Geometric Scaling

David Šálek
Charles University, Prague

in collaboration with
G. Beuf, R. Peschanski, Ch. Royon
arXiv: 0803.2186

Low-x Workshop
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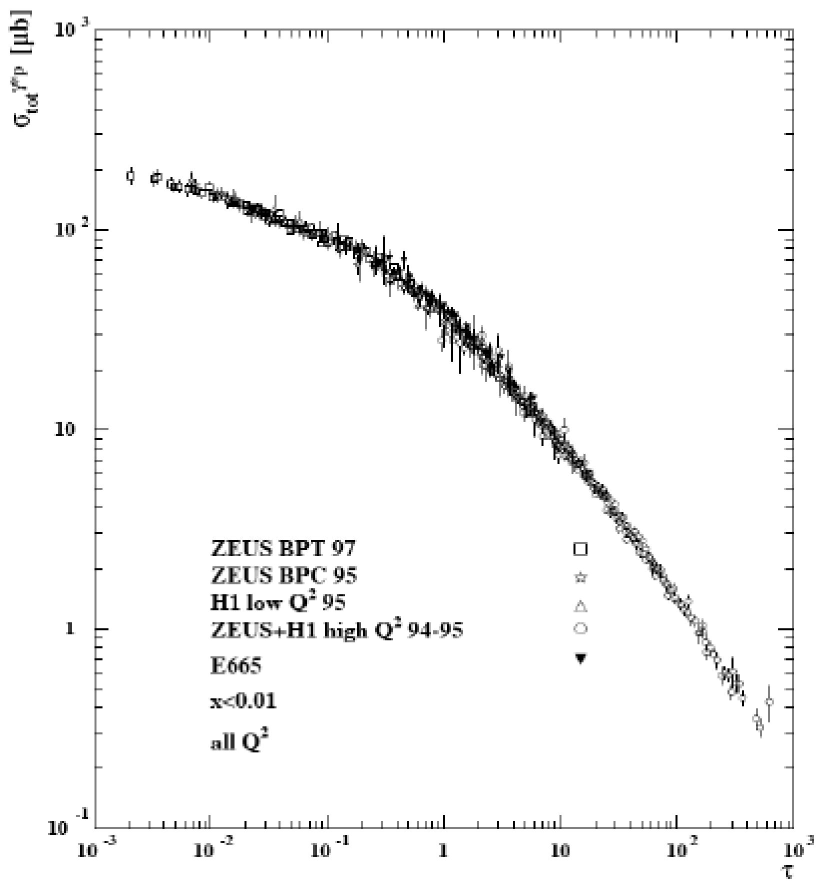
First Observation

$$\tau = Q^2 (x / x_0)^\lambda$$

- $\sigma(\gamma^* p)$ as a function of τ
- *A. M. Stasto, K. Golec-Biernat, J. Kwiecinski, Phys. Rev. Let. 86 (2001) 596*



Quality Factor Method

- compare the data $\sigma = \sigma(Q^2, x)$ and the scaling laws $\tau = \tau(Q^2, x; \lambda)$ using the quality factor method:
  - normalise data $v_i = \log(\sigma)$ and scalings $u_i = \tau(\lambda)$ between 0 and 1
  - order in u_i
  - define the Quality Factor: (ε in case two data points have the same Q^2 and x)
  - fit λ to maximise the QF

$$QF(\lambda) = \left[\sum_i \frac{(v_i - v_{i-1})^2}{(u_i - u_{i-1})^2 + \epsilon^2} \right]^{-1}$$

