Wilson Loops and the AGT Correspondence

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Introduction

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- For circular loops the perturbative series for $< W >$ was conjectured to be resummable giving the number of planar graphs
- The same result can be recovered from matrix models evaluating

$$< \frac{1}{N} \mathrm{tr} e^M >= \frac{1}{Z} \int \mathcal{D}M \frac{1}{N} \mathrm{tr} e^M e^{-\frac{2N}{\lambda} \mathrm{tr} M^2}$$

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$$< W_R(C) > \approx \frac{1}{Z_{S^4}} \int \mathcal{D}a e^{-\frac{8\pi^2 r^2 a^2}{g^2}} \mathrm{tr}_R e^{2\pi i r a}$$

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- This result extends the "standard" computations in $\mathbb{R}^4$ or for complex manifolds in many respects:
  - Since W is real we must account for instantons and anti-instantons
  - For complex manifolds the contributions of different patches are multiplied

Localization

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- Introducing $\Omega = \begin{pmatrix} \epsilon_1\sigma_1 & 0 \\ 0 & \epsilon_2\sigma_1 \end{pmatrix}$ the original $N = 2$ SUSY theory gets deformed
- The e.o.m. for the scalar field becomes $D^2\varphi = \Omega_{\mu\nu}F^{\mu\nu} + \mathrm{ferm.}$ and the zero modes

$$\nabla_{[\mu}Z^a_{\nu]} = (\nabla_{[\mu}Z^a_{\nu]})^{\mathrm{dual}} \quad \nabla^\mu Z^a_\mu = 0$$

lead to $Z_\mu = D_\mu\varphi - \Omega^\nu_\lambda x^\lambda F_{\nu\mu}$ from which $\tilde\varphi = \varphi + \delta x^\mu A_\mu$

The scalar field

- Furthermore the moduli space of the solutions needs to be compactified and made smooth. This makes the theory non commutative
- In turn the scalar field in ADHM is $\tilde{\varphi} = \bar{U}\delta U$ where the space spanned from U is isomorphic to the ideal $\mathcal{I} = \{z_1^{k-1} z_2^{l-1} | k, l \neq Y\}$

z_2^4			
z_2^3	$z_1 z_2^3$	$z_1^2 z_2^3$	
z_2^2	$z_1 z_2^2$	$z_1^2 z_2^2$	
z_2	$z_1 z_2$	$z_1^2 z_2$	$z_1^3 z_2$
1	z_1	z_1^2	z_1^3

The scalar field

- The eigenvalues of U can now be computed $\Rightarrow$ $\lambda_{k,l} = a_u + (k-1)\epsilon_1 + (l-1)\epsilon_2$ and the character

$$\mathrm{tr}\, e^{z\lambda}\big|_Y = \mathcal{V} \sum_{u=1}^{N} \sum_{(k,l)\notin Y_u} e^{z\chi_{(k,l)}}$$

$$= \sum_u \left(e^{za_u} - (1 - e^{z\,\epsilon_1})(1 - e^{z\,\epsilon_2}) \sum_{(i,j)\in Y_u} e^{z\lambda_{(i,j)}} \right)$$

- An interesting way to think of $\tilde{\varphi}$ is to define $\mathcal{F} = \tilde{\varphi} + \lambda + F$ and $\Phi = \tilde{\varphi} + \lambda_m \theta^m + \frac{1}{2} F_{mn}\theta^m\theta^n + \ldots$

Back to WL

- Let $z_\ell(s) = r_\ell e^{\mathrm{i}\epsilon_\ell s}$ and $\delta z_\ell = \dot{z}_\ell = i\epsilon_\ell e^{\mathrm{i}\epsilon_\ell s}$. If $x^m = (z_1, z_2, \bar{z}_1, \bar{z}_2)$ then $1 = |\dot{x}| = \epsilon_1 |r_1|^2 + \epsilon_2 |r_2|^2$
- Then the WL is

$$\mathcal{C} = \mathrm{i} \int_0^L (A_m \dot{x}^m + |\dot{x}| \varphi_1)\, ds = \frac{\mathrm{i}}{2} \int_0^L \tilde{\varphi}(s) ds - \mathrm{h.c.}$$

