Saturation and phase transitions in DIS and in hadron collisions

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"Saturation" in different reactions (models); the S-matrix theory:

1) DIS – from individual parton interactions to collective effects;

2) hh (AA) – will be the "black disc limit" saturated at the LHC?

3) EOS and phase transitions from the scattering amplitude (S-matrix) = data

a) Lepton-hadron inclusive (DIS) and excluive (e.g. DVCS);

b) From DIS SF (F_2(x,Q^2)~ImA(s,t=0,Q^2) to the scattering amplitude A(s,t,Q^2);

c) From γ*p→ γ*p to Vp→Vp (or hh—hh. AA);

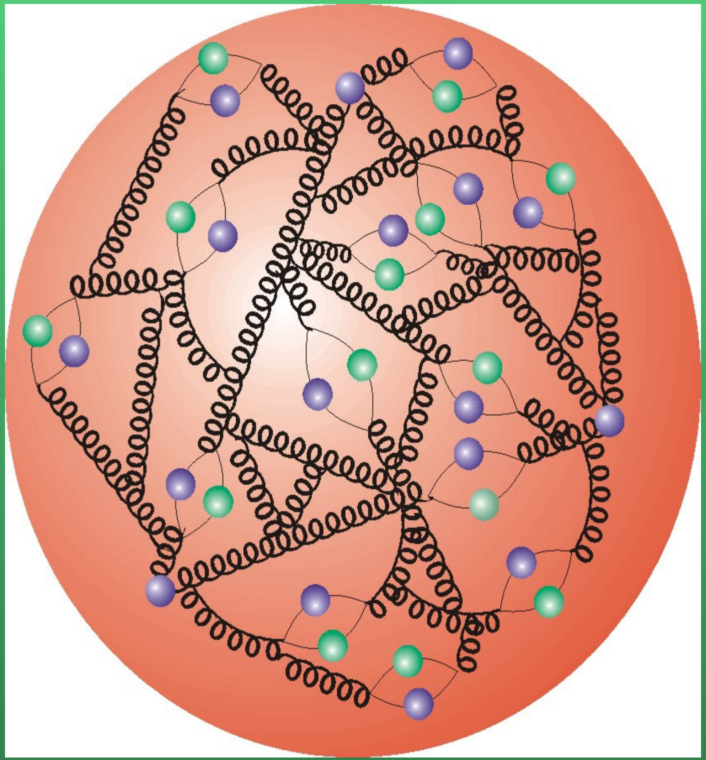
d) Thermodymics (EoS, phase transicions) from the S-matrix (scattering amplitude)

