NLO BFKL in $\gamma^*\gamma^*$ collisions

A. Papa

Università della Calabria & INFN - Cosenza

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In collaboration with F. Caporale and D.Yu. Ivanov

Outline

1. The $\gamma^*\gamma^*$ total cross section
   - Introduction and Motivations
   - The BFKL approach
2. Theoretical background
   - Build-up of the amplitude
   - Series representation
3. Numerical results
4. Discussion and conclusions

Outline

Introduction and Motivations

$\sigma_{\gamma^*\gamma^*}$ for large photon virtualities is fully under the control of perturbative QCD

- Fixed-order calculations
  - LO (quark box) [V.M. Budnev et al. (1974)] [I. Schienbein (2002)]
  - NLO [M. Cacciari et al. (2001)]

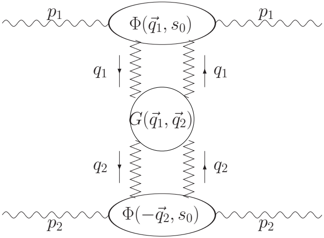
- All-order resummations
  - Double logs [J. Bartels and M. Lublinsky (2003), (2004)]
  - Leading log BFKL [J. Bartels, A. De Roeck and H. Lotter (1996)]
    [A. Bialas, W. Czyz and W. Florkowski (1998)]
    [S.J. Brodsky, F. Hautmann and D.E. Soper (1997)]
    [J. Kwiecinski and L. Motyka (1999), (2000)]
    [M. Boonekamp et al. (1999)]
    [J. Bartels, C. Ewerz and R. Staritzbichler (2000)]
    [N.N. Nikolaev, J. Speth and V.R. Zoller (2001), (2002)]
  - Next-to-leading log BFKL [S.J. Brodsky et al. (1999), (2002)]

Outline

The BFKL approach

Forward scattering $\gamma^* + \gamma^*$ for $s \to \infty$

- BFKL approach: convolution of the Green's function of two interacting Reggeized gluons and of the impact factors for the $\gamma^* \to \gamma^*$ transition



- Valid both in
  LLA (resummation of all terms $(\alpha_s \ln s)^n$)
  NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$)

- The Green's function is determined through the BFKL equation
  [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- The forward kernel of the BFKL equation is completely known in the NLA
  [V.S. Fadin, L.N. Lipatov (1998)] [G. Camici, M. Ciafaloni (1998)]

- The calculation of the $\gamma^* \to \gamma^*$ impact factors has been completed
  [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
  [J. Bartels et al. (2001-2004)]

Outline

Build-up of the amplitude

$$\mathcal{I}m_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^2}\int\frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\Phi_1(\vec{q}_1, s_0)\int\frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\Phi_2(-\vec{q}_2, s_0)\int\limits_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}G_\omega(\vec{q}_1, \vec{q}_2)$$

BFKL equation: $\delta^2(\vec{q}_1 - \vec{q}_2) = \omega\, G_\omega(\vec{q}_1, \vec{q}_2) - \int d^2\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_2)$

With operator notation in the transverse momentum space:

$$\hat{1} = (\omega - \hat{K})\hat{G}_\omega \qquad \longrightarrow \qquad \hat{G}_\omega = (\omega - \hat{K})^{-1}$$

$$\hat{K} = \bar{\alpha}_s \hat{K}^0 + \bar{\alpha}_s^2 \hat{K}^1 \ , \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

LO eigenfunctions of the LLA kernel: $\{|\nu\rangle\}$

$$\hat{K}^0|\nu\rangle = \chi(\nu)|\nu\rangle \ , \qquad \chi(\nu) = 2\psi(1) - \psi\left(\frac{1}{2} + i\nu\right) - \psi\left(\frac{1}{2} - i\nu\right)$$

$$\langle\vec{q}|\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu - \frac{1}{2}} \ , \qquad \langle\nu'|\nu\rangle = \int\frac{d^2\vec{q}}{2\pi^2}\left(\vec{q}^{\,2}\right)^{i\nu - i\nu' - 1} = \delta(\nu - \nu')$$

$\sigma_{\gamma^*\gamma^*}$ in leading log BFKL

$$\sigma_{\gamma^*\gamma^*}(s, Q_1, Q_2) = \sum_{i,k=T,L} \frac{1}{(2\pi)^2 Q_1 Q_2} \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2}\right)^{i\nu} F_i(\nu) F_k(-\nu) \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s \chi(\nu)}$$

$$F_T(\nu) = F_T(-\nu) = \alpha\, \alpha_s \left(\sum_q e_q^2\right) \frac{\pi}{2} \frac{\left(\frac{3}{2} - i\nu\right)\left(\frac{3}{2} + i\nu\right) \Gamma\left(\frac{1}{2} - i\nu\right)^2 \Gamma\left(\frac{1}{2} + i\nu\right)^2}{\Gamma(2 - i\nu)\Gamma(2 + i\nu)}$$

$$F_L(\nu) = F_L(-\nu) = \alpha\, \alpha_s \left(\sum_q e_q^2\right) \pi \frac{\Gamma\left(\frac{3}{2} - i\nu\right)\Gamma\left(\frac{3}{2} + i\nu\right)\Gamma(\frac{1}{2} - i\nu)\Gamma(\frac{1}{2} + i\nu)}{\Gamma(2 - i\nu)\Gamma(2 + i\nu)}$$

