Combined H1 & ZEUS data and HERAPDF0.1

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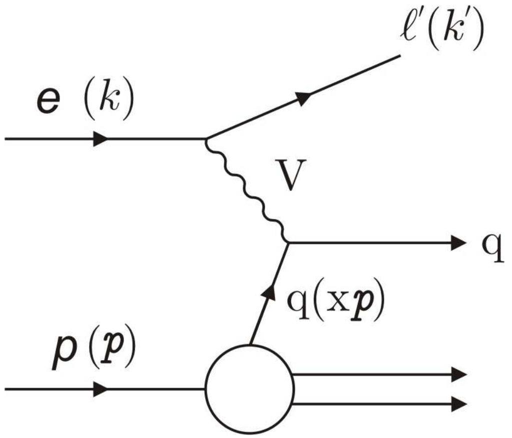
On behalf of the HERA Structure Function Working Group

- Combined deep inelastic data
- NLO QCD fit to the combined data
- Outlook

Deep Inelastic Scattering at HERA



Neutral Current $ep \to eX$, V= γ or Z^0

Charged Current $ep \to \nu X$, V= $W^\pm$

Kinematics

$$Q^2 = -(k-k')^2 \qquad x = \frac{Q^2}{2p.q} \qquad y = \frac{p.q}{p.k} \qquad q = k - k'$$

$$s = (k+p)^2 \qquad Q^2 = sxy \qquad Y_\pm = 1 \pm (1-y)^2$$

$$\frac{d^2\sigma^{NC}(e^\pm p)}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+ F_2^{NC}(x,Q^2) \mp Y_- xF_3^{NC}(x,Q^2)\right]^\dagger$$

$$\frac{d^2\sigma^{CC}(e^\pm p)}{dxdQ^2} = \frac{G_F^2}{4\pi x}\frac{M_W^4}{(Q^2+M_W^2)^2}\left[Y_+ F_2^{CC}(x,Q^2) \mp Y_- xF_3^{CC}(x,Q^2)\right]^\dagger$$

$$F_2^{NC} \simeq \sum_i e_i^2 x(q_i + \overline{q}_i)\ \ (\gamma \text{ only}); \quad F_2^{CC} = \sum_i x(q_i + \overline{q}_i); \quad xF_3^{CC} = \sum_i x(q_i - \overline{q}_i)$$

$q_i(x,Q^2)$ - momentum density of quark flavour i in proton

† F_L has been ignored

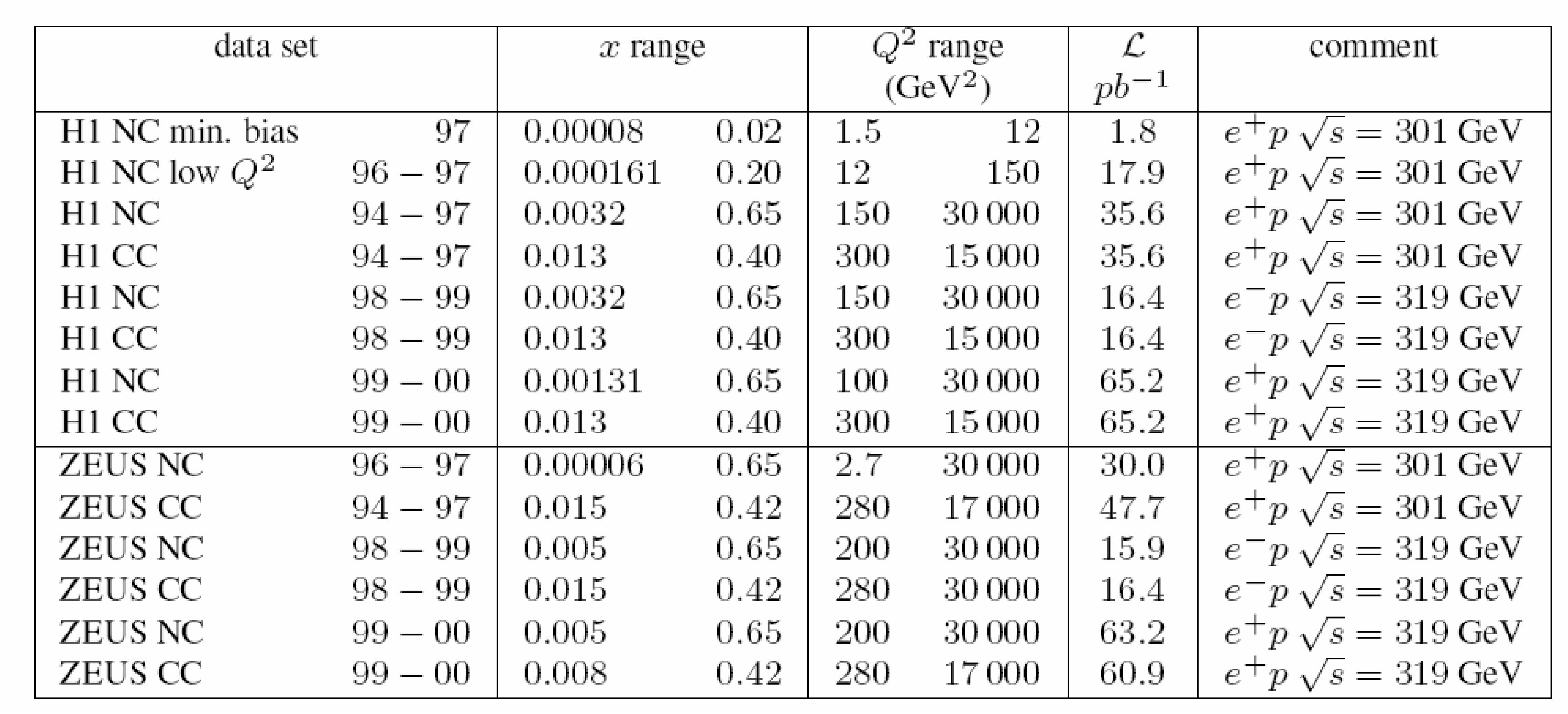
Combined deep inelastic data

- Scope of the project
- Data
- Method
- Results

Scope of the project

- Combination of HERA-I (1994-2000) inclusive DIS cross-sections
  - more precisely reduced cross-sections (the terms in [ ] on slide 2)
- Exploit the different technology of the H1 and ZEUS detectors to ‘cross-calibrate’, and hence reduce the systematic uncertainties
- The basic assumption is that the two experiments are measuring the same cross-sections at the same (x, Q^2) point.
- The method (developed by A. Glazov) uses an iterative χ^2 minimisation which takes full account of error correlations
  - first discussed at DIS2005 and then at the HERA-LHC Workshop
- Preliminary results for the combined data as submitted to LP2007 and presented at DIS2008 (Feltesse)

Input NC & CC data sets: $1.5 < Q^2 < 30000$ GeV^2, 240 pb^{-1}



data set		x range		Q^2 range (GeV^2)		$\mathcal{L}$ pb^{-1}	comment
H1 NC min. bias	97	0.00008	0.02	1.5	12	1.8	e^+p $\sqrt{s} = 301$ GeV
H1 NC low Q^2	96 – 97	0.000161	0.20	12	150	17.9	e^+p $\sqrt{s} = 301$ GeV
H1 NC	94 – 97	0.0032	0.65	150	30 000	35.6	e^+p $\sqrt{s} = 301$ GeV
H1 CC	94 – 97	0.013	0.40	300	15 000	35.6	e^+p $\sqrt{s} = 301$ GeV
H1 NC	98 – 99	0.0032	0.65	150	30 000	16.4	e^-p $\sqrt{s} = 319$ GeV
H1 CC	98 – 99	0.013	0.40	300	15 000	16.4	e^-p $\sqrt{s} = 319$ GeV
H1 NC	99 – 00	0.00131	0.65	100	30 000	65.2	e^+p $\sqrt{s} = 319$ GeV
H1 CC	99 – 00	0.013	0.40	300	15 000	65.2	e^+p $\sqrt{s} = 319$ GeV
ZEUS NC	96 – 97	0.00006	0.65	2.7	30 000	30.0	e^+p $\sqrt{s} = 301$ GeV
ZEUS CC	94 – 97	0.015	0.42	280	17 000	47.7	e^+p $\sqrt{s} = 301$ GeV
ZEUS NC	98 – 99	0.005	0.65	200	30 000	15.9	e^-p $\sqrt{s} = 319$ GeV
ZEUS CC	98 – 99	0.015	0.42	280	30 000	16.4	e^-p $\sqrt{s} = 319$ GeV
ZEUS NC	99 – 00	0.005	0.65	200	30 000	63.2	e^+p $\sqrt{s} = 319$ GeV
ZEUS CC	99 – 00	0.008	0.42	280	17 000	60.9	e^+p $\sqrt{s} = 319$ GeV

NB: H1 NC min. bias ($Q^2 < 12$ GeV^2) moved up by 3.4 % after re-analysis of luminosity

Some details

- Common (x, Q^2) bins: H1 x; ZEUS Q^2
- Shift measured data by simple interpolation using H1PDF2k
  - checked using ZEUS-Jets, NC shift factors agree within a few permille, some CC < 2%. - differences much less than statistical errors.

- Move data to 920 GeV E_p beam energy
  - simple interpolation for CC
  - additive for NC
  - systematic uncertainty from F_L: compare $F_L = 0$ and $F_L = F_L$(H1PDF2k), up to 5% at high y.
  - treat as a correlated ‘procedural’ systematic uncertainty

χ^2 for a single data set

$$\chi^2_{\exp}(M^{i,true},\Delta\alpha_j)=\sum_i\frac{\left[M^{i,true}-\left(M^i+\sum_j\frac{\partial M^i}{\partial\alpha_j}\Delta\alpha_j\right)\right]^2}{\sigma_i^2}+\sum_j\frac{(\Delta\alpha_j)^2}{\sigma^2_{\alpha_j}}$$

M^i	measured central values
σ_i	statistical and uncorrelated systematic uncertainties
σ_{α_j}	correlated systematic uncertainties
$\frac{\partial M^i}{\partial\alpha_j}$	sensitivity of datum i to systematic j
$M^{i,true}$	fitted combined H1 - ZEUS data
$\Delta\alpha_j$	fitted shifts of correlated uncertainties

By definition $\chi^2=0$ for $M^{i,true}$ and $\Delta\alpha_j=0$;

$Cov(M^{i,true},M^{j,true})$ gives the error matrix for the combined data

Caveat

- In principle a nice simple χ^2 which allows minimisation by linear equations
- Unbiased for uncertainties independent of the central value (additive)
- However, for cross-sections, many uncertainties are proportional to the central value (multiplicative)
- This introduces a bias, as a smaller M^i will have a smaller relative error and hence give a smaller overall χ^2

- Modify χ^2 - translate multiplicative to additive uncertainty using $M^{i,true}$, common to all measurements

Revised χ^2 for a single data set

$$\chi^2_{\exp}(M^{i,true},\Delta\alpha_j)=\sum_i \frac{\left[M^{i,true}-\left(M^i+\sum_j \frac{\partial M^i}{\partial\alpha_j}\frac{M^{i,true}}{M^i}\Delta\alpha_j\right)\right]^2}{\left(\sigma_i\frac{M^{i,true}}{M^i}\right)^2}+\sum_j\frac{(\Delta\alpha_j)^2}{\sigma^2_{\alpha_j}}$$

Minimisation is now non - linear, use an iterative procedure

1. Minimise original χ^2 to find an initial approximation to $\{M^{i,true}\}$
2. Scale errors $\sigma_i \to \sigma_i \dfrac{M^{i,true}}{M^i}$
3. Repeat step 1

