

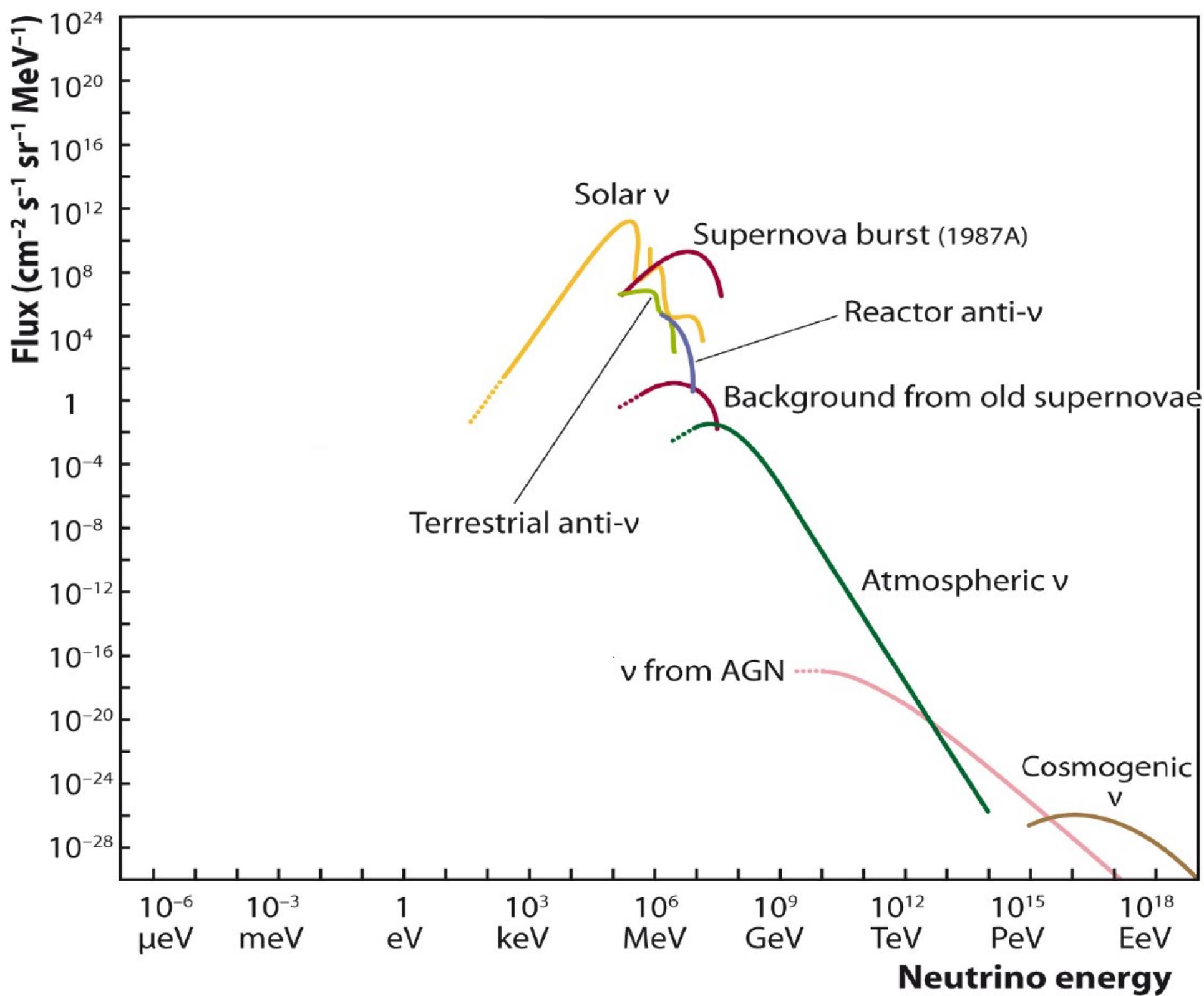
Neutrino Cosmology (I)

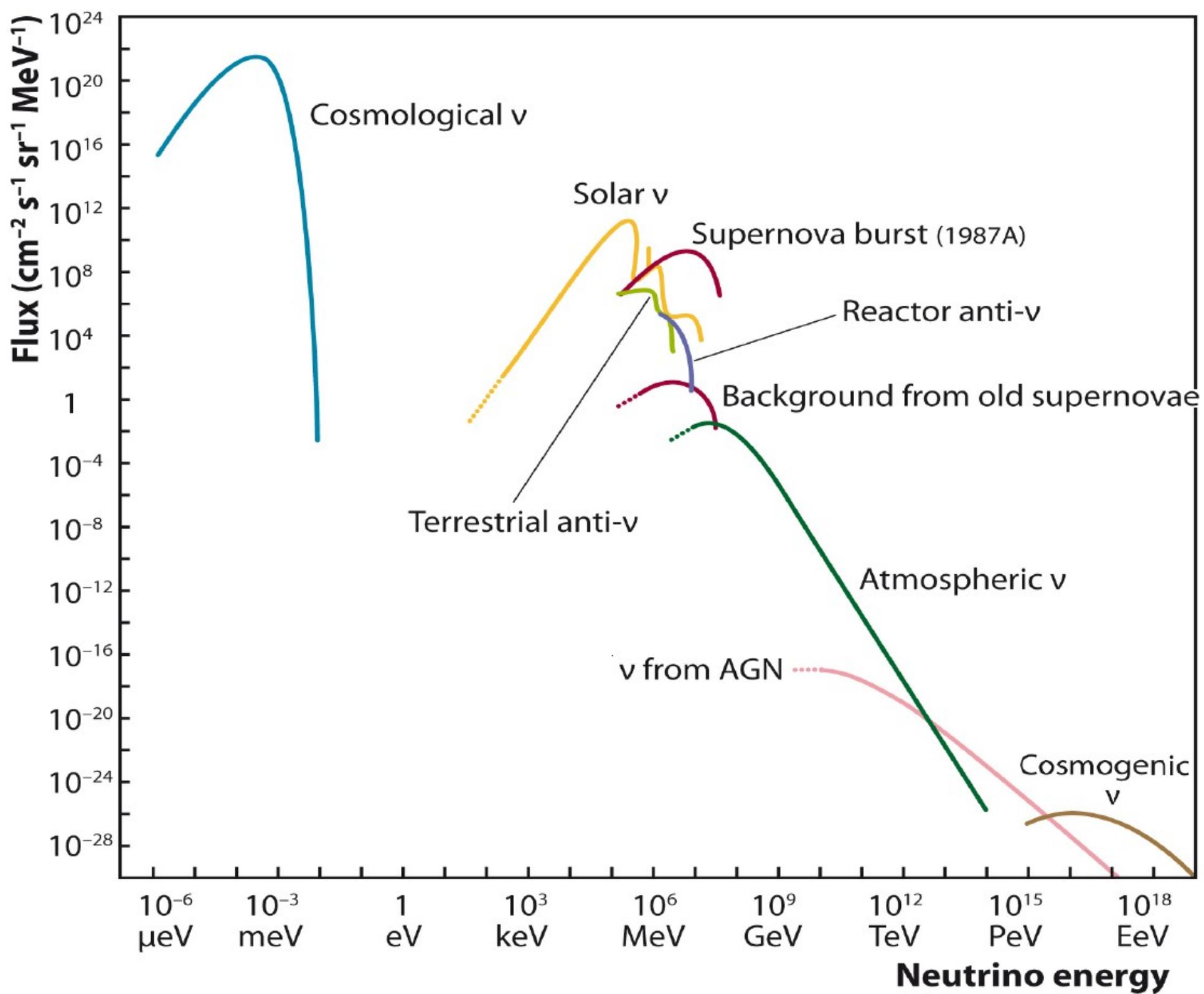
Stanford, August 11th 2015

Alessandro Melchiorri

University of Rome “La Sapienza”

(Disclaimer: many thanks to Eleonora Di Valentino and Sergio Pastor for some of the slides)





Cosmology: Basics

Isotropic and homogeneous expansion of the Universe (at zeroth order) determined by the scale factor $a(t)$, that just depends on time.

The scale factor solves the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}(\rho_R + \rho_M + \rho_\Lambda)$$

We have just 3 types of Energy components !

Energy density in RADIATION (as CMB photons) scales as:

$$\rho_R = \frac{\rho_R^0}{a^4} \longrightarrow a(t) = \sqrt{\frac{t}{t_0}}$$

Energy density in MATTER (CDM and baryons) scales as:

$$\rho_M = \frac{\rho_M^0}{a^3} \longrightarrow a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

Energy density in cosmological constant scales as:

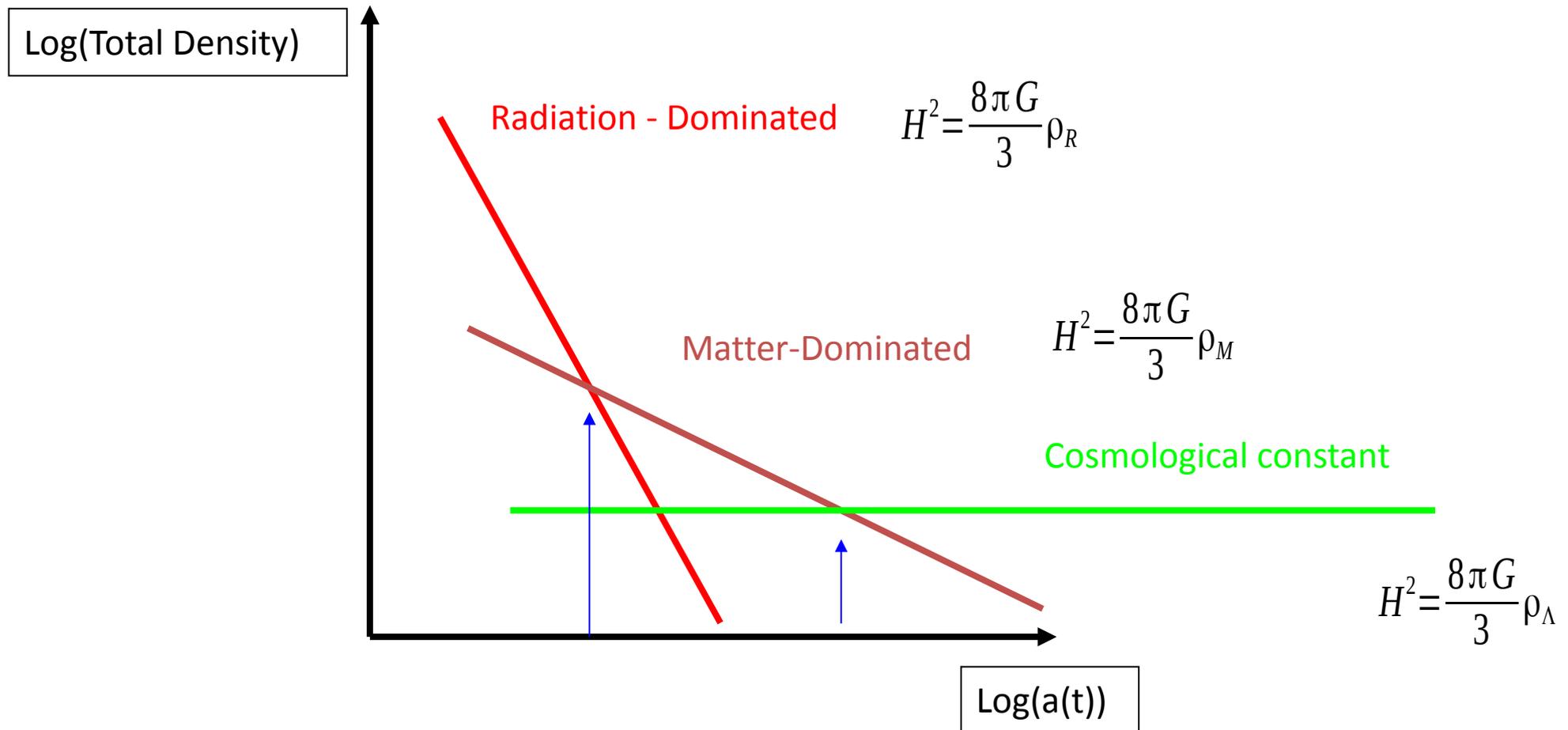
$$\rho_\Lambda = \rho_\Lambda^0 \longrightarrow a(t) = e^{(H_0(t-t_0))}$$

Neutrinos are the only particle in the standard cosmological model that change status: they move from radiation to matter during the evolution of the universe !