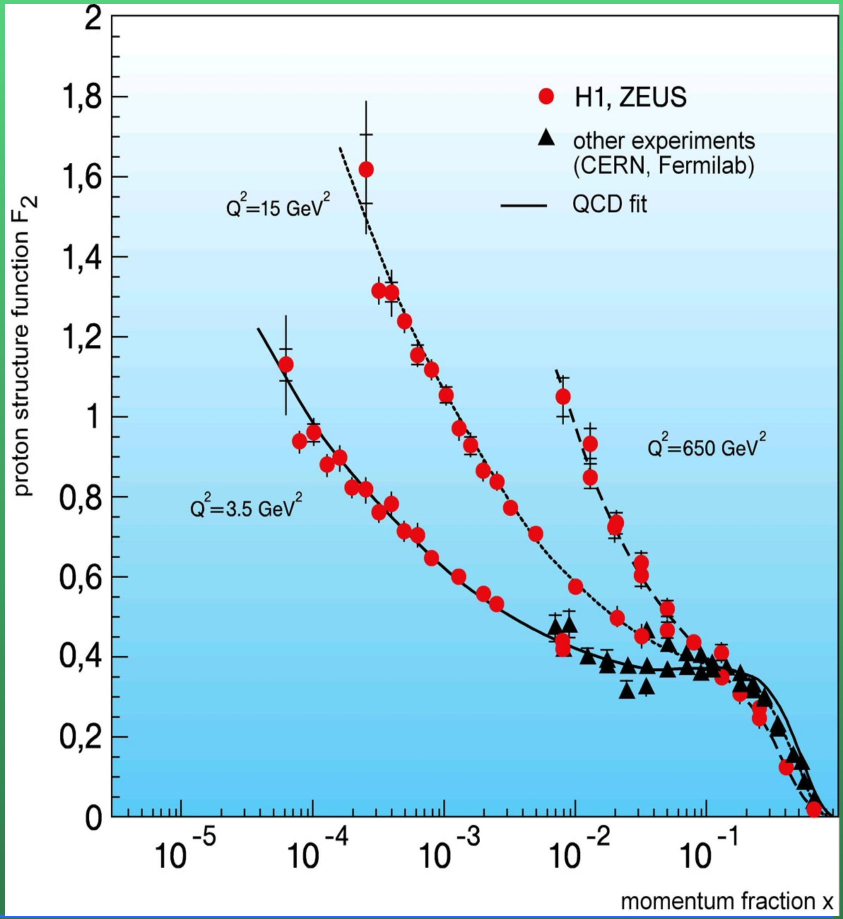
Structure of the Proton

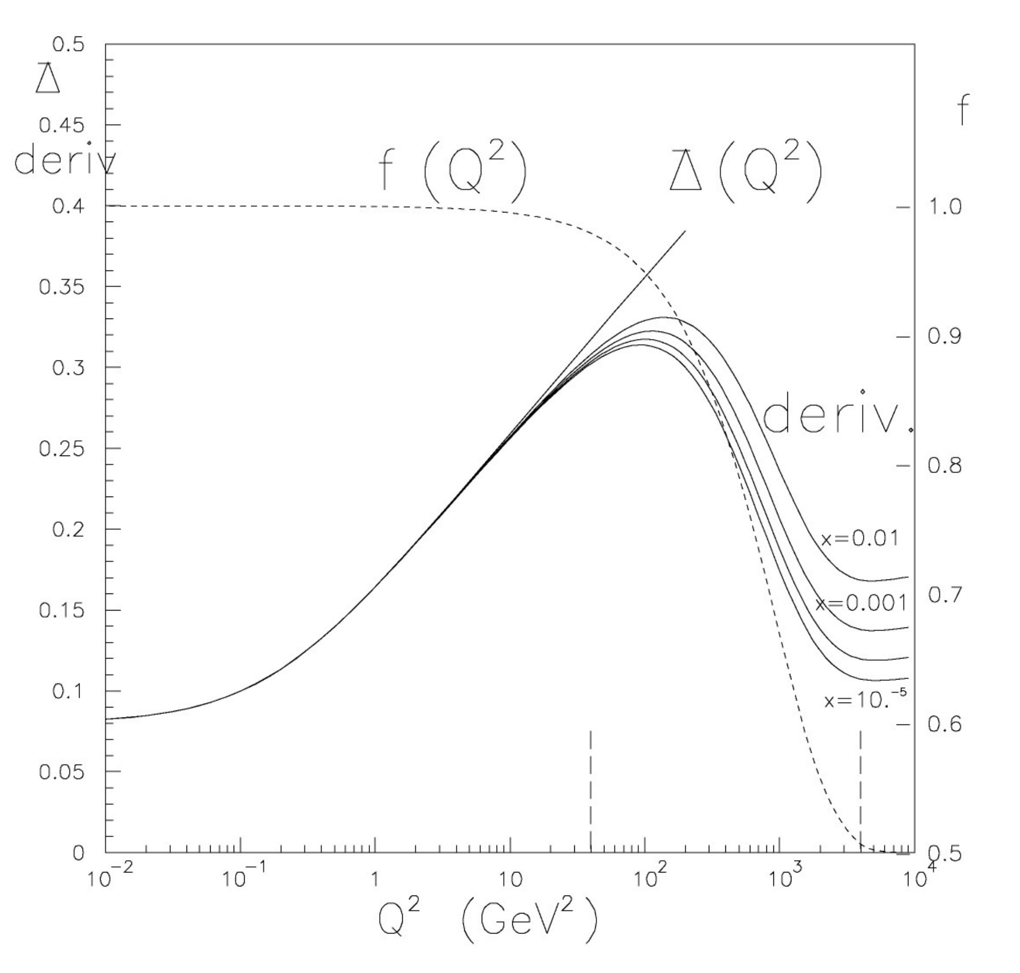
3 "valence quarks"

+

"sea" of gluons and short lived $q\bar{q}$ pairs







Δ
deriv
f
f (Q²)
Δ (Q²)
deriv.
x=0.01
x=0.001
x=10.⁻⁵
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1.0
0.9
0.8
0.7
0.6
0.5
10⁻²
10⁻¹
1
10
10²
10³
10⁴
Q² (GeV²)

$$F_2^{(S,0)}(x,Q^2) = A\left(\frac{Q^2}{Q^2+a}\right)^{1+\widetilde{\Delta}(Q^2)} e^{\Delta(x,Q^2)},$$

with the "effective power"

$$\widetilde{\Delta}(Q^2) = \epsilon + \gamma_1 \ell n\left(1+\gamma_2 \ell n\left[1+\frac{Q^2}{Q_0^2}\right]\right),$$

and

$$\Delta(x,Q^2) = \left(\widetilde{\Delta}(Q^2)\ell n\frac{x_0}{x}\right)^{f(Q^2)},$$

where

$$f(Q^2) = \frac{1}{2}\left(1+e^{-Q^2/Q_1^2}\right).$$

a) Large Q^2, fixed x:

$$F_2^{(S,0)}(x, Q^2 \to \infty) \to A \;\; \exp\sqrt{\gamma_1 \ell n \ell n \frac{Q^2}{Q_0^2} \;\; \ell n \frac{x_0}{x}} \;\; ,$$