- The path is closed for $L = 2\pi n_1/\epsilon_1 = 2\pi n_2/\epsilon_2$ and $\epsilon_1/\epsilon_2 = n_1/n_2$. Therefore $\sum_u e^{\frac{2\pi \mathrm{i} n_1}{\epsilon_1} \tilde{\varphi}_u}\Big|_Y = \sum_u e^{\frac{2\pi \mathrm{i} n_1}{\epsilon_1} a_u}$

$$\langle \mathrm{tr}\, W \rangle_{S^4} = \frac{1}{Z} \int_\gamma d^N a\, \mathrm{tr}\, e^{\frac{2\pi \mathrm{i} n_1\, a}{\epsilon_1}} \left| Z_{\mathrm{one-loop}}(a) Z_{\mathrm{inst}}(a, \vec{\tau}) \right|^2$$

Deformed $N = 2^*$

- It is now natural to introduce a deformed $N = 2^*$ given by $S_{\rm class} = \int d^4 d^4\theta\, \mathcal{F}_{\rm class}(\Phi) + {\rm h.c.}$ where only the scalar in Φ gets a v.e.v. with

$$\mathcal{F}_{\rm class}(\Phi) = \sum_{J=2}^{p} \frac{{\rm i}\tau_J}{2\pi J!}\,{\rm tr}\,\Phi^J$$

- The partition function thus defined is the generating function of $\langle {\rm tr}\,\tilde{\varphi}^{J_1}\,{\rm tr}\,\tilde{\varphi}^{J_2}\,\ldots\rangle_{\rm undeformed}$ given that

$$\frac{1}{J!}\langle {\rm tr}\,\tilde{\varphi}^J\rangle = \frac{{\rm i}\epsilon_1\epsilon_2}{2\pi}\,\partial_{\tau_J}\ln Z(\vec{\tau})$$

Deformed $N = 4$

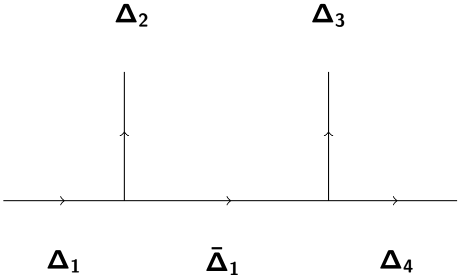
- In the limit $m = \epsilon_1$ we go back to a $N = 4$ theory with potential $V(a, \vec{\tau}) = \frac{4\pi}{\epsilon_1 \epsilon_2 N} \sum_{J=2}^{p} \frac{\tau_J}{J!} \mathrm{tr} a^J$

$$\langle W \rangle = \int d^N a \Delta(a) \mathrm{tr} e^{ia} e^{-NV(a,\vec{\tau})}$$

- Computations can be easily carried out in particular cases. In presence of a quartic terms $g_4 a^4$ one gets

$$W = \frac{1}{N} \sum_{n=0}^{\infty} \left\langle \frac{\mathrm{tr}\, a^{2n}}{(2n)!} \right\rangle = 1 + \sum_{n,k} \frac{(-12 g_4 \lambda)^k \lambda^n (2k + n - 1)!}{n!(n-1)!k!(k+n+1)!}$$
$$= 1 + \sum_{k=0}^{\infty} \frac{\lambda(-12\lambda^2 g_4)^k \, (2k)!\, {}_1F_2\,(2, k+3; 2k+1; \lambda)}{k!\,(k+2)!}$$

AGT dual



- A correlator $\langle \Phi(z_1, \bar{z}_1)\Phi(z_2, \bar{z}_2)\Phi(z_3, \bar{z}_3)\Phi(z_4, \bar{z}_4)\rangle$ gets contributions from the conformal blocks which are holomorphic

$$Z = 1 + \sum_k q^k Z_k = (1-q)^{\Delta(\alpha_2)}(1 + \sum_k q^k \mathcal{F}_k(q|\Delta_i))$$

AGT dual

- There is a natural basis $a_{-\vec{k}} L_{-\vec{\ell}} |P\rangle$ with $L_n |P\rangle = a_n |P\rangle = 0$ for $n > 0$