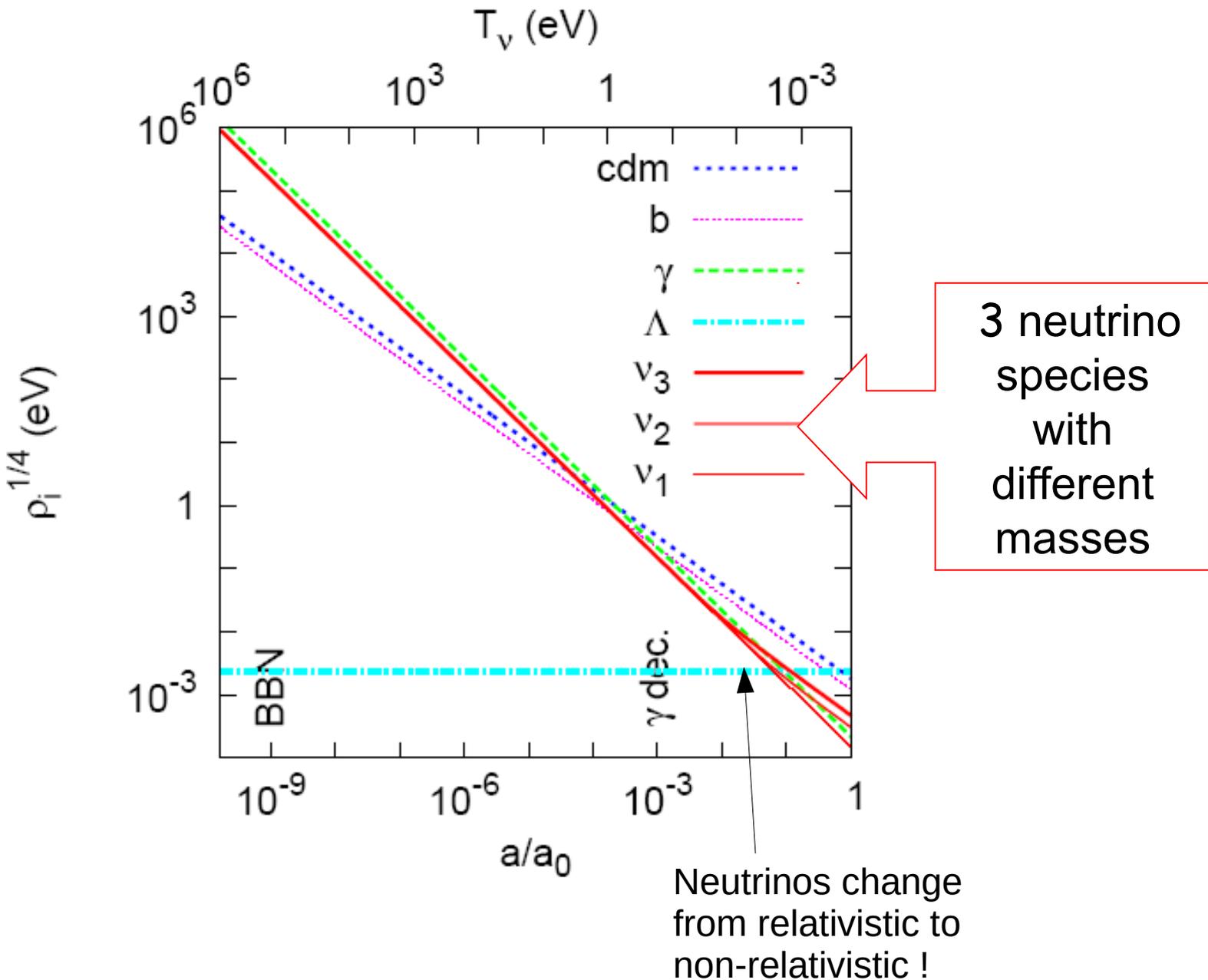
Multiple-Component Universe

If the different components do not interact between themselves then each energy component evolves independently

Thus the total energy density of the Universe is dominated by each component at different epochs: radiation (early universe), matter (late universe), c.c. (today).



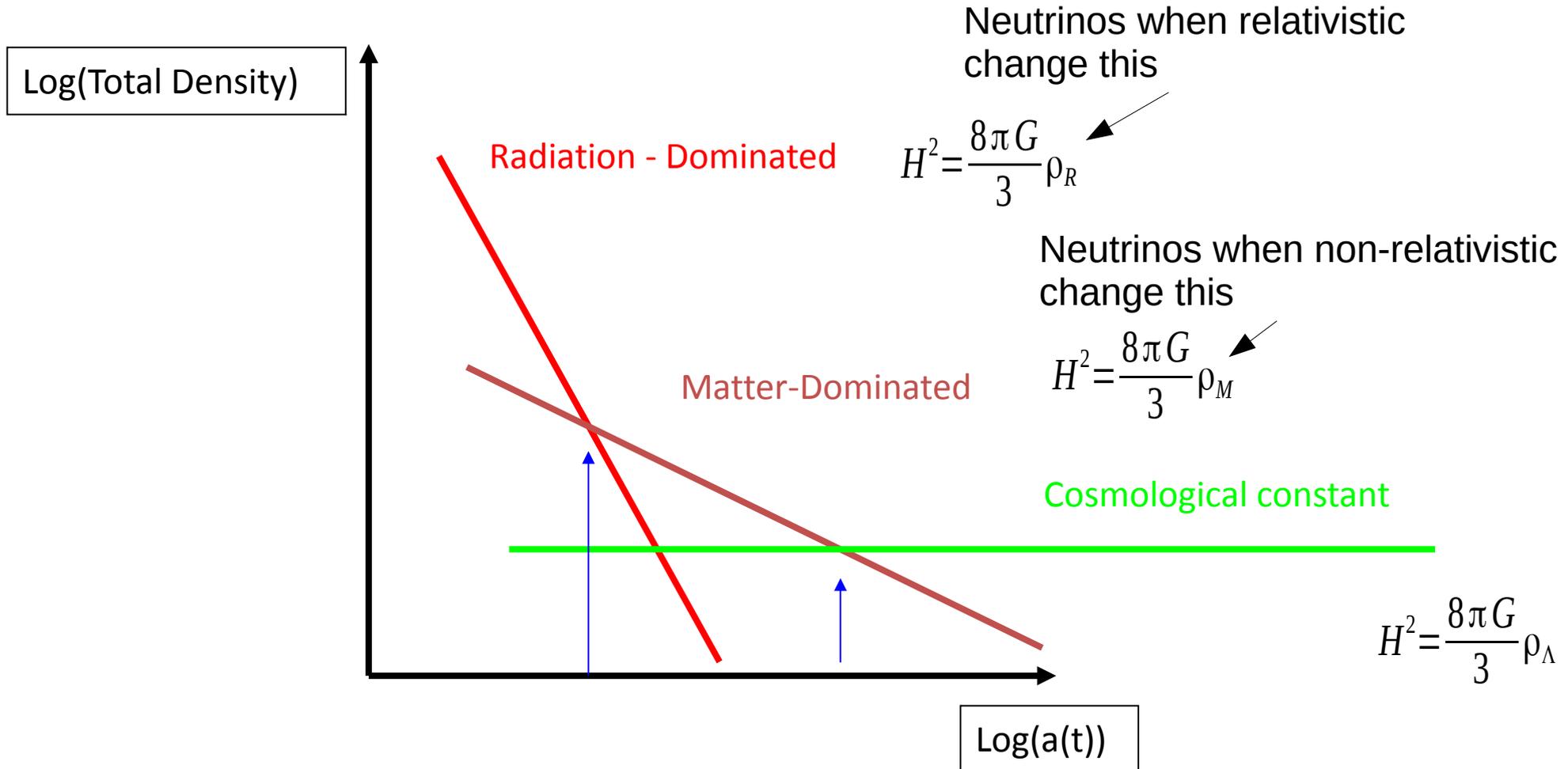
Evolution of the background densities: 1 MeV \rightarrow now



Multiple-Component Universe

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Relativistic Neutrinos $T \gg m_\nu$

Neutrinos in the Early Universe

- Neutrinos at temperature $T > 1 \text{ MeV}$ are in equilibrium with the primordial plasma made by photons, electrons and positrons through weak interactions. Neutrinos follow a **Fermi-Dirac** distribution with the **same temperature T of the plasma.**

$$\nu_\alpha \nu_\beta \leftrightarrow \bar{\nu}_\alpha \bar{\nu}_\beta$$

$$\nu_\alpha \bar{\nu}_\beta \leftrightarrow \bar{\nu}_\alpha \nu_\beta$$

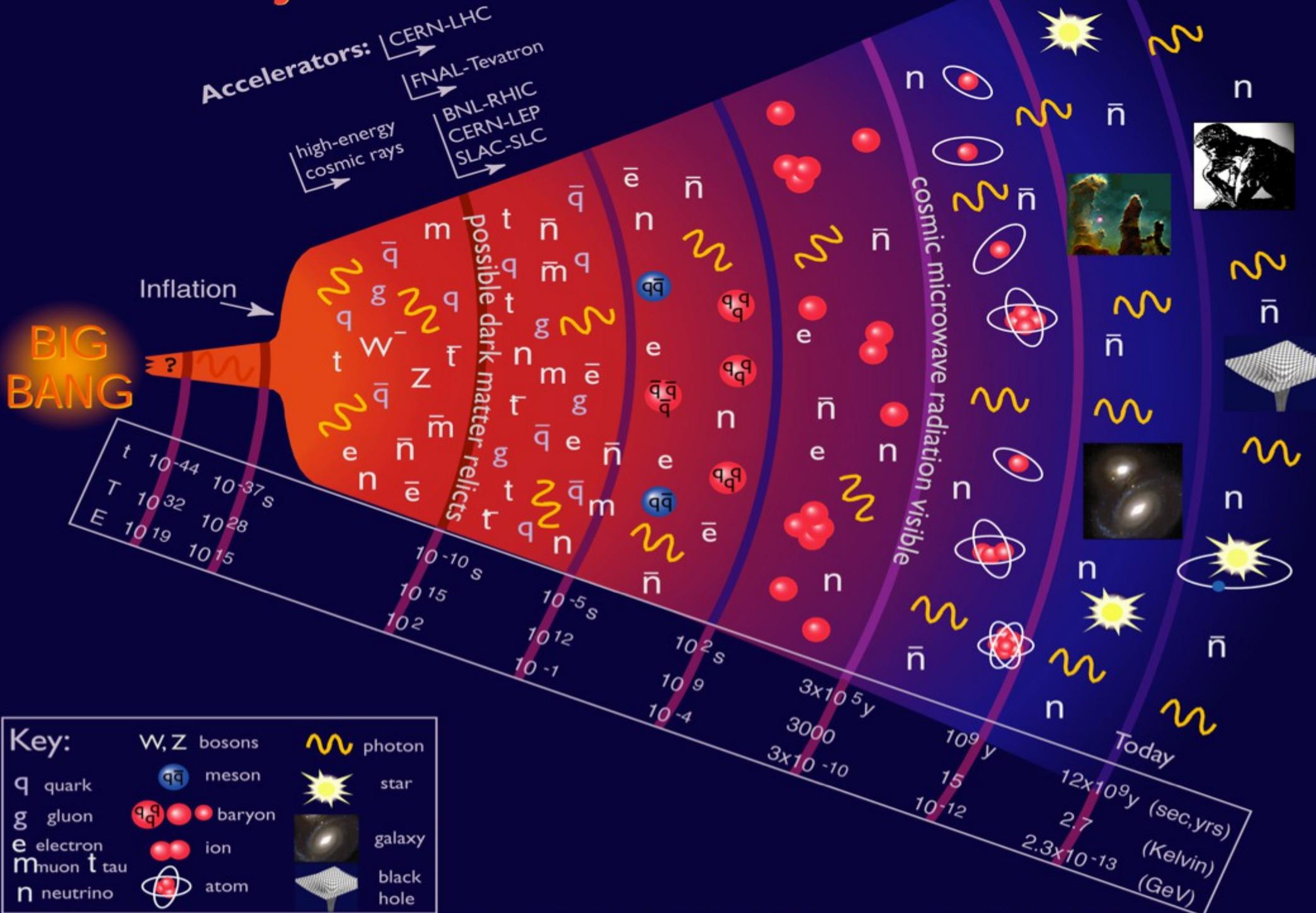
$$\nu_\alpha e^- \leftrightarrow \bar{\nu}_\alpha e^-$$

