PDF uncertainties using a Monte Carlo method

Voica A. Radescu

In collaboration with A. Glazov

DESY

- Motivation
- Method
- Cross check of the method:
  - Comparison with the standard error estimation
- Test various assumptions for the error distributions:
  - Gauss, Log-normal, Uniform
- Summary

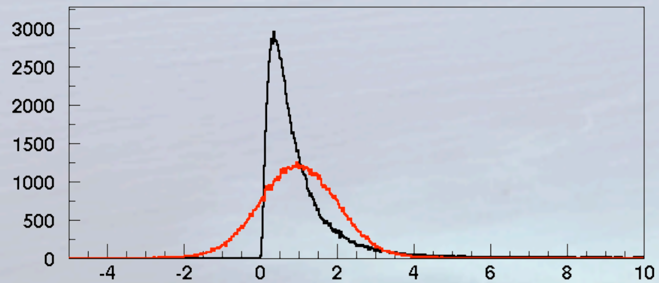
Motivation

- **The idea is to use a Monte Carlo method to estimate PDF uncertainties under various assumptions for the error distributions.**

- **Standard error estimation of PDFs relies on the assumption that all errors follow Gauss statistics.**
  ✔ **Monte Carlo method can provide an independent cross check of it.**
- **However, Gaussian assumption is not always correct:**

$$\sigma \sim \frac{N}{\mathcal{L}\mathcal{A}}$$

  - **Some systematic uncertainties follow Log-Normal Distribution:**
    - **lumi ,detector acceptance,etc.**



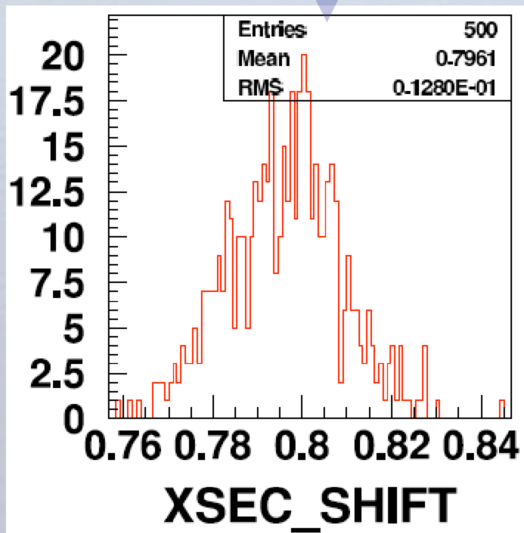
Gauss and Log-normal distributions:
° **same mean**
° **shifted peaks**
How is that affecting PDF's errors?

  - **Some systematic uncertainties follow a Uniform Distribution:**
    - **"upper" limit uncertainties**

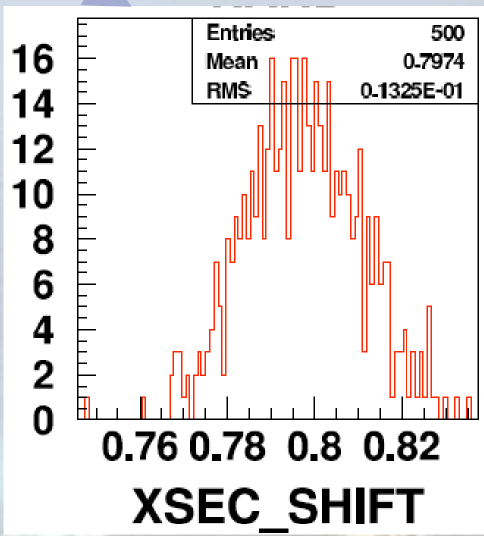
✔ **Monte Carlo method allows to test the various assumptions.**

Method (I)

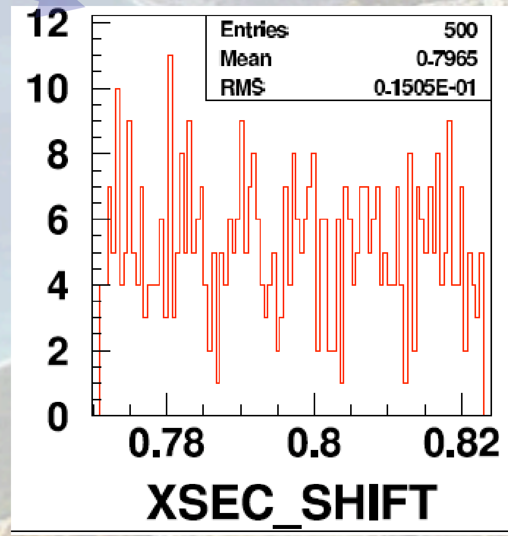
- **Prepare “shifted data sets”:**
  - **Allow the central value of the cross sections (σ_i) to fluctuate within its systematic and statistical uncertainties taking into account all the correlations:**
    - **Various assumptions can be considered for the error distributions:**
      - **Gauss, Lognormal, Uniform**



$$\sigma_i = \sigma_i(1 + \delta_i^{uncorr} RAND_i)$$



$$\sigma_i = \sigma_i R_i^{log}(1, \delta_i^{uncorr})$$



$$\sigma_i = \sigma_i - a + 2aR_j^{unif}$$

Method (II)

- **Shifts for Statistical errors:**
  - **allow each data point to randomly fluctuate within its statistical uncertainty assuming either Gauss, Log-normal, or uniform distributions**

- **Shifts for Systematic errors:**
  - **For each systematic source j uniformly select "fluctuation probability" P_j**
  - **For each data point shift the central value of cross section such that probability of this shift for systematic source j is equal P_j (or $(1-P_j)$)**

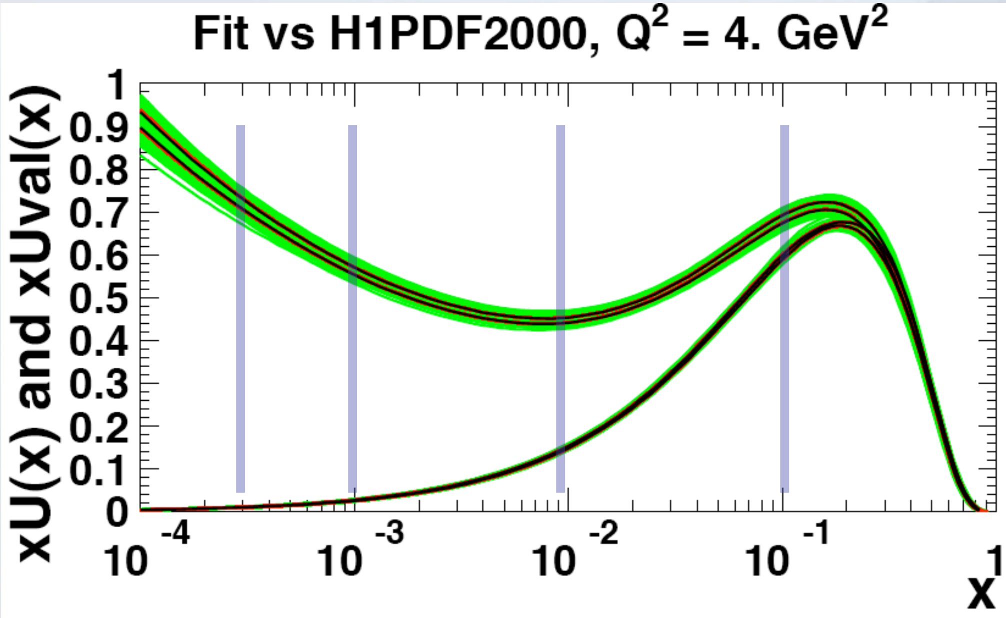
Method (III)