(asymptotic solution of the GLAP evolution equation).

b) Low Q^2, fixed x:

$$F_2^{(S,0)}(x, Q^2 \to 0) \to A \; e^{\Delta(x, Q^2 \to 0)} \;\; \left(\frac{Q^2}{a}\right)^{1+\widetilde{\Delta}(Q^2 \to 0)}$$

with

$$\widetilde{\Delta}(Q^2 \to 0) \to \epsilon + \gamma_1 \gamma_2 \left(\frac{Q^2}{Q_0^2}\right) \;\; \to \;\; \epsilon,$$

$$f(Q^2 \to 0) \to 1,$$

whence

$$F_2^{(S,0)}(x, Q^2 \to 0) \to A \;\; \left(\frac{x_0}{x}\right)^{\epsilon} \;\; \left(\frac{Q^2}{a}\right)^{1+\epsilon} \;\; \propto (Q^2)^{1+\epsilon} \;\; \to 0 \; ,$$

as required by gauge invariance.

c) Low x, fixed Q^2:

$$F_2^{(S,0)}(x \to 0, Q^2) = A \left(\frac{Q^2}{Q^2 + a} \right)^{1+\widetilde{\Delta}(Q^2)} e^{\Delta(x \to 0, Q^2)}.$$

If

$$f(Q^2) \sim 1 \ ,$$

i.e. when $Q^2 \ll Q_1^2$, we get the standard (Pomeron-dominated) Regge behavior (with a Q^2 dependence in the effective Pomeron intercept)

$$F_2^{(S,0)}(x \to 0, Q^2) \to A \left(\frac{Q^2}{Q^2 + a} \right)^{1+\widetilde{\Delta}(Q^2)} \left(\frac{x_0}{x} \right)^{\widetilde{\Delta}(Q^2)} \propto x^{-\widetilde{\Delta}(Q^2)}.$$

SF for both low- and high x, S+NS:

$$F_2(x,Q^2) = F_2^{(S)}(x,Q^2) + F_2^{(NS)}(x,Q^2)$$

where

$$F_2^{(S)}(x,Q^2) = F_2^{(S,0)}(x,Q^2)\ (1-x)^{n(Q^2)},$$

with

$$n(Q^2) = \frac{3}{2}\left(1 + \frac{Q^2}{Q^2+c}\right),$$

$c = 3.5489$ GeV^2.

$$F_2^{(NS)}(x,Q^2) = B\ (1-x)^{n(Q^2)}\ x^{1-\alpha_r}\ \left(\frac{Q^2}{Q^2+b}\right)^{\alpha_r}.$$

Slopes :

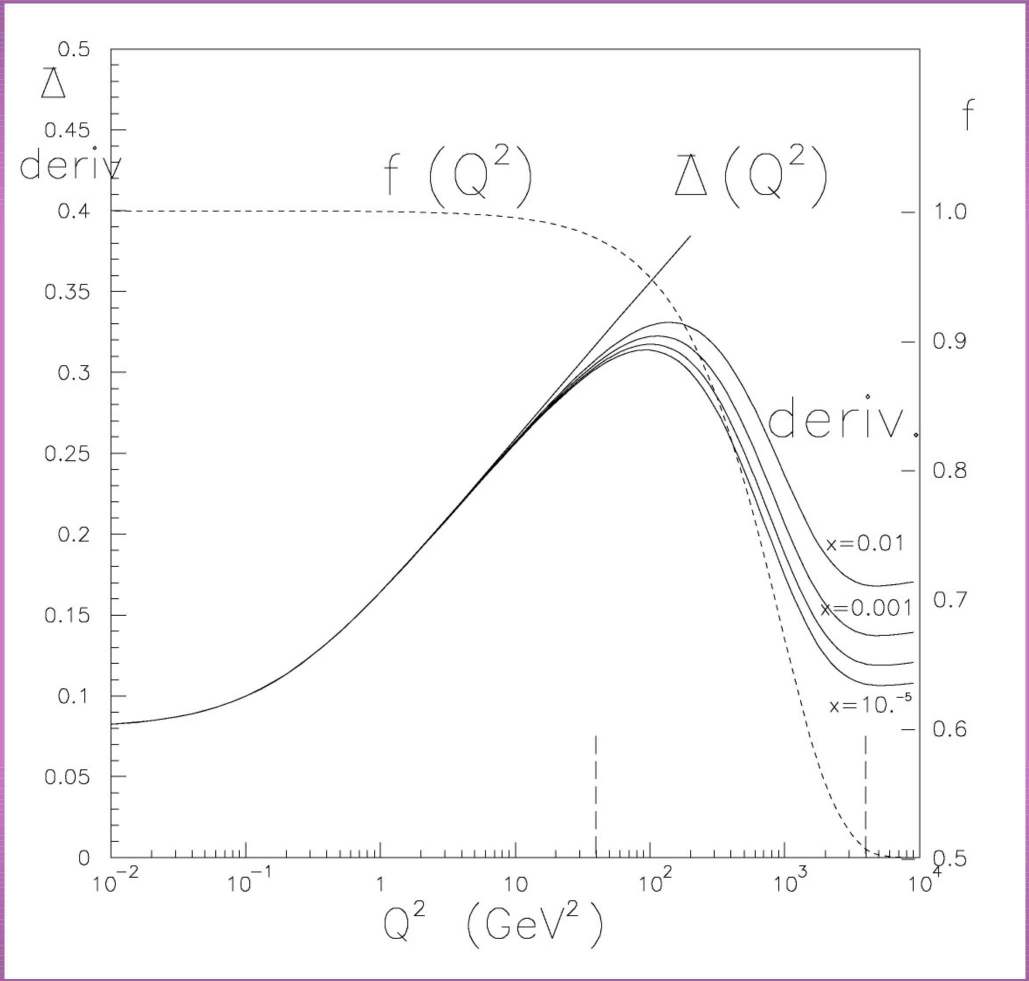
$$\frac{\partial F_2}{\partial(\ell n Q^2)}$$