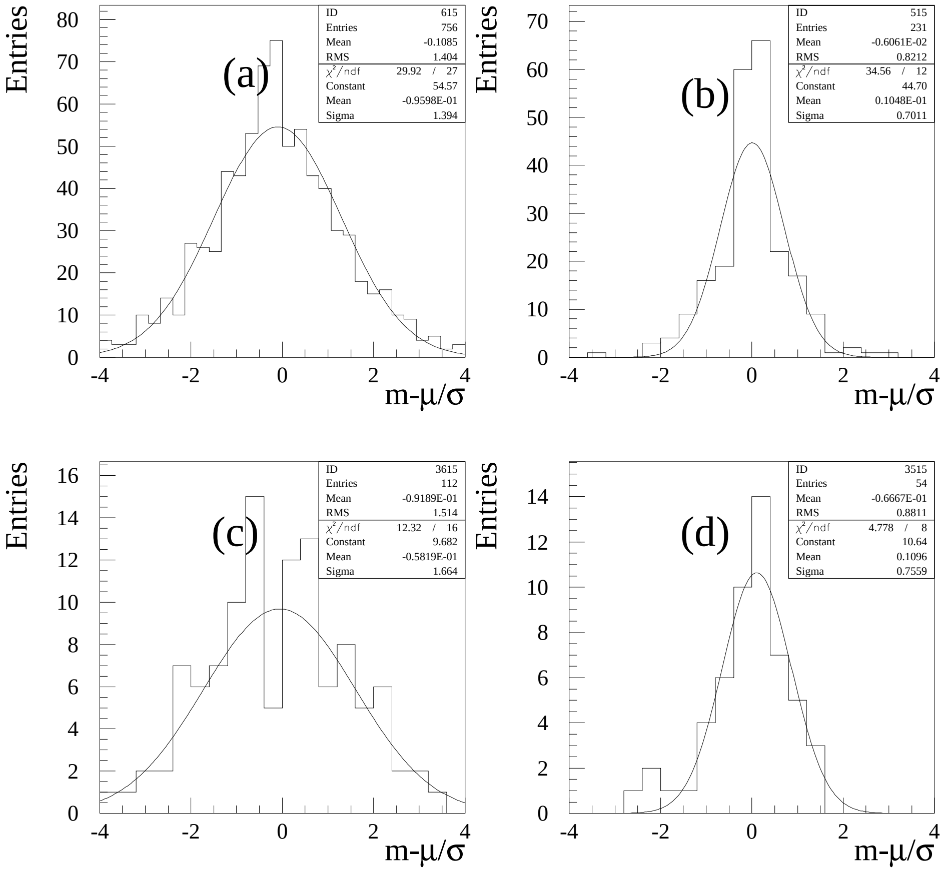
Convergence is usually after two iterations

Full χ^2 is the sum over all $\chi^2_{\exp}$.

Uncertainties

- Statistical uncertainties are uncorrelated
- Systematic uncertainties:
  - point-to-point uncorrelated, added in quadrature to statistical giving a total point-to-point uncorrelated uncertainty
  - point-to-point correlated errors, (e.g. energy scales), often common for CC and NC measurements for a given experiment and run period
    - multiplicative or additive? Try both – gives additional uncertainty < 1% for low Q^2 rising to 1.5% at large Q^2
  - overall normalisation uncertainty, similarly common for a given experiment and run period (clearly multiplicative)
- Correlations between H1 and ZEUS, (e.g. MC simulations, calibration methods..), 12 possible sources identified
  - compare 2^{12}-1 averages taking all pairs as correlated or uncorrelated in turn to give deviation from central values
  - largest (~1%) from photoproduction MC and hadronic energy scales
  - treat these as procedural uncertainties

Quality of the fit



1153 individual NC, CC data averaged to 573 points

$\chi^2 = 510$

	Pulls	
	Mean	Sigma
(a) NC e+p	-0.09	1.4
(b) NC e-p	0.01	0.7
(c) CC e+p	-0.05	1.7
(c) CC e-p	0.1	0.8

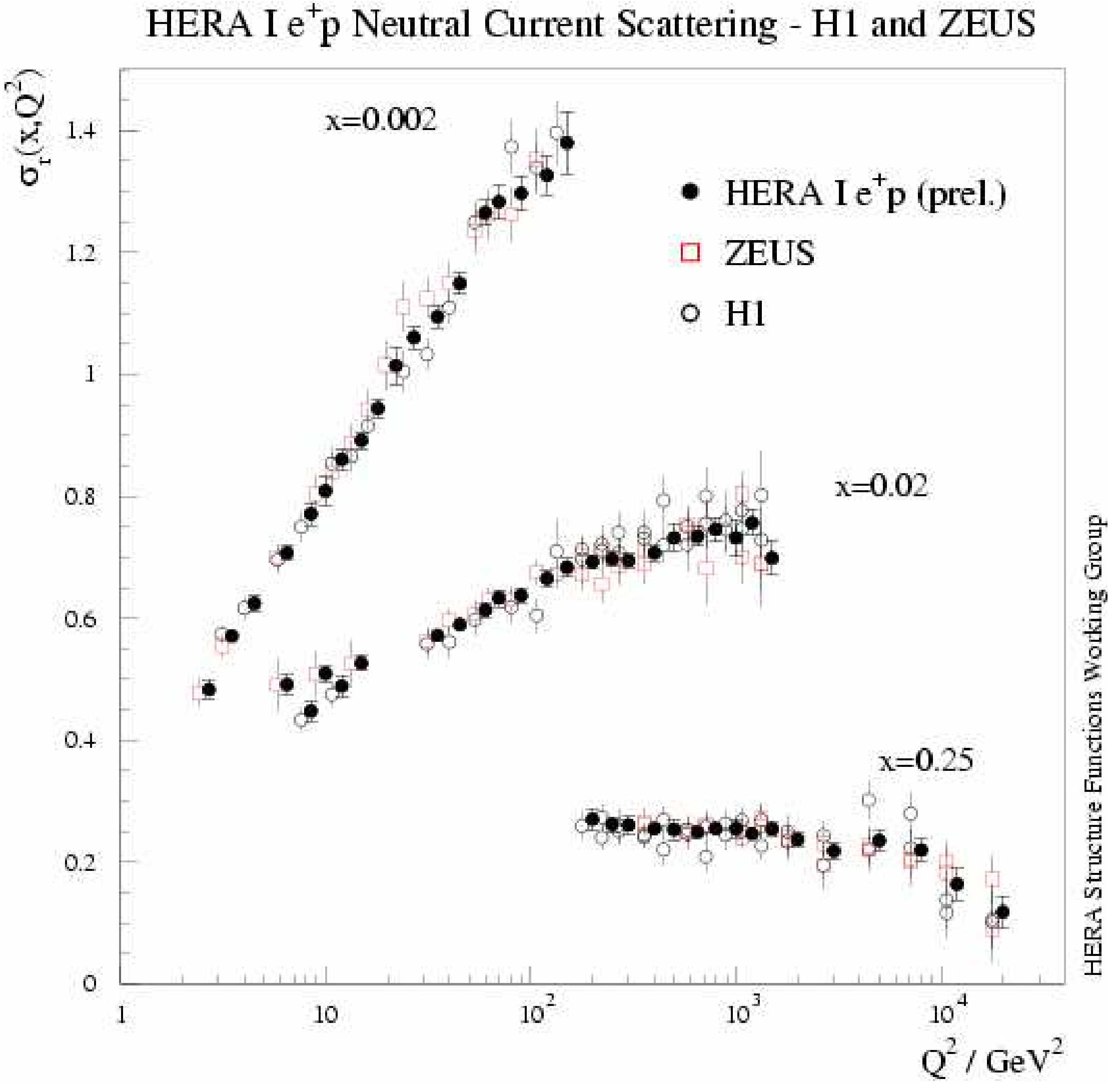
A total of 43 systematic uncertainties from the data and 4 from the averaging procedure

Comments on the results

- All uncertainties lie within 1σ of the central value of published data
  - except the normalisation of H1 NC low Q^2 (1996-7), up by 1.6σ
- Almost all systematic uncertainties reduced, eg
  - H1 rear calorimeter energy scale by a factor of 3
  - ZEUS forward energy flow modelling by a factor of 4
- Overall precision improved
  - $Q^2 < 12$ GeV^2, separately 2-3%, combined better than 2%
  - medium Q^2, 1.5% achieved
  - highest Q^2, 10% achieved, increased statistics now important

- Both H1PDF2k and ZEUS-Jets PDFs describe the combined data well

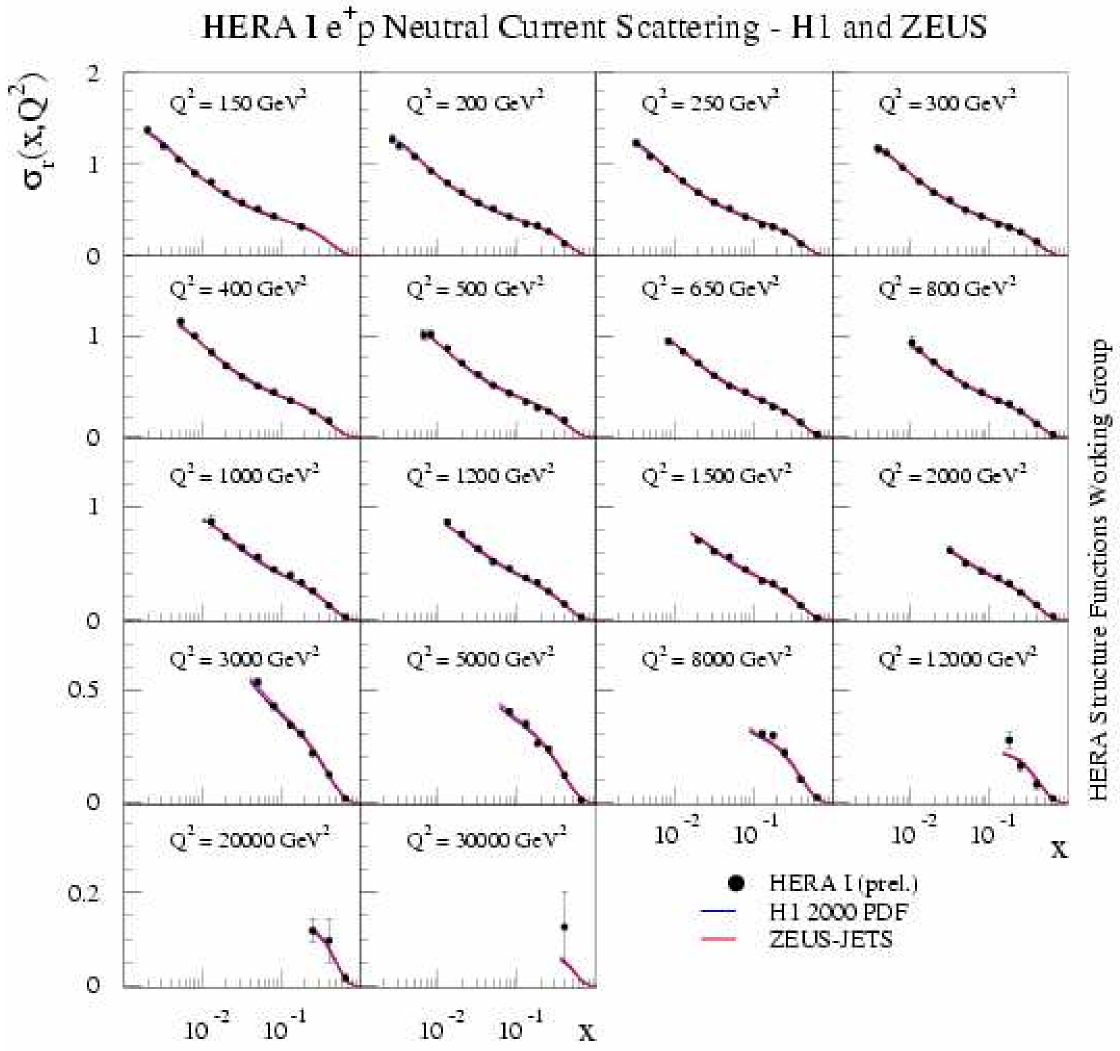
Examples (I): NC e^+p, at fixed *x*



Combined data
is smoother than
that of either H1
or ZEUS – with
significantly smaller
uncertainties