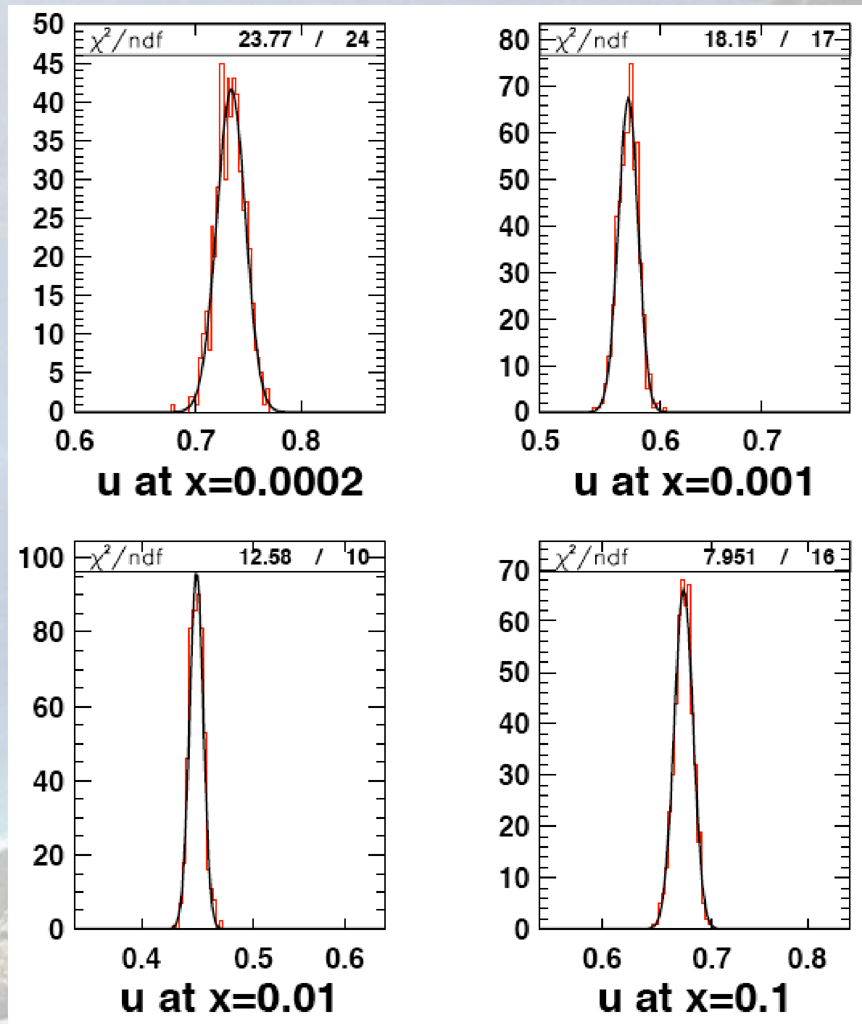
- Repeat the preparations for N times (here N ≥ 100)
- Perform the NLO QCD fit N times to extract PDFs
- PDF uncertainties =>from the RMS of the spread

- This study is performed using:
  - published H1-HERA I data of NC and CC $e^{\pm}p$ scattering cross sections [ref: Eur. Phys. J. C 30, 1-32 (2003)]
  - fit program H1 QCDNUM implementation at NLO:
    - MSbar renormalisation scheme, DGLAP evolution at NLO, massless quarks, polynomial form for PDF parametrisation a' la H1PDF2000

Test of the Method: Gauss Distribution (xU)

- To test the method, assume all errors follow Gauss distribution and compare the results to standard error estimation



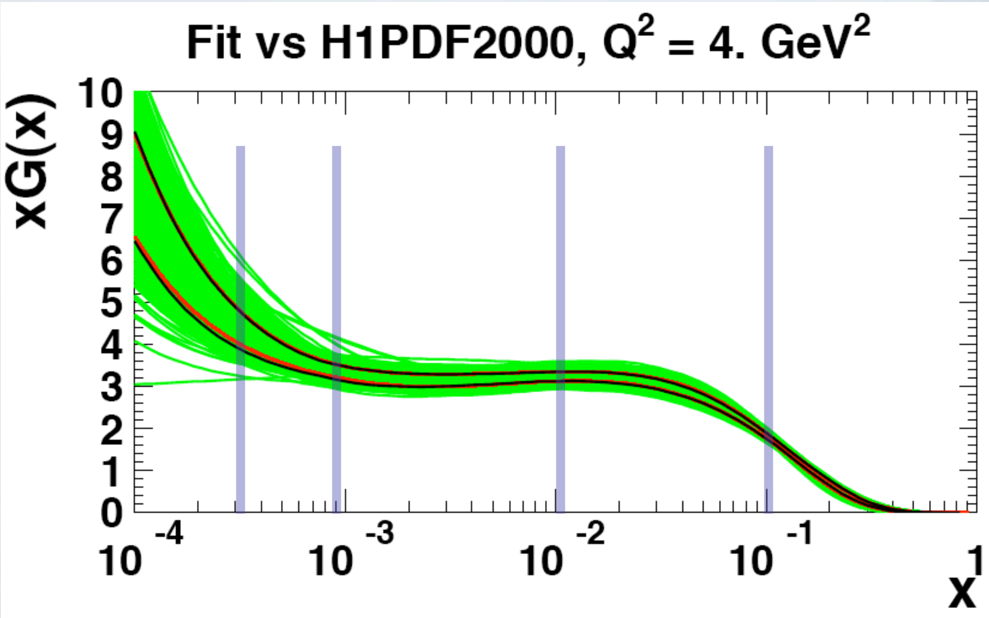


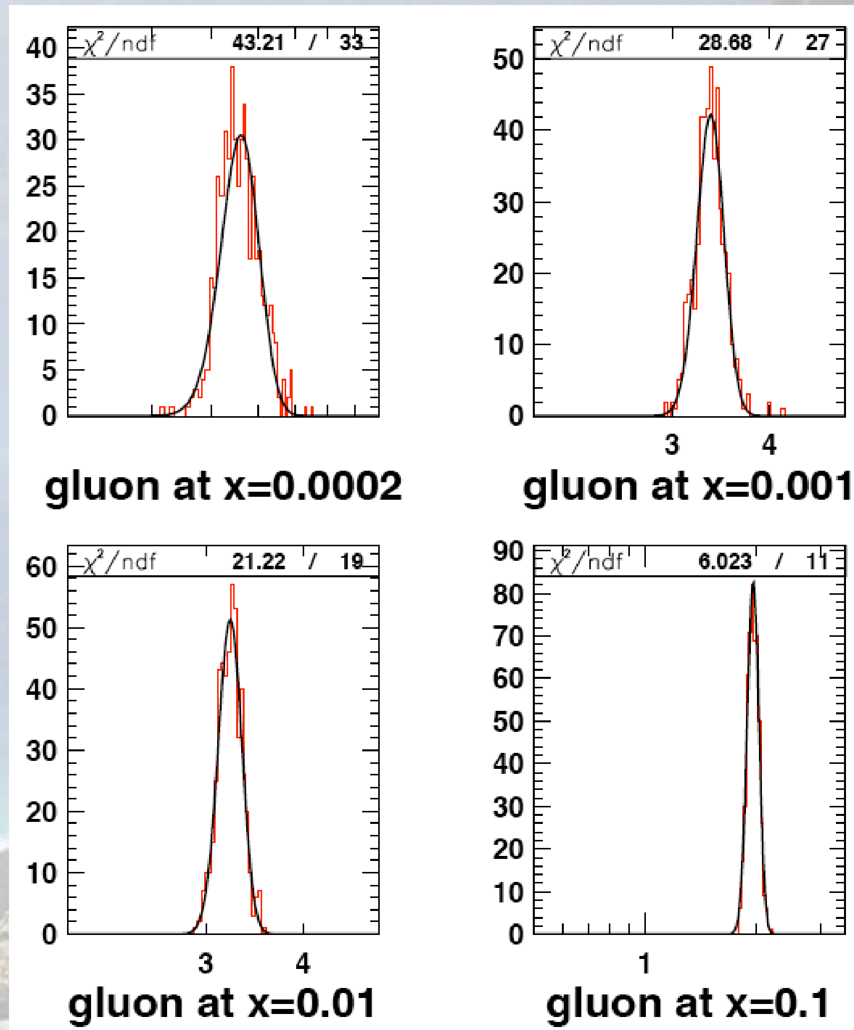
- 500 Green lines
- Red lines: PDF uncertainties from RMS
- Black lines: Hessian errors

- Projections at various x values are shown:
  - Good Gauss Fit

Test of the Method: Gauss Distribution (xG)

- **To test the method, assume all errors follow Gauss distribution and compare the results to standard error estimation**





- 500 Green lines
- Red lines: PDF uncertainties from RMS
- Black lines: Hessian errors

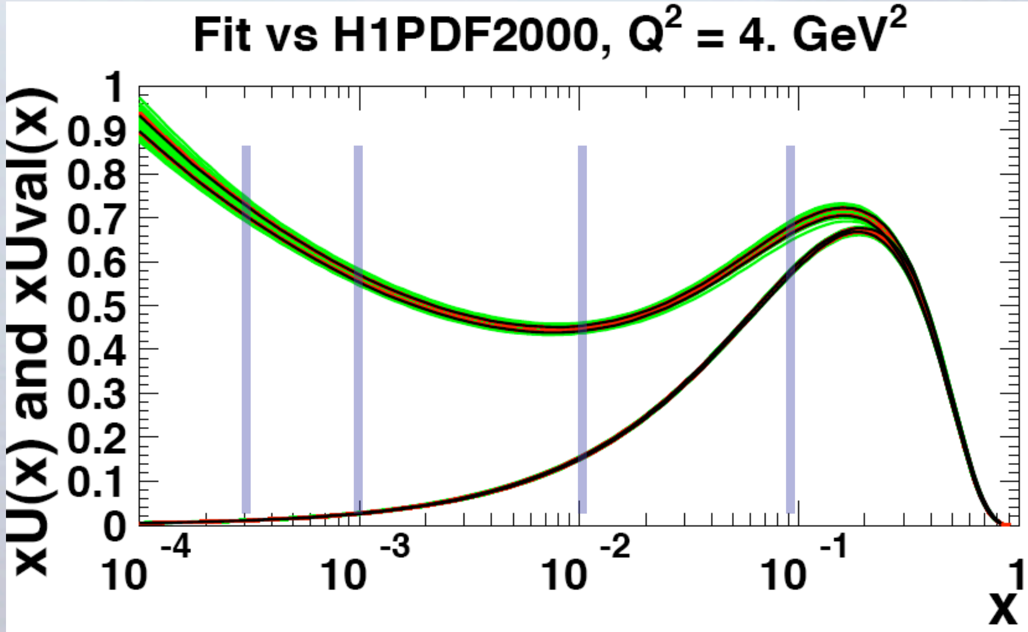
Good agreement with the standard error estimation!