as a function of x and Q^2;

$$\frac{\partial \ell n F_2}{\partial(\ell n(1/x))}$$

as a function of Q^2 for fixed x values.

(P. Desgrolard, L. Jenkovszky, F. Paccanoni, Eur. Phys. J. **C7** (1999) 263.)



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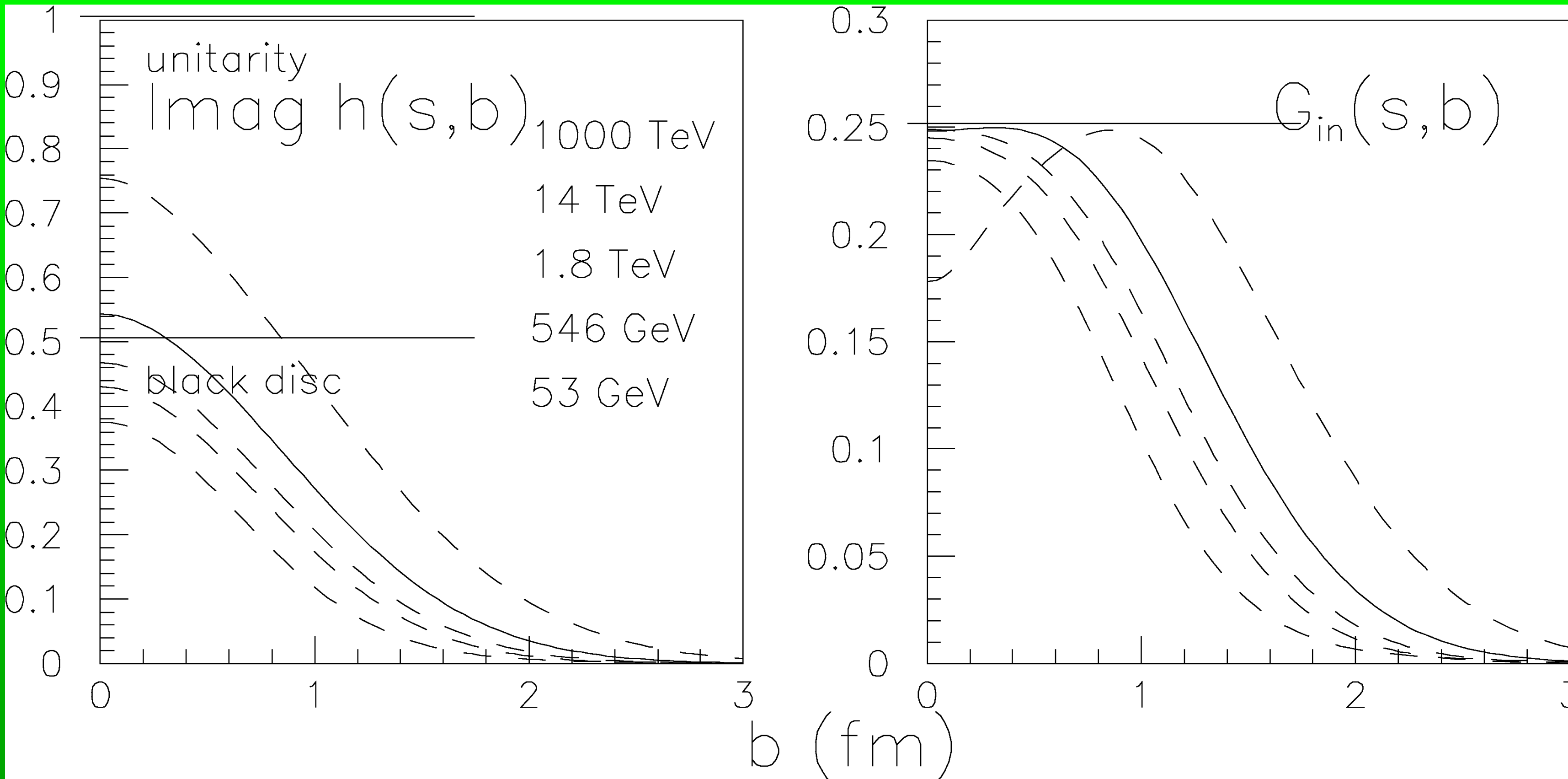