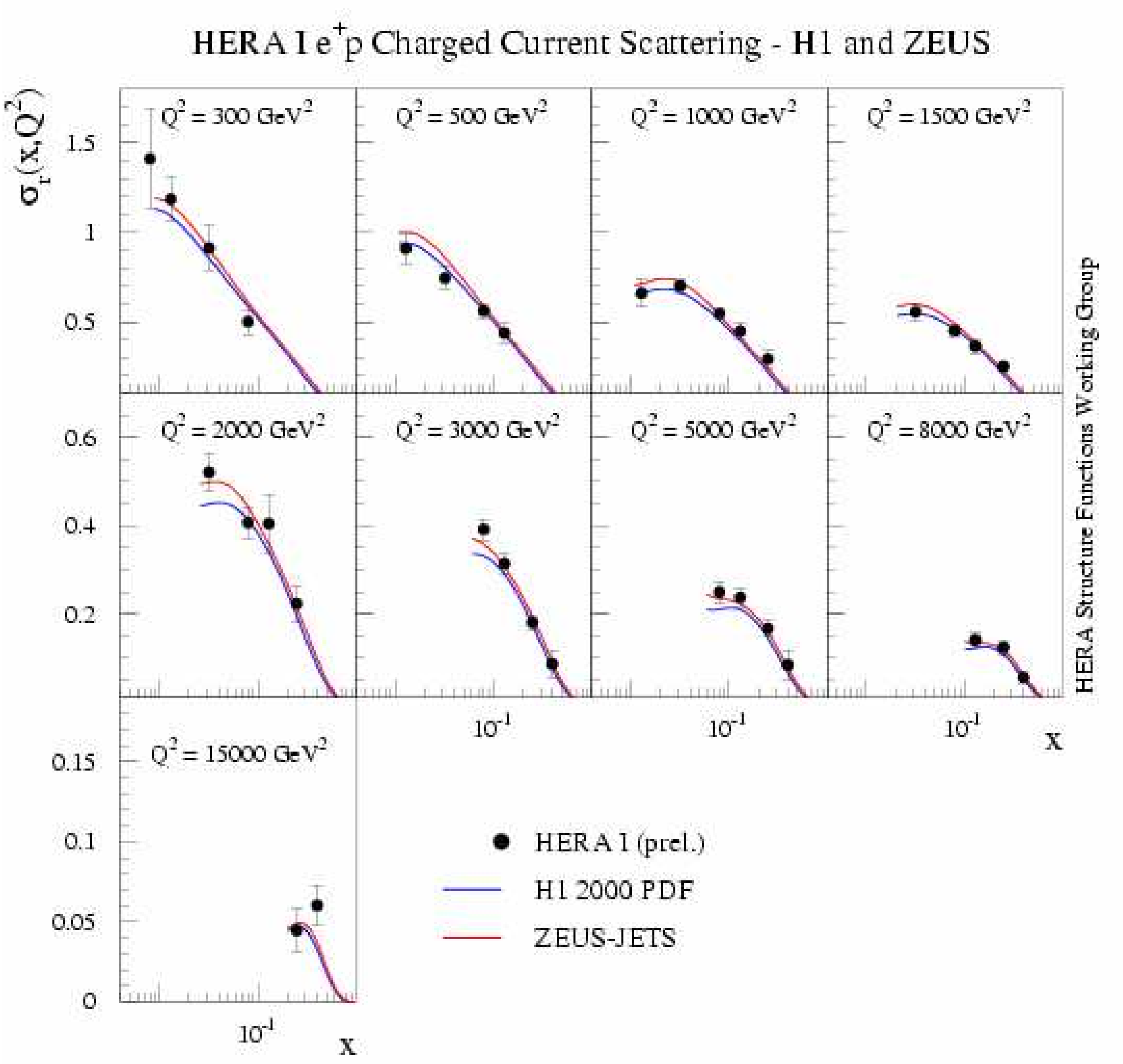
Examples (II): NC e^+p, high Q^2

Combined data compared to H1PDF2k & ZEUS-Jets calculations



Examples (III): CC e^+p

Combined data compared to H1PDF2k & ZEUS-Jets calculations



Combined data – summary & outlook

- A robust procedure has been developed for combining the H1 and ZEUS NC and CC reduced cross-section data
- The experiments cross calibrate each other, leading to a significant reduction in systematic uncertainties across the kinematic plane, in addition at large Q^2 there is a reduction in statistical error
- It is hoped to publish the combined data later this year (H1 has a couple of HERA-I NC data sets still to be published)
- HERA-II data on NC and CC cross-sections with polarised e+ and e- beams are being extracted by H1 and ZEUS
- Once the individual results are published, the combined HERA-II data will be produced

NLO QCD fit to the combined HERA data

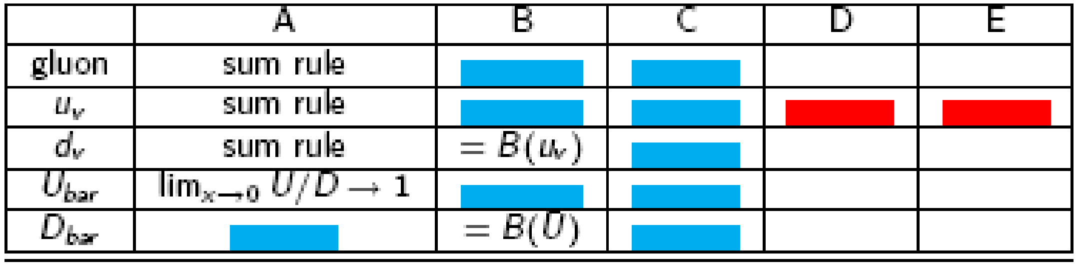
- Context & Scope
- Form of the PDF parameterisation
- Error/uncertainty treatment
- Model assumptions
- HERAPDF0.1
- Comparisons
- LHAPDF
- Summary

Context & Scope

- H1PDF2k and ZEUS-Jets, most recent PDF sets from H1 and ZEUS
  - differ in many details (parameterisation and choice of partons, uncertainty treatment, input data)
  - results broadly compatible, but the gluon PDFs in particular are different
- Goal is an NLO PDF fit to the combined HERA-I data alone
- A lot of preliminary and ongoing work undertaken by the H1-ZEUS team, e.g.
  - try each other's approaches on own data and combined data
  - try both hessian and offset methods for uncertainty estimates
  - try different flavour break-ups and heavy flavour schemes
  - etc
- The outcome (HERAPDF0.1) should be viewed as work in progress

HERA PDF parameterisation at Q_0^2

$$xf(x,Q_0^2) = Ax^B(1-x)^C(1+Dx+Ex^2+Fx^3+\cdots)$$



	A	B	C	D	E
gluon	sum rule	🟦	🟦		
u_v	sum rule	🟦	🟦	🟥	🟥
d_v	sum rule	$= B(u_v)$	🟦		
U_{bar}	$\lim_{x\to 0} U/D \to 1$	🟦	🟦		
D_{bar}	🟦	$= B(\bar{U})$	🟦		

Optimisation and constraints on parameters

Partons fitted : $xg, xu_v, xd_v, x\bar{U} = x\bar{u} + x\bar{c}, x\bar{D} = x\bar{d} + x\bar{s} + x\bar{b}$

Sea flavour break - up at Q_0 : $s = f_s D, c = f_c U, A_{\bar{U}} = (1-f_s)A_{\bar{D}}/(1-f_c)$

with $f_s = 0.33, f_c = 0.15$

Parameter optimisation: start with A, B, C (BLUE) add D, E ,F... until no χ^2 advantage – find only D & E (red) non zero for xu_v

This form is derived from the H1 and ZEUS parameterisations

less model dependence for B parameters than H1 form

no additional x(ubar-dbar) input as used in the ZEUS form

More details

- NLO DGLAP framework for evolving PDFs to arbitrary Q^2
- Zero-mass variable-number heavy flavour scheme
- Renormalisation and factorisation scales: Q^2
- Fit 573 combined HERA-I NC & CC data
- A total of 11 free parameters

Further fixed parameters:

$Q_0^2 = 4\,\mathrm{GeV}^2$ (input scale)

$\mathrm{Q}_{\mathrm{min}}^2 = 3.5\,\mathrm{GeV}^2$ (minimum for data)

$m_c = 1.4\,\mathrm{GeV}$(charm mass), $m_b = 4.75\,\mathrm{GeV}$(beauty mass)

$\alpha_\mathrm{S}(M_Z) = 0.1176$ (PDG 2006 value)

Error/uncertainty treatment

- Combined data have much reduced errors, systematic uncertainties smaller than statistical across most of (x, Q^2) plane
- Combine 43 systematic uncertainties of the data with their statistical uncertainties in quadrature, then offset the 4 combination systematic uncertainties. Gives $\chi^2/dof = 476.7/562$
- Checks:
  - taking 47 systematics in quadrature gives $\chi^2/dof = 428/562$
  - taking all systematics as correlated gives $\chi^2/dof = 553.1/562$
  - all three methods give very similar PDF central values and uncertainties
- The self consistency and small systematics of the combined data allows the use of $\Delta\chi^2 = 1$ to calculate PDF parameter uncertainties

Model uncertainties

- To be added to total PDF uncertainty

m_c (1.45): $1.3 \rightarrow 1.55$ GeV

m_b (4.75): $4.3 \rightarrow 5.0$ GeV

f_s (0.33): $0.25 \rightarrow 0.40$

f_c (0.15): $0.10 \rightarrow 0.20$

Q_0^2 (4.0): $2.0 \rightarrow 6.0\ \mathrm{GeV}^2$

$Q_{\min}^2$ (3.5): $2.5 \rightarrow 5.0\ \mathrm{GeV}^2$

- To be compared with the results

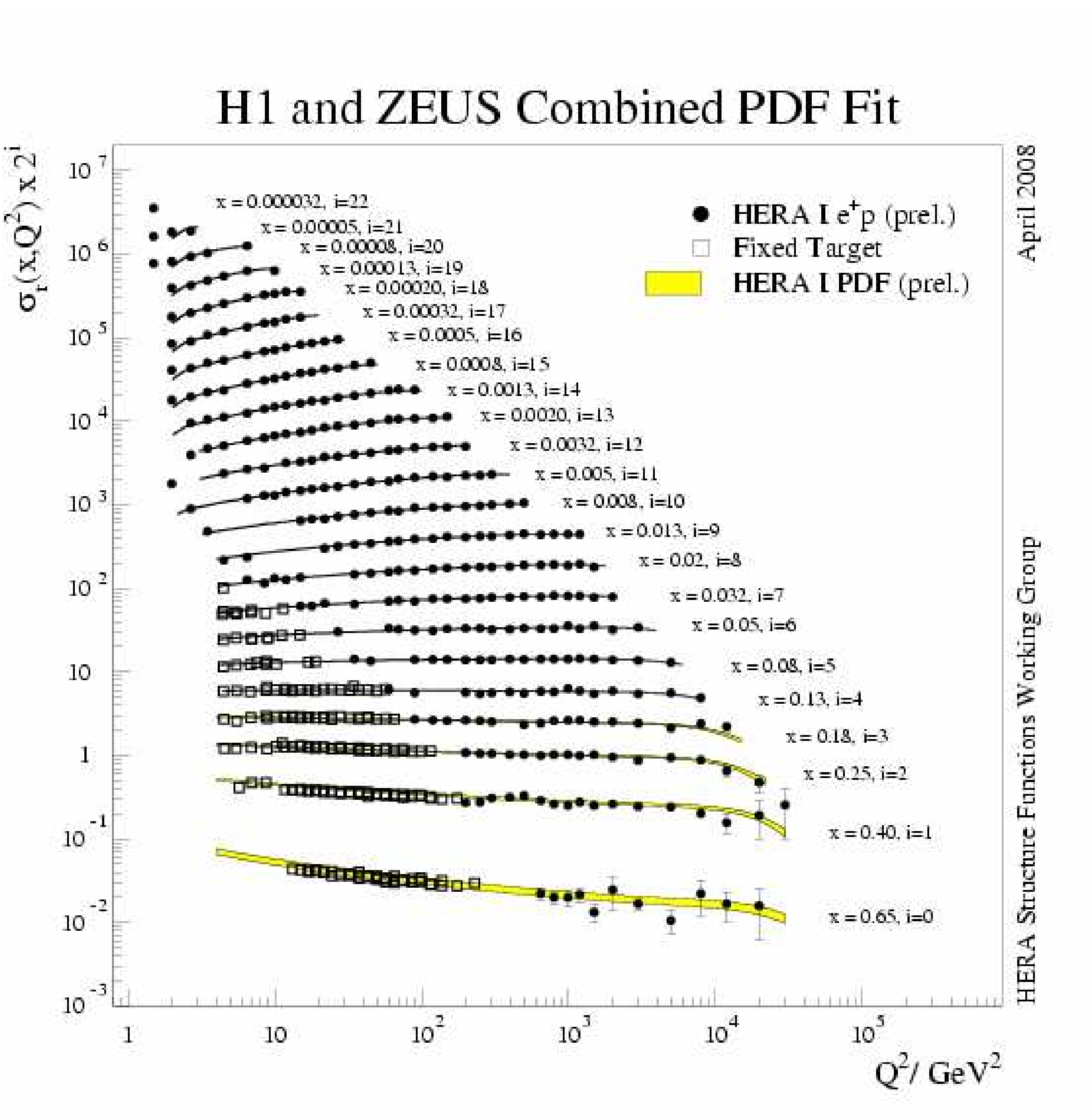
Vary $\alpha_S(M_Z)$ (0.1176): $0.1156 \rightarrow 1196$