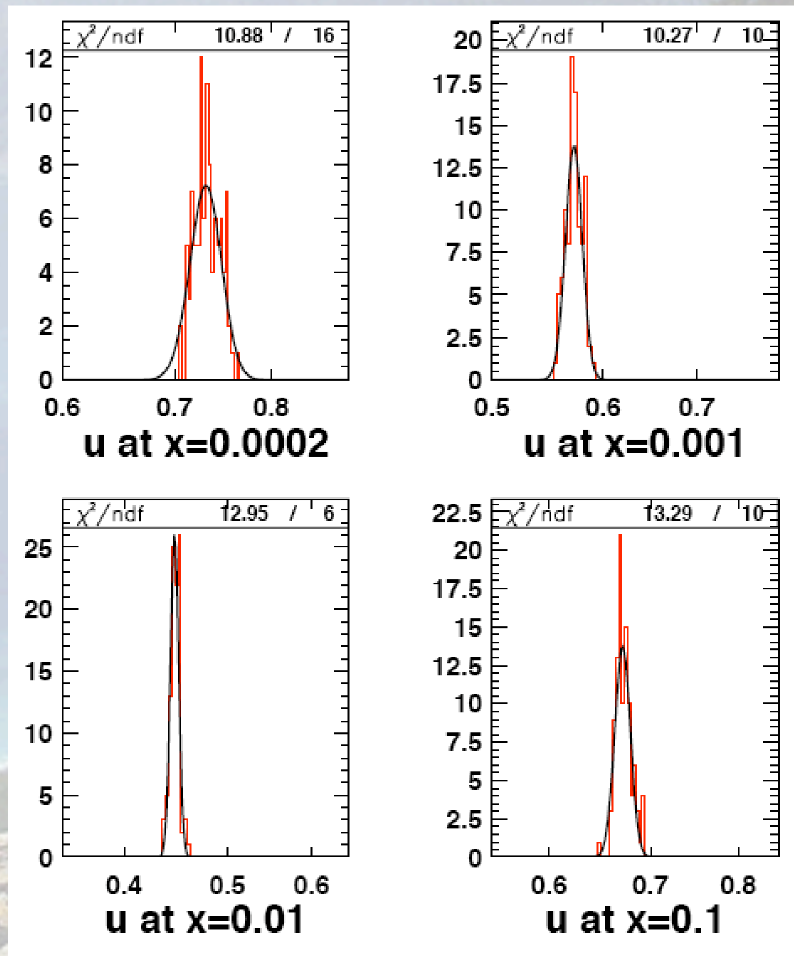
Test various assumptions for the errors

- Now that the method is cross checked, we are ready to test other assumptions:
  1. Log-normal for lumi, all the rest set to Gauss
  2. Log-normal for all systematic errors, Gauss for statistical uncertainty
  3. Uniform for all errors

1. Log-normal dist. for Lumi (xU)

- Assume that all errors, apart from Lumi, follow Gauss
  - Test the effect of log-normal assumption for Lumi uncertainty



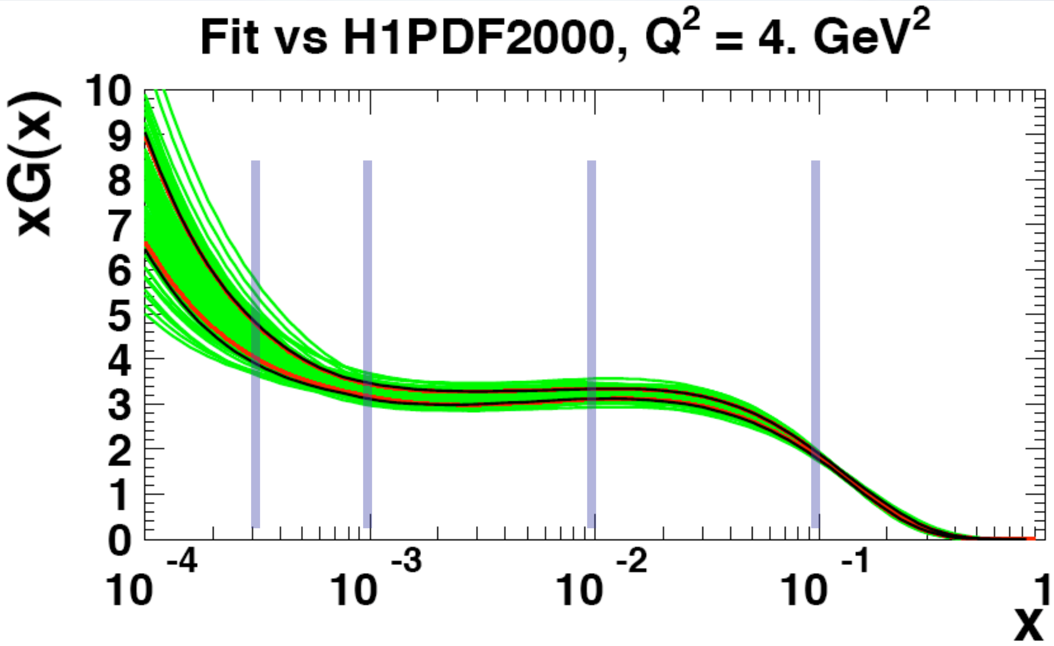


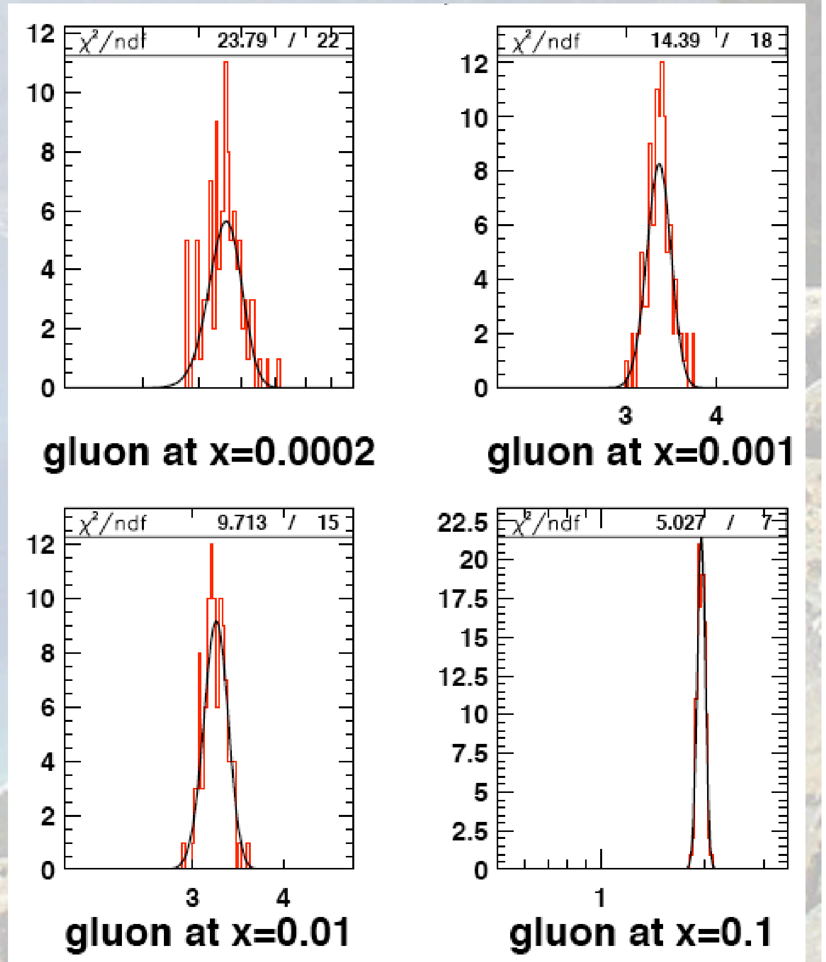
- 100 Green lines
- Red lines: PDF uncertainties from RMS
- Black lines: Hessian errors

Similar effect to pure gaussian case!