$$\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$$

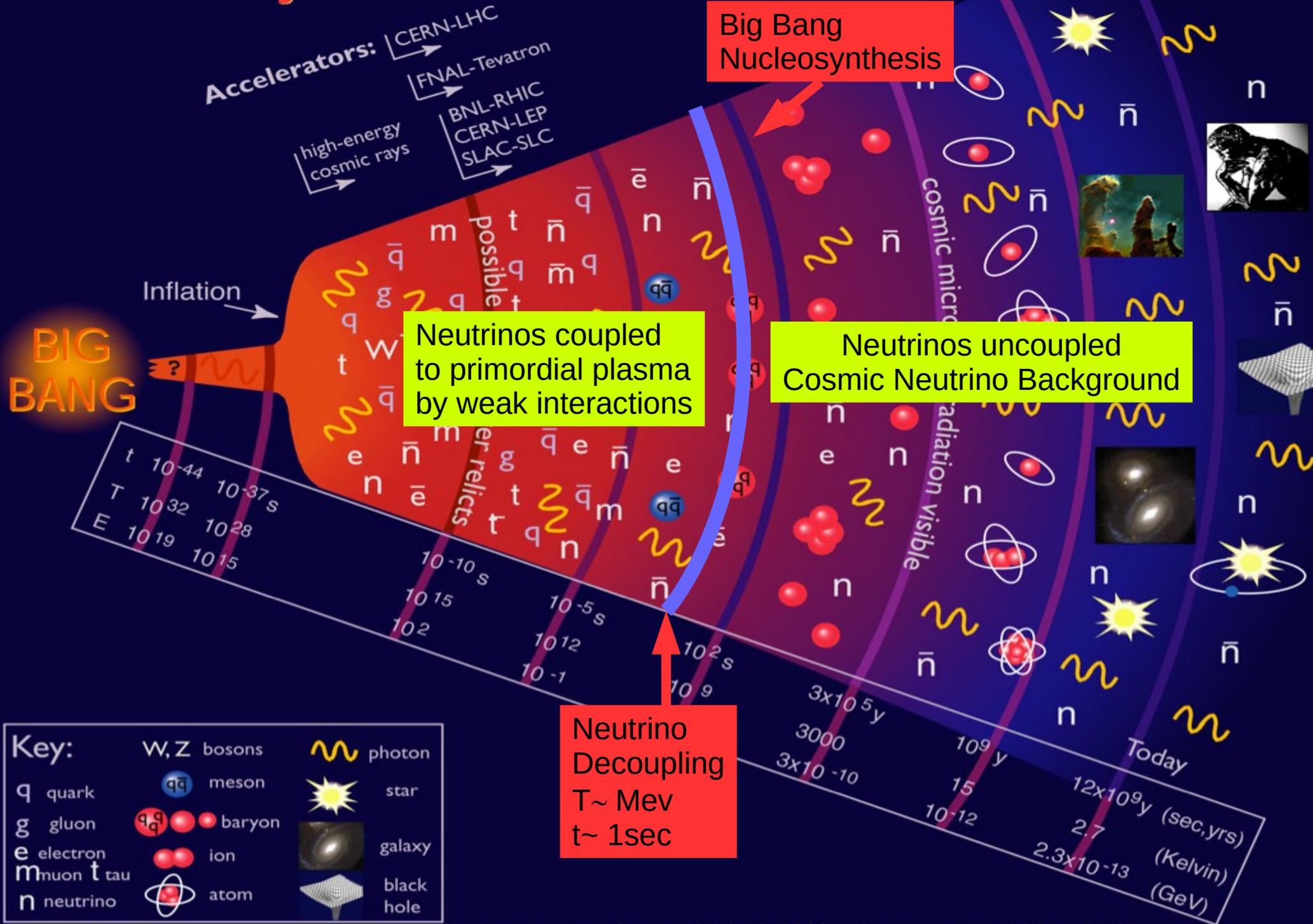
$$f_\nu(p, T) = \frac{1}{e^{p/T} + 1}$$

- At those temperatures neutrino are **relativistic** !

History of the Universe



History of the Universe



Neutrino Decoupling

As the Universe expands, particle densities are diluted and temperatures fall. Weak interactions become ineffective to keep neutrinos in good thermal contact with the e.m. plasma

Rough, but quite accurate estimate of the decoupling temperature

Rate of weak processes \sim Hubble expansion rate

$$\Gamma_w \approx \sigma_w |v| n, H^2 = \frac{8\pi\rho_R}{3M_p^2} \rightarrow G_F^2 T^5 \approx \sqrt{\frac{8\pi\rho_R}{3M_p^2}} \rightarrow T_{dec}^v \approx 1 \text{ MeV}$$

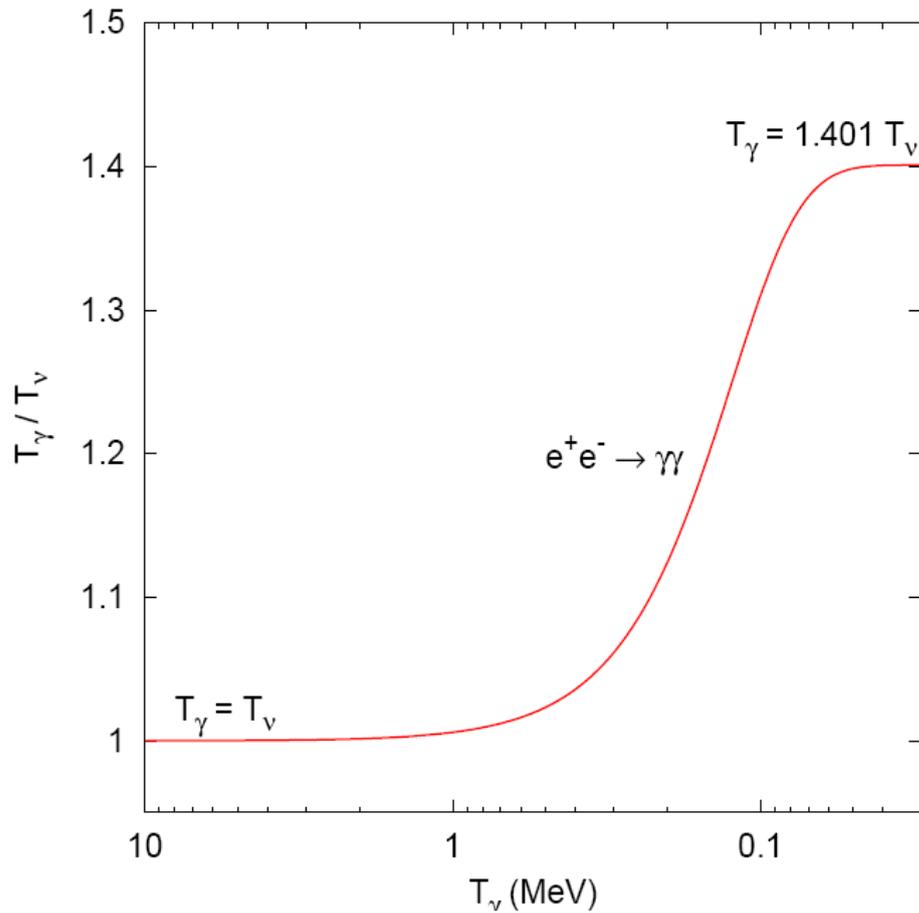
Neutrino Decoupling

- After decoupling, neutrinos keep the Fermi-Dirac distribution with temperature scaling as $1/a(t)$.
- Since also the e.m. plasma temperature scales as $1/a(t)$, e.m. plasma and the neutrino background share the same temperature even if they are decoupled.
- However at $T \sim m_e$ **electron-positron pairs annihilate** heating photons but not the decoupled neutrinos.

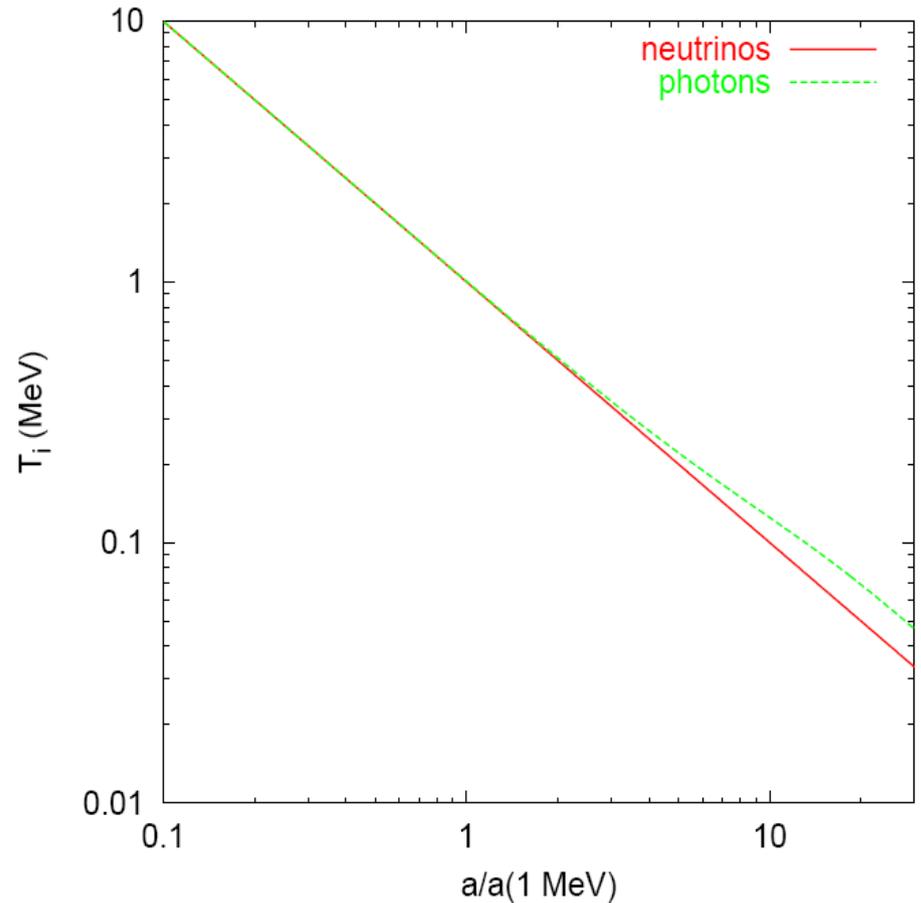
$$e^+ e^- \rightarrow \gamma\gamma$$

CMB and CNB temperatures

The photon temperature **increases** respect to neutrino temperature.



The photon temperature **falls slower** than $1/a(t)$.



CMB and CNB temperatures

- A classic result that assumes instantaneous neutrino decoupling (see, for example, Dodelson Modern Cosmology) gives:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

- Since for a relativistic gas of fermions and bosons we have:

$$\rho_\nu = \frac{7}{8} \frac{\pi^2}{30} g_\nu T_\nu^4 \quad \rho_\gamma = \frac{\pi^2}{30} g_\gamma T_\gamma^4$$

- and, since $g_\nu = 2 \cdot N_\nu$, we have:

$$\rho_{rad} = \rho_\gamma + \rho_\nu = \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_\nu\right] \rho_\gamma$$

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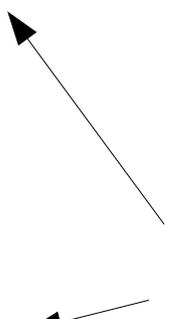
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We know this !

