

# Hard Multi-Jet Predictions using High Energy Factorisation

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in collaboration with  
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Low-x workshop  
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# What, Why, How?

## What?

Develop a framework for reliably calculating many-parton rates inclusively (ensemble of 2, 3, 4, ... parton rates) and in a flexible way (jets, W+jets, Higgs+jets, ...)

## Why?

$(n + 1)$ -jet rate not necessarily small compared to  $n$ -jet rate  
Partons not in observed jets can masquerade as  $\cancel{p}_\perp$

## How?

Factorisation of QCD Amplitudes in the High Energy Limit.  
New Technique. Validation.

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# Why Study Multi-jet Observables?

## Just a few important examples

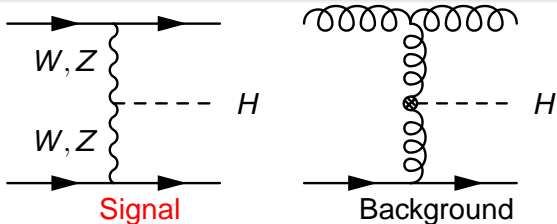
- 1 Pure Multi-jets
- 2  $W + (n \geq 2)$  jets
- 3 **Higgs + 2 jets**

Will discuss how all these observables can be described in a framework tailored to the description of multiple, also hard gluon emission

# Why Study Multi-jet Observables?

## $H + (n \geq 2)$ jets

- 1 When (!) a fundamental scalar has been found at the LHC we need to determine whether this one is responsible for the observed EWSB
- 2 Determine the couplings to  $Z$  or  $W$  by studying the angular distribution of the jets

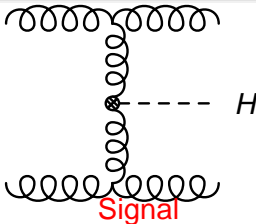
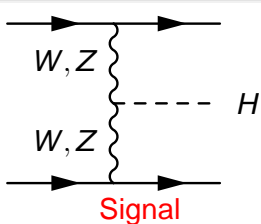


**Important to understand** the behaviour of the **QCD process** in order to separate **the two channels**

# Why Study Multi-jet Observables?

## $H + (n \geq 2)$ jets

**Search** for the **Higgs Boson!** May **relax** traditional Weak Boson Fusion cuts; then the QCD process can dominate. The two jets may help **give significance over background** compared to fully inclusive Higgs boson production.



**Important to understand** the behaviour of the **QCD process** in order to separate **from non-Higgs boson related background**.

# Do we need a new approach?

## Already know how to calculate. . .

- Shower MC: at most  $2 \rightarrow 2$  "hard" processes with additional parton shower
- Flexible Tree level calculators:  
MadGraph, AlpGen, SHERPA, . . .  
Allow most  $2 \rightarrow 4$ , some  $2 \rightarrow 5$  processes (and 6 constrained) to be calculated at tree level.  
Interfaced with Shower MC makes for a powerful mix!
- MCFM: Many relevant  $2 \rightarrow 3$  processes at up to NLO (i.e. including  $2 \rightarrow 4$ -contribution).
- . . . ⟨your favourite method here⟩

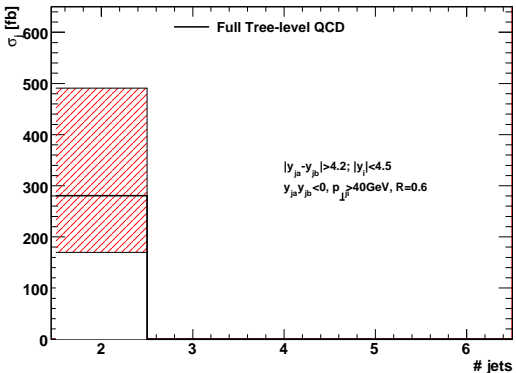
Could all be labelled "Standard Model contribution", but give vastly different results depending on the question asked!



# All Order Resummation Necessary?

Are tree-level (or generally fixed order) calculation always sufficient?