1. Log-normal dist. for Lumi (xG)

- **Assume that all errors, apart from Lumi, follow Gauss**
  - **Test the effect of log-normal assumption for Lumi uncertainty**



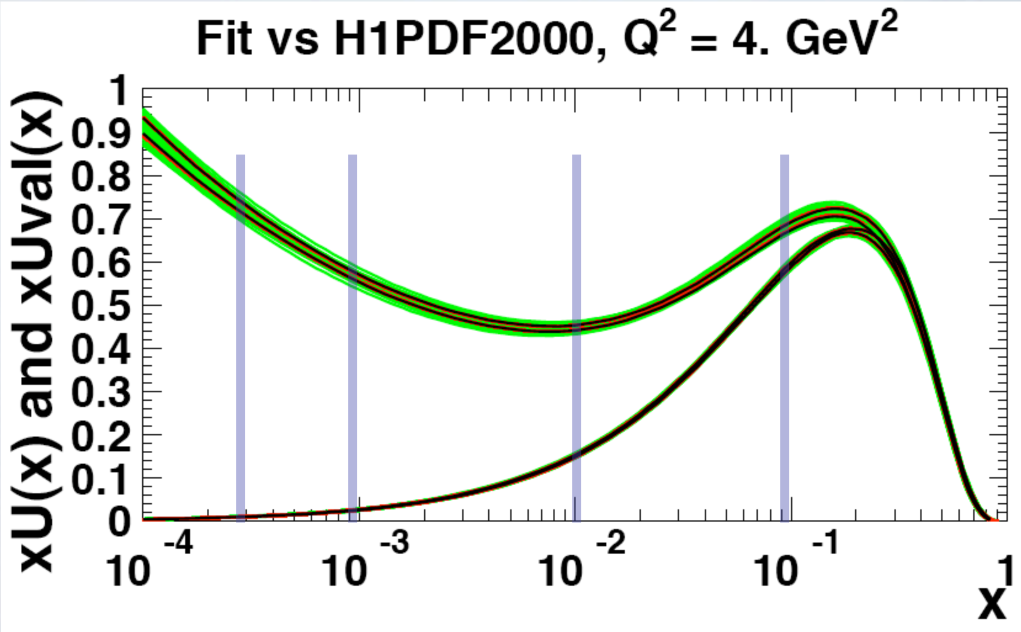


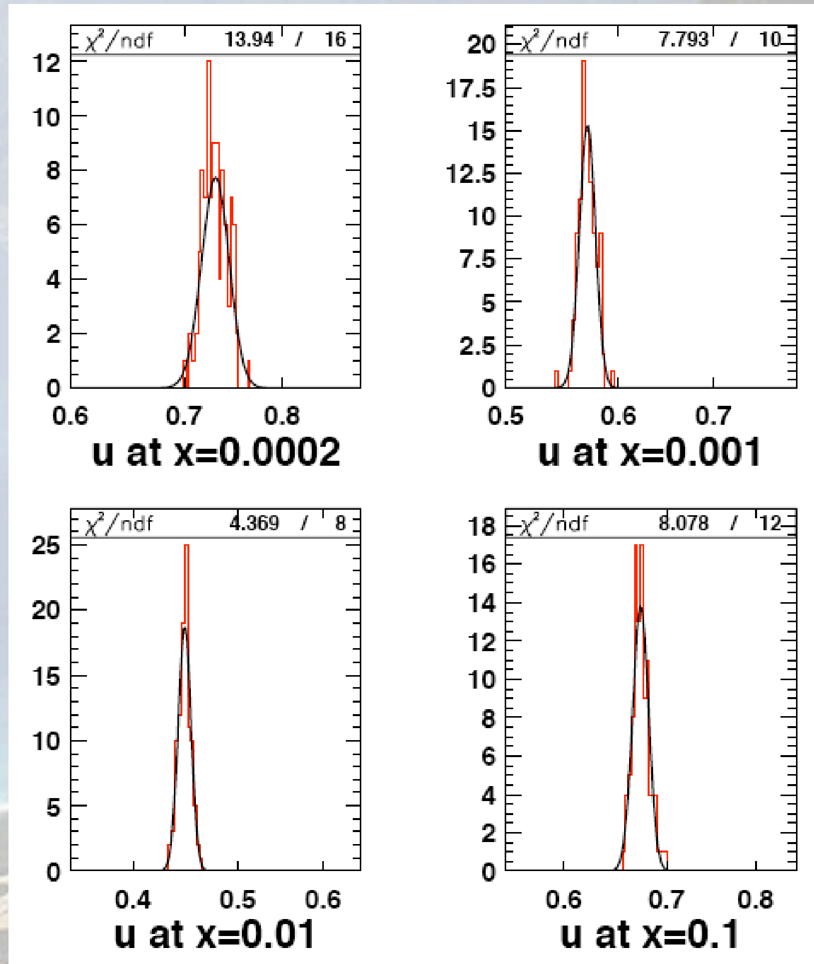
- 100 Green lines
- Red lines: PDF uncertainties from RMS
- Black lines: Hessian errors

Similar effect to pure gaussian case!

2. Log-normal dist. for all systematic (xU)

- Assume that statistical errors follow Gauss distribution and and all systematic errors follow Log-normal distribution



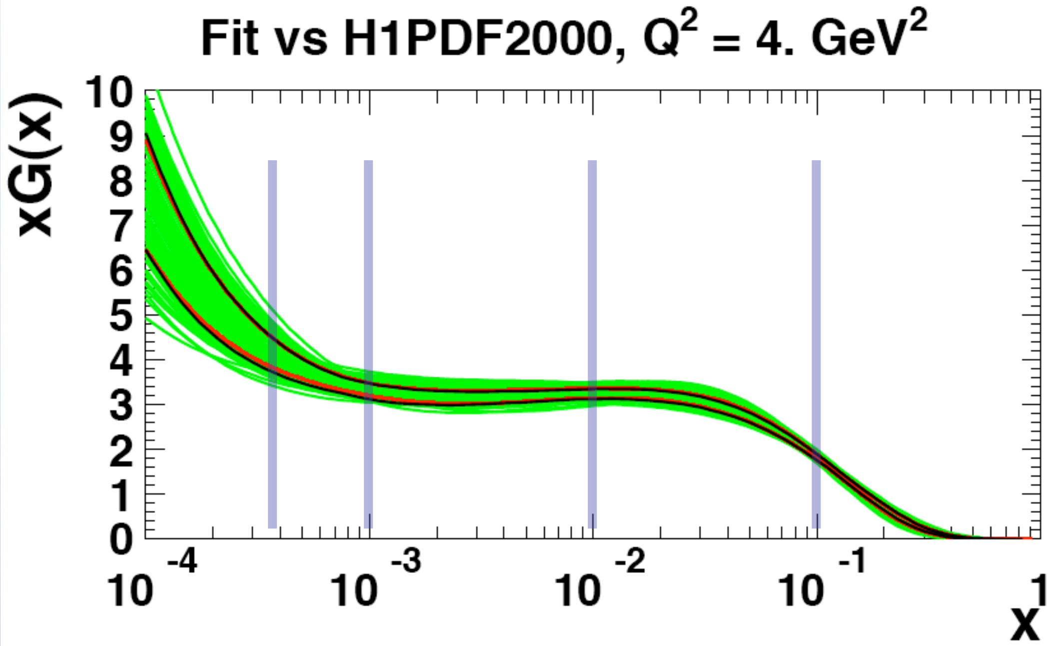


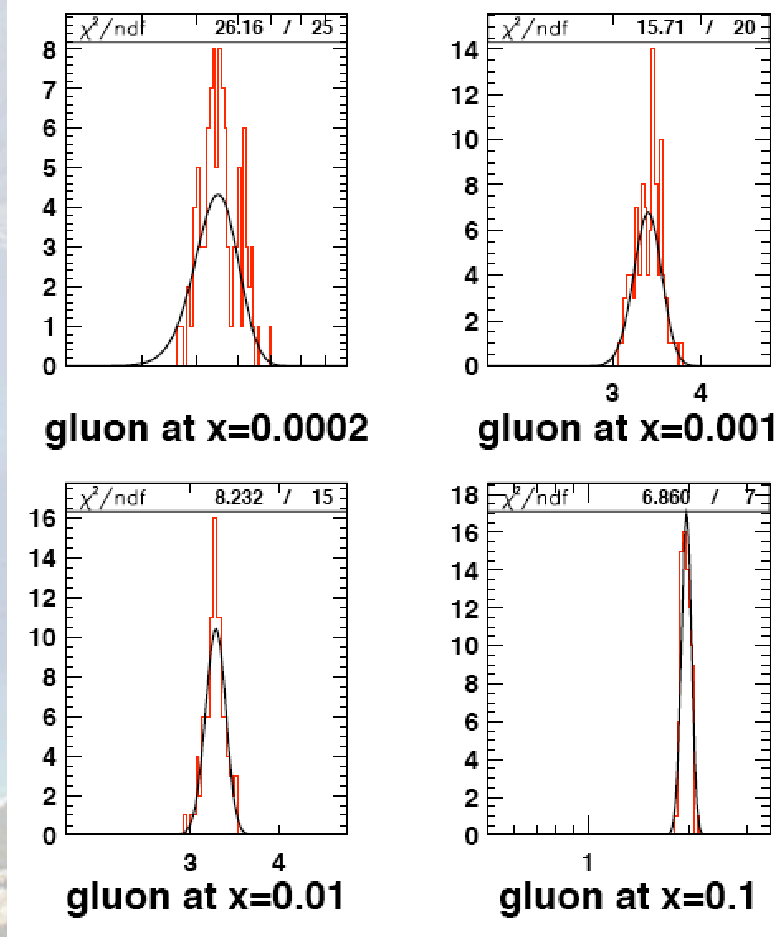
- 100 Green lines
- Red lines: PDF uncertainties from RMS
- Black lines: Hessian errors

Similar effect to pure gaussian case!