unitarity
Imag h(s,b)
1000 TeV
14 TeV
1.8 TeV
546 GeV
53 GeV
black disc
$G_{in}(s,b)$
b (fm)

EOS, high energies

$$\mu = 0; \quad p(T), \quad s(T) = p'(T);$$

$$\epsilon(T) = p'(T)T - p(T) = s(T)T - p(T).$$

Collective properties of the nuclear matter vs. the S matrics, or how can the EOS (equation of state) be inferred from the scattering amplitude (data)?

The answer was given in the paper *R. Dashen, S.Ma, H.J. Bernstein, Phys. Rev.* **187** *(1969) 345.*

$$\beta(\Omega - \Omega_0) = -\frac{1}{4\pi}\sum_{n=2}^{\infty} z^n \int_{nm}^{\infty} dE e^{-\beta E} (Tr_n A S^{-1} \frac{d}{dE} S),$$

where Ω is the thermodynamical potential, $z = e^{\beta\mu}$, $\beta = 1/T$.

The S matrix can be saturated either by experimental data points or by a model for the scattering amplitude.

For the latter a direct-channel resonance model was used by *P. Fre and L. Sertorio (Nuovo Cim.* **28A** *(1975) 538;* **31A** *(1076) 365).*

At high energies, the S matrix (scattering amplitude) is Regge behaved:

$$A(s,t) = \sum_i \xi_i(t)\beta_i(t)(-is/s_0)^{\alpha_i(t)}, \quad i = P, f, ...$$

$$p(T) = p_0(T) + p_1(T) + p_2(T),$$

$$p_1(T) = \frac{T^2}{2(2\pi)^4}\int_{2m}^{\infty} dEK_2(\beta E)E^2\frac{d}{dE}[ReA(s,0)(1-\frac{4m^2}{E^2})^{1/2}],$$

$$p_2(T) = \frac{T^2}{8(2\pi)^5}\int_{2m}^{\infty} dEK_2(\beta E)\int_{4m^2-s}^{infty}[ReA(s,t)\frac{d}{dE}ImA(s,t)],$$

where $K_2(z)$ is the Bessel funcion of imaginary argument.

L.L. Jenkovszky and A.A. Trushevsky (Nuovo Cim. **34A** *(1976) 369)* saturated the scattering amplitude with a Pomeron exchange in the t channel, resulting in:

L.L. Jenkovszky and A.A. Trushevsky (Nuovo Cim. **34A** *(1976) 369)* saturated the scattering amplitude with a Pomeron exchange in the t channel, resulting in:

$$p(T) \sim k(\sigma_t, \alpha')T^6, \quad T \gg m, \quad p = \epsilon/5.$$

This *heretic* result resides on two basic and firm properties of the strong interaction, namely the existence of the forward cone in the differential cross section and the non-decreasing total cross sections.