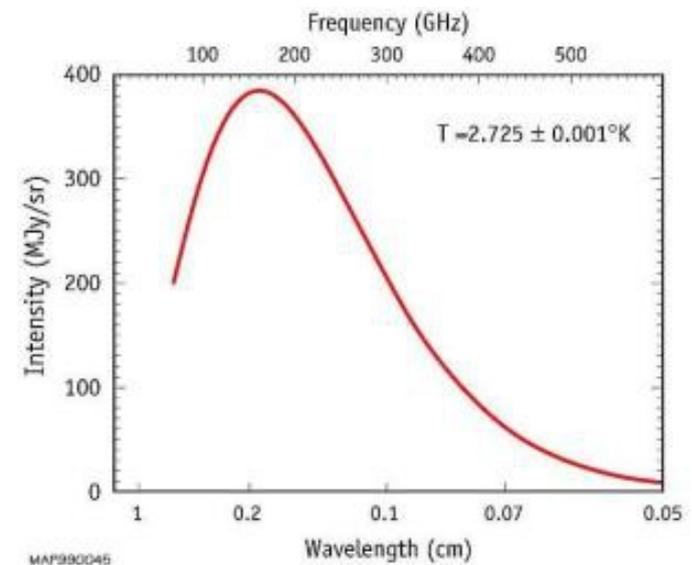


The Cosmic Microwave Background

Discovered By Penzias and Wilson in 1965.

It is an image of the universe at the time of recombination (near baryon-photons decoupling), when the universe was just a few thousand years old ($z \sim 1000$).

The CMB frequency spectrum is a perfect blackbody at $T=2.726$ K: this is an outstanding confirmation of the hot big bang model.



CMB and CNB temperatures

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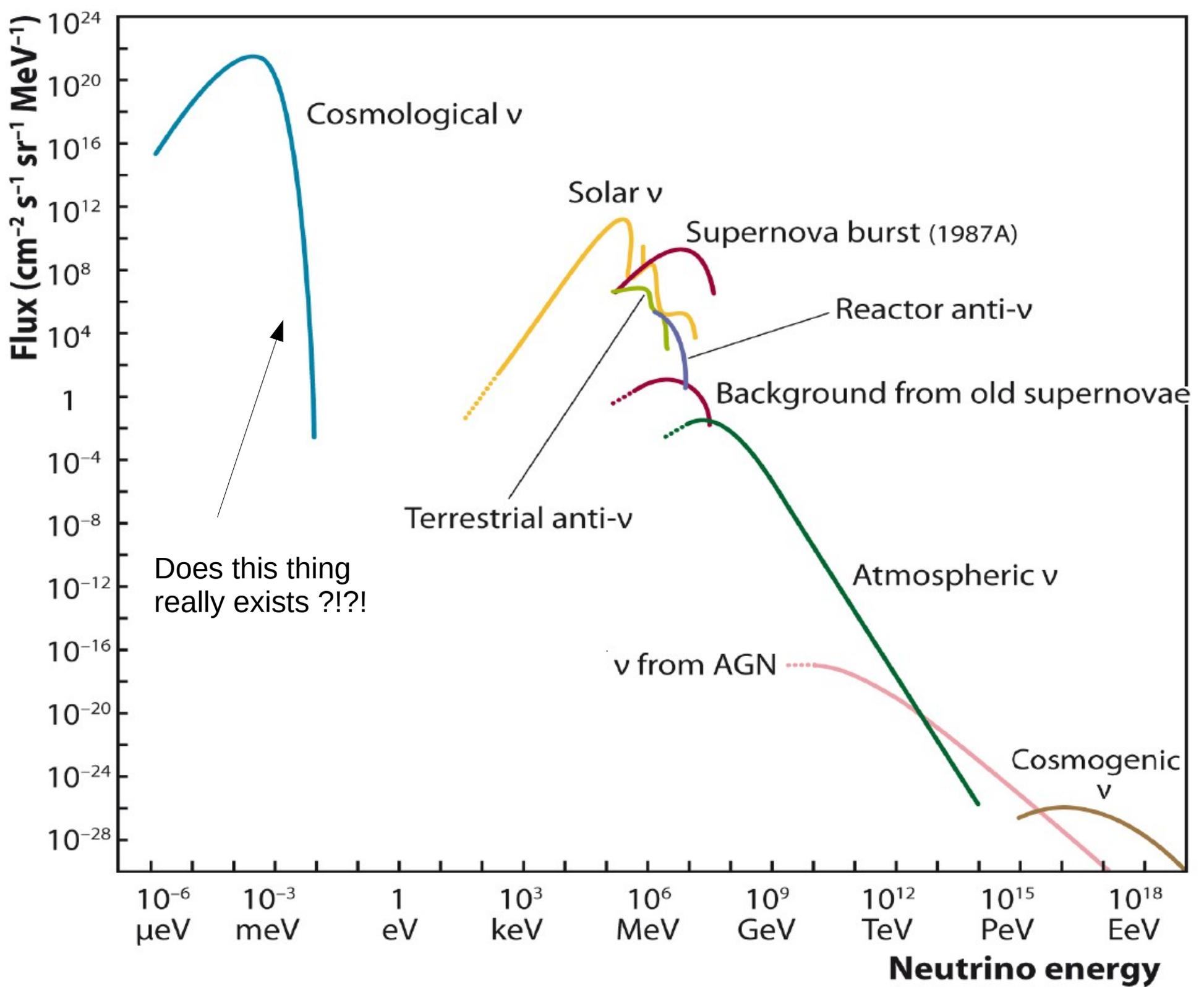
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Neutrino number, if zero NO CNB

We know this !



Neutrinos and Hubble rate

Neutrinos change the Hubble rate during the radiation dominated epoch (or when the radiation component is still not negligible respect to matter):

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_\nu\right] \rho_\gamma$$

NO CNB means **smaller** Hubble rate,
more neutrinos means **larger** Hubble rate.
This has important implications:

- If we change N_ν during BBN we change the Hubble rate and the final (post-BBN) abundance of primordial nuclides.
- If we change N_ν at CMB, the relativistic component is non-negligible, we change the Hubble rate and two important scales to the creation of CMB anisotropies: the sound horizon and the damping scales.
- Moreover, if we change N_ν we change the epoch of matter-radiation equality and structure formation.

Caveat

- Neutrino decoupling is, in reality, non-instantaneous.
a more correct formula is given by:

$$\rho_{rad} = \rho_\gamma + \rho_\nu = \left[1 + \frac{7}{8} \left(\frac{46}{125} \right)^{4/3} N_\nu \right] \rho_\gamma$$

- But since cosmologists are conservative we assume the standard equation:

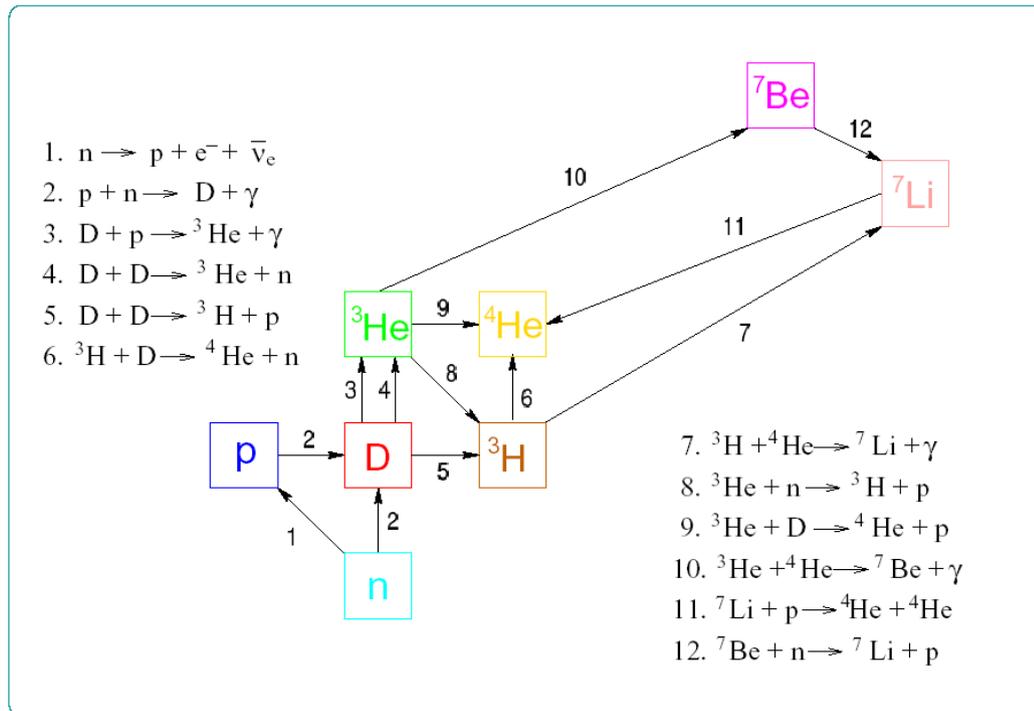
$$\rho_{rad} = \rho_\gamma + \rho_\nu = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{eff} \right] \rho_\gamma$$

- But where, for the standard 3 neutrinos framework, we have:

$$N_{eff} = 3.046$$

Testing N_{eff} with cosmology

Big Bang Nucleosynthesis

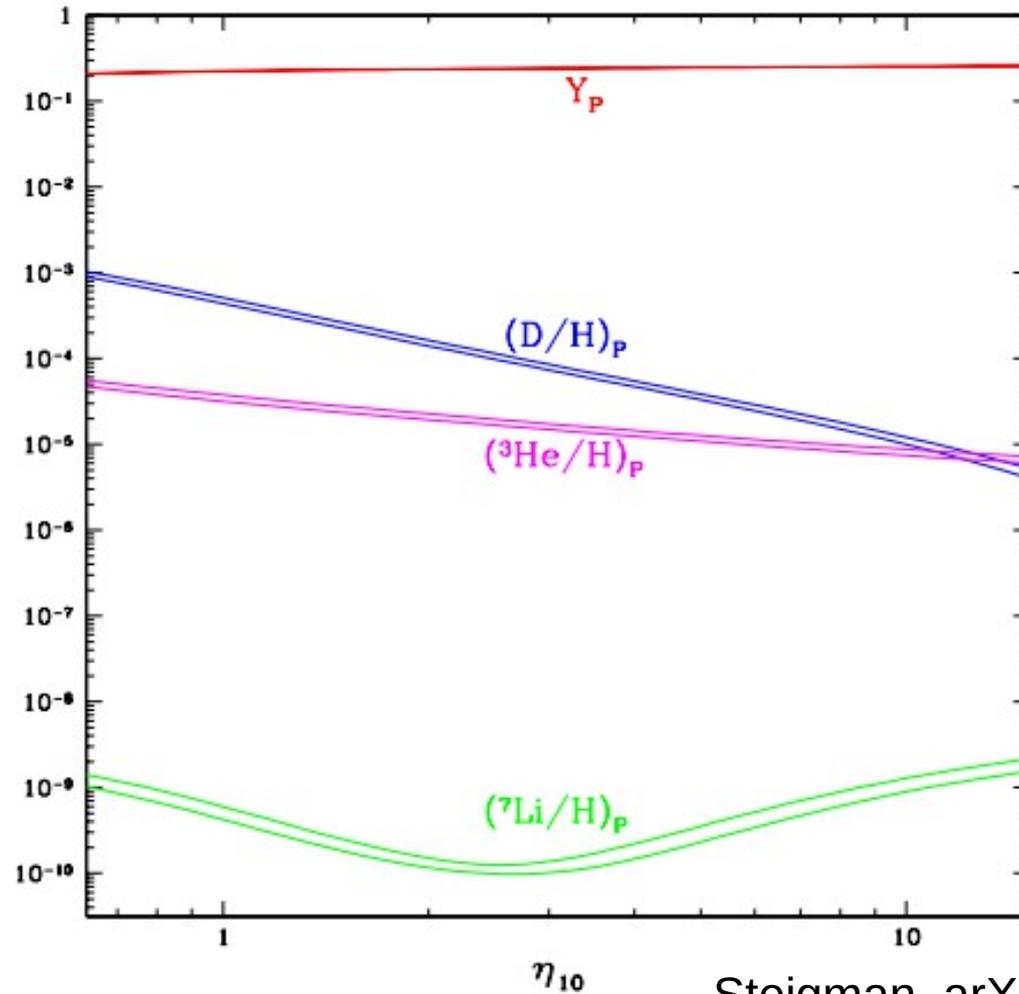


Produced elements: D, ${}^3\text{He}$, ${}^4\text{He}$, ${}^7\text{Li}$ and small abundances of others

Theoretical inputs:

- τ_n , the neutron lifetime;
- G_N , the Newton gravitational constant;
- η , the baryon to photon number density ratio;
- the nuclear rates.