2. Log-normal dist. for all systematic (xG)

- **Assume that statistical errors follow Gauss distribution, all systematic errors follow Log-normal distribution**



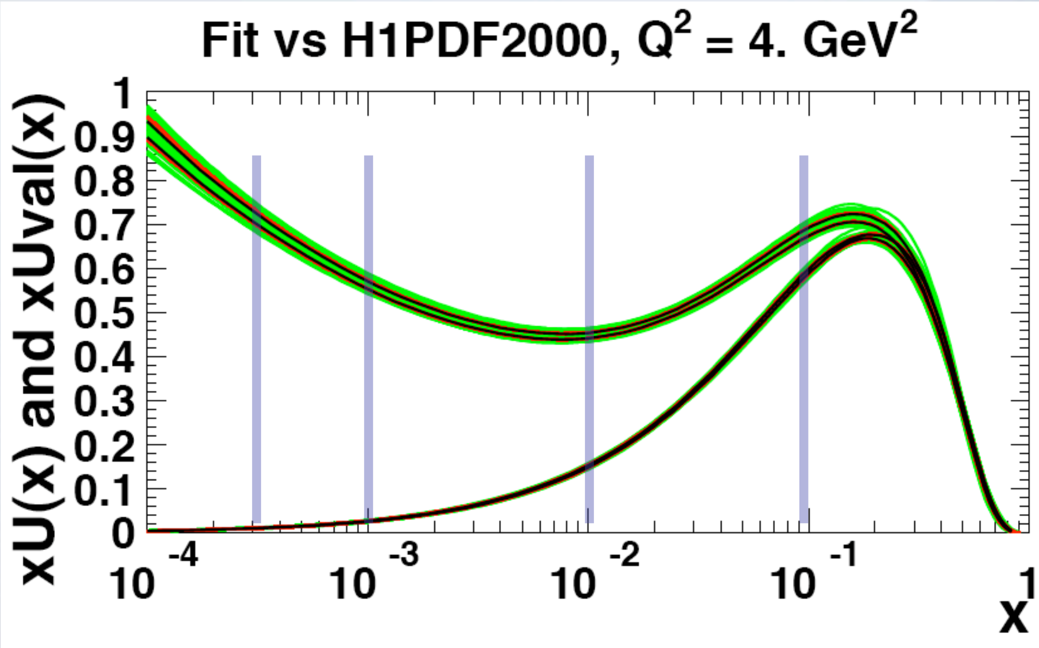


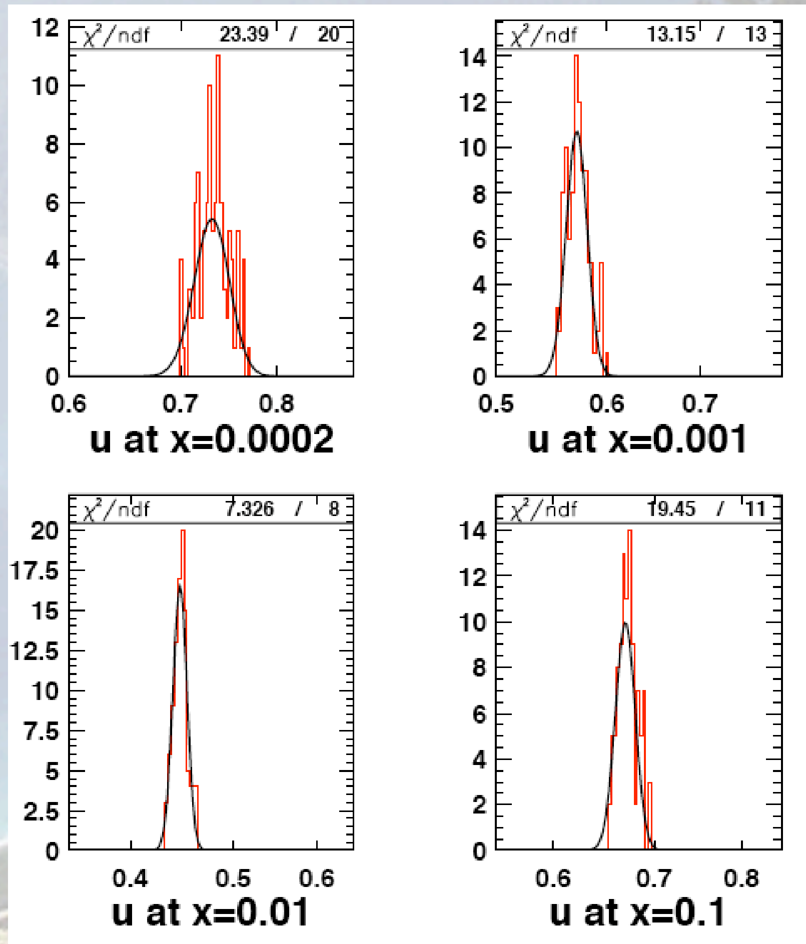
- 100 Green lines
- Red lines: PDF uncertainties from RMS
- Black lines: Hessian errors

Similar effect to pure gaussian case!

3. Uniform dist. for all errors (xU)

- **Assume all errors follow uniform distribution.**



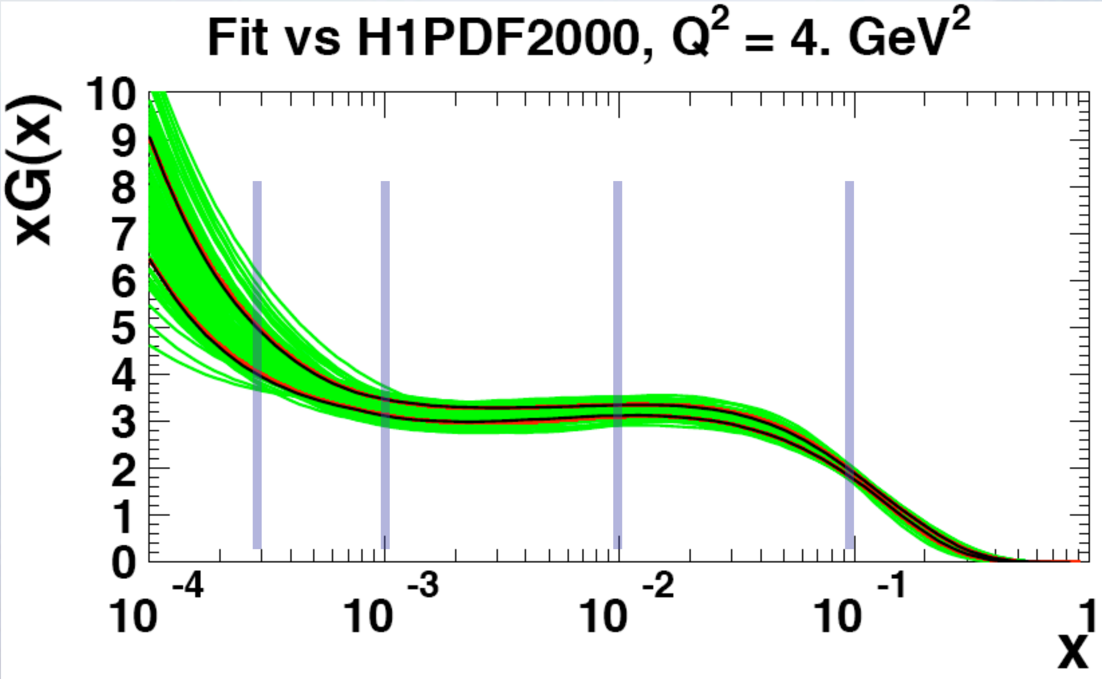


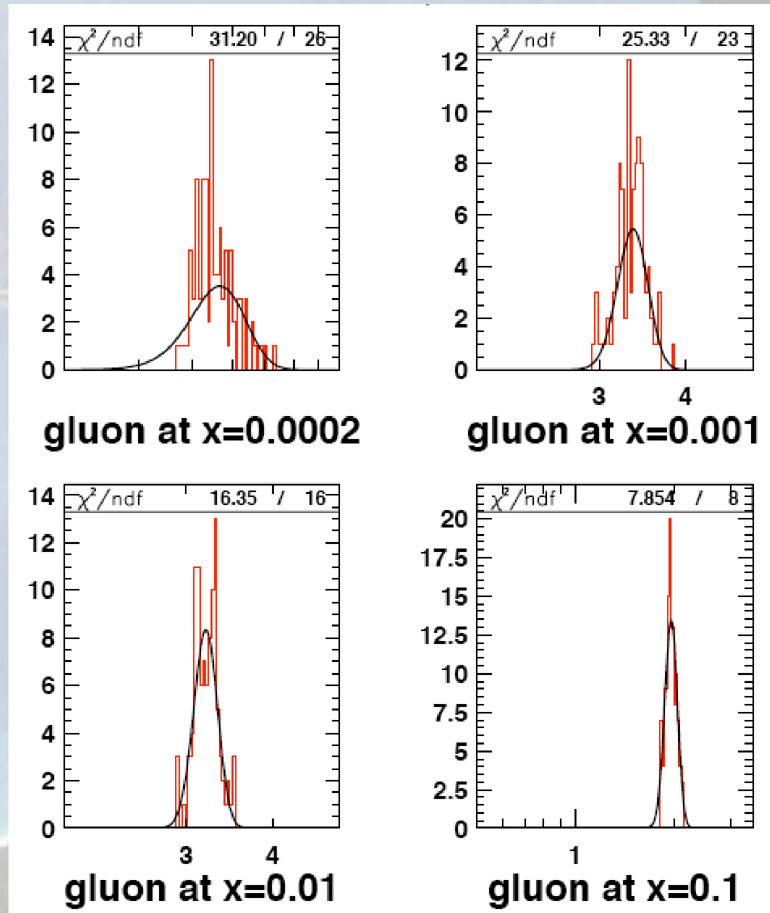
  - 100 Green lines
  - Red lines: PDF uncertainties from RMS
  - Black lines: Hessian errors

Similar effect to pure gaussian case!