- F. Gelis, R. Peschanski, L. Schoeffel, G. Soyez, arXiv: hep-ph/0610435

Scaling Laws

- fixed coupling
- running coupling I
- running coupling II
- running coupling II extended
  - more parameters fitted (λ, Y_0, Λ_{QCD})
- diffusive scaling

$$\tau = \log Q^2 - \lambda Y$$

$$\tau = \log Q^2 - \lambda \sqrt{Y}$$

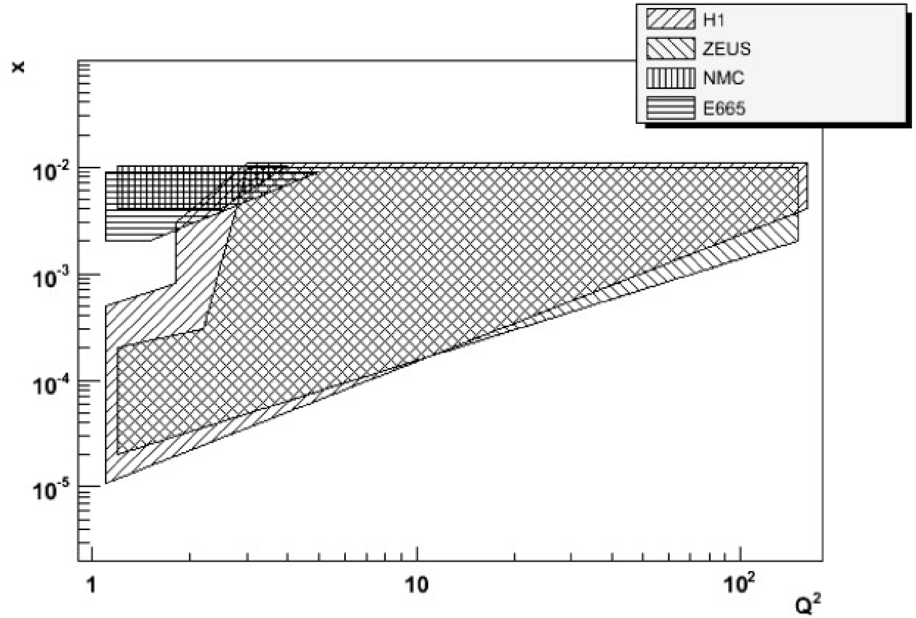
$$\tau = \log(Q^2/\Lambda_{QCD}) - \lambda \frac{Y}{\log(Q^2/\Lambda_{QCD})}$$

$$\tau = \log(Q^2/\Lambda_{QCD}) - \lambda \frac{Y - Y_0}{\log(Q^2/\Lambda_{QCD})}$$

$$\tau = \frac{\log Q^2 - \lambda Y}{\sqrt{Y}}$$

F_2 Data Sets

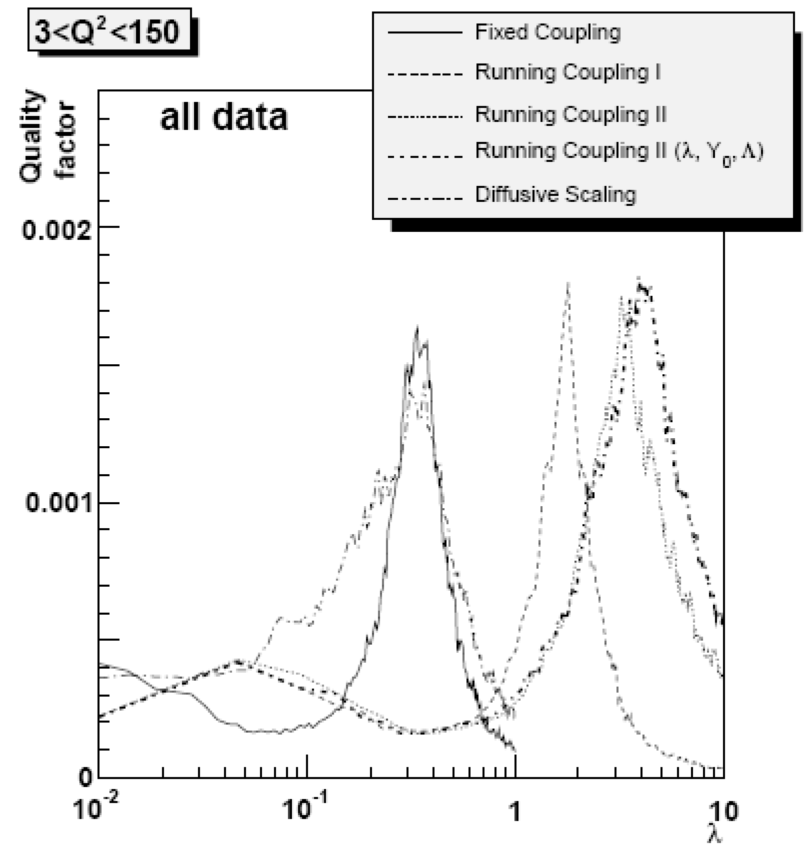
- data from H1, ZEUS, NMC, E665
- $3 < Q^2 < 150$, $x < 10^{-2}$
  - stay in perturbative region
  - avoid photoproduction
  - region where gluons dominate
  - 217 data points
- $1 < Q^2$
  - try to go to lower Q^2 (saturation region)
  - 308 data points



