Vector mesons and DVCS in the Dipole Cascade Picture

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Work done with
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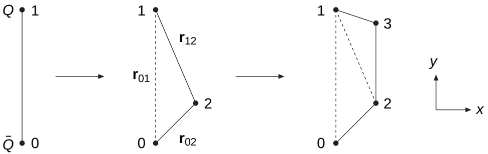
Outline

Mueller Dipole model

- Evolution in rapidity of dipoles in transverse coordinate space.



- Emission probability

$$\frac{d\mathcal{P}}{dy} = \frac{\bar{\alpha}}{2\pi} d^2\boldsymbol{r}_2 \frac{r_{01}^2}{r_{02}^2\, r_{12}^2}$$

- Eg. $\gamma^\star\gamma^\star$, each $\gamma^\star$ splits into $q\bar{q}$ dipoles which evolve through dipole splitting $\Leftrightarrow$ gluon emission.
- Interaction through dipole–dipole scattering probability

$$f = \frac{\alpha_s^2}{2}\left\{\log\left[\frac{|\boldsymbol{x}_1 - \boldsymbol{x}_2|\cdot|\boldsymbol{y}_1 - \boldsymbol{y}_2|}{|\boldsymbol{x}_1 - \boldsymbol{y}_2|\cdot|\boldsymbol{y}_1 - \boldsymbol{x}_2|}\right]\right\}^2$$

- Equivalent to LO BFKL
- Easy to include multiple dipole–dipole scatterings with scattering amplitude

$$T = 1 - e^{-\sum_{ij} f_{ij}}$$

- Less easy to include saturation in the evolution

MC implementation of Mueller Dipoles

- First done by Salam: OEDIPUS
- Dipole splitting is divergent for small dipole sizes
- Final result independent of cutoff because small dipoles has small interaction probability
- Problem for MC implementation — too many dipoles

The [*insert name here*] MC implementation

- Small size dipoles correspond to high $p_\perp \propto 1/r$ gluons
- We may have infinitely many small *virtual* dipoles but there is not enough energy for all of them to interact.
- Our MC tracks each parton: y, $\boldsymbol{x}$, $p_\perp$
- Require each emission to be ordered in p_+ and p_-
- Take into account recoils
- Neighboring dipoles are correlated

- A right-moving dipole with negative p_{-} must collide with left-moving dipole with enough positive p_{-} so that all partons can be put on-shell (+vv.)
- Energy-momentum conservation gives a dynamical cutoff for small dipoles in the evolution
- Also non-interacting (virtual) side chains takes energy, energy conservation effects are somewhat over estimated.
- Hopefully, the MC can be used to also study final-state properties

The Dipole Swing

- Each dipole carries a colour index
- Two dipoles with the same index are allowed to reconnect $(\boldsymbol{x}_1, \boldsymbol{y}_1), (\boldsymbol{x}_2, \boldsymbol{y}_2) \rightarrow (\boldsymbol{x}_1, \boldsymbol{y}_2), (\boldsymbol{x}_2, \boldsymbol{y}_1)$ with probability

$$\propto \frac{(\boldsymbol{x}_1 - \boldsymbol{y}_1)^2(\boldsymbol{x}_2 - \boldsymbol{y}_2)^2}{(\boldsymbol{x}_1 - \boldsymbol{y}_2)^2(\boldsymbol{x}_2 - \boldsymbol{y}_1)^2}$$

- Can be interpreted as gluon exchange, but also as modeling the quadrupole as two combinations of dipoles.
- This gives saturation in the evolution (the number of dipoles are not decreased, but smaller dipoles are preferred).
- The cross sections becomes independent of Lorentz frame

Modeling the proton

- The virtual photon wave functions are well known
- How do we describe the initial dipole state of a proton?
- We have at three valence quarks $\Rightarrow$ three dipole ends.
- Three independent dipoles didn't work very well
- Three (correlated) dipoles in Δ-geometry worked very well.
- Previous investigation of diffraction and elastic scattering in *pp* indicates little room for fluctuations in the proton wavefunction.
- $|\Psi|^2 = C\,e^{-(r-R_p)^2/w^2}$

Confinement effects

Mueller used a simple Coulomb potential in deriving dipole emissions and dipole–dipole scattering.