Sometimes the  $(n + 1)$ -jet rate is as large as the  $n$ -jet rate  
Higgs Boson plus  $n$  jets at the LHC at leading order

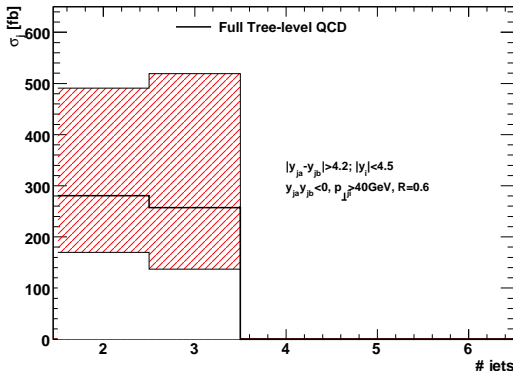


C.D. White, JRA

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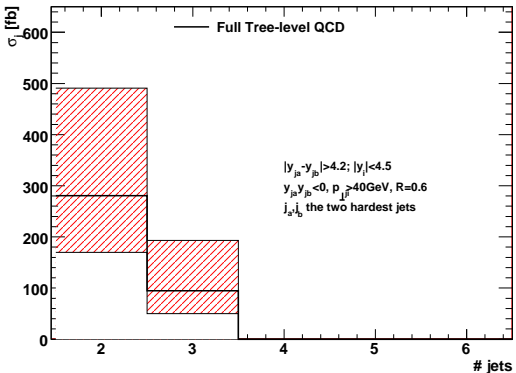
C.D. White, JRA

Indication that we need to go further! However, fixed order tools **exhausted** (full  $2 \rightarrow 3$  with a massive leg at two loops **untenable!**).

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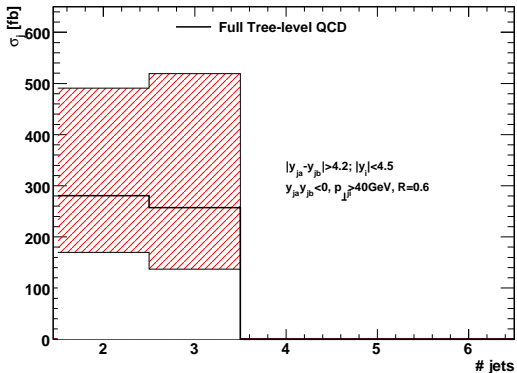
C.D. White, JRA

Could require that the two jets passing the cuts are also the two hardest jets. This reduces the three-jet phase space and the higher order corrections. Sensitivity to pert. corrections?

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C.D. White, JRA

The method we develop will be applicable to both set of cuts, but crucially will allow a **stabilisation** of the perturbative series by **resumma-  
tion**

# Resummation and Matching

Consider the **perturbative expansion** of an observable

$$R = r_0 + r_1 \alpha_s + r_2 \alpha_s^2 + r_3 \alpha_s^3 + r_4 \alpha_s^4 + \dots$$

**Fixed order** pert. QCD will calculate a fixed number of terms in this expansion.  $r_n$  may contain **large logarithms** so that  $\alpha_s \ln(\dots)$  is large.

$$\begin{aligned} R &= r_0 + (r_1^{LL} \ln(\dots) + r_1^{NLL}) \alpha_s + \left( r_2^{LL} \ln^2(\dots) + r_2^{NLL} \ln(\dots) + r_2^{SL} \right) \alpha_s^2 + \dots \\ &= r_0 + \sum_n r_n^{LL} (\alpha_s \ln(\dots))^n + \sum_n r_n^{NLL} \alpha_s (\alpha_s \ln(\dots))^n + \text{sub-leading terms} \end{aligned}$$

Useful **iff the terms** really do describe **the dominant part** of the **full pert. series** and can be **summed to all orders** in the pert. expansion (LLA). **Matching** combines **best of both worlds**:

$$R = r_0 + r_1 \alpha_s + r_2 \alpha_s^2 + \left( r_3^{LL} \ln^3(\dots) + r_3^{NLL} \ln^2(\dots) + r_3^{SL} \right) \alpha_s^3 + \dots$$

## Factorisation of QCD Matrix Elements

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It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:

Soft limit → **eikonal approximation** → enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider. . .

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# To boldly go...

In a previous episode of a CERN seminar series:  
A wise man said...

“Use known results to gain deeper insights...”

young\* postdoc

“Use insight to gain yet unknown results...”