Results

- running coupling II extended favoured
- diffusive scaling disfavoured

scaling	parameter	value	QF
FC	λ	0.33	1.63
RC I	λ^2	3.39	1.62
	(λ	1.84)	
RC II	λ	3.44	1.69
RC II ext.	λ	3.90	1.82
	Y_0	-1.2	
	Λ_{QCD}	0.30	
DS	λ	0.36	1.44



non-normalised QF = QF(λ):
compare different scaling laws on one data set

Family of Scalings

- λ parameter fit results in running coupling I and running coupling II not similar just by chance

$$\tau = L - \frac{(\lambda Y)^{\delta}}{L^{2\delta-1}} \qquad L = \log(Q^2/\Lambda)$$

- $\delta = ½$ ⟶ running coupling I $\qquad \tau = \log Q^2 - \lambda\sqrt{Y}$

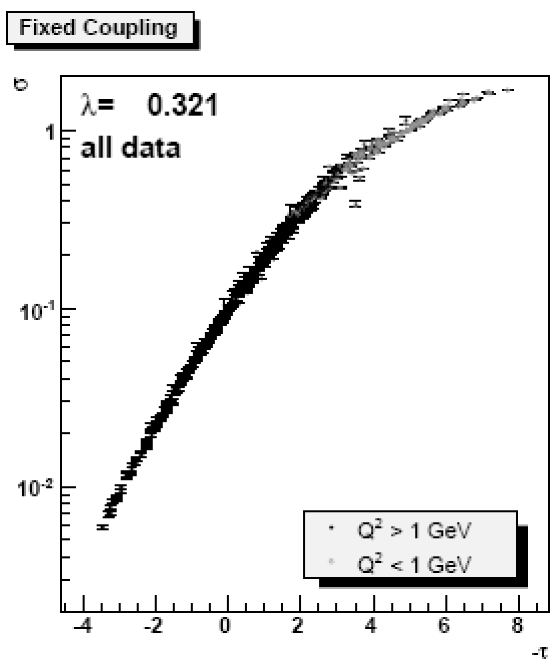
- $\delta = 1$ ⟶ running coupling II $\qquad \tau = \log(Q^2/\Lambda_{QCD}) - \lambda \frac{Y}{\log(Q^2/\Lambda_{QCD})}$

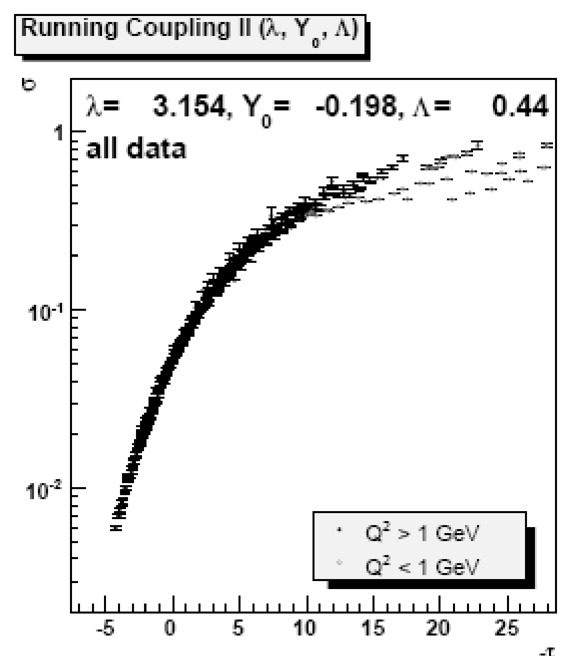
- we get similar λ whatever the δ parameter is