Next-to-leading log BFKL Green's function

$$\hat{G}_\omega = (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} \left(\bar{\alpha}_s^2 \hat{K}^1\right) (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2 \hat{K}^1\right)^2\right]$$

Action of the full NLA kernel on the LLA eigenfunctions:

$$\begin{aligned}\hat{K}|\nu\rangle &= \bar{\alpha}_s(\mu_R)\chi(\nu)|\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(\nu) + \frac{\beta_0}{4N_c}\chi(\nu)\ln(\mu_R^2)\right)|\nu\rangle \\ &+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(\nu)\left(i\frac{\partial}{\partial\nu}\right)|\nu\rangle\,, \qquad \beta_0 = \frac{11\,N_c}{3} - \frac{2n_f}{3}\end{aligned}$$

$$\chi^{(1)}(\nu) = -\frac{\beta_0}{8\,N_c}\left(\chi^2(\nu) - \frac{10}{3}\chi(\nu) - i\chi'(\nu)\right) + \bar{\chi}(\nu)$$

$$\begin{aligned}\bar{\chi}(\nu) &= -\frac{1}{4}\left[\frac{\pi^2-4}{3}\chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)}\right. \\ &+ \left.\frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3 + \left(1 + \frac{n_f}{N_c^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right) + 4\phi(\nu)\right]\end{aligned}$$

$$\phi(\nu) = 2\int_0^1 dx\,\frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right]$$

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&+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(\nu)\left(i\frac{\partial}{\partial\nu}\right)|\nu\rangle\,, \qquad \beta_0 = \frac{11\,N_c}{3} - \frac{2n_f}{3}
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$\sigma_{\gamma^*\gamma^*}$ with next-to-leading log BFKL Green's function

$$\sigma_{\gamma^*\gamma^*} = \frac{1}{(2\pi)^2 Q_1 Q_2} \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2}\right)^{i\nu} \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)} \sum_{i,k=T,L} F_i(\nu) F_k(-\nu)$$
$$\times \left\{1 + \bar{\alpha}_s^2(\mu_R) \ln\left(\frac{s}{s_0}\right)\left[\bar{\chi}(\nu) + \frac{\beta_0}{8N_c}\chi(\nu)\left(-\chi(\nu) + \frac{10}{3} + 2\ln\frac{\mu_R^2}{Q_1 Q_2}\right)\right]\right\}$$

Invariance under renormalization group and under change of the energy scale s_0 in the NLA $\longrightarrow$

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$$\times \left\{1 + \bar{\alpha}_s(\mu_R) A(s_0) + \bar{\alpha}_s(\mu_R) B(\mu_R)\right.$$
$$\left. + \bar{\alpha}_s^2(\mu_R) \ln\left(\frac{s}{s_0}\right)\left[\bar{\chi}(\nu) + \frac{\beta_0}{8N_c}\chi(\nu)\left(-\chi(\nu) + \frac{10}{3} + 2\ln\frac{\mu_R^2}{Q_1 Q_2}\right)\right]\right\}$$

$$A(s_0) = \chi(\nu)\ln\frac{s_0}{Q_1 Q_2} \qquad B(\mu_R) = \frac{\beta_0}{2N_c}\ln\frac{\mu_R^2}{Q_1 Q_2}$$

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$$\times \left\{1 + \bar{\alpha}_s(\mu_R) A(s_0) + \bar{\alpha}_s(\mu_R) B(\mu_R)\right.$$
$$\left. + \bar{\alpha}_s^2(\mu_R) \ln\left(\frac{s}{s_0}\right)\left[\bar{\chi}(\nu) + \frac{\beta_0}{8N_c}\chi(\nu)\left(-\chi(\nu) + \frac{10}{3} + 2\ln\frac{\mu_R^2}{Q_1 Q_2}\right)\right]\right\}$$