It has become very fashionable to **claim** (my favourite method) can predict observables important for the LHC programme. Will actually **validate\*\*** the claims

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# Regge and High Energy Factorisation

In the High Energy Limit,  $2 \rightarrow 2$  **scattering amplitudes** are **dominated** by the  **$t$ -channel exchange** of the particle of the **highest spin** allowed by the scattering theory

$$\mathcal{M}^{p_a p_b \rightarrow p_1 p_2} \xrightarrow{\text{Regge limit}} \hat{s}^{\hat{\alpha}(\hat{t})} \gamma(\hat{t})$$

Regge (1959)

$\hat{s} = (p_a + p_b)^2$ ,  $\hat{t} = (p_a - p_1)^2$ , Regge limit:  $\hat{s} \rightarrow \infty$ ,  $\hat{t}$  fixed.

**Multi-particle generalisation?**

$$\mathcal{M}^{p_a p_c \rightarrow p_{a'} p_b p_{c'}} \xrightarrow{\text{Multi Regge limit}}$$

$$\hat{s}_1^{\hat{\alpha}(\hat{t}_1)} \hat{s}_2^{\hat{\alpha}(\hat{t}_2)} \gamma(\hat{t}_1, \hat{t}_2, \frac{S_{12}}{S_1 S_2})$$

MRK:  $\hat{s}_{12}, \hat{s}_1, \hat{s}_2 \rightarrow \infty$ ,  $t_1, t_2$  fixed

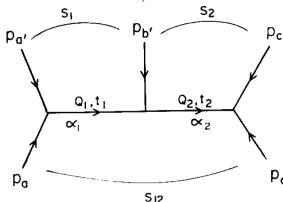
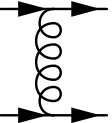
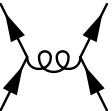
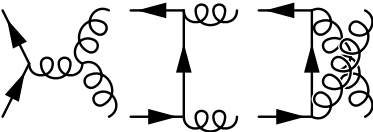


Fig. 2.1. Five-particle diagram showing notation.

Brower, DeTAR, Weis (1974)

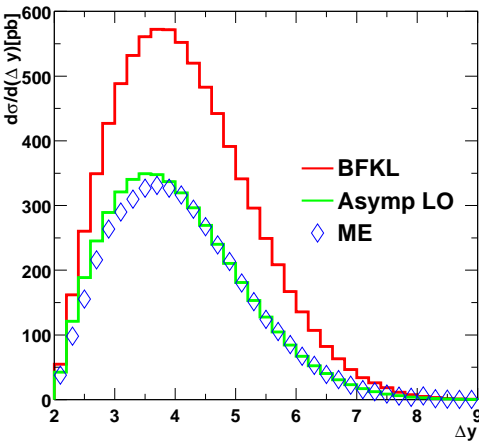
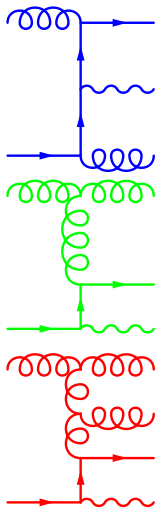
# High Energy Factorisation - $t$ -channel dominance

Process	Diagrams	$\overline{\sum}  \mathcal{M} ^2 / g^4$
$qq' \rightarrow qq'$		$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q\bar{q} \rightarrow q'\bar{q}'$		$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$q\bar{q} \rightarrow gg$		$\frac{32}{27} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$

High Energy Limit:  $|\hat{t}|$  fixed,  $\hat{s} \rightarrow \infty$

# $t$ -channel dominance

Example:  $W+n$ -jet production at the LHC



$$\Delta y = y_{j_2} - y_{j_1}, y_W, y_{j_2} \geq 1, y_{j_1} \leq -1$$

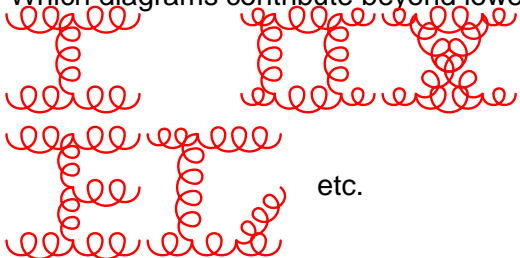




# FKL at Leading Logarithmic Accuracy

Fadin, Kuraev, Lipatov

Which diagrams contribute beyond lowest order?