3. Uniform dist. for all errors (xG)

- Assume all errors follow uniform distribution.





- 100 Green lines
- Red lines: PDF uncertainties from RMS
- Black lines: Hessian errors

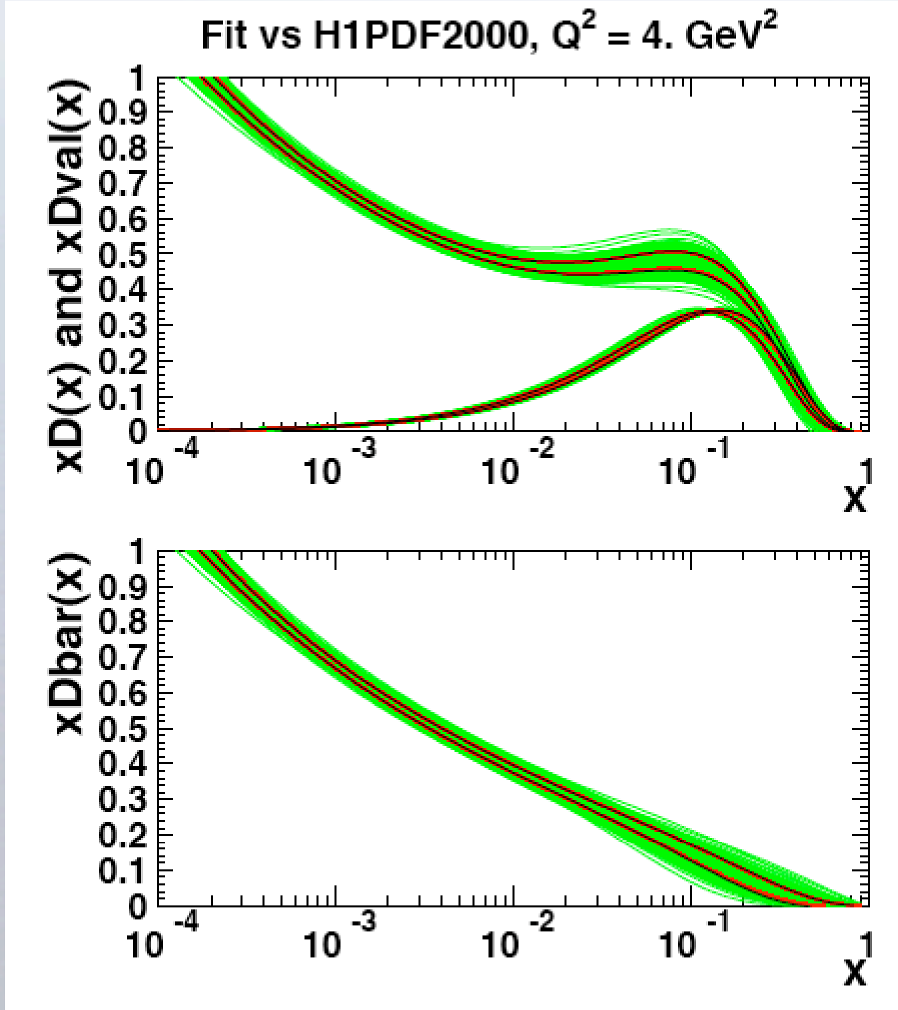
Similar effect to pure gaussian case!

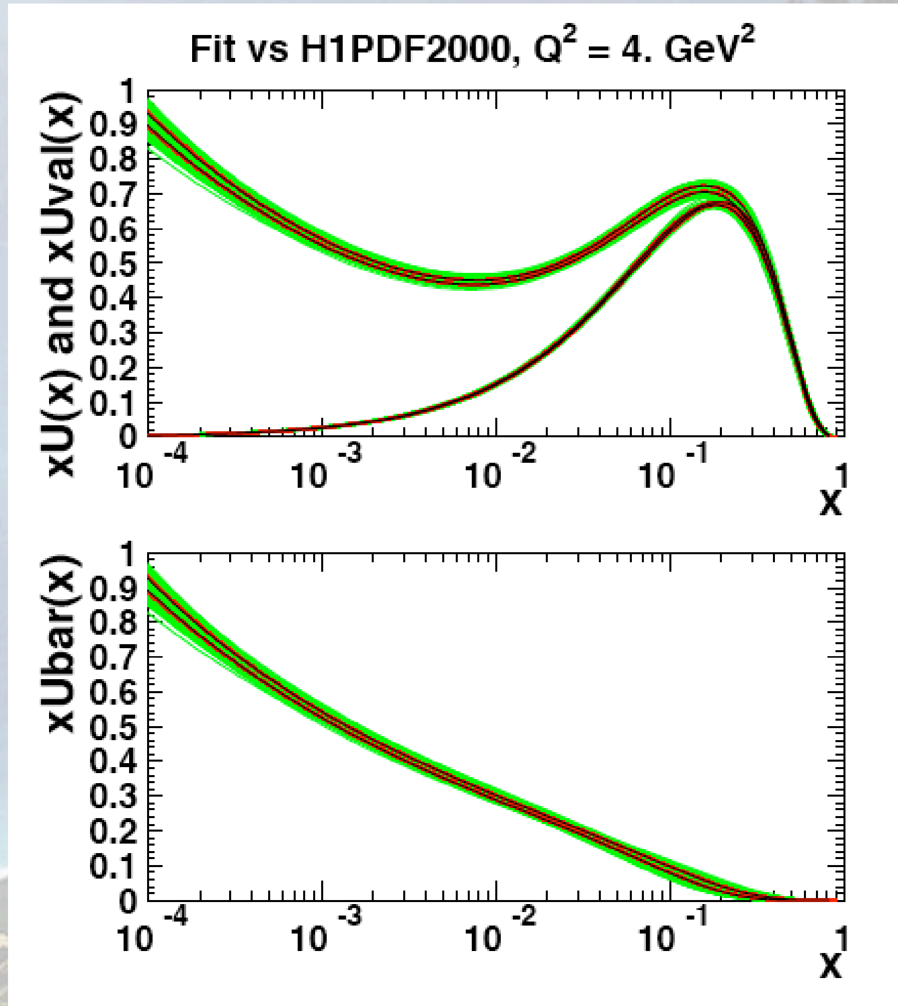
Summary

- **A simple method to estimate PDF uncertainties has been built within QCD Fit framework:**
  - Assuming only Gaussian distribution of all errors, the results agree well with the standard error estimation
- **This method allows to check the effect of non-Gaussian assumptions for distributions of the experimental uncertainties:**
  - For the H1 data, results are similar to the Gaussian case when using Log-normal and Uniform distributions of the uncertainties
- **The method could be extended for other physical variables (i.e. cross sections) for cross checks with the standard error evaluation**

