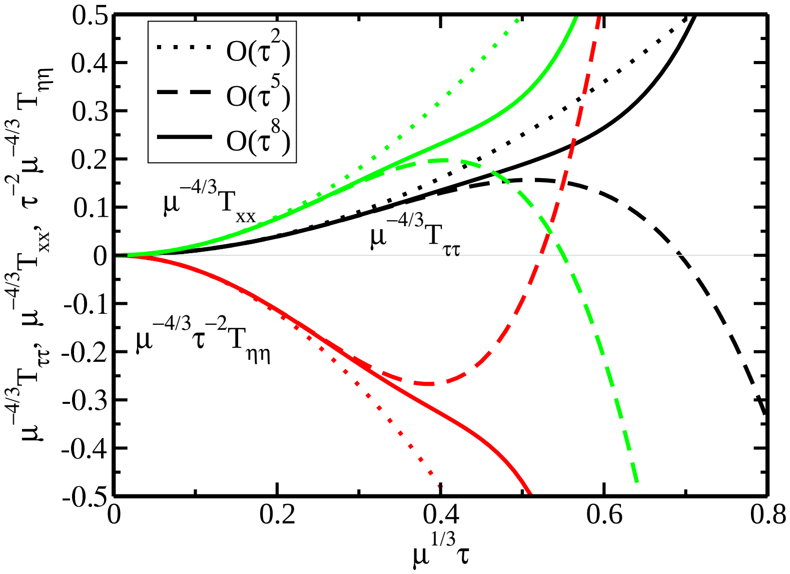


$$ds^2 = \frac{L^2}{z^2} \left\{ -2\, dx^+ \, dx^- + dx_\perp^2 + dz^2 + \frac{2\,\pi^2}{N_c^2} \, \langle T_{1\,--}(x^-) \rangle \, z^4 \, dx^{-\,2} + \frac{2\,\pi^2}{N_c^2} \, \langle T_{2\,++}(x^+) \rangle \, z^4 \, dx^{+\,2} \right.$$
$$\left. + \text{higher order graviton exchanges} \right\} \qquad (1)$$

From: Albacete, Kovchegov, Taliotis

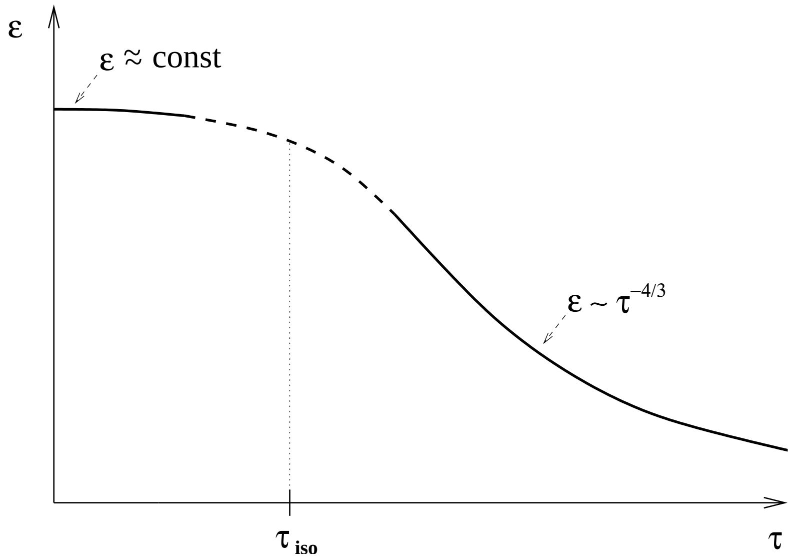
Initial Conditions: Shock Waves (3)

- Strong Coupling: Full Stopping $\sim$ Landau



From: Grumiller,Romatschke

- Weak Coupling



Calculation of the Moving Duals

- Boost-Invariant 5-d F-G metric:

$$ds^2 = \frac{-e^{a(\tau,z)}d\tau^2 + \tau^2 e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2}{z^2} + \frac{dz^2}{z^2}$$

- Scaling : $v = \dfrac{z}{\tau^{s/4}}$

$$[a(\tau,z), b(\tau,z), c(\tau,z)] = [a(v), b(v), c(v)] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)$$

$$v(2a'(v)c'(v) + a'(v)b'(v) + 2b'(v)c'(v)) - 6a'(v) - 6b'(v) - 12c'(v) + vc'(v)^2 = 0$$

$$3vc'(v)^2 + vb'(v)^2 + 2vb''(v) + 4vc''(v) - 6b'(v) - 12c'(v) + 2vb'(v)c'(v) = 0$$

$$2vsb''(v) + 2sb'(v) + 8a'(v) - vsa'(v)b'(v) - 8b'(v) + vsb'(v)^2 +$$
$$4vsc''(v) + 4sc'(v) - 2vsa'(v)c'(v) + 2vsc'(v)^2 = 0\ .$$