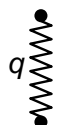
All these contributions can be calculated using **effective vertices** and propagators for the **reggeized gluon**.



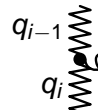
General form proved using s-channel unitarity and a set of bootstrap relations [NLL: Fadin, Fiore, Kozlov, Reznichenko](#)

# FKL formalism (Fadin, Kuraev, Lipatov)

**FKL:** Identification of the **dominant contributions** to the **perturbative series** for processes with two large (perturbative) and disparate energy scales  $\hat{s} \gg |\hat{t}|$  ( $\hat{s}: E_{\text{cm}}^2, \hat{t}: p_{\perp}^2$ )



$$\frac{1}{q^2} \exp(\hat{\alpha}(q)\Delta y)$$



$$C_L^\mu(q_{i-1}, q_i)$$

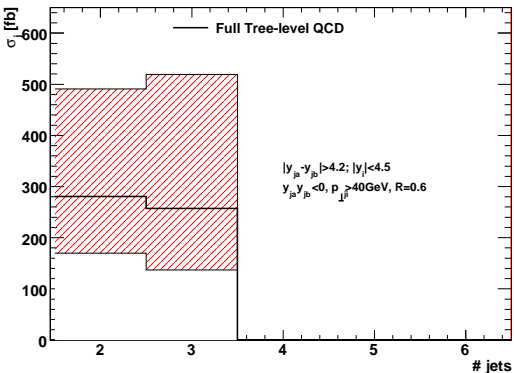
$$C_L^\mu(q_{i-1}, q_i) = \left[ (q_i + q_{i+1})_{\perp}^{\mu i} - \left( \frac{\hat{s}_{ai}}{\hat{s}} + 2 \frac{\hat{t}_{i+1}}{\hat{s}_{bi}} \right) p_b^{\mu i} + \left( \frac{\hat{s}_{bi}}{\hat{s}} + 2 \frac{\hat{t}_i}{\hat{s}_{ai}} \right) p_a^{\mu i} \right]$$

Framework exact in the limit of Multi Regge Kinematic (MRK)

$$s_{ij} \rightarrow \infty, \quad |k_{i\perp}| \approx |k_{j\perp}|, \quad q_i^2 \approx q_j^2$$

## Case study and Validation

# Tree level results for $pp \rightarrow \text{Higgs} + \text{jets}$



Necessary to understand multi-emission topologies in order to

- cleanly extract WBF signal (c. jet veto, angular dist. of jets, ...)
- use H+jets as a discovery channel

# Previous studies of Higgs Boson plus jets

- $h_{jj}$ @full NLO: Increase in cross section over LO estimate of factors 1.2-1.3 or 1.7-1.8 depending on cuts (note: discussion of  $K$ -factors not really useful for a multi-scale problem).

J. Campbell, K. Ellis, G. Zanderighi

- $h_{jj}$ @LO+parton showers: Focus on effects of soft and collinear radiation to all orders. Find significant effects beyond NLO.

V. Del Duca, G. Klämke, D. Zeppenfeld, M.L. Mangano, M. Moretti, F. Piccinini, R. Pittau, A.D. Polosa

Will focus on **developing a framework** which captures an alternative part of the **perturbative series to all orders (not relying** on soft and collinear factorisation) - and **compare it order by order** to the full result where known.

# Higgs Boson plus $n \geq 2$ jets in the HE limit



Extract the effective Higgs Boson vertex using the method of VDD

Only two diagrams contribute to the process Higgs Boson plus 3 jets in the High Energy Limit!

# Some contributions have vanishing HE limit. . .

## $pp \rightarrow h + \text{jets}$ with vanishing HE limit

sub-processes **not contributing at all** in the HE limit:

$$u\bar{u} \rightarrow ghg(g), gg \rightarrow uh\bar{u}(g)$$

or not in **special rapidity configurations** (at LL):

$$gu \rightarrow uhg, ud \rightarrow dhu, gu \rightarrow ghug, \dots$$

Total contribution from full QCD ME of these contributions:

$$\sigma_{hj}^{\text{non-FKL-conf.}} = 0.8 \text{ fb} (< 0.3\%)$$

$$\sigma_{hjj}^{\text{non-FKL-conf.}} = 24 \text{ fb} (< 10\%) \text{ (most will be captured by NLL corrections)}$$

Contributes **less than 10%** of the cross section (**FKL gains one point!**).