To simulate confinement effects we use instead a screened Yakawa potential

$$1/\boldsymbol{k}^2 \rightarrow 1/(\boldsymbol{k}^2 + 1/r_{\mathrm{max}}^2)$$

resulting in exponential suppression of emitting gluons far away, and of scattering between far away dipoles.

r_{max} can be related to the size of the proton.

- The size of the proton and the confinement scale $r_{\max}$.
- Λ_{QCD} in the running coupling
- The width of the proton wave function ($\to 0$)
- The strength of the dipole swing (saturates)

hep-ph/0503181, hep-ph/0610157, hep-ph/0702087

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Tuning of parameters

Total *pp* cross section:

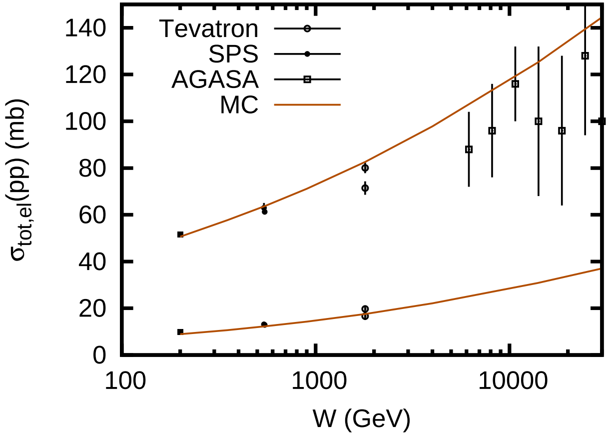
$$\sigma_{\text{tot}} = 2\int d^2\boldsymbol{b} d^2\boldsymbol{r}_{p1} d^2\boldsymbol{r}_{p2} \left|\Psi_p(\boldsymbol{r}_{p1})\right|^2 \left|\Psi_p(\boldsymbol{r}_{p2})\right|^2 \langle 1 - e^{-F}\rangle_{12},$$

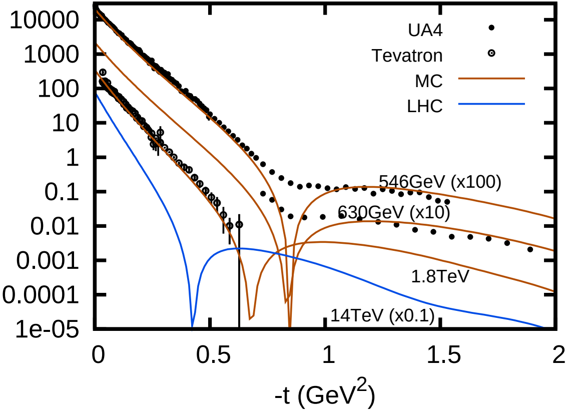
Elastic *pp* cross section:

$$\sigma_{\text{el}} = \int d^2\boldsymbol{b} \left| \int d^2\boldsymbol{r}_{p1} d^2\boldsymbol{r}_{p2} \left|\Psi_p(\boldsymbol{r}_{p1})\right|^2 \left|\Psi_p(\boldsymbol{r}_{p2})\right|^2 \langle 1 - e^{-F}\rangle_{12} \right|^2.$$

t-dependence:

$$\frac{d\sigma_{\text{el}}}{dt} = \frac{1}{4\pi} \left| \int d^2\boldsymbol{b} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} d^2\boldsymbol{r}_{p1} d^2\boldsymbol{r}_{p2} \left|\Psi_p(\boldsymbol{r}_{p1})\right|^2 \left|\Psi_p(\boldsymbol{r}_{p2})\right|^2 \langle 1 - e^{-F}\rangle_{12} \right|^2$$


Tevatron
SPS
AGASA
MC
140
120
100
80
60
40
20
0
100
1000
10000
W (GeV)
σtot,el(pp) (mb)


10000
1000
100
10
1
0.1
0.01
0.001
0.0001
1e-05
0
0.5
1
1.5
2
-t (GeV^2)
UA4
Tevatron
MC
LHC
546GeV (x100)
630GeV (x10)
1.8TeV
14TeV (x0.1)

The energy dependence is very much dictated by the perturbative evolution.