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- A basis $|P\rangle_{\vec{\lambda}} = \sum_{|\vec{\mu}|=|\vec{\lambda}|} C^{\mu_1,\mu_2}_{\vec{\lambda}} a_{-\mu_1}L_{-\mu_2}|P\rangle$ can be defined such that

$$Z_k \approx \sum_{\vec{\lambda}} {}_{\emptyset}\langle P|\mathcal{V}_{\alpha_2}|P'\rangle_{\vec{\lambda}}\ {}_{\vec{\lambda}}\langle P'|\mathcal{V}_{\alpha_3}|P\rangle_{\emptyset}$$
$$= \langle 0|\mathcal{V}_{\alpha_1}(\infty)\mathcal{V}_{\alpha_2}(1)\mathcal{V}_{\alpha_3}(q)\mathcal{V}_{\alpha_4}(0)|\rangle$$

AGT dual

- It also happens that the states $|P\rangle_{\vec{\lambda}}$ are the eigenstates for the system of mutually commuting integrals of motions

$$I_2 = L_0 - \frac{c}{24} + 2\sum_{k=1}^{\infty} a_{-k}a_k$$

$$I_3 = \sum_{k\neq 0} a_{-k}\, L_k + 2iQ\sum_{k=1}^{\infty} k a_{-k}\, a_k + \frac{1}{3}\sum_{i,j} a_i a_j\, a_{-i-j}$$

$$I_4 = 2\sum_{k=1}^{\infty} L_{-k}L_k + L_0^2 - \frac{c+2}{12} + 6\sum_{i+j\neq 0} L_{-i-j}a_i a_j +$$

$$12(L_0 - \frac{c}{24})\sum_{k=1}^{\infty} a_{-k}a_k 6iQ\sum_{k\neq 0} |k| a_{-k}\, L_k + 2(1-5Q^2)$$

$$\sum_{k=1}^{\infty} k^2 a_{-k}a_k + 6iQ\sum_{i,j}^{\infty} |i| a_i a_j a_{-i-j} + \sum_{i,j,k}^{\infty} : a_i a_j a_k a_{-i-j-k} :$$

AGT dual

- This is no surprise since the basis $|P\rangle_{\vec{\lambda}}$ can be written in terms of generalized Jack polynomials
- In turn these Jack polynomials are the eigenfunctions of the hamiltonian of the Calogero-Sutherland model
- This is an aspect of a correspondence between the Hilbert schemes of n points introduced before and Jack poly. The number $n = n_1 + \ldots n_k$ can be partitioned and corresponds to the element $p_{n_1} p_{n_2} \ldots p_{n_k} \in \mathbb{C}[p_1, p_2, \ldots]$ The cohomological degree is $\deg(p_k) = 2(k-1)$. Ex.$Hilb_4 \Longrightarrow H^6(Hilb_4) = 1, H^4(Hilb_4) = 2, H^2(Hilb_4) = 1, H^0(Hilb_4) = 1$

$$(4,0,0,0) \Longrightarrow \deg(p_4) = 6;\ (3,1,0,0) \Longrightarrow \deg(p_3 p_1) = 4$$

$$(2,2,0,0) \Longrightarrow \deg(p_2^2) = 4;\ (2,1,1,0) \Longrightarrow \deg(p_2 p_1^2) = 4$$

$$(1,1,1,1) \Longrightarrow \deg(p_1^4) = 0$$

AGT dual

- The eigenvalues are exactly those of $\mathrm{tr}\varphi^J$. We then computed

$$\mathcal{G}_n(\alpha_i|q)\langle 0|\mathcal{V}_{\alpha_1}(\infty)\mathcal{V}_{\alpha_2}(1)I_n\mathcal{V}_{\alpha_3}(q)\mathcal{V}_{\alpha_4}(0)|\rangle$$

to find

$$\mathcal{G}_n(\alpha_i,\alpha|q) = \mathcal{L}_n\mathcal{G}(\alpha_i,\alpha|q)$$