Vary PDF parameterisation (HERA): H1 → ZEUS

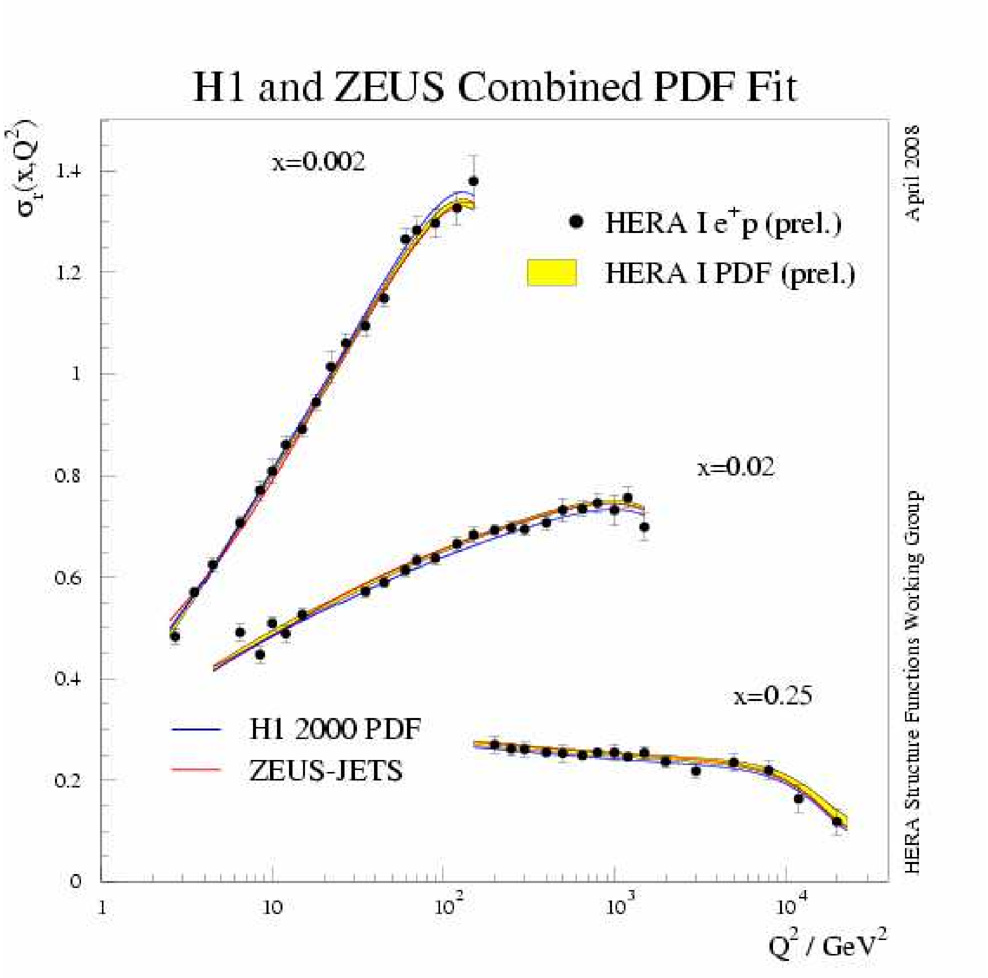
PDF fit results I

HERAPDF0.1 fit quality to the combined HERA-I data for NC e+p

uncertainties on both data and fit are included



PDF fit results II



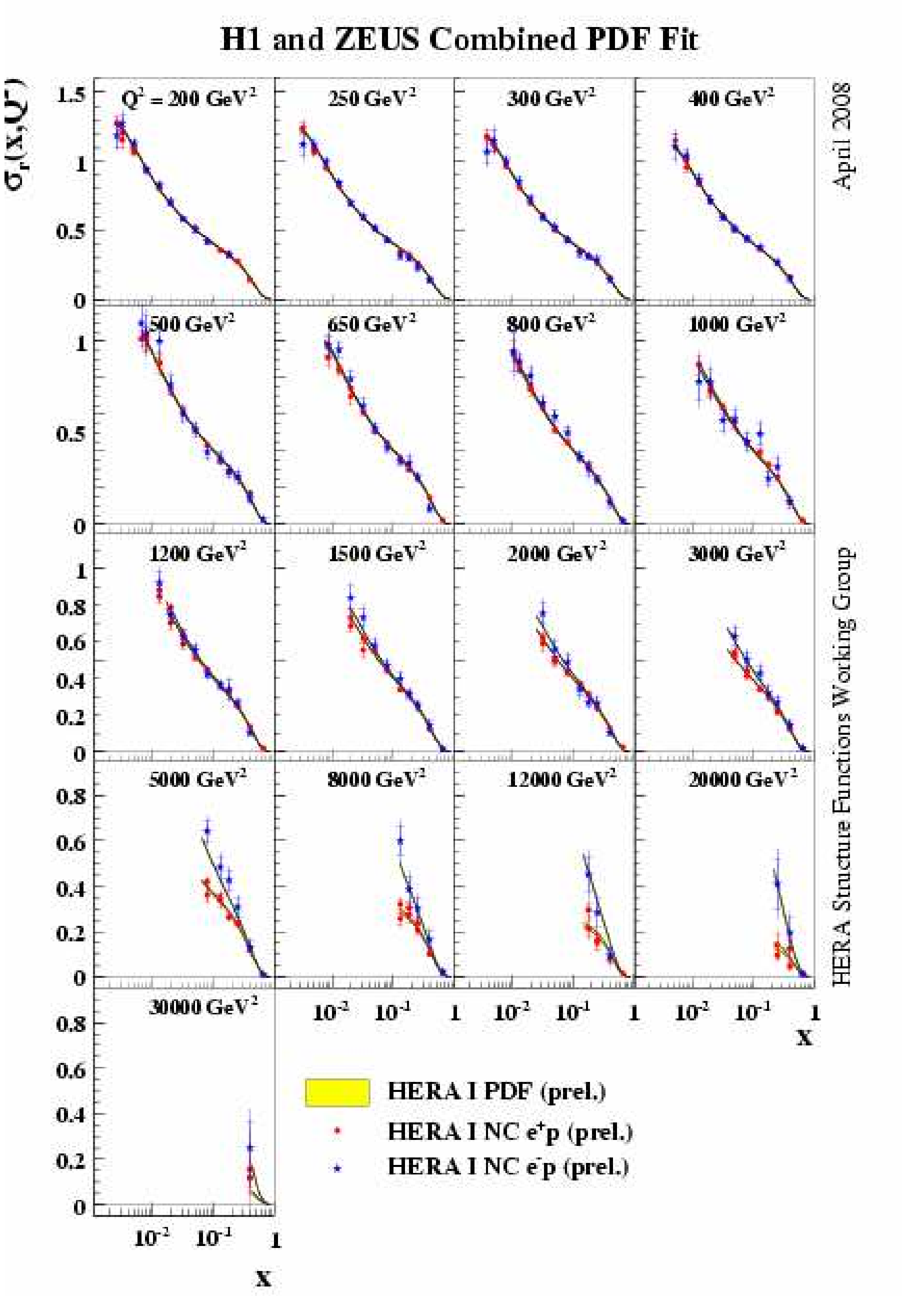
In more detail, for the three *x* values shown on p 13

scaling violation thru' DGLAP eqns gives tight constraint on gluon

PDF fit results III

High Q^2 NC

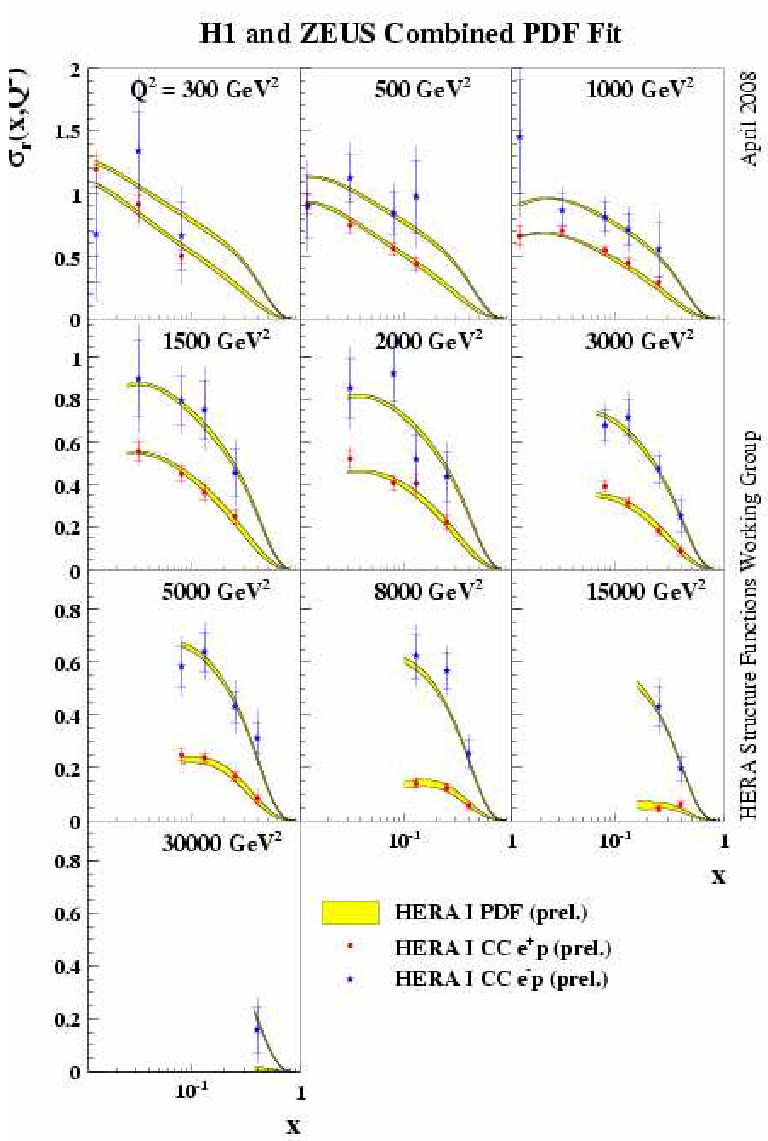
Precision is crucial for the extraction and exploitation of xF_3 and its valence quark dependence