Backup

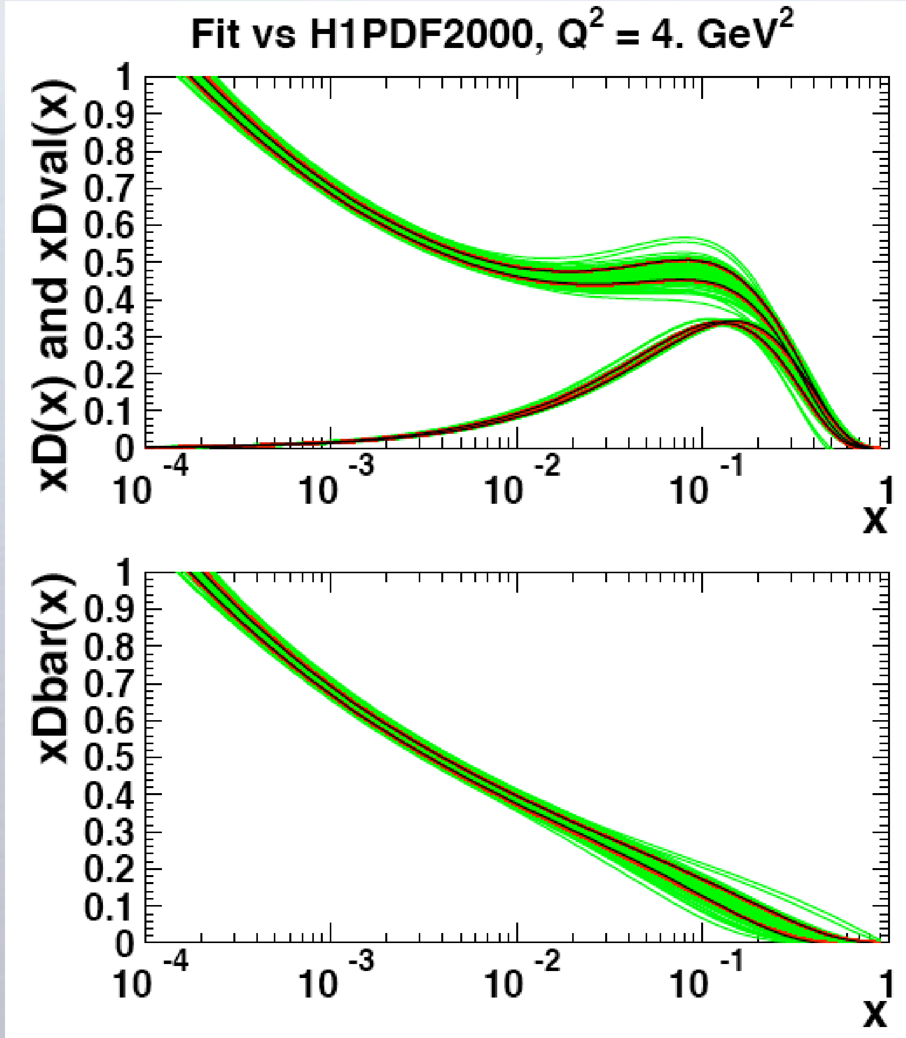
- **Gauss Errors: more distributions**

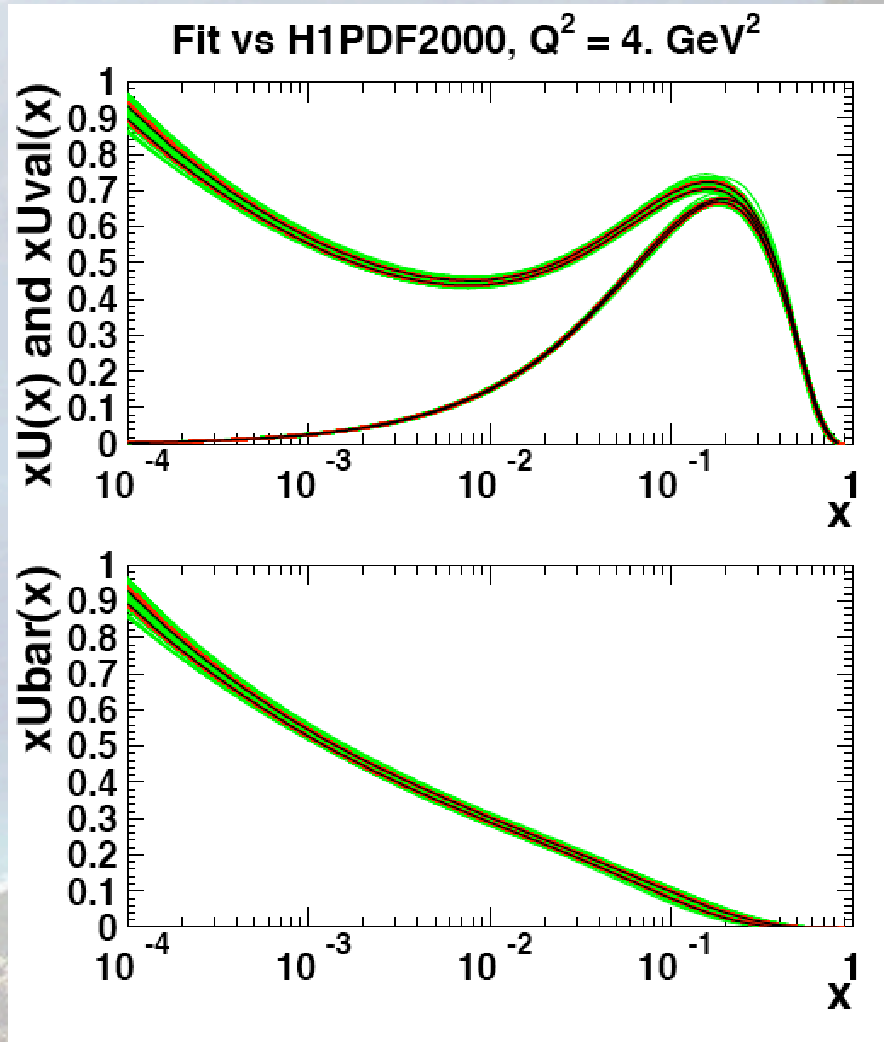




Backup

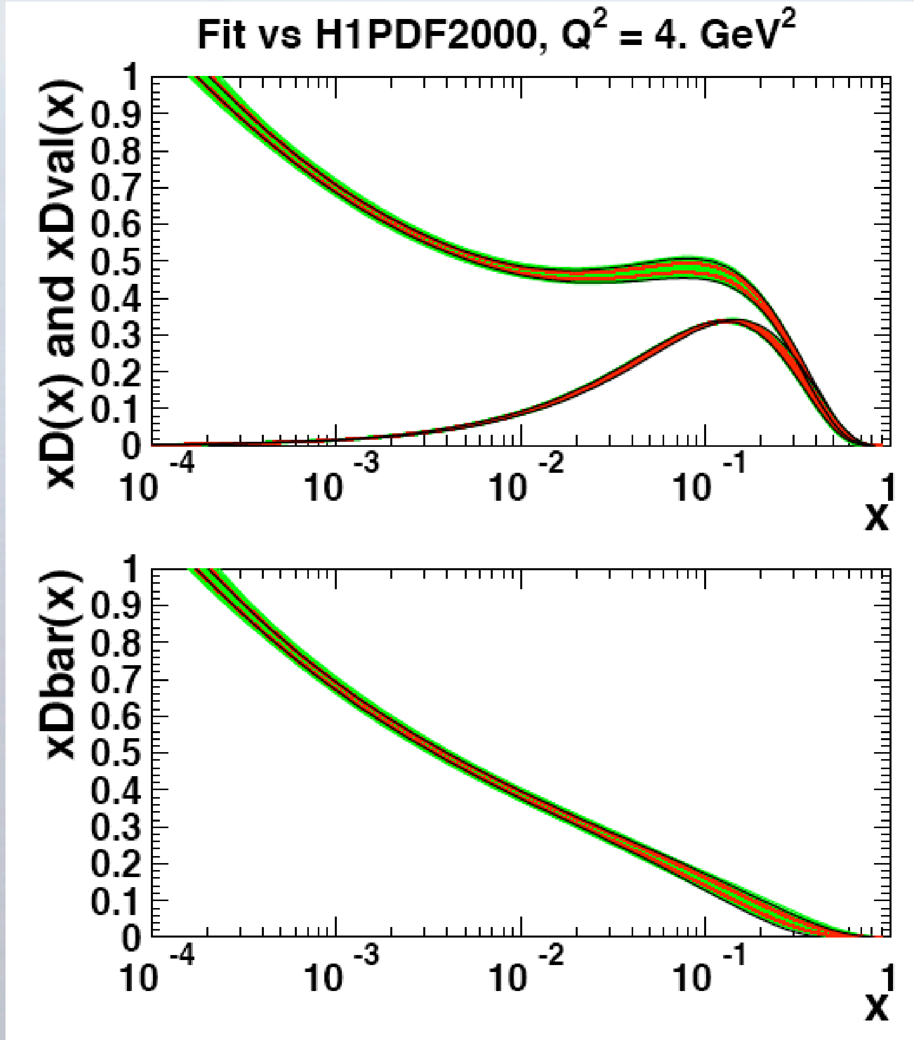
- **Uniform Errors: more distributions**