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Outline

Series representation

$$Q_1 Q_2 \, \sigma_{\gamma^* \gamma^*} = \frac{1}{(2\pi)^2} \left\{ b_0 + \sum_{n=1}^{\infty} \bar{\alpha}_s(\mu_R)^n \, b_n \left[\ln^n \left(\frac{s}{s_0} \right) + d_n(s_0, \mu_R) \ln^{n-1} \left(\frac{s}{s_0} \right) \right] \right\}$$

$$b_n = \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \sum_{i,k=T,L} F_i(\nu) F_k(-\nu) \, \frac{\chi^n(\nu)}{n!}$$

$$d_n = n \ln \frac{s_0}{Q_1 Q_2} + \frac{\beta_0}{4N_c} \left[\frac{b_{n-1}}{b_n} \left((n+1) \ln \frac{\mu_R^2}{Q_1 Q_2} + \frac{5}{3}(n-1) \right) - \frac{n(n-1)}{2} \right]$$
$$+ \frac{1}{b_n} \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \sum_{i,k=T,L} F_i(\nu) F_k(-\nu) \, \frac{\chi^{n-2}(\nu)}{(n-2)!} \, \bar{\chi}(\nu)$$

Series representation

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$$\left. + \frac{1}{b_n} \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \sum_{i,k=T,L} F_i(\nu) F_k(-\nu) \, \frac{\chi^{n-2}(\nu)}{(n-2)!} \, \bar{\chi}(\nu) \right]$$

Series representation

$$Q_1 Q_2\, \sigma_{\gamma^*\gamma^*} = \frac{1}{(2\pi)^2} \left\{ b_0 + \sum_{n=1}^{\infty} \bar{\alpha}_s(\mu_R)^n\, b_n \left[\ln^n \left(\frac{s}{s_0} \right) + d_n(s_0, \mu_R) \ln^{n-1} \left(\frac{s}{s_0} \right) \right] \right\}$$

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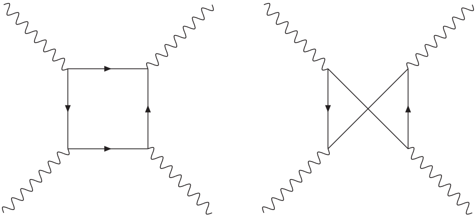
Inclusion of low-energy terms



$$\sigma^{\gamma^*\gamma^*}_{QBOX}(s, Q_1, Q_2) = \sum_q e_q^4 \left(\sigma_{TT} + 2\sigma_{TS} + \sigma_{SS} \right)$$

[V.M. Budnev et al. (1974)]

$$\sigma^{\gamma^*\gamma^*}_{QBOX}(s, Q_1, Q_2) \sim \alpha_s^2 (\ln s)/s$$

Numerical analysis - The “pure” BFKL regime

$$Q_1 = Q_2 \equiv Q \qquad \text{“pure” BFKL regime}$$

Series representation

LLA: b_n coefficients (Q-independent)

$$\begin{array}{lllll} b_0 = 875.90 & b_1 = 1977.90 & b_2 = 2400.76 & b_3 = 1997.37 & b_4 = 1270.78 \\ & b_5 = 654.99 & b_6 = 284.05 & b_7 = 106.34 & b_8 = 35.04 \end{array}$$

NLA: $d_n(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, $n_f = 4$)

$$\begin{array}{llll} d_1 = 0. & d_2 = -4.24 & d_3 = -13.16 & d_4 = -26.78 \\ d_5 = -45.15 & d_6 = -68.28 & d_7 = -96.20 & d_8 = -128.91 \end{array}$$

Large NLA corrections!

d_n coefficients negative and increasingly large in absolute value.

Optimization of the perturbative expansion needed!

PMS method

- Principle of minimal sensitivity (PMS) [P.M. Stevenson (1981)]: require the minimal sensitivity to the change of both s_0 and μ_R.

- Strategy: for each fixed s calculate the amplitude for varying s_0 and μ_R, up to finding the optimal values for which the amplitude is least sensitive to variations of them.

- In practice, there are wide regions in s_0 and μ_R where the amplitude is very weekly dependent on s_0 and μ_R; the stationary point in the (s_0, μ_R)-plane is typically a (local) maximum.

BLM method

- [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)] optimization method: perform a finite renormalization to a physical scheme and then choose the renormalization scale in order to remove the β_0-dependent part.

Strategy:

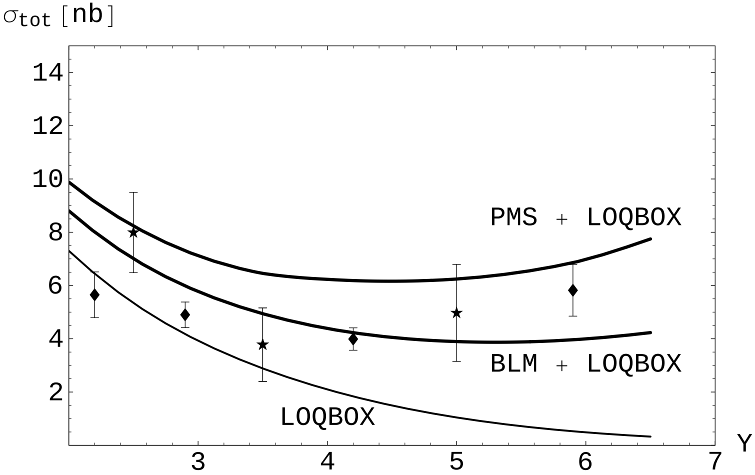
- finite renormalization to the MOM-scheme (ξ=0)