The fluctuations in the dipole evolution is so large there is no room for fluctuations in the proton wavefunction.

The slope for small t comes for free, the position of the dip can be tuned a bit.

The total $\gamma^\star p$ cross section

We need the virtual photon wavefunction:

$$\begin{aligned}
\Psi^{\gamma 0}_{fh\bar{h}}(Q,r,z) &= \frac{\sqrt{\alpha_{EM}N_C}}{\pi} Qz(1-z)e_f K_0(r\epsilon_f)\delta_{h\bar{h}} \\
\Psi^{\gamma +}_{fh\bar{h}}(Q,r,z) &= \frac{\sqrt{\alpha_{EM}N_C/2}}{\pi} e_f \times \\
&\quad \Big(ie^{i\theta}\left(z\delta_{h+}\delta_{\bar{h}-} - (1-z)\delta_{h-}\delta_{\bar{h}+}\right)\epsilon_f K_1(r\epsilon_f) + \\
&\quad + \delta_{h+}\delta_{\bar{h}+} m_f K_0(r\epsilon_f) \Big)
\end{aligned}$$

For small Q^2 we need to worry about VMD and confinement effects.

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$$\Psi^{\gamma +}_{fh\bar{h}}(Q,r,z) = \frac{\sqrt{\alpha_{EM}N_C/2}}{\pi} e_f \times \Big(ie^{i\theta}\left(z\delta_{h+}\delta_{\bar{h}-} - (1-z)\delta_{h-}\delta_{\bar{h}+}\right)\epsilon_f K_1(r\epsilon_f) + +\delta_{h+}\delta_{\bar{h}+} m_f K_0(r\epsilon_f)\Big)$$

For small Q^2 we need to worry about VMD and confinement effects.

We use the prescription from Forshaw et al., where the VMD component is modelled by multiplying with a function

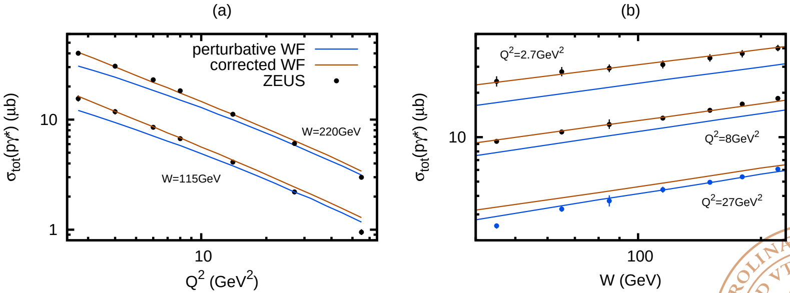
$$f(r) = \frac{1 + B_V \exp(-(r - R_V)^2/w_V^2)}{1 + B_V \exp(-R_V^2/w_V^2)}$$

(cf. our proton wf.)