Low Q^2 Data

- fits to $Q^2 > 1$ data (308 points) and fits to $Q^2 > 3$ data (217 points) give similar results
- do the low Q^2 data points ($Q^2 < 1$) satisfy scaling?
- scaling curves plotted using the parameters obtained in the fit to $Q^2 > 1$ data

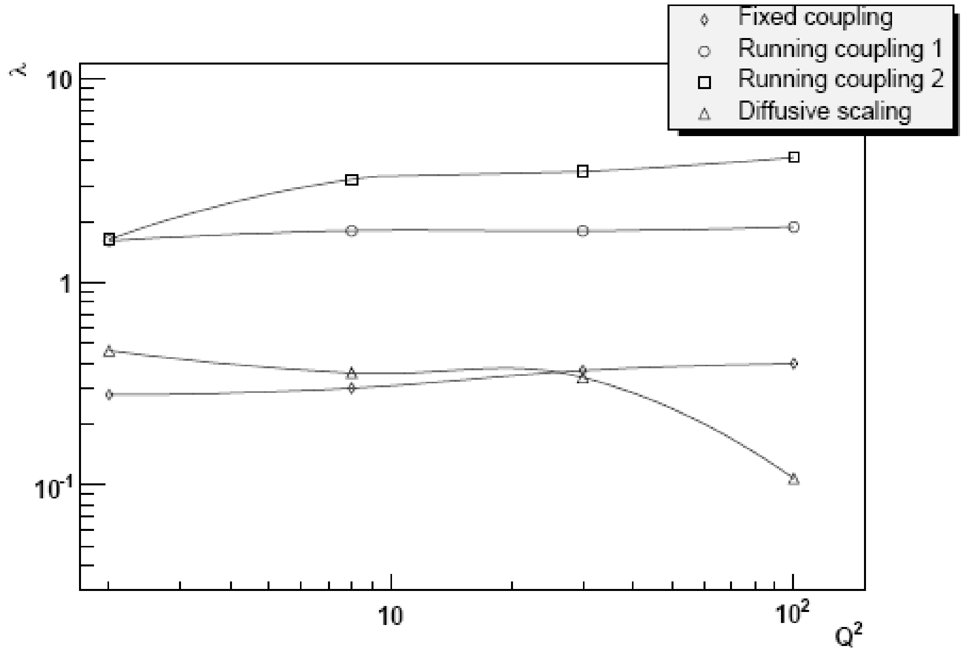
scaling curves





Q^2 Dependence of λ

- λ fitted in four different Q^2 bins: [1, 3], [3, 10], [3, 35], [35, 150] (similar numbers of data points)
- diffusive scaling not stable
- fixed coupling changing more than running coupling I (because it does not depend on Q^2)
- running coupling II not very good at low Q^2 (non-perturbative effects)