- The $\mathcal{L}_n$ are given by

$$\begin{aligned}\mathcal{L}_2 =& z\partial_z - \Delta - \tfrac{c}{24}\\ \mathcal{L}_3 =& \frac{z}{1-z}\left[(Q+\alpha_2-\alpha_3)\,z\,\partial_z + (Q-\alpha_3)(\Delta+\Delta_2-\Delta_1)\right.\\ &\left.-2\alpha_2(Q-\alpha_3)^2 + \alpha_2(\Delta-\Delta_3-\Delta_4)\right]\end{aligned}$$

AGT dual

- Using the gauge theory/CFT dictionary

$$\alpha_1 = \frac{\epsilon}{2} + \frac{1}{2}(m_1 - m_2) \qquad \alpha_2 = -\frac{1}{2}(m_1 + m_2)$$
$$\alpha_3 = \epsilon - \frac{1}{2}(m_1 + m_2) \qquad \alpha_4 = \frac{\epsilon}{2} + \frac{1}{2}(m_1 - m_2)$$
$$\alpha = \frac{\epsilon}{2} + a \qquad \epsilon = \epsilon_1 + \epsilon_2 = Q \qquad \epsilon_1 = b^{-1} \qquad \epsilon_2 = b$$

AGT dual

- We finally find (M_i are Casimirs)

$$\langle \mathrm{tr}\tilde{\varphi}^2\rangle = -2\epsilon_1\epsilon_2 q\partial_q \ln Z$$

$$\langle \mathrm{tr}\tilde{\varphi}^3\rangle = \frac{3q}{1-q}\left(-\frac{M_1}{2}\langle \mathrm{tr}\tilde{\varphi}^2\rangle + M_3\right)$$

$$\langle \mathrm{tr}\tilde{\varphi}^4\rangle = \frac{1+q}{2(1-q)}\langle \mathrm{tr}\tilde{\varphi}^2\rangle^2 + \Big[2a^2 + \epsilon_1\epsilon_2 - 2q^2\left(\frac{\epsilon_1\epsilon_2}{2} - a^2 - M_2 + M_1^2\right) + 2q(\epsilon M_1 + M_2)\Big]\langle \mathrm{tr}\tilde{\varphi}^2\rangle + \frac{4q}{(1-q)^2}[a^4 + a^2(\epsilon M_1 + M_2) + \epsilon M_3 + M_4 - q(a^4 - a^2(M_1^2 - M_2) - M_1M_3 + M_4)]$$

Minimal Models

- Given the localization formula

$$Z_{\text{inst}} = \sum_{Y} q^{|Y|} \prod_{u,v=1}^{N} \frac{Z_{\emptyset,Y_v}(\bar{m}_u - a_v)\, Z_{Y_u,\emptyset}(a_u - m_v)}{Z_{Y_u,Y_v}(a_u - a_v)}$$

where

$$Z_{\emptyset,Y_v}(\bar{m}_u - a_v) = \prod_{(i,j)\in Y_v} (\bar{m}_u - a_v - \epsilon_1(i-1) - \epsilon_2\,(j-1))$$

$$Z_{Y_u,\emptyset}(a_u - m_v) = \prod_{(i,j)\in Y_u} (a_u - m_v + \epsilon_1 i + \epsilon_2\, j)$$

- It is easy to realize that these functions are zero for $m_u = a_u + p_u\epsilon_1 + q_u\epsilon_2$ or $\bar{m}_u = a_u + (p_u - 1)\epsilon_1 + (q_u - 1)\epsilon_2$

Minimal Models

- In particular for the choice $m_u = a_u + \epsilon + \epsilon_2$ one gets

$$Z_{\text{inst}} = {}_N F_{N-1}\left(\begin{smallmatrix}\mathbf{A}\\ \mathbf{B}\end{smallmatrix}\middle| q\right)$$

where

$$\mathbf{A}_v = \frac{a_1 - \bar{m}_v}{\epsilon_1} = \frac{m_1 - \bar{m}_v - 2\epsilon_2}{\epsilon_1} - 1 \qquad v = 1, \dots N$$

$$\mathbf{B}_v = \frac{a_1 - a_v + \epsilon_2}{\epsilon_1} + 1 = \frac{m_1 - m_v}{\epsilon_1} + 1 \qquad v = 2, \dots N$$

- On the AGT side this corresponds to degenerated primary fields ϕ_{nm} leading to null states $L_{nm}\phi_{nm}$