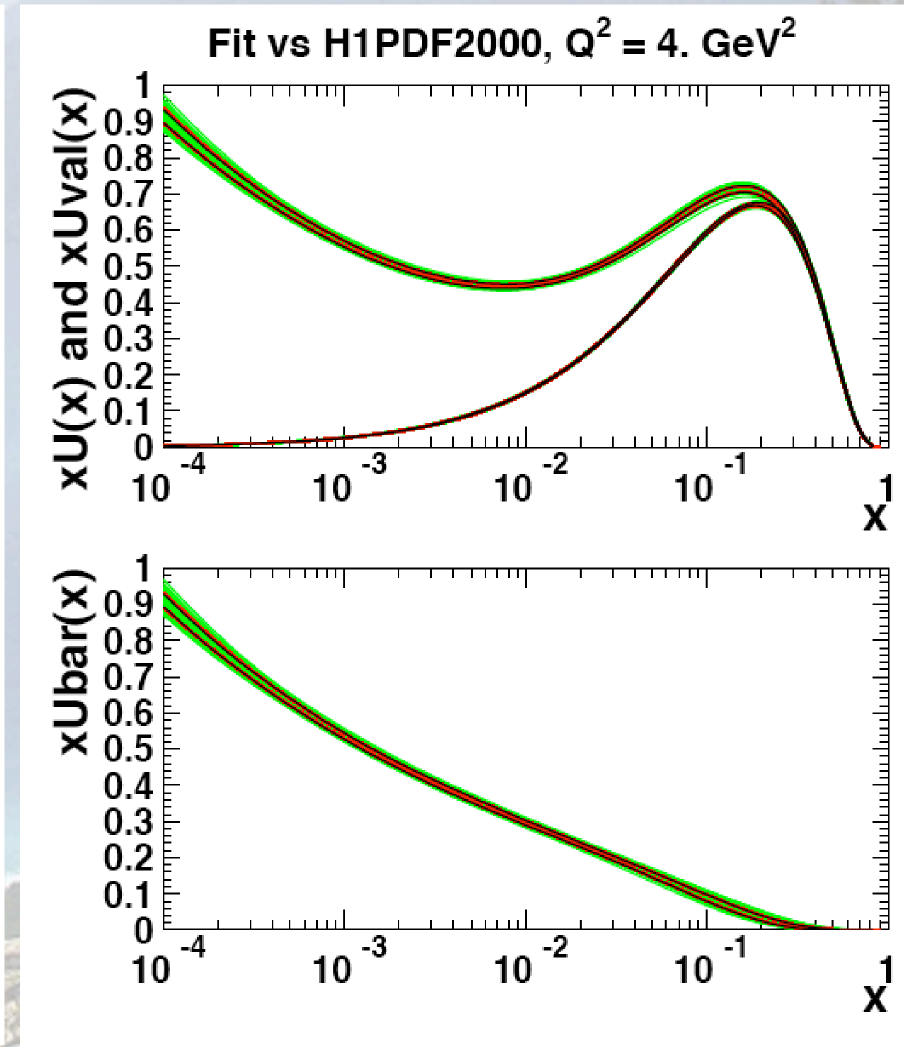




Backup

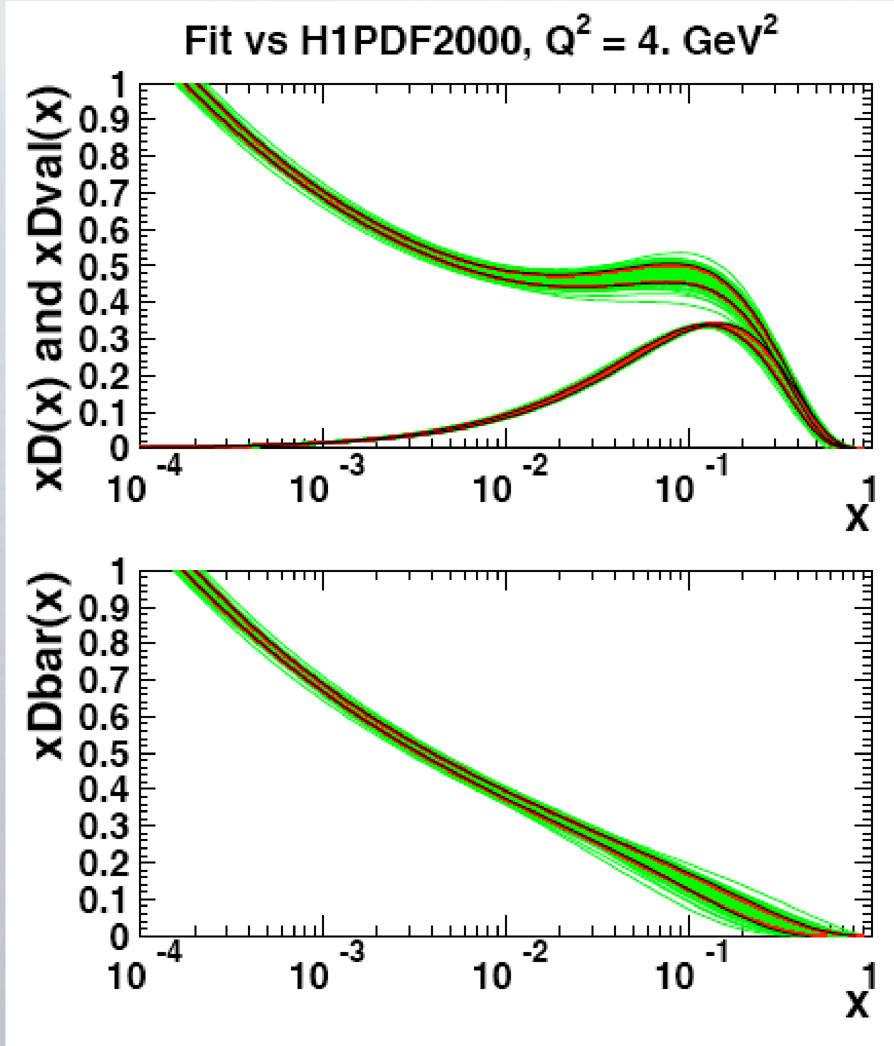
- All gauss, apart from Lumi (log-normal):

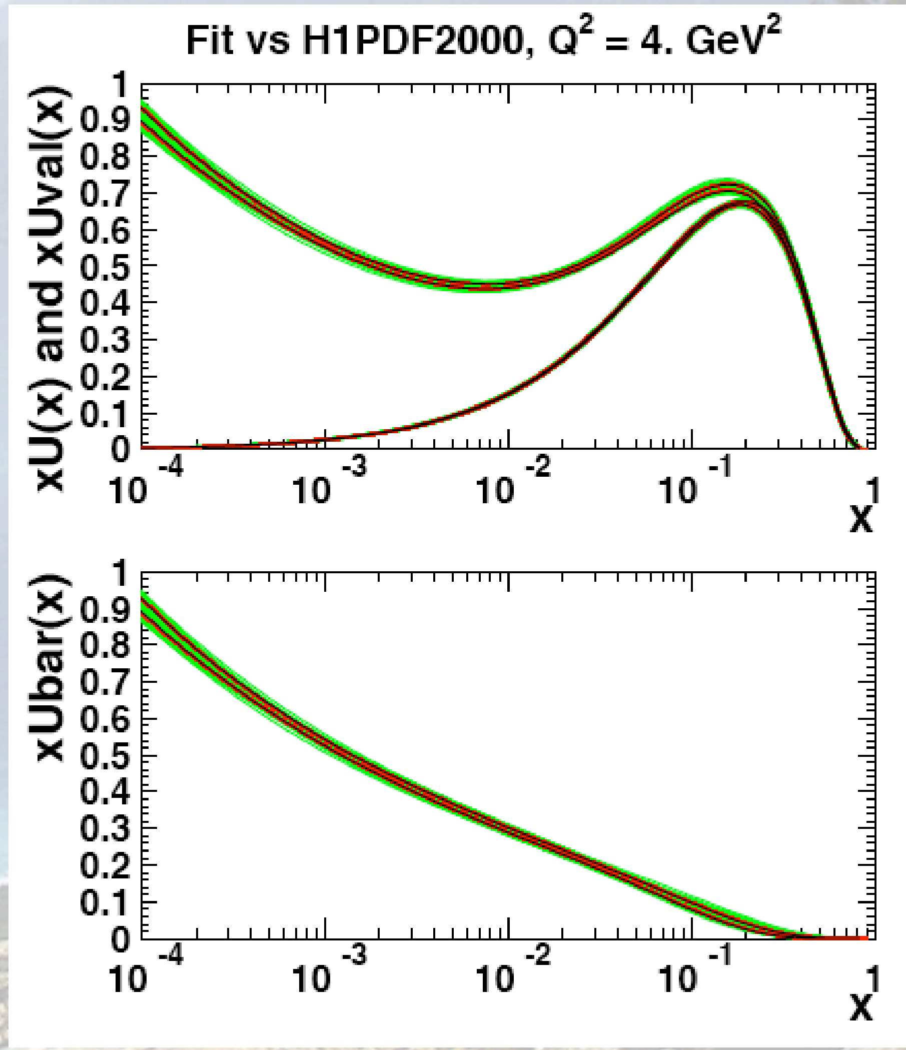




Backup

- **Stat. gauss, sys log-normal:**





Method (IV): more details …

- **Use uniform random number to select the "fluctuation probability" P_j**
- **Cross section point *i* has a sensitivity to systematic *j* δ_{ij}**
  - **If δ_{ij} > 0, select cross section shifts $\delta\sigma_{ij}$ such that:**

$$\int_{-\infty}^{\delta\sigma_{ij}} dx P(x, \delta_{ij}) = P_j$$

  - **If δ_{ij} > 0, select cross section shifts $\delta\sigma_{ij}$ such that:**

$$\int_{\delta\sigma_{ij}}^{+\infty} dx P(x, \delta_{ij}) = P_j$$

Where P is either Gauss, Log-normal, or Uniform distribution