Big Bang Nucleosynthesis



$$\eta \equiv \frac{N_b}{N_\gamma} \simeq 2.75 \times 10^{-8} \Omega_b h^2$$

Steigman, arXiv:astro-ph/0307244

In the standard 3 neutrinos framework Big Bang Nucleosynthesis essentially relates the primordial abundance of light elements to the baryon density parameter Ω_b .

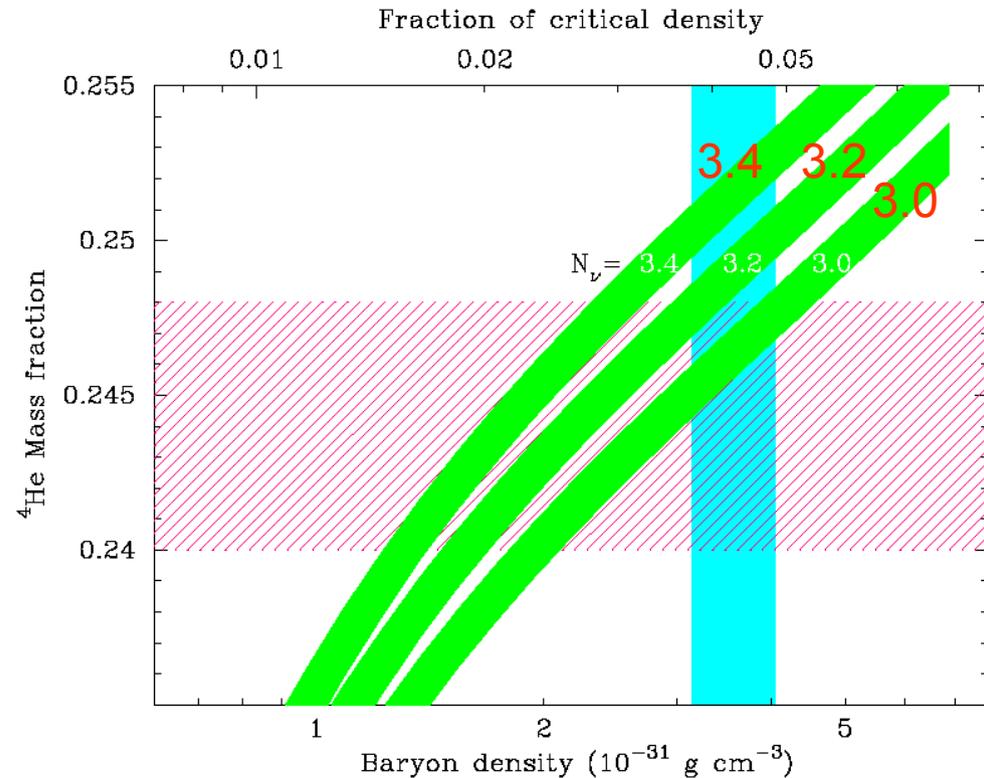
Constraining N_{eff} with BBN

1. N_{eff} fixes the expansion rate during BBN

$$H = \sqrt{\frac{8\pi\rho}{3M_p^2}}$$

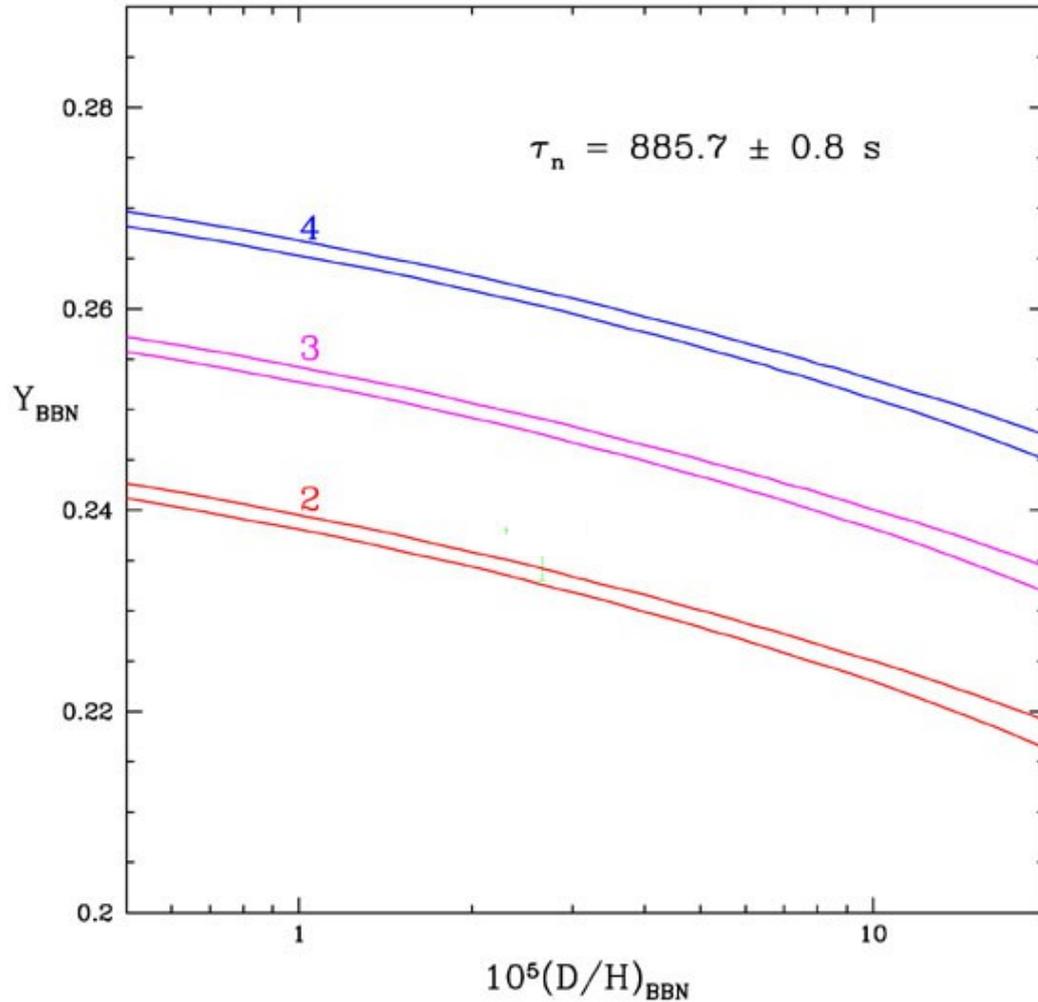
$\rho(N_{\text{eff}}) > \rho_0 \rightarrow \uparrow {}^4\text{He}$

Burles, Nollett & Turner 1999



For the same value of the baryon abundance, increasing the effective neutrino number increases the Helium abundance.

Constraining N_{eff} with BBN



Clearly, measuring Helium abundance alone is not enough to determine N_{eff} since we have to also determine the baryon abundance.

We can however combine with Deuterium measurement and have a BBN only determination.

Steigman, arXiv:astro-ph/0307244

BBN: Measurements of Primordial Abundances

Difficult task: search in astrophysical systems with chemical evolution as small as possible

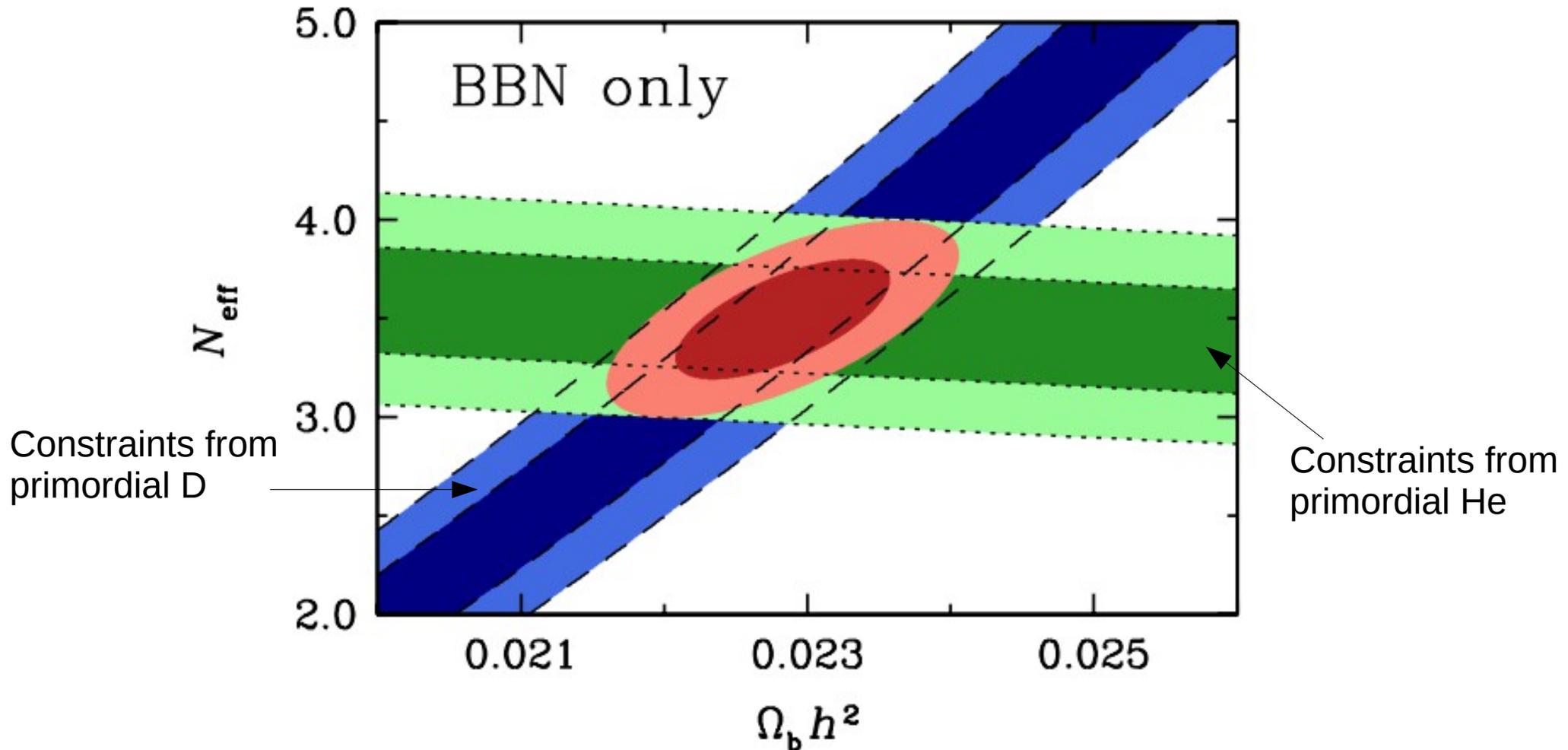
Deuterium: destroyed in stars. Any observed abundance of D is a *lower* limit to the primordial abundance. Data from high-*z*, low metallicity QSO absorption line systems

Helium-3: produced and destroyed in stars (complicated evolution) Data from solar system and galaxies but not used in BBN analysis

Helium-4: primordial abundance increased by H burning in stars. Data from low metallicity, extragalactic HII regions

Lithium-7: destroyed in stars, produced in cosmic ray reactions. Data from oldest, most metal-poor stars in the Galaxy