Minimal Models

- Given our previous choice $n_1 = -\ell p$, $n_2 = \ell q$ we get $\epsilon_1/\epsilon_2 = -p/q$ the central charges

$$c = 1 - \frac{6(p-q)^2}{pq}$$

and the dimension $\Delta_{n,m} = \alpha_{n,m}(Q - \alpha_{n,m})$ of the primary fields of the minimal models with $Q = b + 1/b$, $b = i\sqrt{p/q}$ and

$$\alpha_{n,m} = b\frac{1-n}{2} + \frac{1-m}{2b}$$

- The vev $a = Q/2 - \alpha$ and the correlators follow

Chiral ring from curves

- In the SW theory a crucial role is played by $u_l = \mathrm{tr}\varphi^l = P(\mathrm{tr}\varphi, \dots, \mathrm{tr}\varphi^N)$ for $l > N$ and $SU(N)$.
  Ex.$\mathrm{tr}\varphi^3 = (\mathrm{tr}\varphi)^3 - 3/2\mathrm{tr}\varphi[(\mathrm{tr}\varphi)^2 - \mathrm{tr}\varphi^2]$
- Given $P_N(z) = \det(z - \varphi)$ then classically

$$\mathrm{tr}\frac{1}{z-\varphi} = \frac{P'_N(z)}{P_N(z)}$$

- At the quantum level using the Konishi anomaly we get (for $SU(2)$, $\quad P(z) = (z^2 - a^2)$)

$$\langle \mathrm{tr}\frac{1}{z-\varphi}\rangle = \frac{1}{z} + \frac{\mathrm{tr}\varphi^2}{z^3} + \frac{\mathrm{tr}\varphi^4}{z^5} + \dots =$$

$$\frac{P'_N(z)}{\sqrt{P_N^2(z) - 4q^{N/2}}} = \frac{2}{z} + \frac{2a^2}{z^3} + \frac{2(a^4+2q)}{z^5} + \dots$$

Chiral ring from curves

- From which given $u = 2a^2$

$$\langle \mathrm{tr}\varphi^2 \rangle = u$$

$$\langle \mathrm{tr}\varphi^4 \rangle = \frac{\langle (\mathrm{tr}\varphi^2 \rangle)^2}{2} + 4q$$

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$$-q\, Q(z-\epsilon)\, y(z)\, y(z+\epsilon) + (1+q)\, P(z) y(z+\epsilon) - 1 = 0$$

with $P(z) = z^2 + u_1 z + u_2 \quad Q(z) = 1 + \sum_{\ell=1}^{4} M_\ell\, z^\ell$

Chiral ring from curves

- Now given $y(z+\epsilon) = y_2/z^2 + y_3/z^3 + \ldots$ we have

$$\partial_z \log y(z+\epsilon) = \left\langle \mathrm{tr} \frac{1}{z-\tilde{\varphi}} \right\rangle = \frac{2}{z} + \left\langle \frac{\mathrm{tr}\tilde{\varphi}}{z^2} \right\rangle + \left\langle \frac{\mathrm{tr}\tilde{\varphi}^2}{z^3} \right\rangle + \ldots$$

$$= \frac{2}{z} + \frac{y_3}{z^2} + \frac{-y_3^2 + 2\,y_4}{z^3} + \frac{y_3^3 - 3\,y_3\,y_4 + 3\,y_5}{z^4}$$

$$+ \frac{-y_3^4 + 4\,y_3^2\,y_4 - 2\,y_4^2 - 4\,y_3\,y_5 + 4\,y_6}{z^5} + \ldots$$

and, from the curve another relation for the y_i's in terms of u_1, u_2. Now $\langle \mathrm{tr}\tilde{\varphi} \rangle = 0$ requires $y_3 = 0$ and determines u_1 while u_2 is solved in terms of $\langle \mathrm{tr}\tilde{\varphi}^2 \rangle$. This leads to the same results we found previously.

Conclusions

- Circular Wilson loops are strongly connected to the equivariant scalar field of $N = 2$ SUSY
- We have studied correlators of $\mathrm{tr}\varphi^J$
- The AGT dual gives a nice framework to compare and further investigate such results
- Extension to non circular geometries?