# Gluon Correlations in small-x Evolution 

Emil Avsar<br>In Collaboration with Yoshitaka Hatta arXiv:0805.0710 [hep-ph]

Institut de Physique Théorique de Saclay

France

## Outline

- Evolution equations and correlations
- Calculating the correlations:
- Analytical approach
- Numerical approach
- Summary and Outlook


## Evolution towards smaller $x$



- Hadron contracted to a "pancake" consisiting of gluons at different transverse positions.


## High Energy Evolution Equations

- Start with dipole $(x, y)$. The evolution of $T$ with $Y \equiv \ln 1 / x$ is given by

$$
\partial_{Y} T_{x y}=\int d^{2} z \frac{d \mathcal{P}}{d Y d^{2} z}\left\{-T_{x y}+T_{x z}+T_{z y}-T_{x z ; z y}^{(2)}\right\}
$$



- No correlations: $T^{(2)}=T \cdot T \Rightarrow$ Hierarchy closes down to single equation; BK-eq.
- Important to know value of $R \equiv T^{(2)} / T^{2}$.


## Dipole Densities

- To estimate $R$ analytically one needs to evaluate the double dipole density $n^{(2)}$.
- $n^{(2)}$ has previously been calculated for spatially disconnected dipoles. Power-like correlations were found, implying $n^{(2)} \gg n^{2}$. This in turn implies $T^{(2)} \gg T^{2}$, i.e. $R \gg 1$.
- To make quantitative predicitions for $R$ however, numerical approach needed.
- In numerical approach, Hadron wavefunction explicitely constructed: One has knowledge of all $n^{(k)}$ on an event-by-event basis.



## Analytic Results

- For $x_{01} \gg b, r(\gamma \approx 0.82$ always $)$ :

$$
R \sim\left(\frac{x_{01}}{r}\right)^{2(2 \tilde{\gamma}-\gamma)} e^{2(\chi(1 / 2)-\chi(\tilde{\gamma})) Y}, \quad \chi^{\prime}(\tilde{\gamma}) Y=\ln \frac{x_{01}^{2}}{r^{2}}
$$

- For $r \sim b \gg x_{01}$ :

$$
R \sim\left(\frac{r}{x_{01}}\right)^{2(2 \tilde{\gamma}-\gamma)} e^{2(\chi(1 / 2)-\chi(\tilde{\gamma})) Y}, \quad \chi^{\prime}(\tilde{\gamma}) Y=\ln \frac{r^{2}}{x_{01}^{2}}
$$

- For $b \gg x_{01}, r$ :

$$
R \sim\left(\frac{b^{2}}{x_{01} r}\right)^{2(2 \tilde{\gamma}-\gamma)} e^{2(\chi(1 / 2)-\chi(\tilde{\gamma})) Y}, \quad \chi^{\prime}(\tilde{\gamma}) Y=\ln \frac{b^{4}}{x_{01}^{2} r^{2}}
$$

## Results for $b=0, Y=6,8$



- Power-like behaviour consistent with analytic estimates. At $Y=8$ and $x_{01} / r=5$ and 20, analytic result: $\omega=0.52$ and 0.76 while numerical result gives 0.60 and 0.80 .


## Results for $b=0, Y=10$



- $Y$ dependence consistent with analytic estimates. For asymmetric configurations $R$ decreases faster. We note that $R \gtrsim 1.5$ always.
- Beyond $Y=10, T^{(2)}>1$ so we cannot go any further.


## $b$ Dependence




- Results for $x_{01}=10 r$ and $x_{01}=20 r$ at $Y=10$. $R$ constant as long as $b<x_{01}$, but it increases fast as $b \gtrsim x_{01}$.


## Running Coupling

- In nature the coupling is running so important to include running $\alpha_{s}$.
- The avaliable NLL studies suggest that $\bar{\alpha}_{s}\left(\min \left(r, r_{1}, r_{2}\right)\right)$ should be used in the splitting $r \rightarrow r_{1}, r_{2}$.
- The inclusion of the running $\alpha_{s}$ is straightforward. In practice, however, simulations are very time consuming and we have not been able to perform as detalied analysis.
- Analytic calculations not easy, postponed for a future study.


## Results for Running $\alpha_{s}$




- Results for $b=0$ and $b=5 r$ at $Y=6$. At $Y=8$, $R: 1.6 \rightarrow 1.5$ at $x_{01}=2 r$ and $b=0$, and $R: 11.6 \rightarrow 9.4$ at $b=5 r$.


## Summary and Outlook

- We find power-like correlations which lead to a strong violation of the factorization assumption $T^{(2)}=T \cdot T$.
- Analytic estimates confirmed by numerical analysis, and we have been able to quantitatively study the value of $R$.
- Physical consequences of the large correlations should be explored.
- More results for running $\alpha_{s}$ desirable.
- Studies of higher order correlations, calculation of $T^{(p)}$ for $p>2$. Few analytical results. Numerics straightforward but can be time consuming.