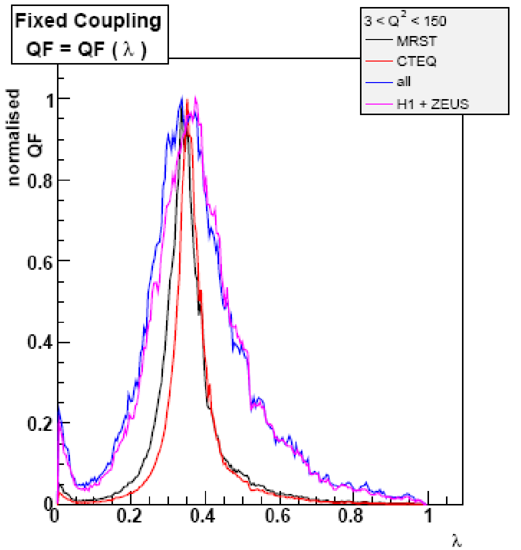


CTEQ, MRST and GRV98 Parametrisations

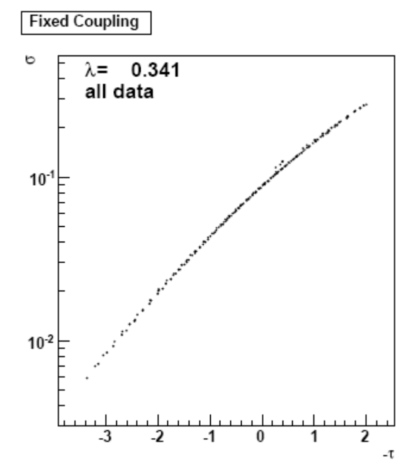
- F_2 from CTEQ, MRST and GRV98 parametrisation tested
- the same x, Q^2 values as $Q^2 > 3$ data (217 points)
- smooth scaling curves
- similar values of λ as in the data
- CTEQ parametrisation gives higher QF than MRST
- GRV98 gives the smallest QF
- fixed coupling is favoured

	data	CTEQ	MRST	GRV98
λ	0.33	0.36	0.34	0.33
QF	1.6	16.3	10.4	5.5

- DGLAP shows scaling but it's not naturally explained (saturation explains the scaling naturally)



normalised QF = QF(λ) plot: compare one scaling law on different data sets



Different CTEQ and MRST Versions

- CTEQ (fixed coupling)

	65 M 300	66 C1 450	66 C2 451	66 C3 452	66 C4 453	66 M4 400
λ	0.36	0.36	0.36	0.33	0.33	0.36
QF	14.3	12.3	12.1	11.7	13.1	12.4

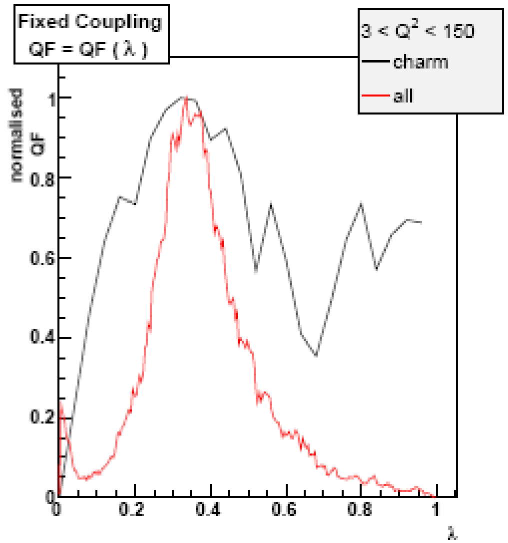
- MRST (fixed coupling)

	2002 NLO	2002 NNLO	2004 NLO	2004 NNLO		data
λ	0.34	0.34	0.34	0.34		0.33
QF	10.4	11.0	11.3	11.2		1.6

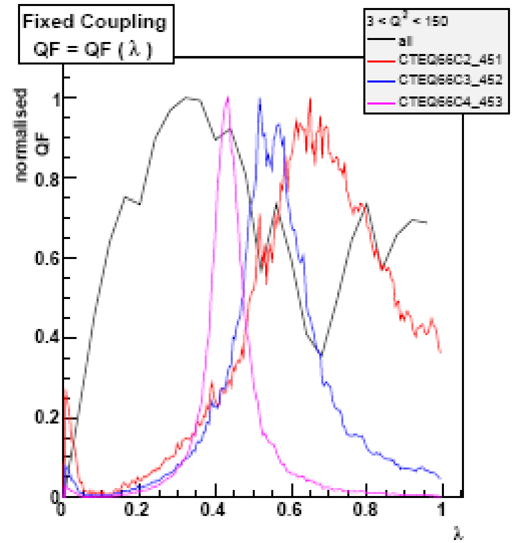
- all the CTEQ and MRST versions give similar results
- CTEQ 66 C4 453 is slighlty favoured

Charm Data

- data from H1, ZEUS, NMC
- 25 data points
- $Q^2 > 3$ (to be away from charm mass effects)
- charm data fit results similar to F_2 data fits