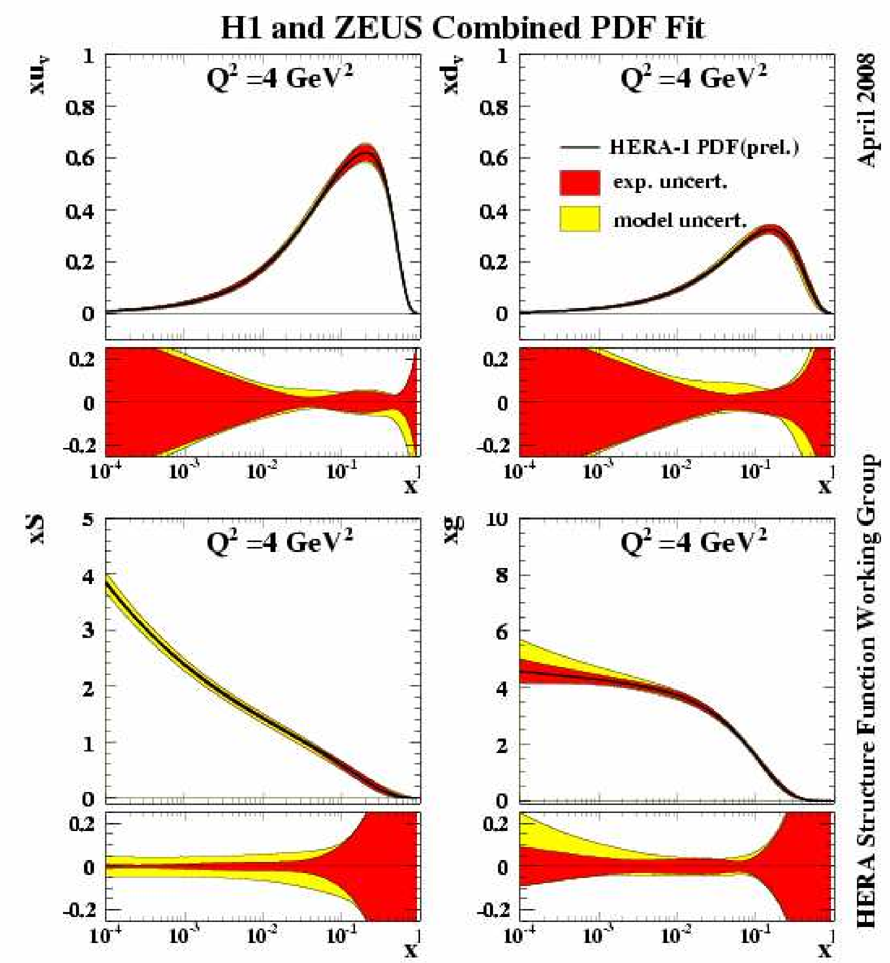
PDF fit results IV

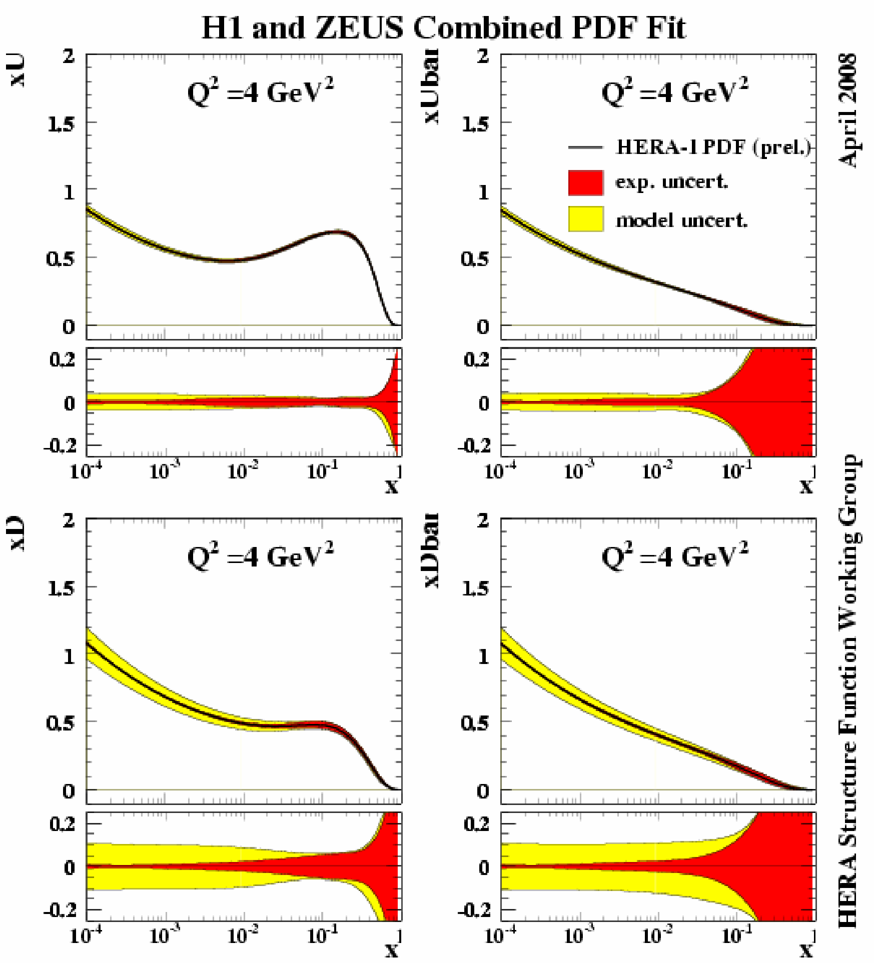
High Q^2 CC

Precision needed to exploit the different flavour dependence of the e+ and e- cross-sections



PDFs at the starting scale $Q_0^2 = 4$ GeV^2

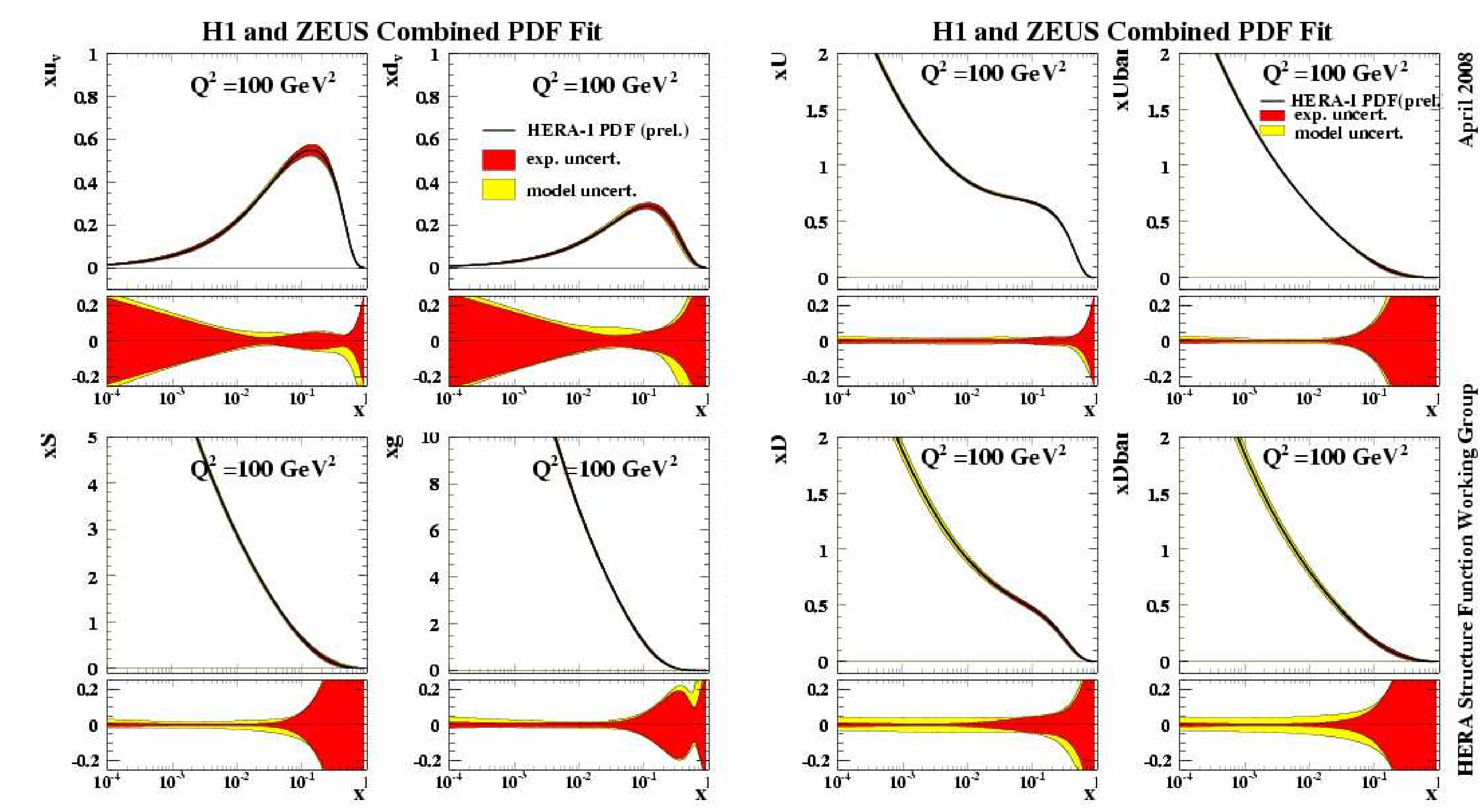




Total exp. uncertainty band (red); model uncertainties (yellow)

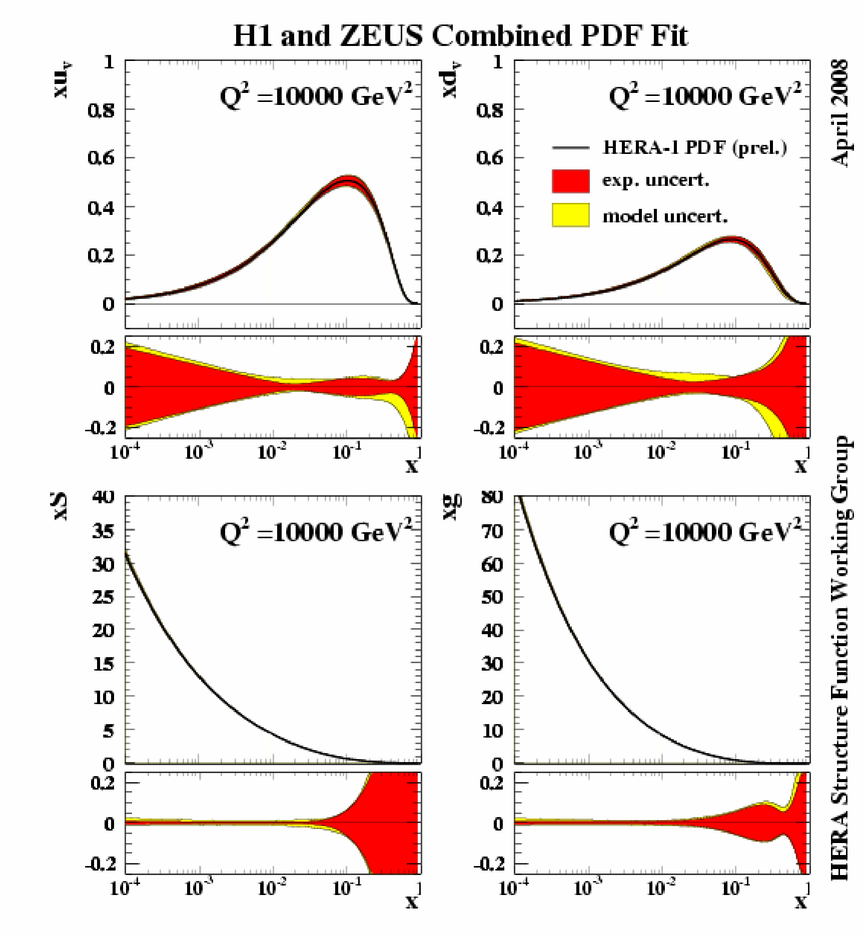
- f_s dominates model uncert. on sea; Q_0^2 & Q_{min}^2 dominate xg & xq_v