The HE limit will **approximate** the remaining configurations (**but how well? . . .**).

Will later add back the missing pieces by **matching to the fixed order results**.



# The Scattering Amplitude

$$\begin{aligned}
 i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} &= 2i\hat{s} \\
 &\cdot \left( ig_s f^{ad_0 c_1} g_{\mu_a \mu_0} \right) \\
 &\cdot \prod_{i=1}^j \left( \frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left( ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \\
 &\cdot \left( \frac{1}{q_h^2} \exp[\hat{\alpha}(q_h^2)(y_j - y_h)] C_H(q_{j+1}, q_h) \right) \\
 &\cdot \prod_{i=j+1}^n \left( \frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y'_{i-1} - y'_i)] \left( ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \\
 &\cdot \frac{1}{q_{n+1}^2} \exp[\hat{\alpha}(q_{n+1}^2)(y'_n - y_b)] \left( ig_s f^{bd_{n+1} c_{n+1}} g_{\mu_b \mu_{n+1}} \right)
 \end{aligned}$$

Have: **exact** result in the **very exclusive limit** of **infinite separation** between **all particles**

Want: **inclusive** cross sections. . .

# The Traditional Implementation Using the BFKL Eqn

Adding one emission  $\rightarrow$  emergence of extra factor in  $|\mathcal{M}|^2$  of

$$\frac{-C^{\mu_i} \cdot C_{\mu_i}}{t_j t_{j+1}} \xrightarrow{\text{Ultimate MRK limit}} \frac{4}{p_{i\perp}^2}$$

Taking into account contraction of colour factors, the addition of an emission leads to the following factor in the colour and spin summed and averaged square of the matrix element

$$\frac{4 g_s^2 C_A}{p_{i\perp}^2}$$

Only **transverse degrees of freedom** left!

# The Traditional Implementation Using the BFKL Eqn\*

$$|\mathcal{M}^{gg \rightarrow hgg}|^2 = \frac{4\hat{s}^2}{N_c^2 - 1} \frac{C_A g_s^2}{p_{0\perp}^2} \left| C_{HEL}^H(-p_{0\perp}, p_{1,\perp}) \right|^2 \frac{C_A g_s^2}{p_{1\perp}^2}$$

$$|\mathcal{M}^{gg \rightarrow hggg}|^2 = \frac{4\hat{s}^2}{N_c^2 - 1} \frac{C_A g_s^2}{p_{0\perp}^2} \left| C_{HEL}^H(q_{a\perp}, q_{b,\perp}) \right|^2 \frac{4}{p_{1\perp}^2} \frac{C_A g_s^2}{p_{2\perp}^2}$$

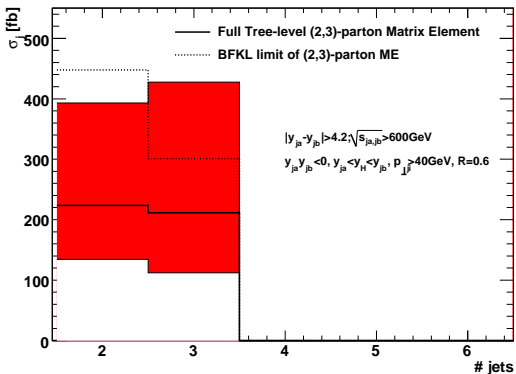
⋮ ⋮ ⋮

$$\frac{d\hat{\sigma}_{gg \rightarrow g \dots h \dots g}}{dp_{a\perp}^2 dy_a dp_{b\perp}^2 dy_b dp_{H\perp}^2 dy_H} = \int d^2 q_{a\perp} d^2 q_{b\perp} \left( \frac{\alpha_s N_c}{p_{a\perp}^2} \right) f(-p_{a\perp}, q_{a,\perp}, \Delta y_{aH}) \cdot \left| C_{HEL}^H(q_{a,\perp}, q_{b,\perp}) \right|^2 f(q_{b\perp}, p_{b,\perp}, \Delta y_{Hb}) \left( \frac{\alpha_s N_c}{p_{b\perp}^2} \right)$$

$$\text{BFKL eqn : } \frac{\partial}{\partial y} f(q_a, q_b, y) = \int d^{D-2} \mathbf{q} K(q_a, \mathbf{q}) f(\mathbf{q}, q_b, y)$$

Applies **factorisation** and **kinematic approximations** in **all of phase space**

# Comparison between BFKL and Full Matrix Element



C.D. White, JRA

Not convincing\*. Can obviously match to FO, but better also improve resum<sup>n</sup>!