$$\alpha_S \to \alpha_S \left[1 + T_{MOM}(\xi = 0)\frac{\alpha_S}{\pi}\right]$$

$$T_{MOM}(\xi = 0) = T^{conf}_{MOM} + T^{\beta}_{MOM}$$

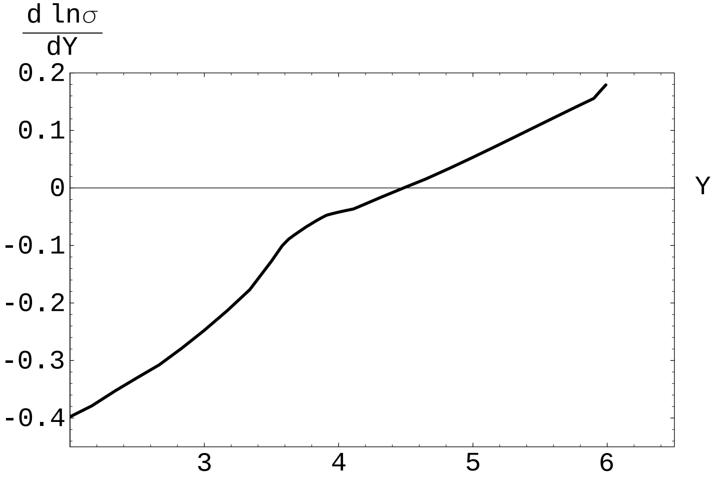
$$T^{conf}_{MOM} = \frac{N_C}{8}\frac{17}{2}I \qquad T^{\beta}_{MOM} = -\frac{\beta_0}{2}\left[1 + \frac{2}{3}I\right] \qquad I \simeq 2.3439$$

- s_0 and μ_R chosen in order to make the term proportional to β_0 in the resulting amplitude vanish (the β_0-dependence in the series representation of the amplitude is hidden into the d_n coefficients)
- optimal values for s_0 and μ_R determined according to “minimum sensitivity”



$$Q^2 = 17\text{GeV}^2 \quad (n_f = 4)\,, \qquad Y \equiv \ln\left(\frac{s}{Q^2}\right)$$

Experimental data: OPAL (stars, Q^2=18 GeV^2), L3 (diamonds, Q^2=16 GeV^2)



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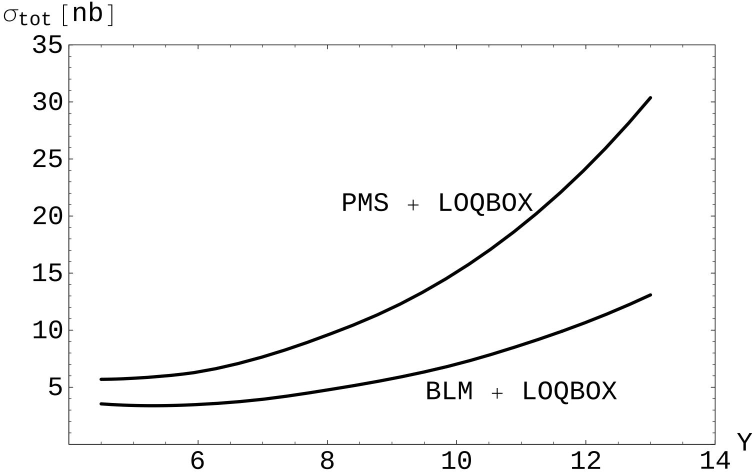

$Q^2 = 20\mathrm{GeV}^2 \;\; (n_f = 5)\,, \qquad Y \equiv \ln\left(\frac{s}{Q^2}\right)$

Discussion and conclusions

The approximation of using LO impact factors + NLO BFKL Green's function is not new:

- $\gamma^*\gamma^* \to VV$, with V a light vector meson [Enberg et al. (2005)]
- $\gamma^*\gamma^*$ total cross section [Brodsky et al. (2002)]

Novelties in our approach w.r.t. the latter:

- optimization procedures performed on the amplitude itself and not on the NLO Pomeron intercept
- LO impact factors with "mandatory" NLO terms
- two optimization methods used to have a control of systematic effects

The reason for being PMS systematic higher than BLM could be that the stationary points in the space of parameters (s_0, μ_R) turn to be always maxima.

Looking forward to $\gamma^*\gamma^*$ collisions in NLO BFKL
(instead of NLO BFKL in $\gamma^*\gamma^*$ collisions!)

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