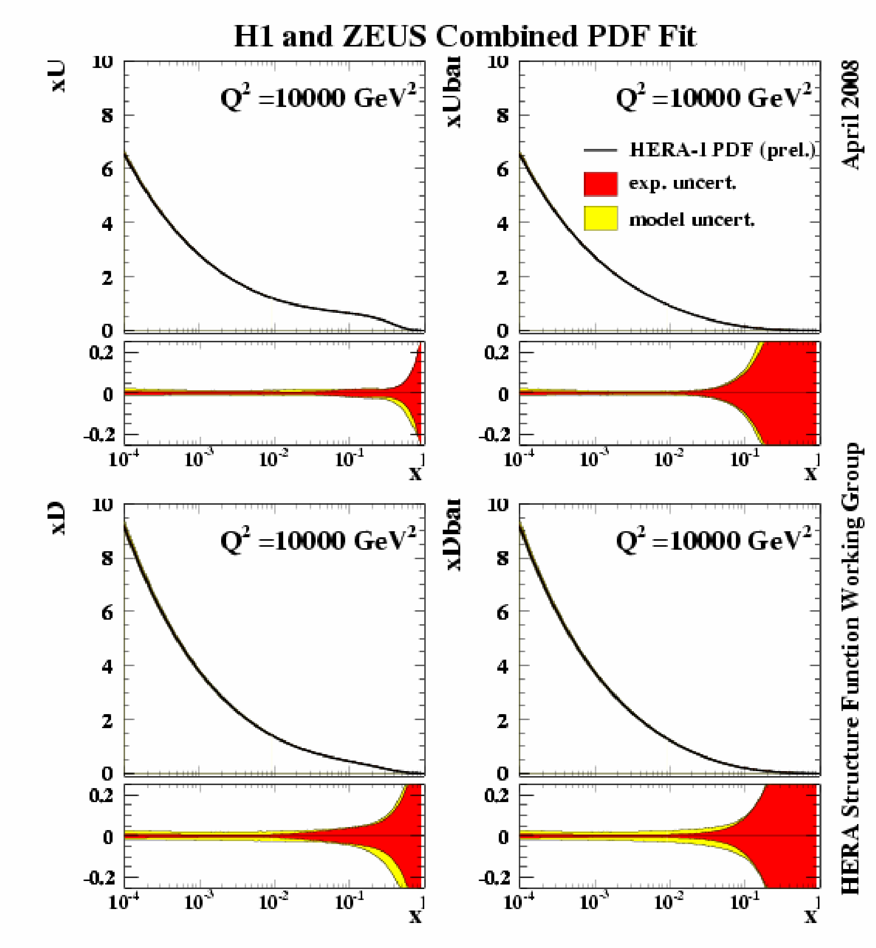
PDFs at $Q^2 = 100\ \mathrm{GeV}^2$



Uncertainties decrease as Q^2 increases

PDFs at Q^2 = 10000 GeV^2



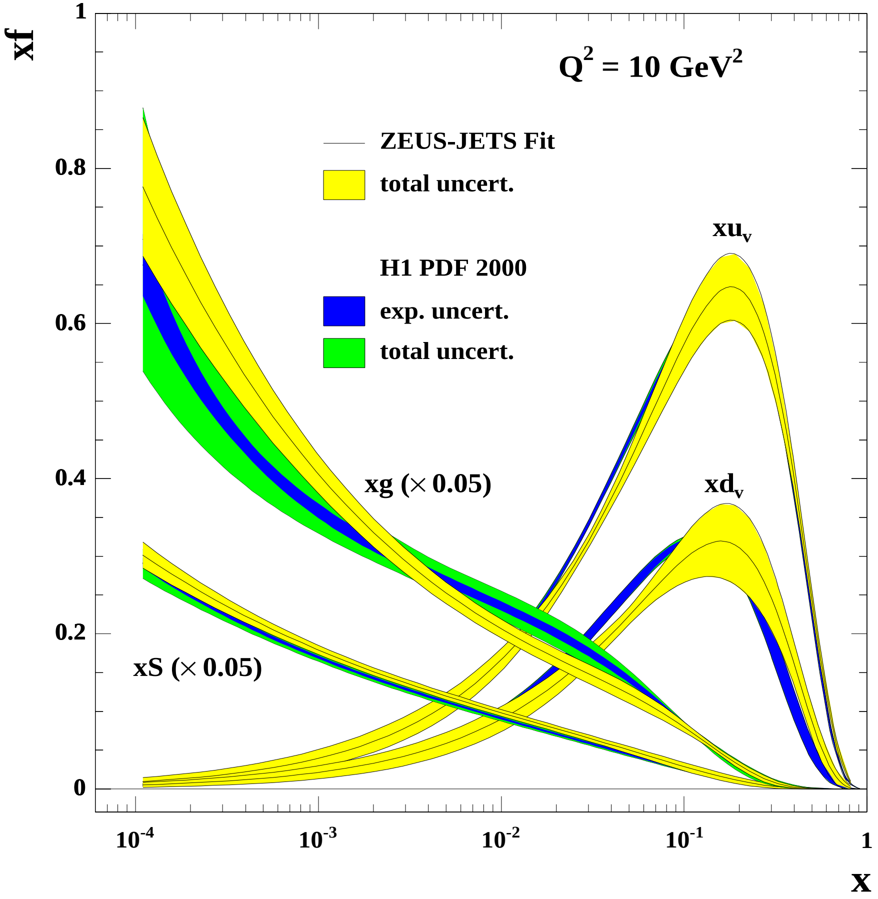


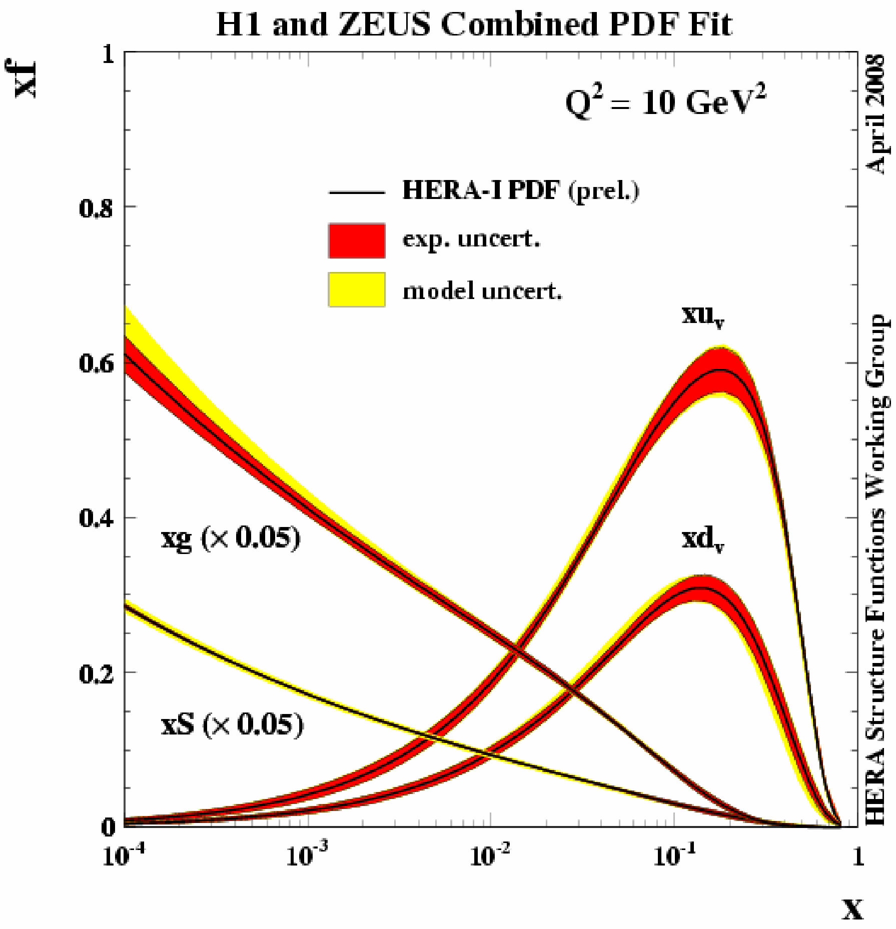
Scale relevant for the LHC – impressively small uncertainties

see Cooper-Sarkar & Perez (talk at HERA-LHC May 08 w/shop, Indico confId = 27458)

Comparisons I: with H1 & ZEUS fits

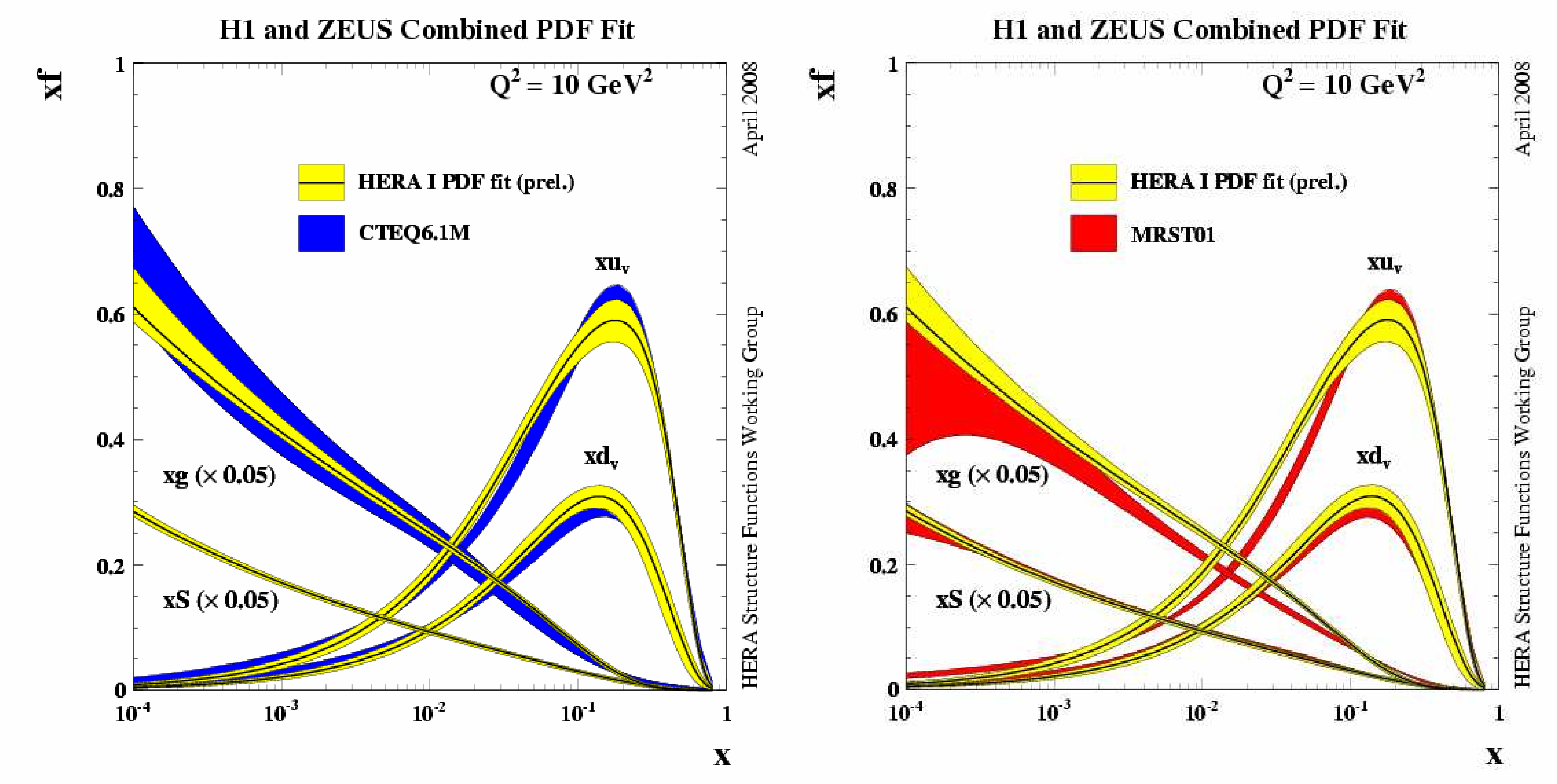
NB: H1PDF2k has α_S variation included in model error, ZEUS-Jets does not.





Improved precision and resolution of a discrepancy

Comparisons II: with CTEQ & MRST



Difference between HERAPDF0.1 and MRST01 *xg* at low *x* is due in part to parameterisation

LHAPDF

- Results shown here are those released at DIS08
- The intention is to release HERAPDF0.1 to LHAPDF ‘soon’
- Quite a few details are being checked and refined, e.g.
  - more work on flavour break-up of the sea
  - ditto on varying Q_0^2 and m_c
  - studies of xg at low and high x wrt other PDFs and other data
- None of the above have produced any significant differences from the results shown here
- There are also technical choices to be made, e.g.
  - input parameters plus evolution code?
  - or PDF values on (x, Q^2) grid?