---

\* And this is even the energy and momentum conserving variant of BFKL - please ask about this point if you want to see something crazy. It is actually a very important point.

# Improving the Framework\*

Start again from the FKL amplitudes:

$$i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} = 2i\hat{s} \dots \prod_{i=1}^j \left( \frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left( ig_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \dots$$

Instead of extending the kinematic approximations valid in the MRK limit to all of phase space, impose the right analytic behaviour away from the MRK limit

- 1 Position of Divergences
- 2 Positivity of  $|\mathcal{M}|^2$

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The full scattering amplitude is divergent for several momentum configurations, for which the BFKL approximations is finite. These divergences obviously lie explicitly outside of MRK. However, we choose to re-instate several of these divergences by using **the full momentum dependence of all invariants**.

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Using full expression for propagators **automatically takes into account the dominant source of NLL corrections** to *any* logarithmic accuracy. NLL corrections to Lipatov Vertex  $C^\mu$  starts to address the dependence on longitudinal momenta between two neighbouring partons. We can restore the **full** propagator between all gluons.

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Start again from the FKL amplitudes:

$$i\mathcal{M}_{\text{HE}}^{ab \rightarrow p_0 \dots p_j h p_{j+1} p_n} = 2i\hat{s} \dots \prod_{i=1}^j \left( \frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] (ig_s t^{c_i d_i c_{i+1}}) C_{\mu_i}(q_i, q_{i+1}) \right) \dots$$

Instead of extending the kinematic approximations valid in the MRK limit to all of phase space, impose the right analytic behaviour away from the MRK limit

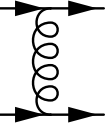
- 1 Position of Divergences
- 2 Positivity of  $|\mathcal{M}|^2$

*Minimal interference:* Insist just  $-C^\mu C_\mu > 0$ . Cuts out only a small region of phase space. Related in idea to the so-called *Kinematical Constraint* (earlier attempts at restricting the region in which the BFKL eqn. was applied)<sub>[actually allows for a check of the kinematic constraint directly on the formalism underpinning the BFKL eqn, instead of assuming the BFKL equation and then repairing with kin. cons.]</sub>



# What basically does this amount to?

Consider again  $qq' \rightarrow qq'$  scattering:

Process	Diagrams	$\overline{\sum}  \mathcal{M} ^2 / g^4$
$qq' \rightarrow qq'$		$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$

$$\hat{s} + \hat{t} + \hat{u} = 0, \text{ i.e. } \hat{u} = -(\hat{s} + \hat{t}), \hat{u}^2 = \hat{s}^2 + \hat{t}^2 + 2\hat{s}\hat{t}.$$

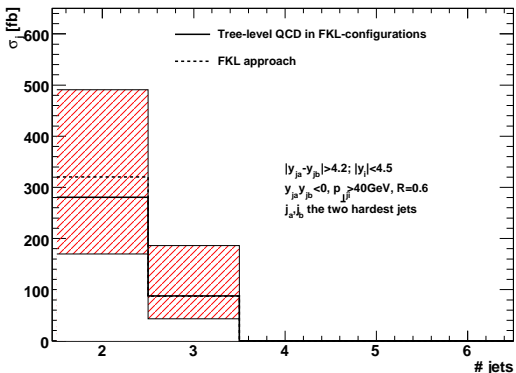
$$\text{BFKL result: } \frac{4}{9} \frac{2\hat{s}^2 + \hat{t}^2 + 2\hat{s}\hat{t}}{\hat{t}^2} \rightarrow \frac{8}{9} \frac{\hat{s}^2}{k_{\perp}^4}$$

$$\text{FKL result: } \frac{4}{9} \frac{2\hat{s}^2 + \hat{t}^2 + 2\hat{s}\hat{t}}{\hat{t}^2} \rightarrow \frac{8}{9} \frac{\hat{s}^2}{\hat{t}^2}$$

Formalism re-introduces the right position of divergences in the sub-MRK region to all orders.