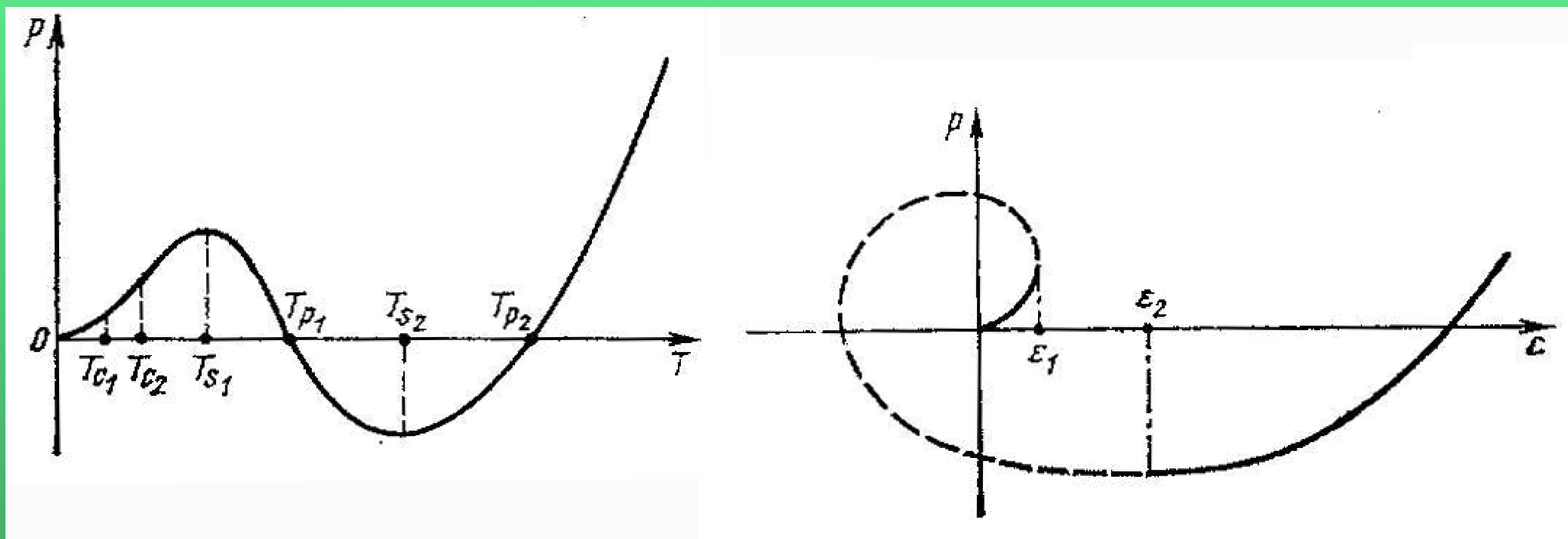
By duality the sum of direct channel resonances is dual to Regge exchanges (L.L. Jekovszky, P. Fre and L. Sertorio, Lett. Nuovo Cim. **15** *(1976) 365.)*

The non-asymptotic behavior of the EOS $p(T)$ was studied by *A.B. Bugrij and A.A. Trushevsky (ZHETP,* **73** *(1977) 3)*, who included in the scattering amplitude non-leading (secondary) trajectories (f, ω etc) with the following (surprising) result:

$$p(T) = AT^4 - BT^5 + CT^6,$$

where the coefficients A, B and C are determined by fits to the data on hadronic (e.g. pp, $\bar{p}p$) scattering data (see: *L.L. Jenkovszky and A.N. Shelkovenko, Nuovo Cim. A* **101** *((1989) 137).*

Remarcably, this EOS exhibits a local maximum and minimum at negative temperature



OS from the S matrix (scattering amplitud

Bugrij, Jenkovszky, Trushevsky

A generalization of the bag EOS: $B \to B(T)$ (C.G. Källman, Phys. Lett. B **134** (1984) 363).

$$p_q((T) = a_q T^4 - AT, \quad p_h(T) = a_h T^4;$$

$$\epsilon_q = 3a_q T^4, \quad \epsilon_h = 3a_h T^4;$$

$$s_q(T) = 4a_q T^3 - A, \quad s_h(T) = 4a_h T^3,$$

where $A = (a_q - a_h)T_c^3$.

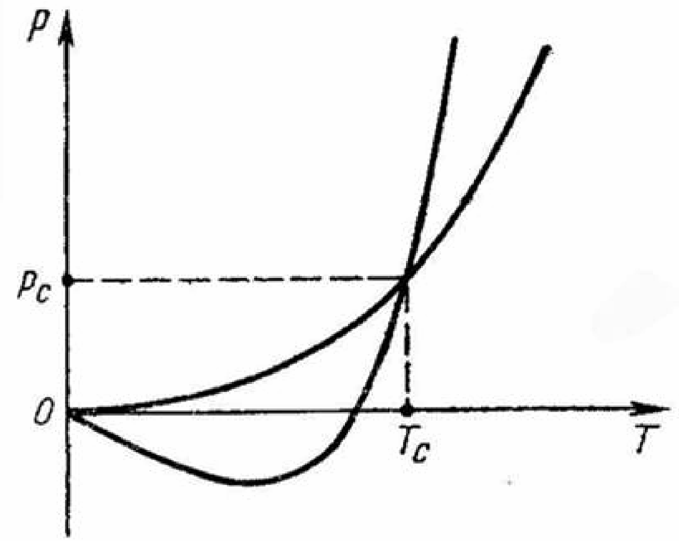
This system of bag equations of state can be written in one line:

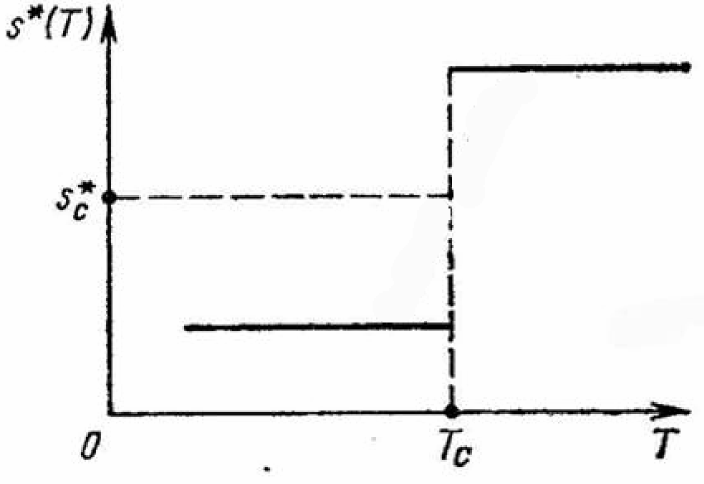
$$s(T) = p'(T) = \frac{2}{45}\pi^2 T^3 \Big(g_h (1 - \Theta(T - T_c)) + g_q \Theta(T - T_c) \Big).$$

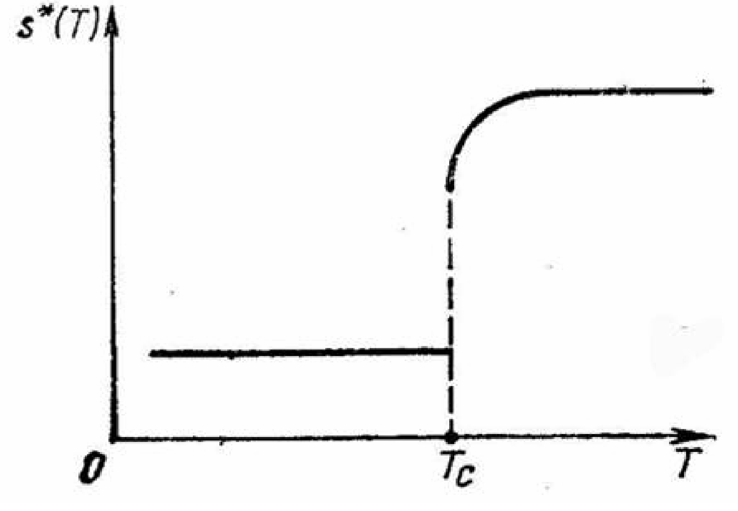
Blaizot and Ollitrault (Phys. Lett. B **191** (1987) 21):