Current Constraints from BBN only



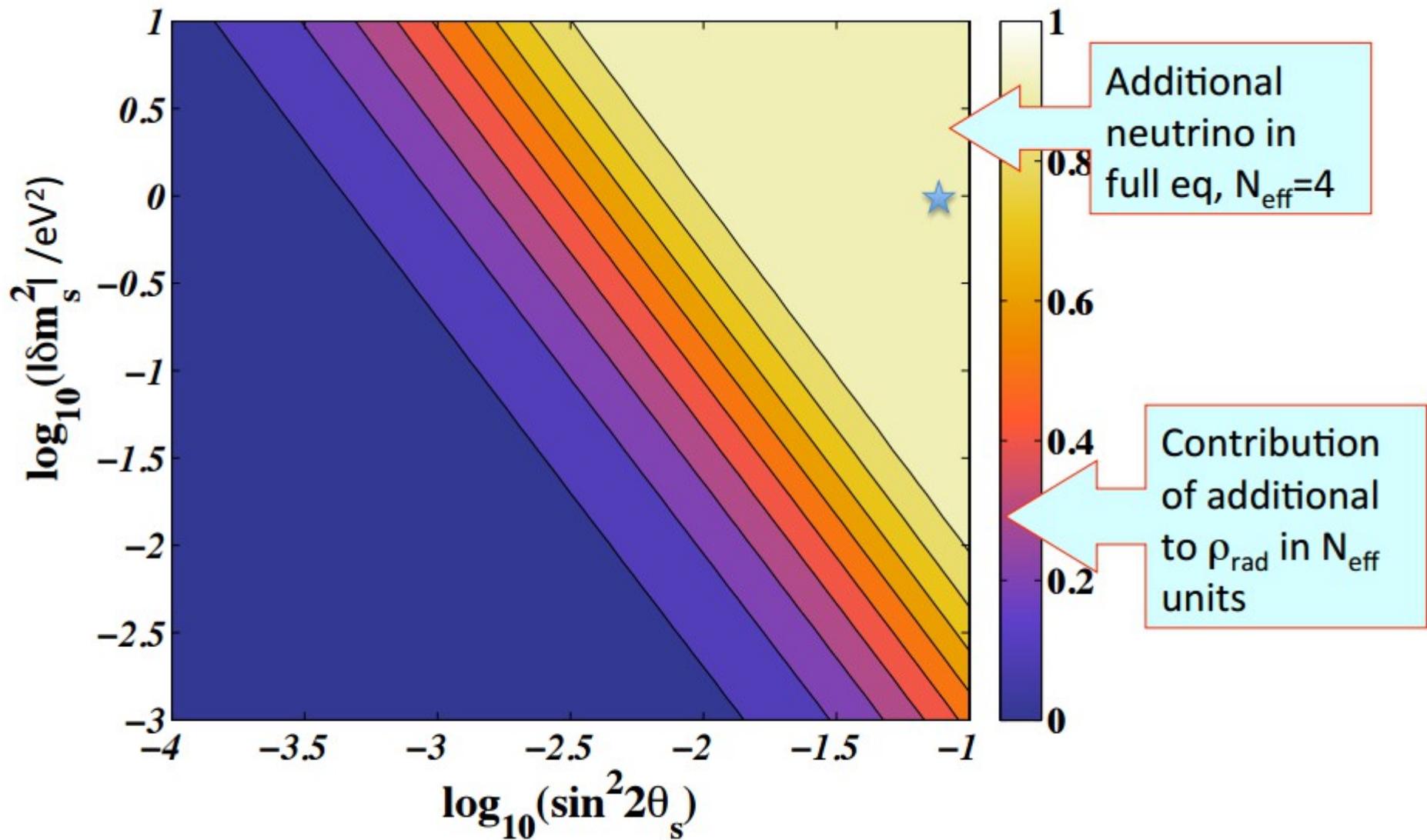
Neutrino Background needed
at about 17 sigmas !

$$100 \Omega_{b,0} h^2 = 2.28 \pm 0.05$$

$$N_{\text{eff}} = 3.50 \pm 0.20 .$$

Mild indication for
 $N_{\text{eff}} > 3.046$

Can this be an extra sterile neutrino ?



Hannestad, Tamborra & Tram, JCAP 07 (2012) 025

Star: preferred sterile neutrino model from:

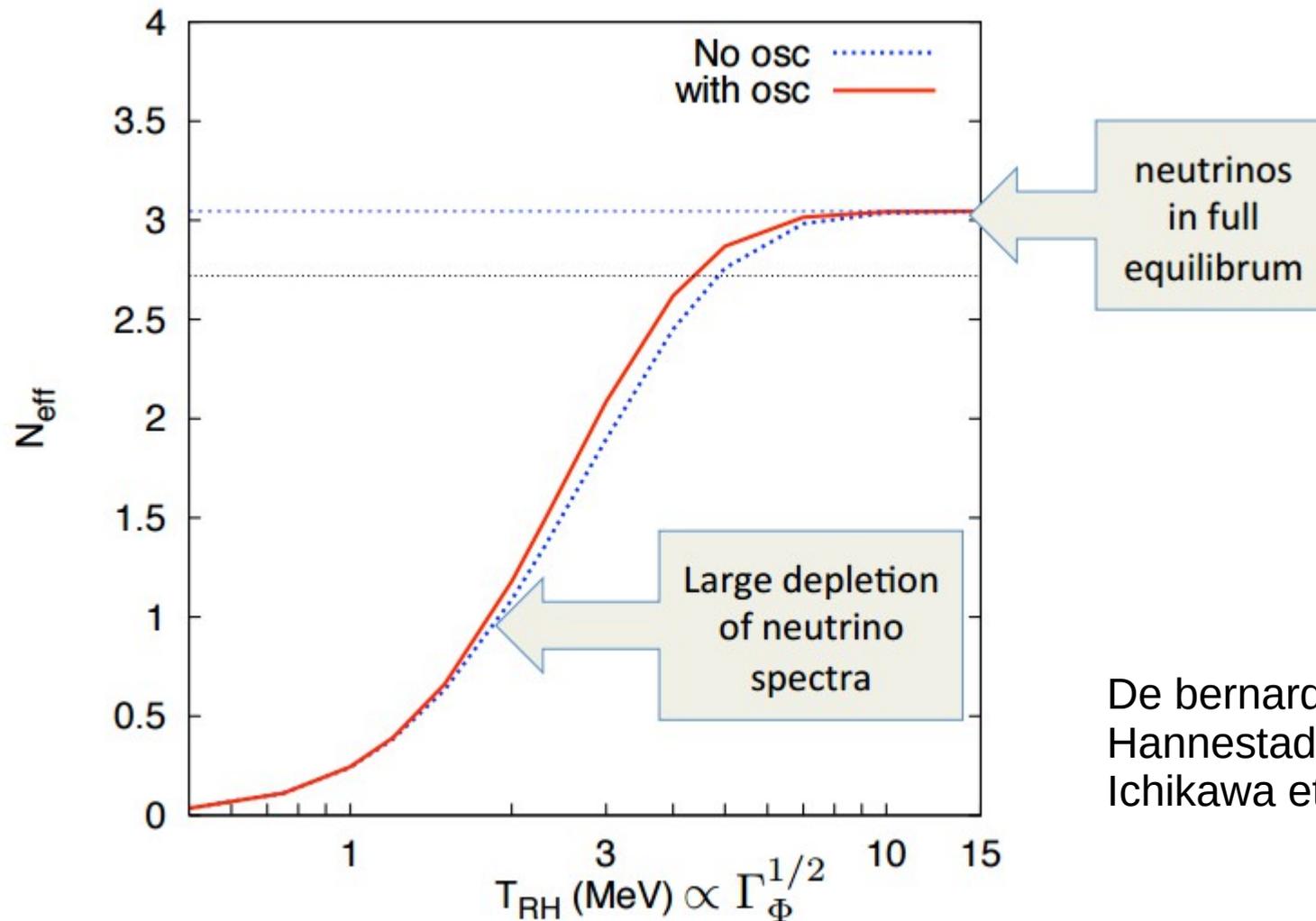
C. Giunti and M. Laveder, Phys.Lett. B706 (2011) 200–207, arXiv:1111.1069

Mechanisms for having $N_{\text{eff}} > 3.046$

- Non-standard decoupling (Mangano et al. 2006)
 $N_{\text{eff}} \sim 3.04 - 3.1$
- Thermal axions (see eg. Di Valentino et al. 2015)
- Gravitational waves (Smith et al., 2006)
- Extra dimensions
-

Mechanisms for having $N_{\text{eff}} < 3.046$

- Inflationary reheating temperature at $\sim \text{MeV}$ scales.

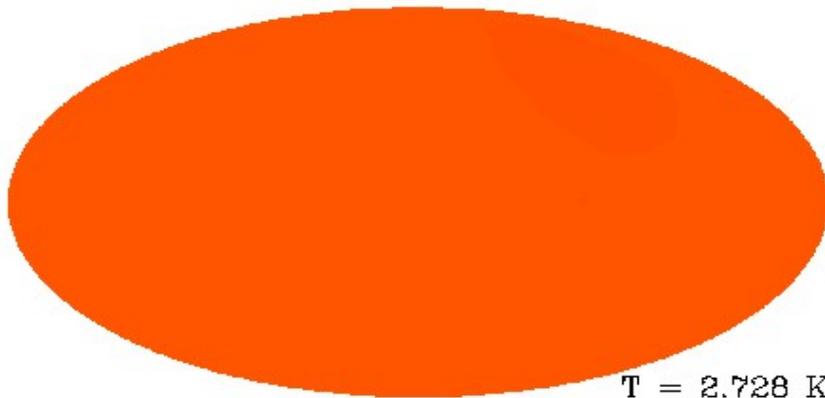


De bernardis et al., 2008
Hannestad, 2005
Ichikawa et al, 2004

Testing Neff with CMB anisotropies

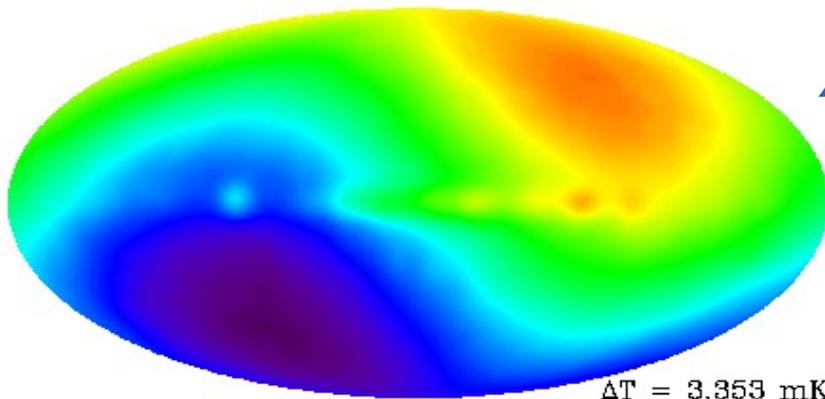
The Microwave Sky

COBE (circa 1995) @90GHz



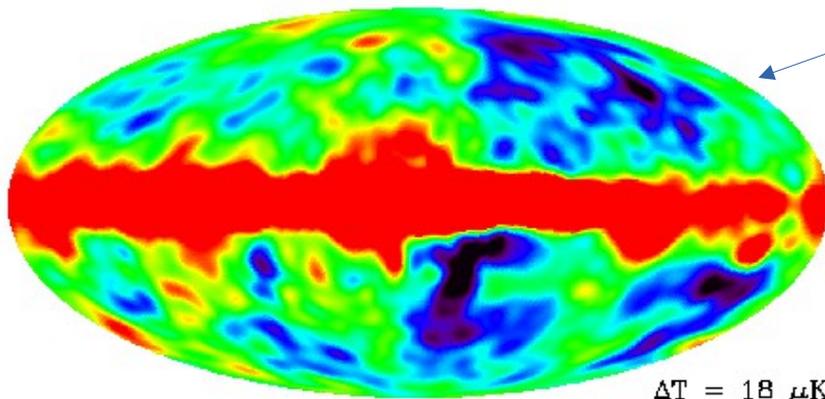
Uniform...