# Comparison between FKL and Full Matrix Element



V. Del Duca, C. White, JRA

Difference between FKL (2 diagrams) and full result (  $10^3$  diagrams) is much less than the renormalisation and factorisation scale uncertainty. Repair with matching corrections.

# Beyond validation. . .

Have so far demonstrated that the terms we can take into account reproduce the full tree level results to within  $\mathcal{O}(10\%)$  where ever these are known.

Can calculate this approximation for the tree-level Higgs Boson plus  $n$ -parton amplitude, and include also some virtual corrections. Can thereby form the inclusive *any*-parton sample (i.e. LO: only H+2 partons, NLO: H+2 and H+3 partons, . . .)

Fully exclusive in all particles - Can perform any analysis using your favourite jet algorithm

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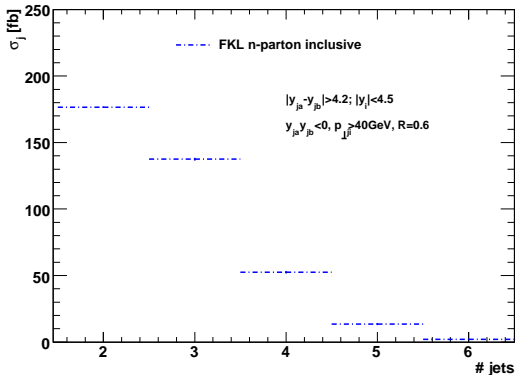
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# FKL All Order Resummation Incl. Matching

$$\sigma_{hjj}^{LO} : 281\text{fb}; \sigma_{hjj}^{\text{resummed+matched}} : 382\text{fb}$$

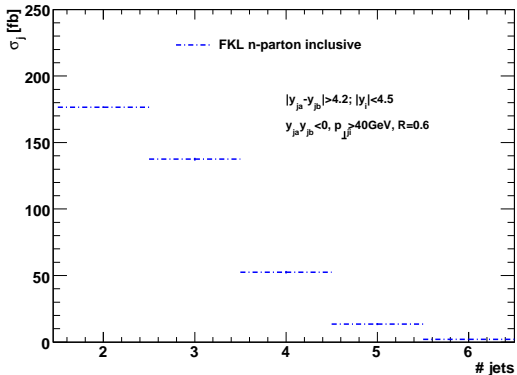


C.D. White, JRA

Can sum over  $n$ -parton inclusive samples (both real and virtual contributions included). Matching to the tree level  $n$ -parton matrix elements.

# FKL All Order Resummation Incl. Matching

$$\sigma_{hjj}^{LO} : 281\text{fb}; \sigma_{hjj}^{\text{resummed+matched}} : 382\text{fb}$$



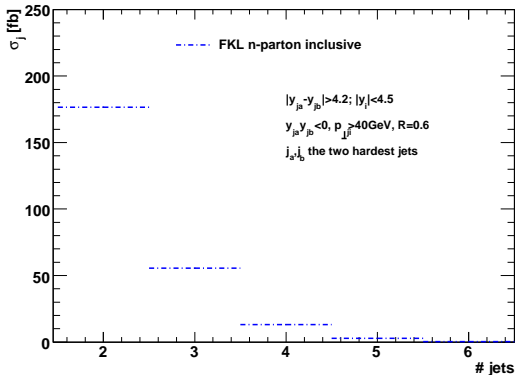
C.D. White, JRA

Any central jet veto will obviously only operate on states with 3 and more jets. According to this calculation, around 50% of total cross section would survive any additional jet veto (i.e.  $p_T < 40\text{GeV}$ ).



# FKL All Order Resummation Incl. Matching

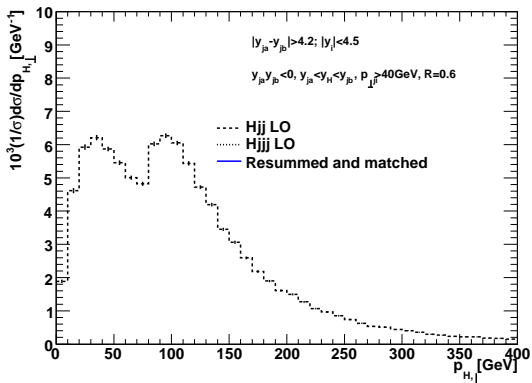
$$\sigma_{hjj}^{LO} : 281\text{fb}; \sigma_{hjj}^{\text{resummed+matched}} : 248\text{fb}$$



C.D. White, JRA

...or and even larger relative part when tagging on the hardest jets.