$$\Theta(x) \to (1/2)[1 + th(\frac{x}{\Delta T})],$$

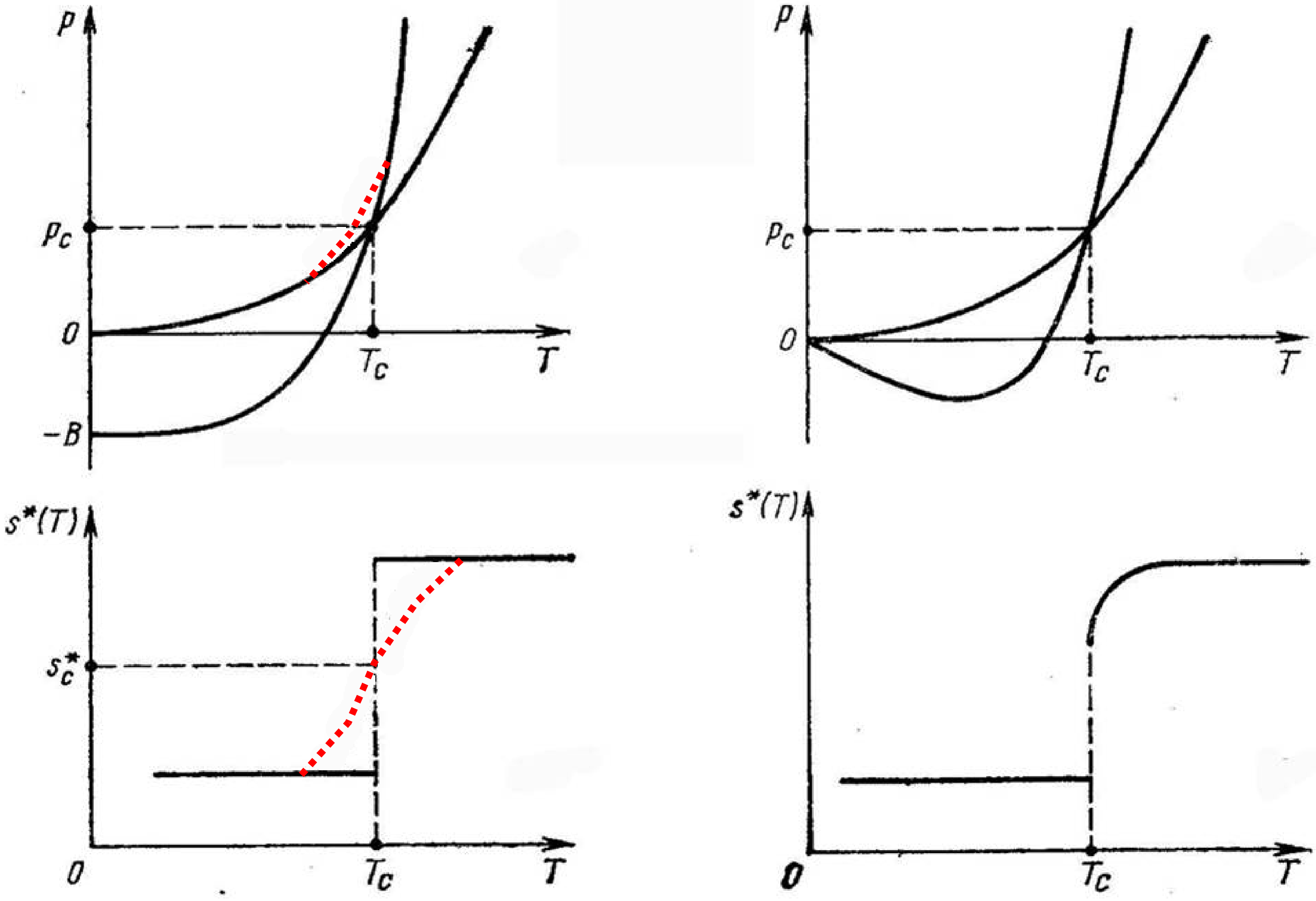
BAG EOS

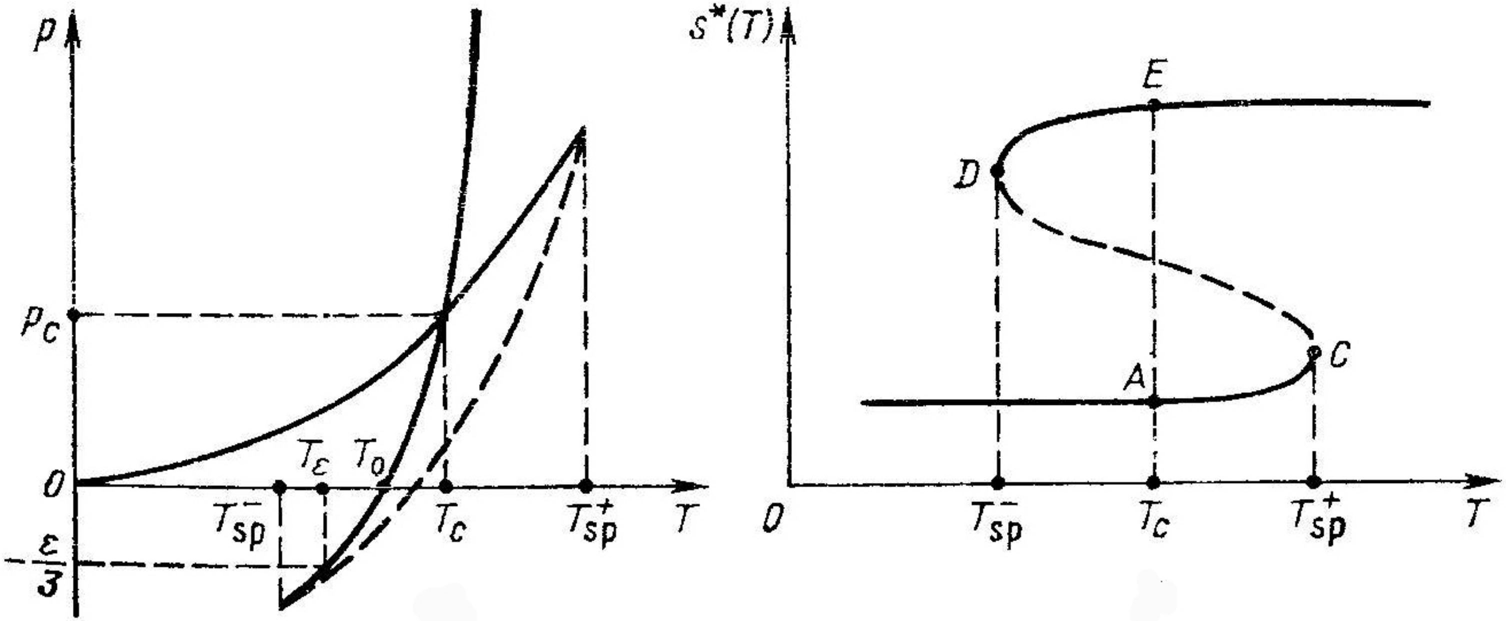






Modified bag EOS





Metastability in the bag EOS (Jenkovszky, Kaemfer, Sysosev)