First Anisotropy we see is a Dipole anisotropy:
Implies solar-system barycenter has velocity $v/c \sim 0.00123$ relative to 'rest-frame' of CMB.

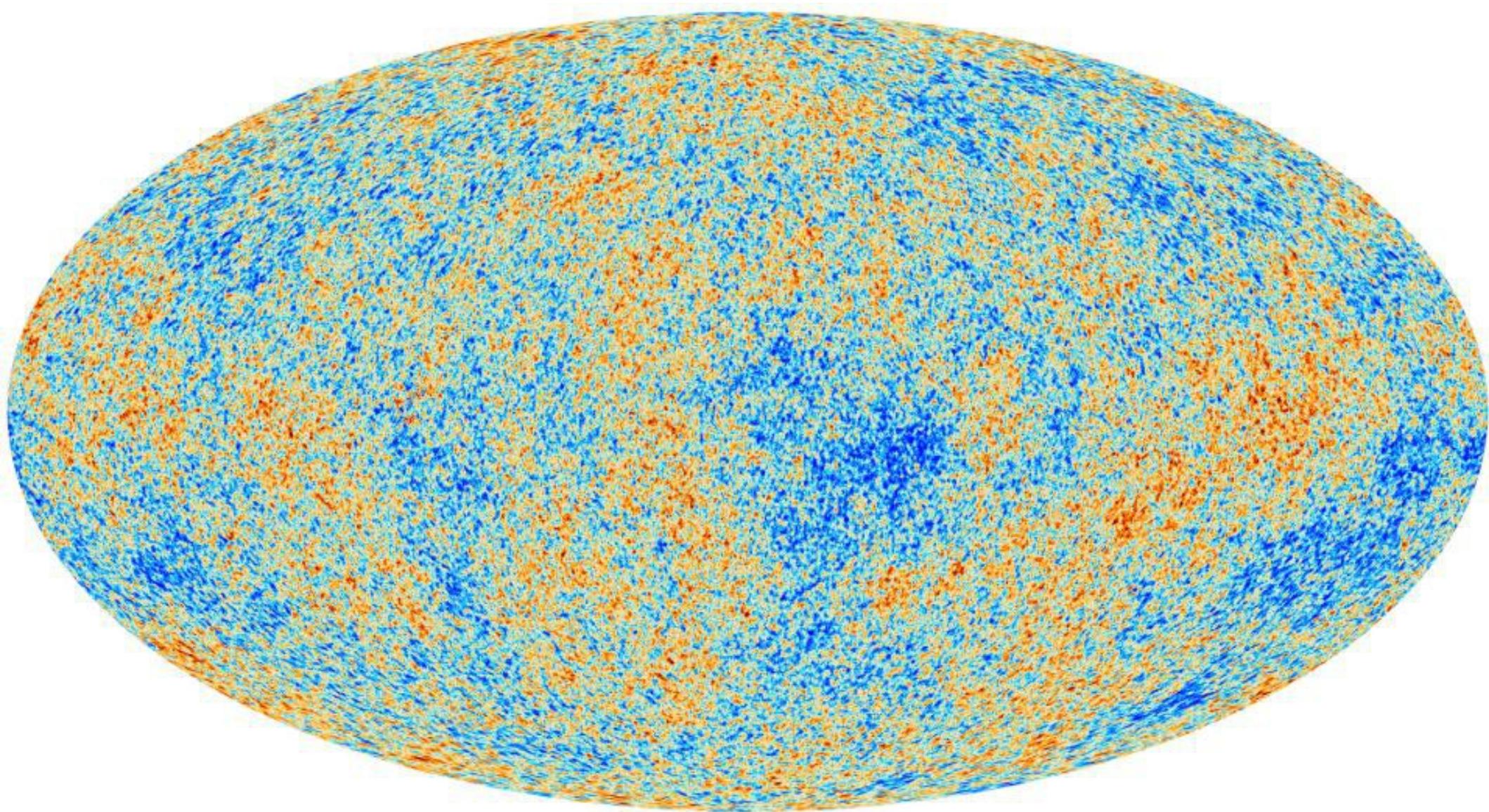


If we remove the Dipole anisotropy and the Galactic emission, we see anisotropies at the level of $(\Delta T/T)_{\text{rms}} \sim 20 \mu\text{K}$ (smoothed on $\sim 7^\circ$ scale).

These anisotropies are the imprint left by primordial tiny density inhomogeneities ($z \sim 1000$)..



Planck 2013 CMB Map

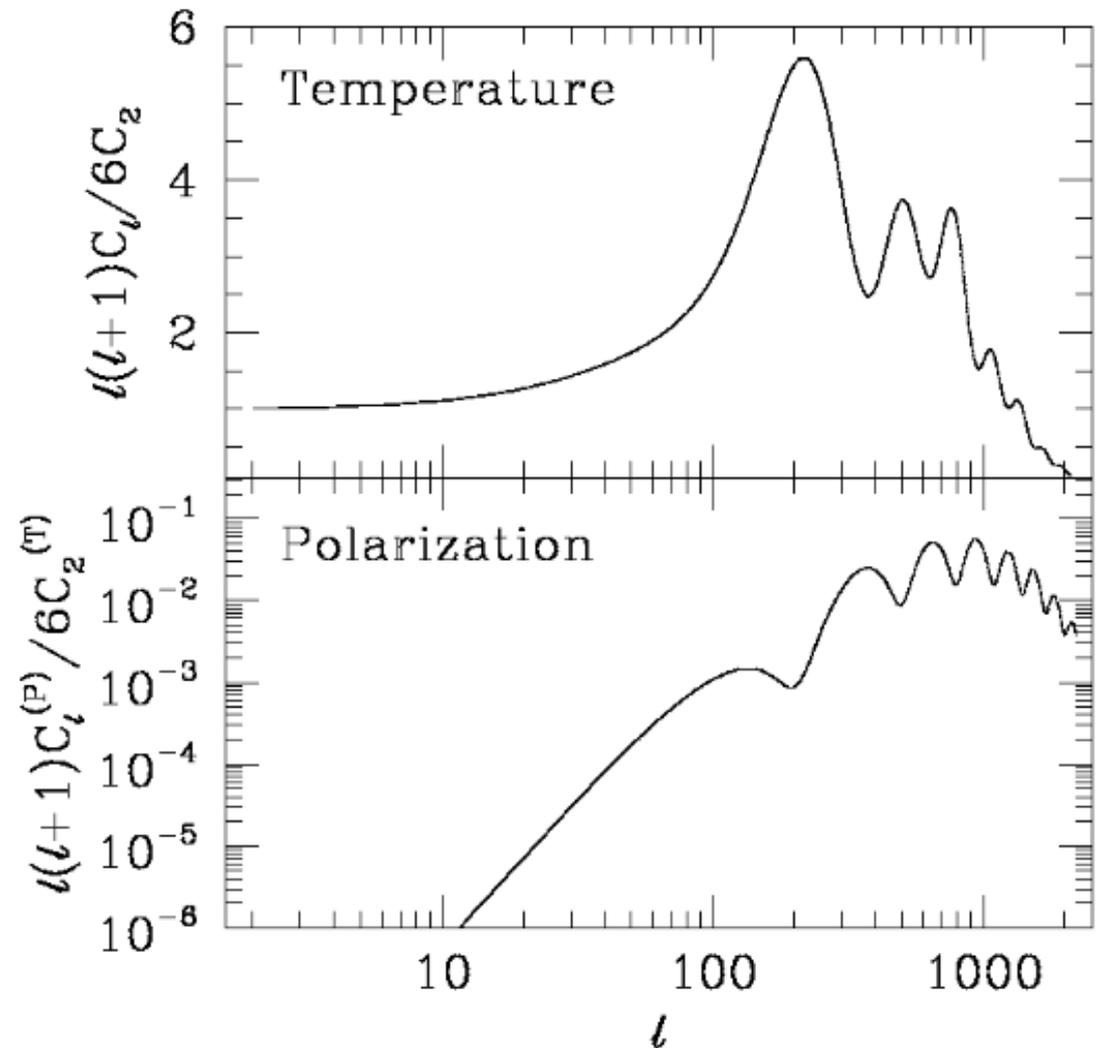


The CMB Angular Power Spectrum

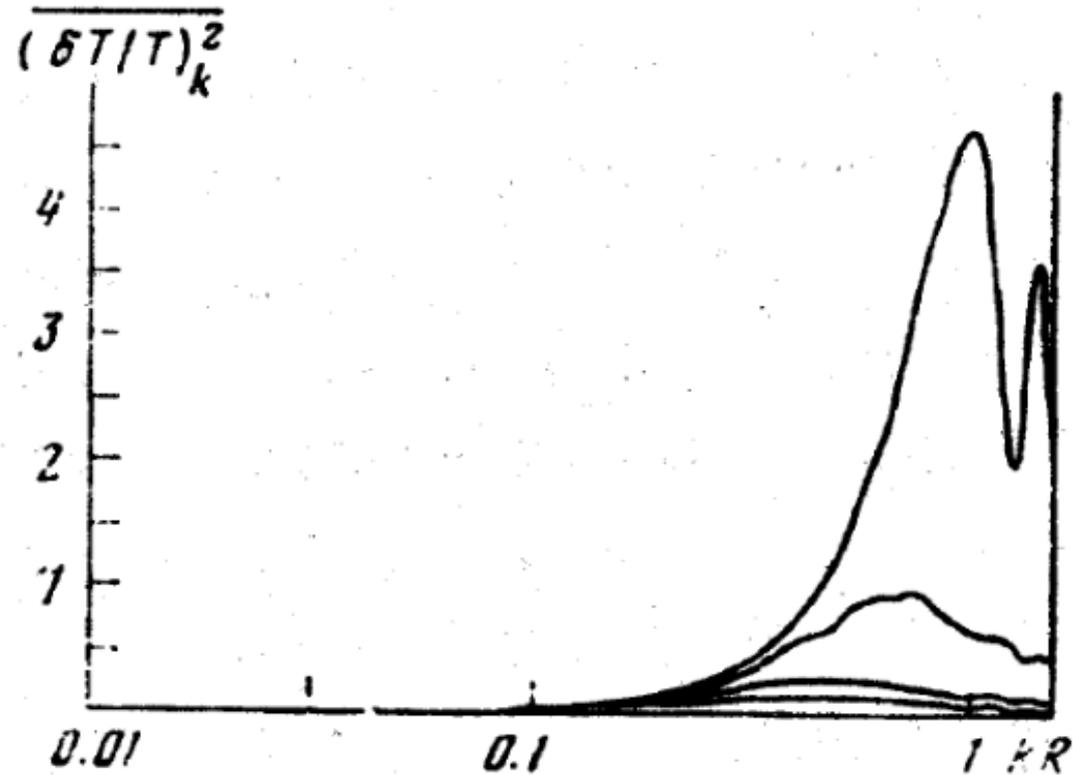
$$\left\langle \frac{\Delta T}{T}(\vec{\gamma}_1) \frac{\Delta T}{T}(\vec{\gamma}_2) \right\rangle = \frac{1}{2\pi} \sum_{\ell} (2\ell + 1) C_{\ell} P_{\ell}(\vec{\gamma}_1 \cdot \vec{\gamma}_2)$$

R.m.s. of $\Delta T/T$ has $l(l+1)C_l/2\pi$ power per decade in l :

$$\langle (\Delta T/T)^2 \rangle_{rms} = \sum_l \frac{(2l+1)}{4\pi} C_l \approx \int \frac{l(l+1)}{2\pi} C_l d \ln l$$