Summary (PDF fit)

- The improved precision of the combined HERA-I is reflected in the improved precision of the HERAPDF0.1 fit
- Experimental and fit-model uncertainties have been studied and allowed for
- Differences between H1PDF2k and ZEUS-Jets understood and resolved
- Note that the HERA fit parameterisation is ‘minimal and optimised’ in form and number of parameters
  - does not require target mass corrections
  - does not require heavy target or deuteron corrections

This is the just the start of the ‘combined HERA data’ programme

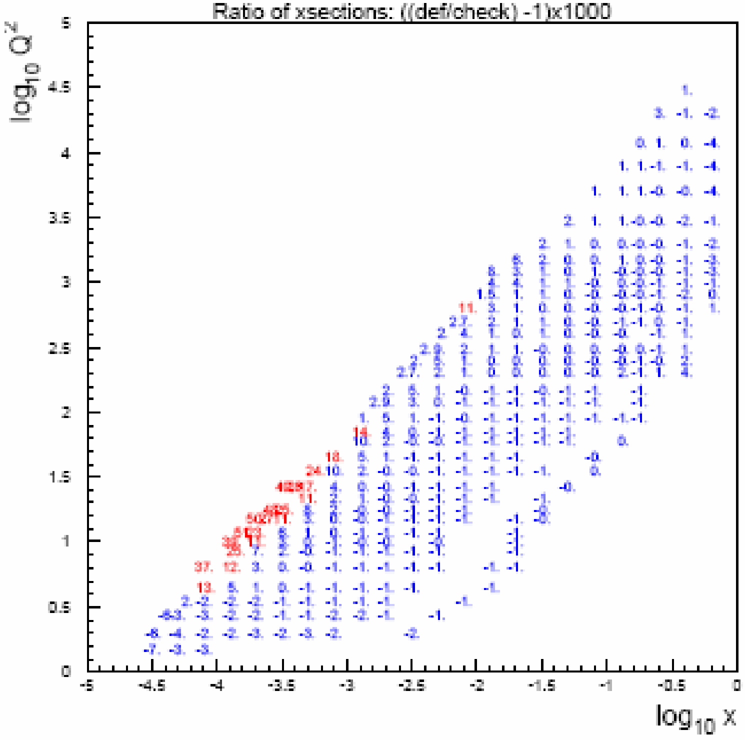
EXTRAS

CME corrections: $F_L = 0$

To test model (F_L) dependence at high-y we have repeated the combination (with F_L=0)

Almost negligible
(but up to 4-5% at high-y)

(Default/NoFL -1)x1000

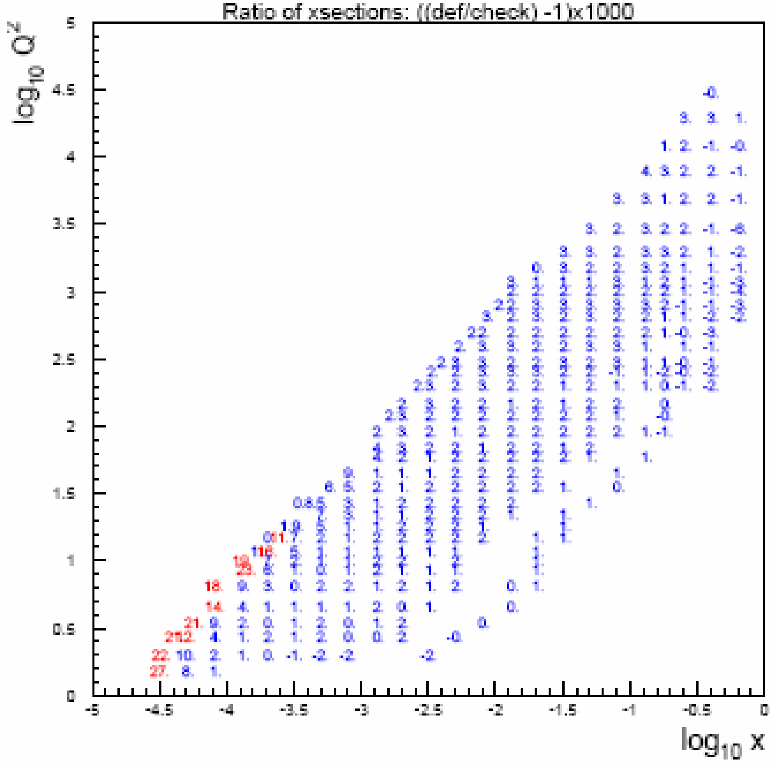


R Devenish Low x 2008

Uncertainty on correlation between data sets

(Uncorrelated/Largest change -1)x1000

Mostly negligible ~0.2 -0.3 %
(but up to 2-3% in particular
regions of the phase space)

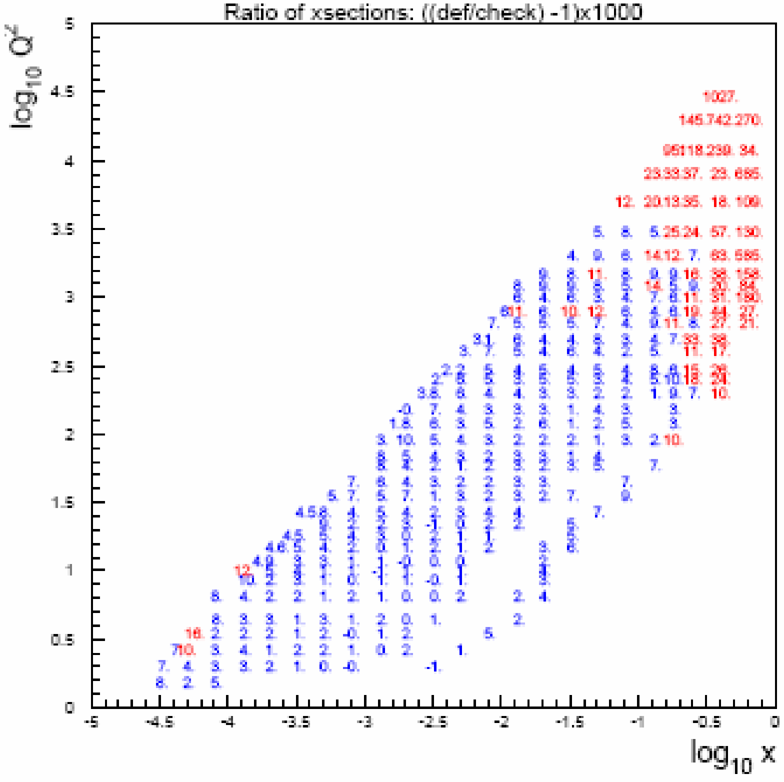


Multiplicative vs Additive

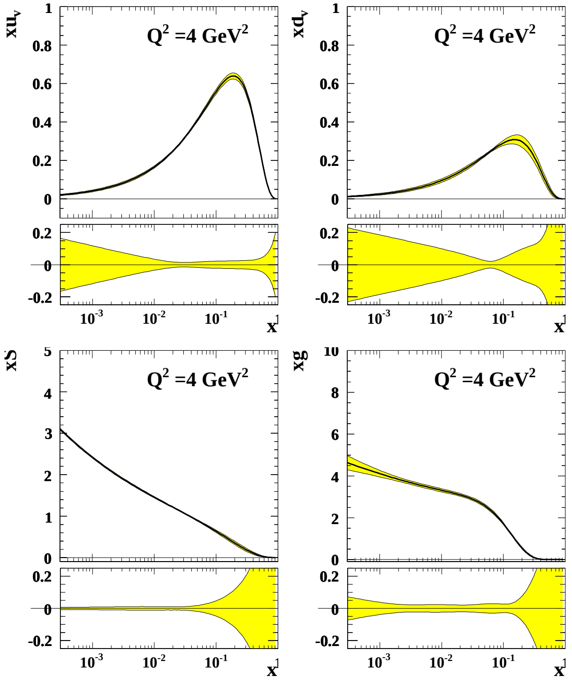
(Uncorrelated/Largest change -1)x1000

Comparison of the averaged cross sections obtained assuming all syst unc. as multiplicative in nature w.r.t. to a combination were only normalizations were treated as multiplicative

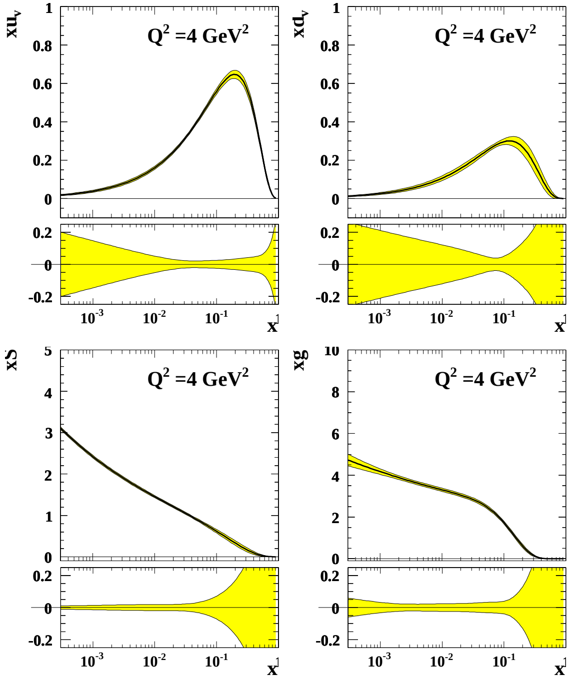
Mostly negligible, except at very large Q^2 and x where statistical errors and fluctuations are the largest.