Also we include a suppression of large r using a confinement scale related to r_{max}

$$r_{\mathrm{soft}}(r_{\mathrm{pert}}) = R_{\mathrm{shrink}} \sqrt{\ln\left(1 + \frac{r_{\mathrm{pert}}^2}{R_{\mathrm{shrink}}^2}\right)}.$$

$$\sigma_{\mathrm{tot}}(\gamma^* p) = \sum_{\lambda f} \int d^2\boldsymbol{b} d^2\boldsymbol{r}_p d^2\boldsymbol{r} dz \left|\Psi_p(\boldsymbol{r}_p)\right|^2 \left|\Psi^{\lambda}_{\gamma,f}(Q,\boldsymbol{r},z)\right|^2 \langle 1 - e^{-F}\rangle_{dp}$$



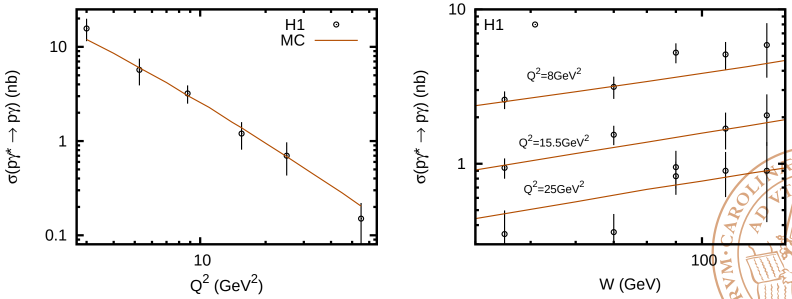
$R_V = 3.0\ \mathrm{GeV}^{-1}$, $w_V = 0.2\ \mathrm{GeV}^{-1}$, $B_V = 9.0$

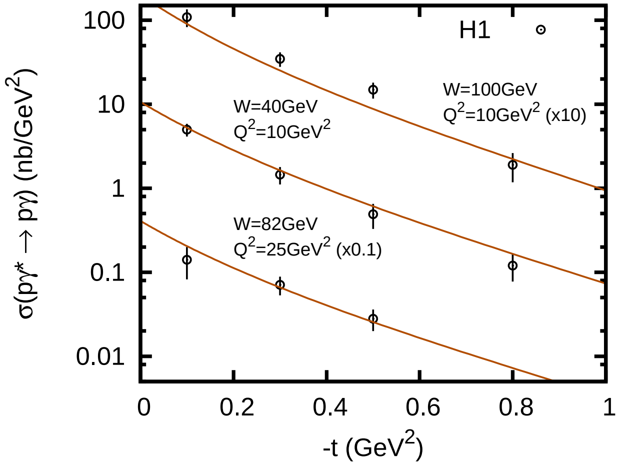
Deeply virtual Compton scattering

Quasi-elastic $\gamma^\star p \rightarrow \gamma p$ scattering

$$\sigma_{\text{DVCS}} =$$

$$\int d^2\boldsymbol{b} \sum_\lambda \left| \int d^2\boldsymbol{r}_p d^2\boldsymbol{r} dz \sum_f |\Psi_p(\boldsymbol{r}_p)|^2 \, \Psi_f^{\star\gamma\lambda}(Q,\boldsymbol{r},z)\Psi_f^{\gamma\lambda}(0,\boldsymbol{r},z)\langle 1 - e^{-F}\rangle_{dp} \right|^2$$




H1
W=100GeV
Q2=10GeV2 (x10)
W=40GeV
Q2=10GeV2
W=82GeV
Q2=25GeV2 (x0.1)
σ(pγ* → pγ) (nb/GeV2)
-t (GeV2)
100
10
1
0.1
0.01
0
0.2
0.4
0.6
0.8
1

Exclusive Vector Meson Produciton

Quasi-elastic $\gamma^\star p \to Vp$ scattering

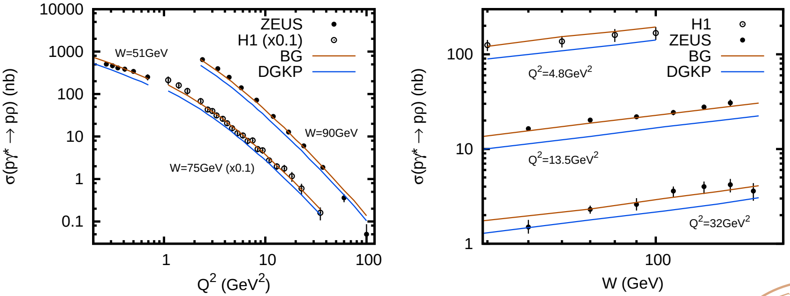
DGKP:

$$\begin{aligned}
\Psi^{V0}_{fh\bar{h}}(r,z) &= \mathcal{N}_0 M_V \delta_{-h\bar{h}} z(1-z)\sqrt{z(1-z)} \times \\
&\exp\left(-\frac{M_V^2(z-0.5)^2}{2\omega_L^2}\right) \exp\left(-\frac{r^2\omega_L^2}{2}\right) \\
\Psi^{V+}_{fh\bar{h}}(r,z) &= \mathcal{N}_+ \left(\omega_T^2 r i e^{i\theta}(z\delta_{h+}\delta_{\bar{h}-} - (1-z)\delta_{h-}\delta_{\bar{h}+}) + m_f\right) \sqrt{z(1-z)} \times \\
&\exp\left(-\frac{M_V^2(z-0.5)^2}{2\omega_T^2}\right) \exp\left(-\frac{r^2\omega_T^2}{2}\right).
\end{aligned}$$

Boosted Gaussian:

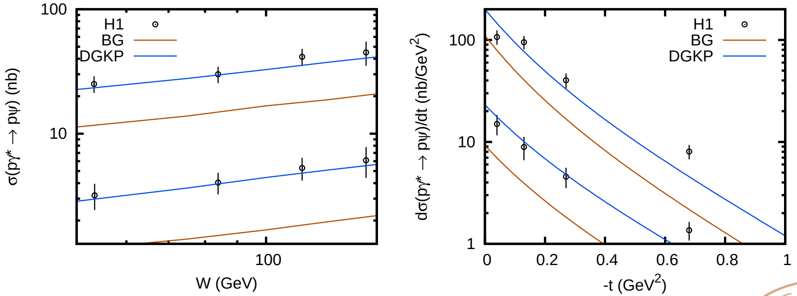
$$\Psi^{V0}_{fh\bar{h}}(r,z) = \frac{\mathcal{N}_0}{M_V}\left(z(1-z)M_V^2 + m_f^2 + 8\frac{z(1-z)}{R^2} - \left(4\frac{z(1-z)r}{R^2}\right)^2\right)\delta_{h\bar{h}} \times$$
$$\exp\left(-\frac{m_f^2R^2}{8z(1-z)}\right)\exp\left(-2z(1-z)\frac{r^2}{R^2}\right)\exp\left(\frac{R^2}{2}m_f^2\right)$$
$$\Psi^{V+}_{fh\bar{h}}(r,z) = \mathcal{N}_+\left(4z(1-z)\frac{r}{R^2}ie^{i\theta}(z\delta_{h+}\delta_{\bar{h}-} - (1-z)\delta_{h-}\delta_{\bar{h}+}) + m_f\delta_{h+}\delta_{\bar{h}+}\right) \times$$
$$\exp\left(-\frac{m_f^2R^2}{8z(1-z)}\right)\exp\left(-2z(1-z)\frac{r^2}{R^2}\right)\exp\left(\frac{R^2}{2}m_f^2\right)$$

$\gamma^\star p \to \rho p$



Also t-dependence look good. As well as ϕ-production .

$\gamma^\star p \rightarrow J/\Psi p$



No attempt to tune m_c or the VMD radius of the photon.

Summary

- We have a Monte Carlo implementation of Mueller Dipoles
- Key ingredients:
  - Energy-momentum conservation
  - Dipole swing
  - Simple proton and photon model
  - Confinement effects
  - ...possibility to study final states
- Reasonable description of data:
  - Total cross sections for *pp* and $\gamma^\star p$
  - (Quasi-) Elastic scattering in *pp* and $\gamma^\star p$
  - Diffraction at HERA
  - ...more to come

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