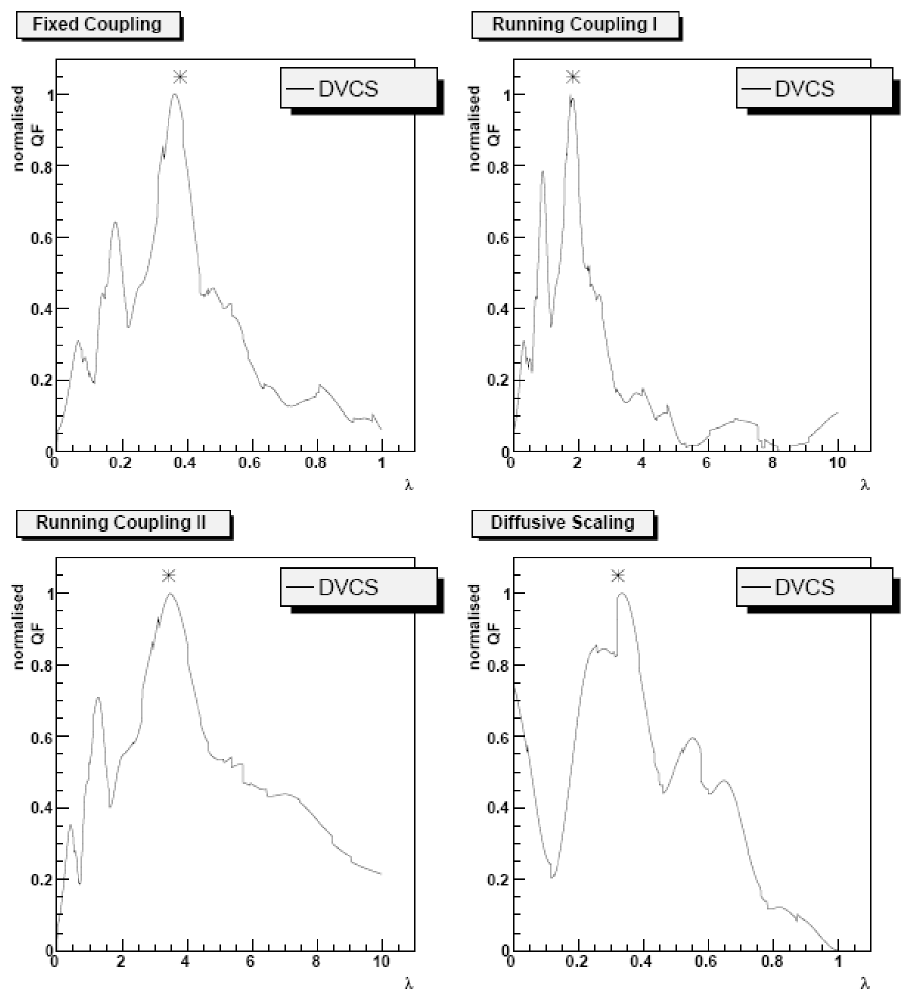


- MRST cannot be fitted
  - fits give too large λ
  - scaling reappears at higher Q^2
- CTEQ 66 C4 453 works the best
  - λ close to data, fits Q^2 stable
- other CTEQ describes the data better than MRST
  - λ values closer to data
  - smooth scaling curves only at higher Q^2



HERA DVCS Data

- 34 data points
- fit results similar to the F_2 fit results (stars)



$$\sigma_{DVCS}^{\gamma^* p \to \gamma p}(x, Q^2) = \sigma_{DVCS}^{\gamma^* p \to \gamma p}(\tau[x, Q^2])$$