- Asymptotic Solution

$$\begin{aligned} a(v) &= A(v) - 2m(v) \\ b(v) &= A(v) + (2s-2)m(v) \\ c(v) &= A(v) + (2-s)m(v) \end{aligned}$$

Free Streaming and general cases

$$\boxed{v = \frac{z}{\tau^{1/4}}}$$

- Asymptotic metric

$$ds^2 = \frac{\left(-(1+\frac{v^4}{\sqrt{8}})^{\frac{1-2\sqrt{2}}{2}}(1-\frac{v^4}{\sqrt{8}})^{\frac{1+2\sqrt{2}}{2}}dt^2 + (1+\frac{v^4}{\sqrt{8}})^{\frac{1}{2}}(1-\frac{v^4}{\sqrt{8}})^{\frac{1}{2}}\tau^2 dy^2 +\right.}{z^2}$$

$$\frac{\left. +(1+\frac{v^4}{\sqrt{8}})^{\frac{1+\sqrt{2}}{2}}(1-\frac{v^4}{\sqrt{8}})^{\frac{1-\sqrt{2}}{2}}dx_\perp^2\right)}{z^2} + \frac{dz^2}{z^2}$$

- True Singularities ?

Ricci scalar:

$$R = -20 + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right) \quad .$$

Riemann tensor squared:

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$$

AdS/CFT: Selection of the Perfect Fluid

- Curvature invariant: $\boxed{\mathfrak{R}^2 = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}}$

$$\mathfrak{R}^2 = \frac{4}{\left(1-\Delta(s)^2 v^8\right)^4} \cdot \Big[10\,\Delta(s)^8 v^{32} - 88\,\Delta(s)^6 v^{24} + 42\,v^{24} s^2 \Delta(s)^4 +$$

$$+112\,v^{24}\Delta(s)^4 - 112\,v^{24}\Delta(s)^4 s + 36\,v^{20} s^3 \Delta(s)^2 - 72\,v^{20} s^2 \Delta(s)^2 +$$

$$+828\,\Delta(s)^4 v^{16} + 288\,v^{16}\Delta(s)^2 s - 288\,v^{16}\Delta(s)^2 - 108\,v^{16} s^2 \Delta(s)^2 +$$

$$-136\,v^{16} s^3 + 27\,v^{16} s^4 - 320\,v^{16} s + 160\,v^{16} + 296\,v^{16} s^2 + 36\,v^{12} s^3 +$$

$$-72\,v^{12} s^2 - 88\,\Delta(s)^2 v^8 + 42\,v^8 s^2 + 112\,v^8 - 112\,v^8 s + 10 \Big] + \mathcal{O}\Big(\frac{1}{\tau^{\#}}\Big)$$

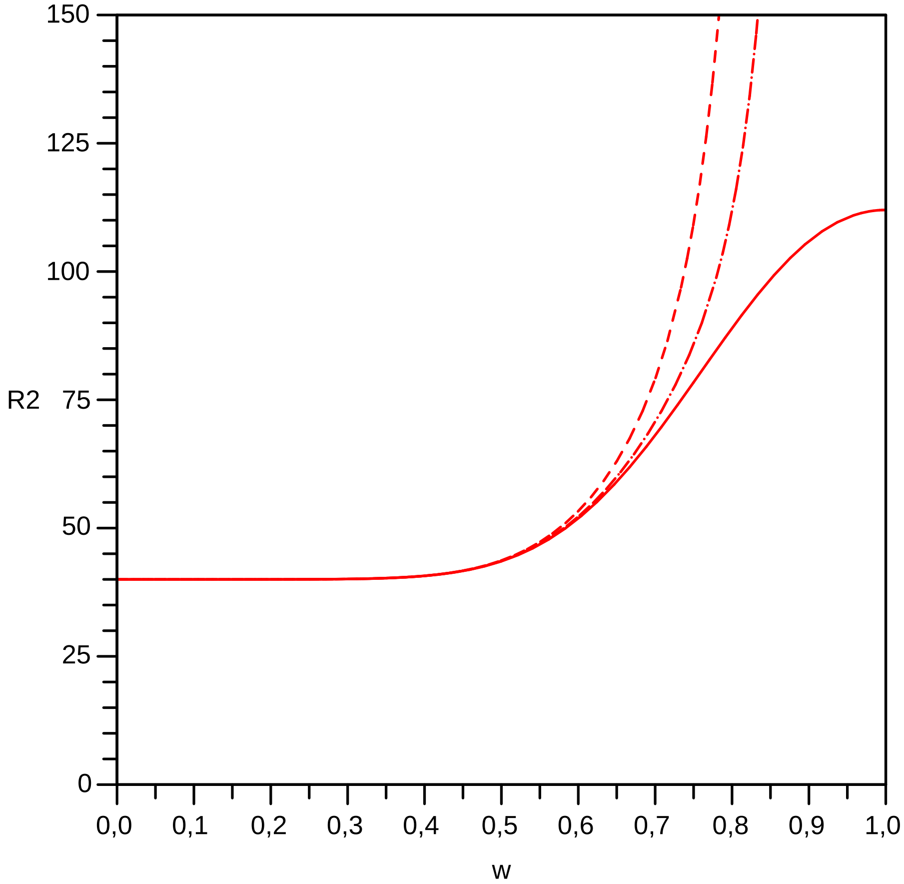
- $\mathfrak{R}^2$ for $s = \frac{4}{3}$:

$$\boxed{{\mathfrak{R}^2}_{\text{perfect fluid}} = \frac{8(5w^{16} + 20w^{12} + 174w^8 + 20w^4 + 5)}{(1+w^4)^4}}$$

where $w = v/\Delta(\frac{4}{3})^{\frac{1}{4}} \equiv \sqrt[4]{3}\,v$.

AdS/CFT: Selection of the Perfect Fluid

$\mathfrak{R}^2$ for $s = \frac{4}{3} \pm .1$



Thermalization response-time of a perfect fluid

- Quasi-normal scalar modes of a static Black Hole in Fefferman-Graham coordinates

$$\boxed{\Delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_i\left(\sqrt{-g}g^{ij}\partial_j\phi\right) = 0}$$

$$-\frac{1}{z^3}\frac{\left(1-\frac{z^4}{z_0^4}\right)^2}{\left(1+\frac{z^4}{z_0^4}\right)}\partial_t^2\phi(t,z)+\partial_z\left(\frac{1}{z^3}\left(1-\frac{z^8}{z_0^8}\right)\partial_z\phi(t,z)\right) = 0\,.$$

- Separation of variables $\phi(t,z) = e^{i\omega t}\phi(z)$

$$\phi''+\frac{1-\tilde{z}^2}{\tilde{z}(1-\tilde{z})(2-\tilde{z})}\,\phi'+\left(\frac{\omega}{\pi T}\right)^2\,\frac{1}{4\tilde{z}(1-\tilde{z})(2-\tilde{z})}\,\phi = 0$$

- Dominant Decay Time

$$\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667\ i$$

- A note on Viscosity *vs.* Q-n modes

Thermalization response-time of a perfect fluid

- Quasinormal scalar modes for the boost-invariant Black Hole geometry

$$\boxed{\Delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_i\left(\sqrt{-g}g^{ij}\partial_j\phi\right) = 0}$$

$$\left[\partial_z \to \tau^{-\frac{1}{3}}\partial_v \ ; \ \partial_\tau \to \partial_\tau - \frac{1}{3}\tau^{-\frac{4}{3}}\partial_v\right]$$

$$-\frac{1}{v^3}\frac{(1+v^4)^2}{1-v^4}\partial_\tau^2\phi(\tau,v) + \tau^{-\frac{2}{3}}\ \partial_v\left(\frac{1}{v^3}(1-v^8)\partial_v\phi(\tau,v)\right) = 0$$

- Separation of variables $\phi(\tau,v) = f(\tau)\phi(v)$

$$\partial_\tau^2 f(\tau) = -\omega^2\tau^{-\frac{2}{3}}f(\tau) \ \Rightarrow \ f(\tau) = \sqrt{\tau}J_{\pm\frac{3}{4}}\left(\frac{3}{2}\omega\tau^{\frac{2}{3}}\right) \ \sim \ \tau^{\frac{1}{6}}\ e^{\frac{3}{2}i\omega\tau^{\frac{2}{3}}}$$

$$\boxed{\partial_v\left(\frac{1}{v^3}(1-v^8)\partial_v\phi(v)\right) + \omega^2\frac{1}{v^3}\frac{(1+v^4)^2}{1-v^4}\phi(v) = 0}$$

- Dominant Decay Proper-Time

$$\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667\ i$$