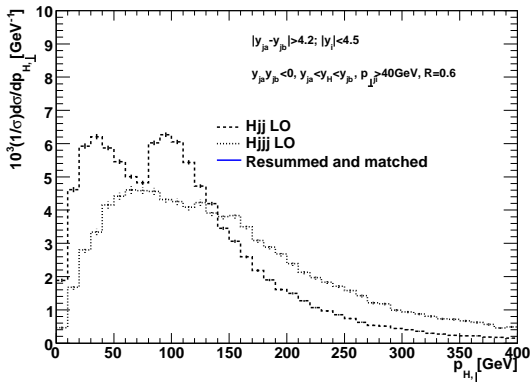
# Impact on Observables



C.D. White, JRA

Strong features of Higgs boson transverse momentum spectrum (caused by strong azimuthal correlation coupled with cuts on jets) disappears at higher orders.

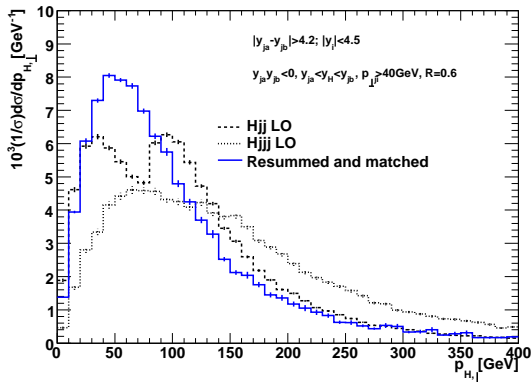
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# Impact on Observables

$$A_\phi = \frac{\sigma(\phi_{j_a j_b} < \pi/4) - \sigma(\pi/4 < \phi_{j_a j_b} < 3\pi/4) + \sigma(\phi_{j_a j_b} > 3\pi/4)}{\sigma(\phi_{j_a j_b} < \pi/4) + \sigma(\pi/4 < \phi_{j_a j_b} < 3\pi/4) + \sigma(\phi_{j_a j_b} > 3\pi/4)}$$

Results from lowest order:

$A_\phi > 0$  (CP-even),  $A_\phi \approx 0$  (CP-blind),  $A_\phi < 0$  (CP-odd)

$A_\phi$ (2p/2j)	0.50
$A_\phi$ (3p/3j)	0.23
$A_\phi$ (FKL/ $\geq 2j$ )	0.16
$A_\phi$ (FKL/ $\equiv 2j$ )	0.27

**Significant azimuthal decorrelation** from higher orders real radiation - even when **not hard enough to be detected as jets!**

# Outlook and Conclusions

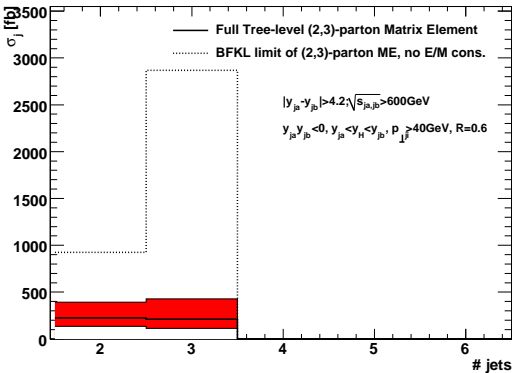
## Conclusions

- Emerging framework for the study of processes with multiple hard jets
- Working implementation, including matching to the known fixed order results
- Impact many studies: jet correlations, missing (transverse) energy, . . .

## Outlook

- H+jets studies being finalised; expect public code soon
- Implement other processes and test against Tevatron Data
- Les Houches Interface to study effects of showering
- Extend Studies to full NLL Accuracy
- . . .

# Thank you for asking that question. . .



Formulation valid for  $\hat{s} \rightarrow \infty, |t|$  fixed. But  $\hat{s} < s$  fixed at any collider! E/M conserv. not just “subleading corrections” in partonic scattering, but stops the evolution all together (even before the strict MRK limit is reached!). This is the level of