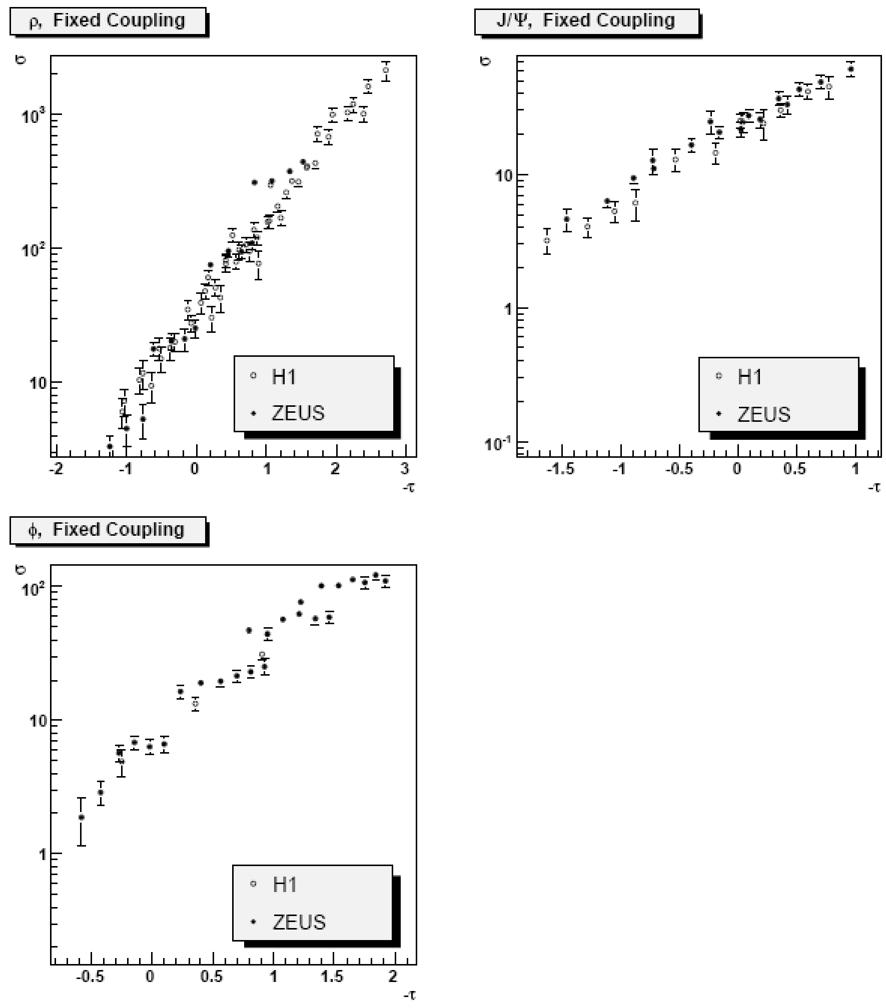
Other HERA Data Sets

- vector meson data, diffractive data
- not precise enough to perform fits
- use the values obtained in fits to H1 + ZEUS F_2 data and see whether the scaling works
- vector meson data: replace Q^2 by $Q^2 + M_V^2$, where M_V is the mass of the vector meson
- diffractive data: use $\beta\sigma_{diff}$ and the same definition of τ replacing x by x_{IP}

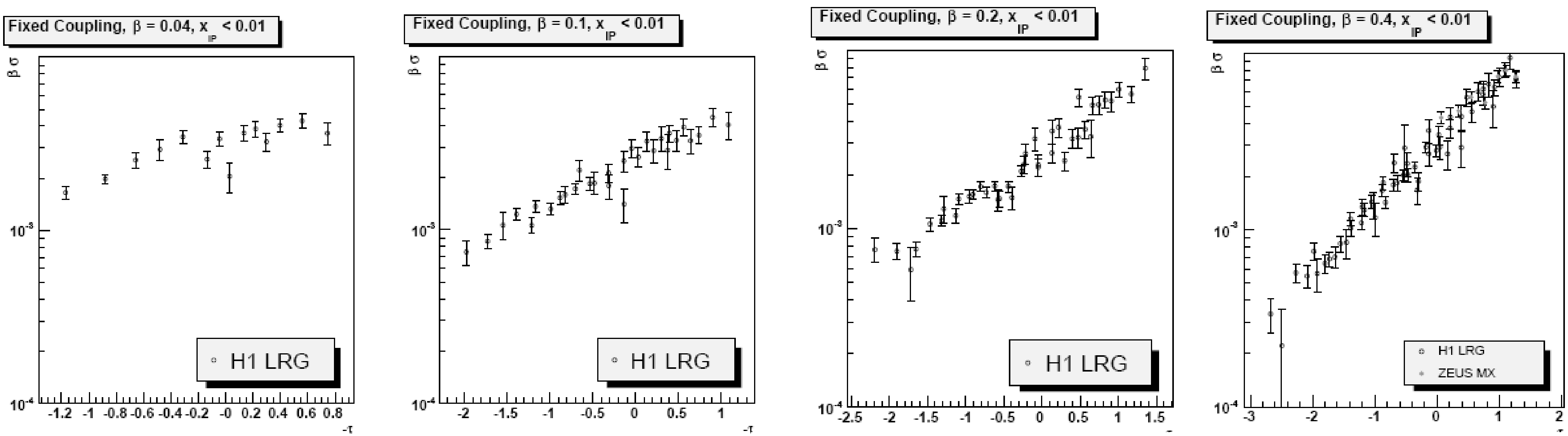
- C. Marquet, L. Schoeffel, arXiv: hep-ph/0606079