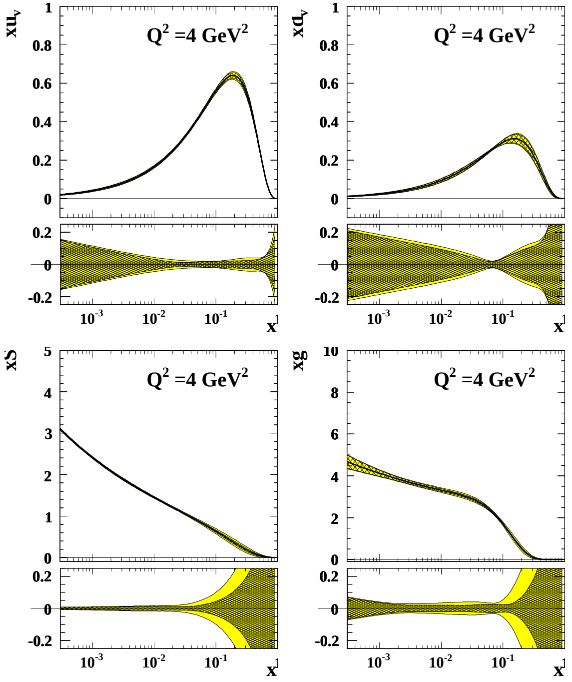
Different error treatments



47 systematic errors added to statistical quadratically χ^2= 428.0

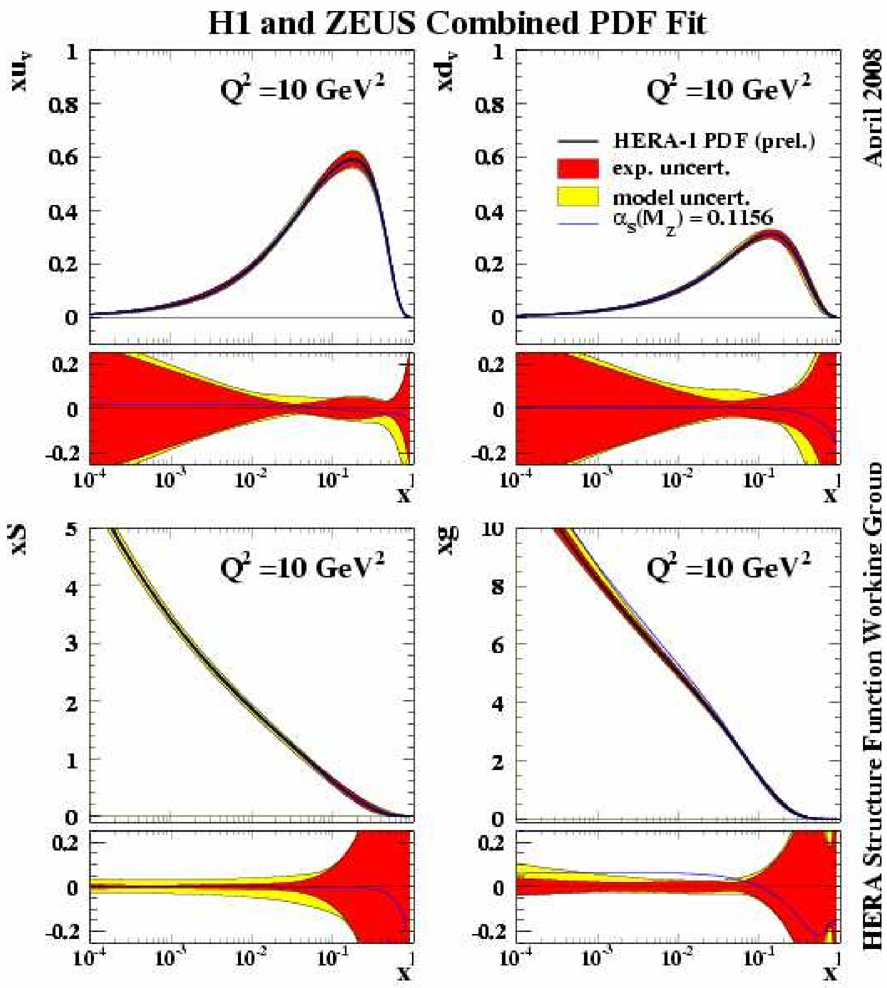


47 systematic errors treated by Hessian method χ^2=553.1

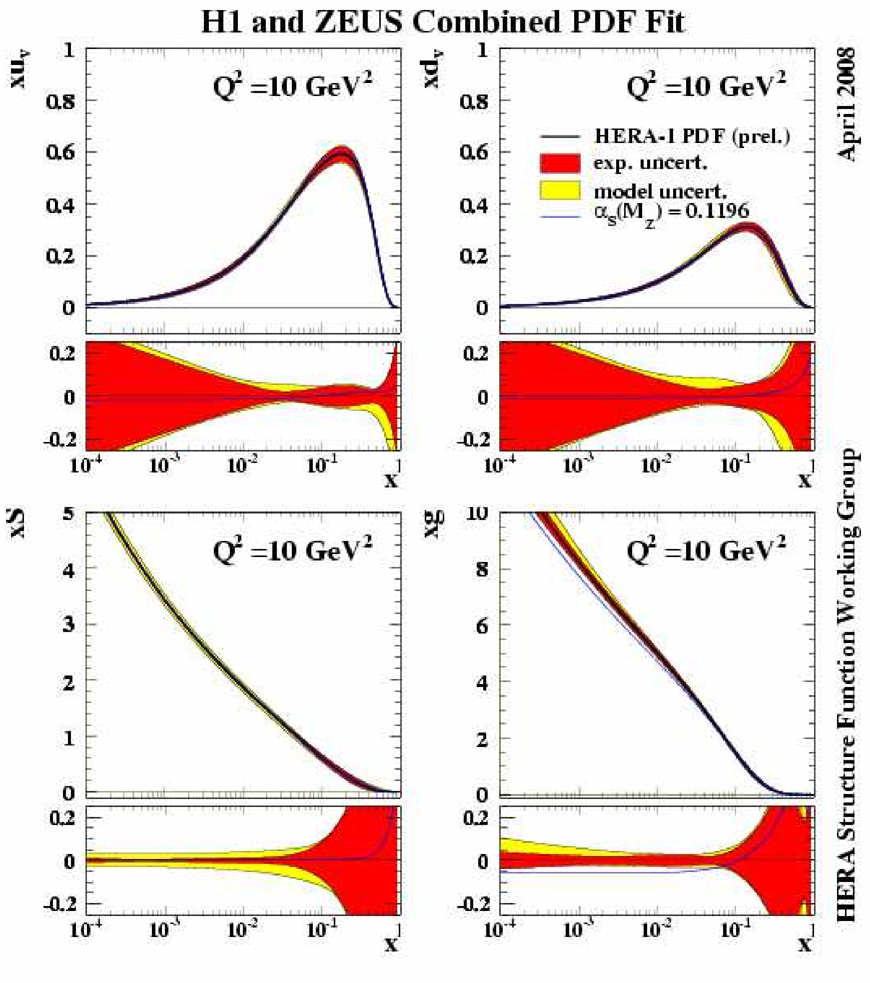


43 original sources of systematic errors added to statistical quadratically and 4 procedural errors Offset χ^2=476.7

Varying α_S



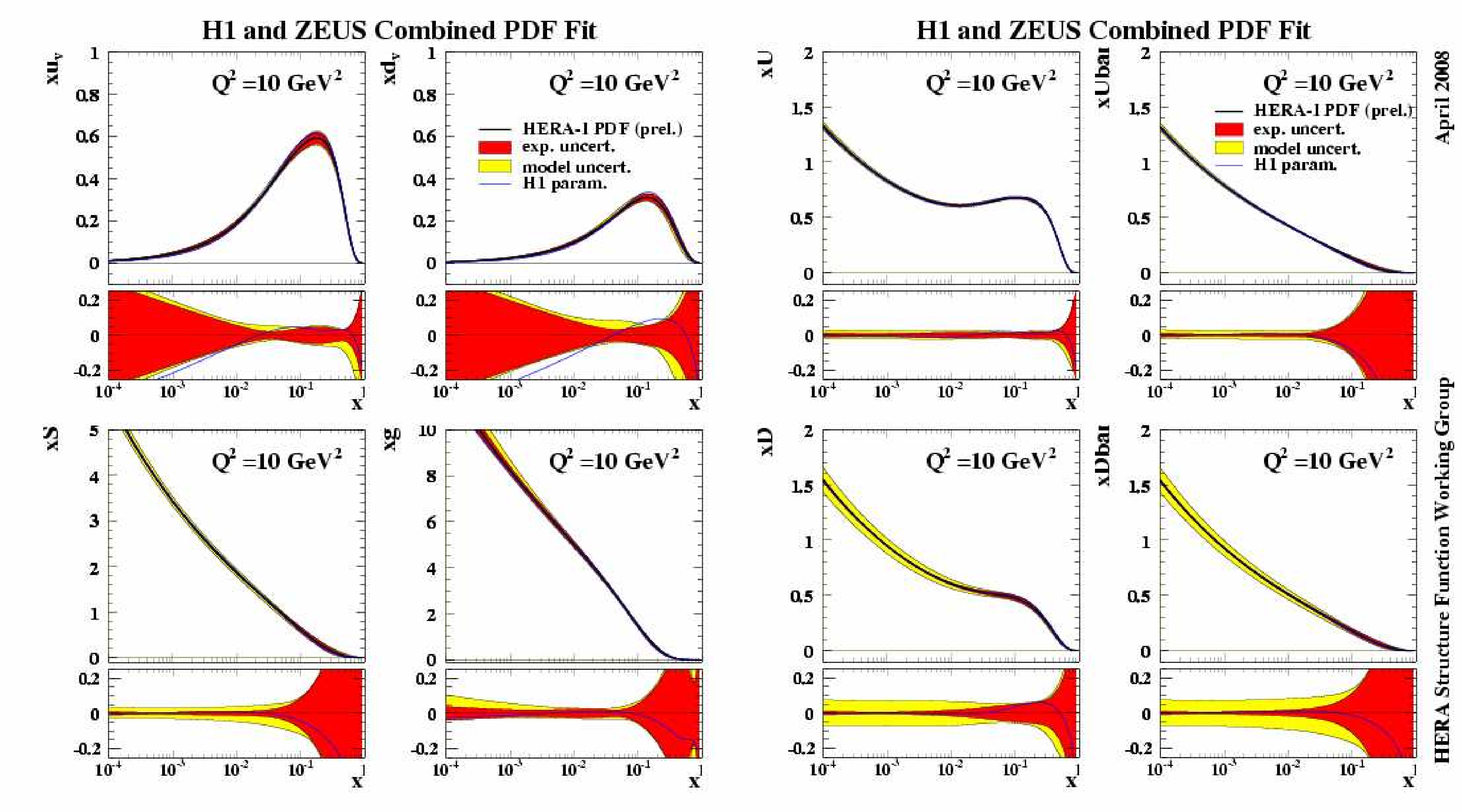
α_S= 0.1156



α_S= 0.1196

Variation is (just) outside the gluon error band

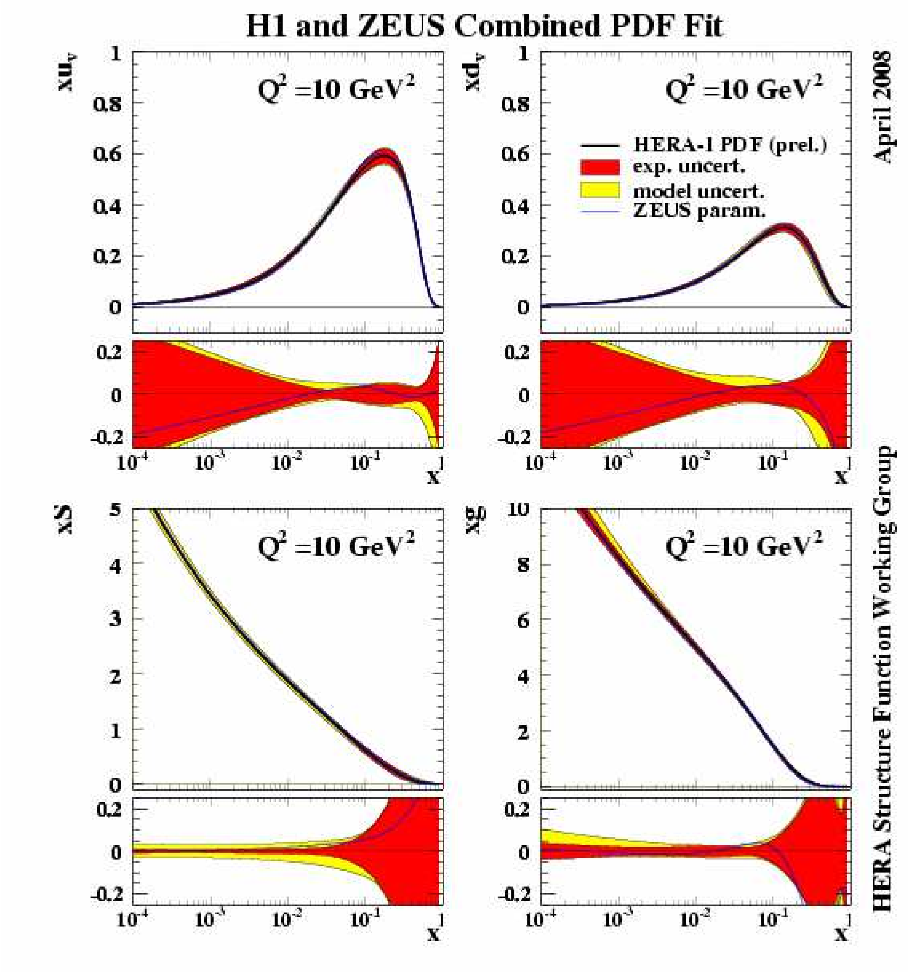
Variation: H1 style parameterisation

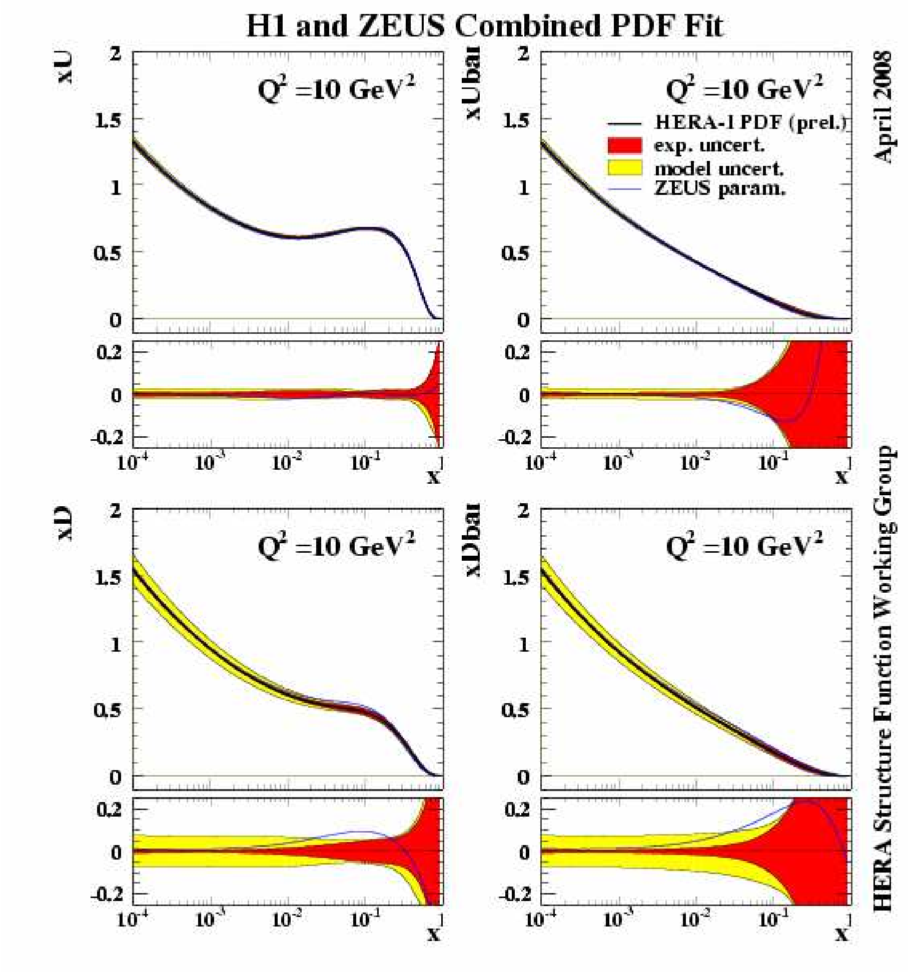


Central HERAPDF0.1 fit compared to H1 style parameteristion (optimised)

Marginally outside error bands for valence quarks at low x

Variation: ZEUS style parameterisation





Central HERAPDF0.1 fit compared to ZEUS style parameteristion (optimised)

just inside error bands if model uncertainty included