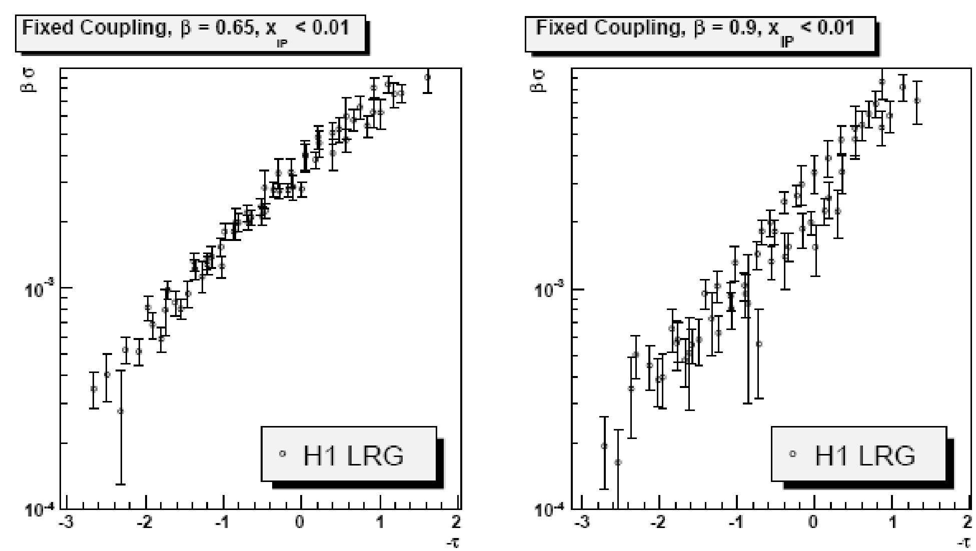
HERA Vector Meson Data

$$\sigma_{VM}^{\gamma^* p \to V p}(x_{I\!P}, Q^2, M_V^2) = \sigma_{VM}^{\gamma^* p \to V p}(\tau_V[x_{I\!P}, Q^2 + M_V^2])$$



HERA Diffractive Data





$$\frac{d\sigma_{diff}^{\gamma^* p \to X p}}{d\beta}(\beta, x_{I\!P}, Q^2) = \frac{d\sigma_{diff}^{\gamma^* p \to X p}}{d\beta}(\beta, \tau_d[x_{I\!P}, Q^2])$$

Summary

- different scalings studied on F_2 and DVCS data
  - fixed coupling, running coupling I and running coupling II lead to a good description of data
  - running coupling I and running coupling II fall into a more general family of scalings
  - diffusive scaling disfavoured
- MRST and CTEQ parametrisations lead to similar results as data
  - CTEQ gives higher QF
- F_2 charm studied
  - similar results as F_2 data
  - MRST and CTEQ parametrisations give larger values of λ
  - MRST shows scaling only at higher Q^2
- diffractive and vector meson data show scaling as well (using the values of λ obtained in F_2 studies)

Outlook

- study the geometric scaling in DGLAP
- try to obtain a parametrisation to fit the data based on different scalings (numerical solution of BK equation with running α_s)