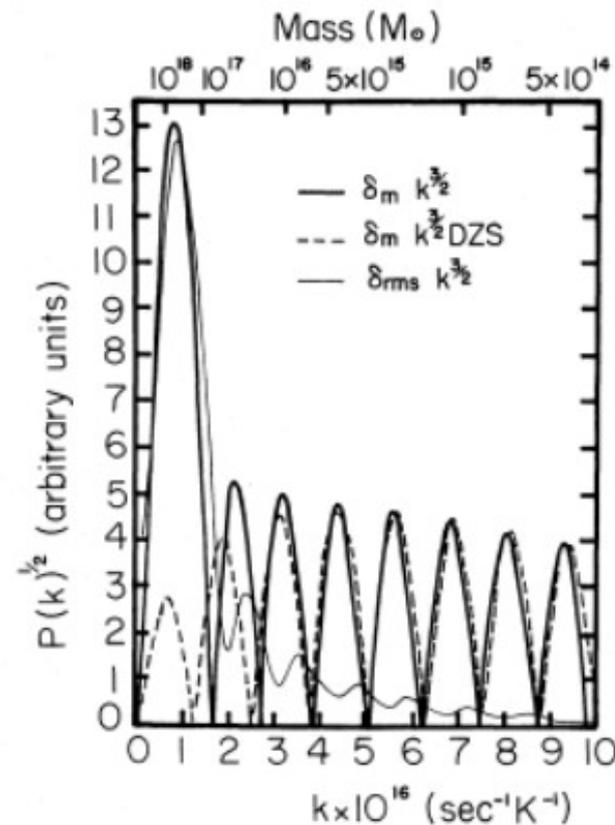
A Brief History of the CMB Anisotropies Angular Spectrum (Theoretical predictions)



[Doroshkevich, A. G.](#); [Zel'Dovich, Ya. B.](#); [Syunyaev, R. A.](#)

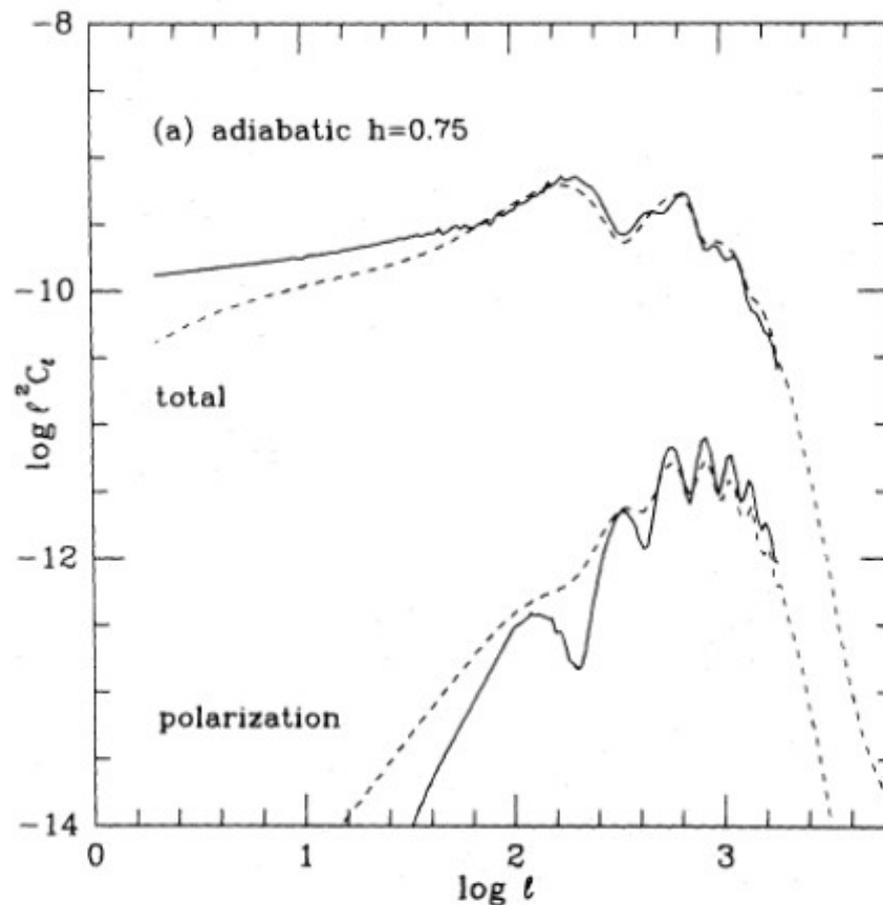
Soviet Astronomy, Vol. 22, p.523, 1978

A Brief History of the CMB Anisotropies Angular Spectrum (Theoretical predictions)



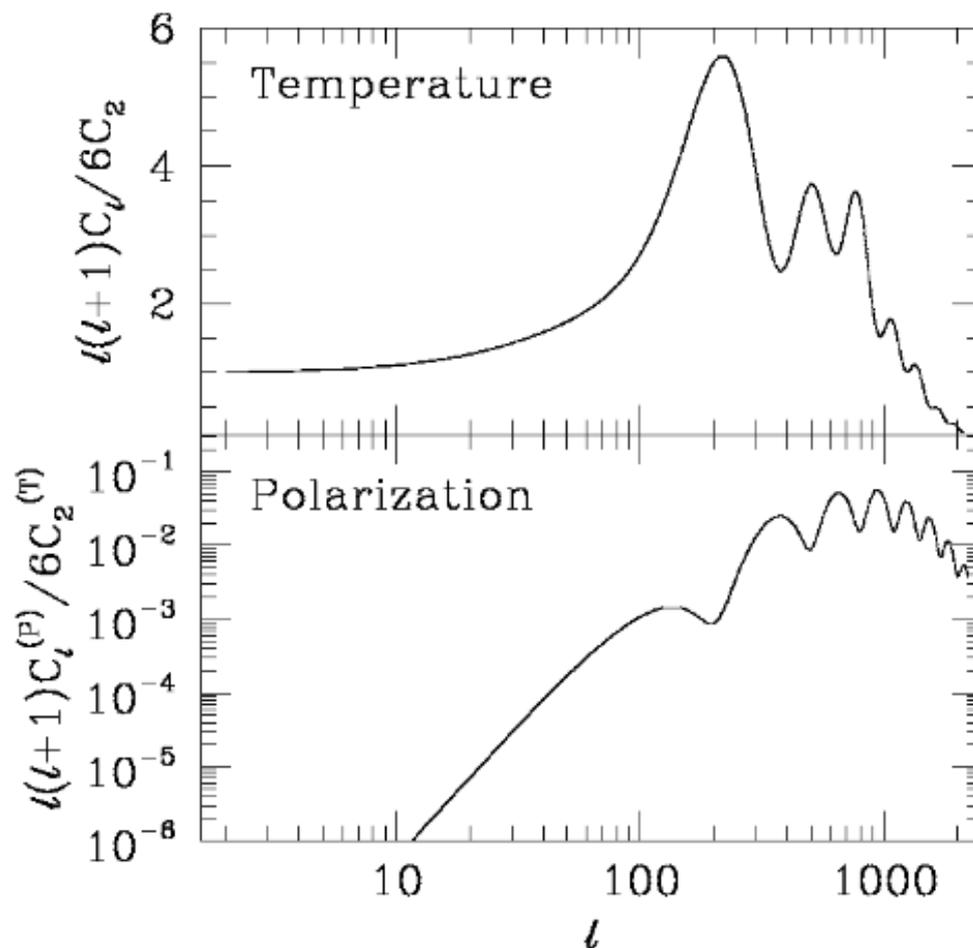
[Wilson, M. L.](#); [Silk, J.](#), *Astrophysical Journal*, Part 1, vol. 243, Jan. 1, 1981, p. 14-25.
1981

A Brief History of the CMB Anisotropies Angular Spectrum (Theoretical predictions)



[Bond, J. R.](#); [Efstathiou, G.](#); Royal Astronomical Society, Monthly Notices (ISSN 0035-8711), vol. 226, June 1, 1987, p. 655-687, 1987

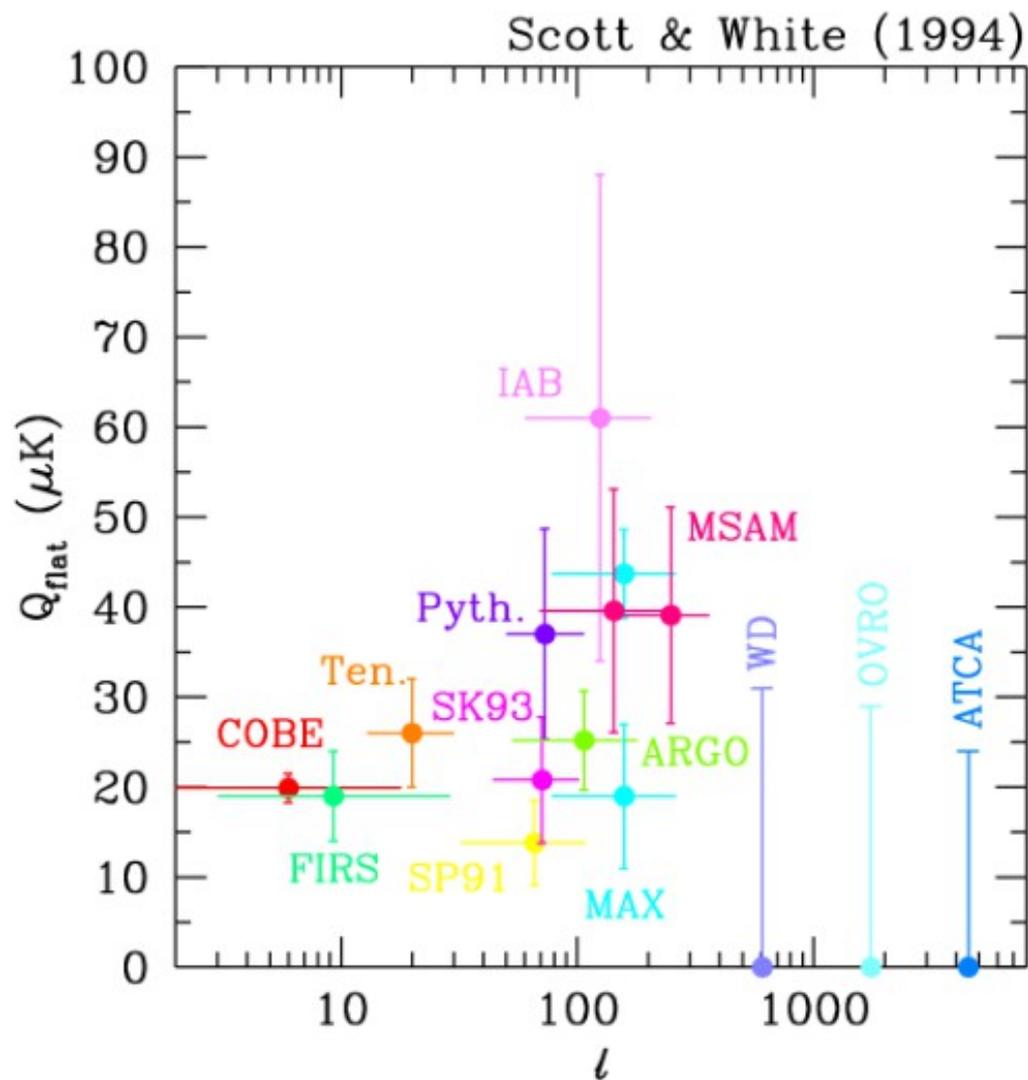
A Brief History of the CMB Anisotropies Angular Spectrum (Theoretical predictions)



[Hu, Wayne](#); [Scott, Douglas](#); [Sugiyama, Naoshi](#); [White, Martin](#).

Physical Review D, Volume 52, Issue 10, 15 November 1995, pp.5498-5515

A Brief History of the CMB Anisotropies Angular Spectrum (Experimental Data)



In 1995 Big Bang Model was nearly dead...

nature

International weekly journal of science

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News and Views

Nature **377**, 99 (14 September 1995) |

Big Bang not yet dead but in decline

John Maddox

The latest measurements of the Hubble constant make the Big Bang account of the origin of the Universe more dependent on the coincidence of numbers than it has so far been. But it remains the only theory in the field. [▲ Top](#)

Is there a crisis in cosmology, or is it that the latest measurement of the Hubble constant is yet another of those numerical disagreements that plague the field from time to time? That is the question inevitably prompted by last week's article by N.

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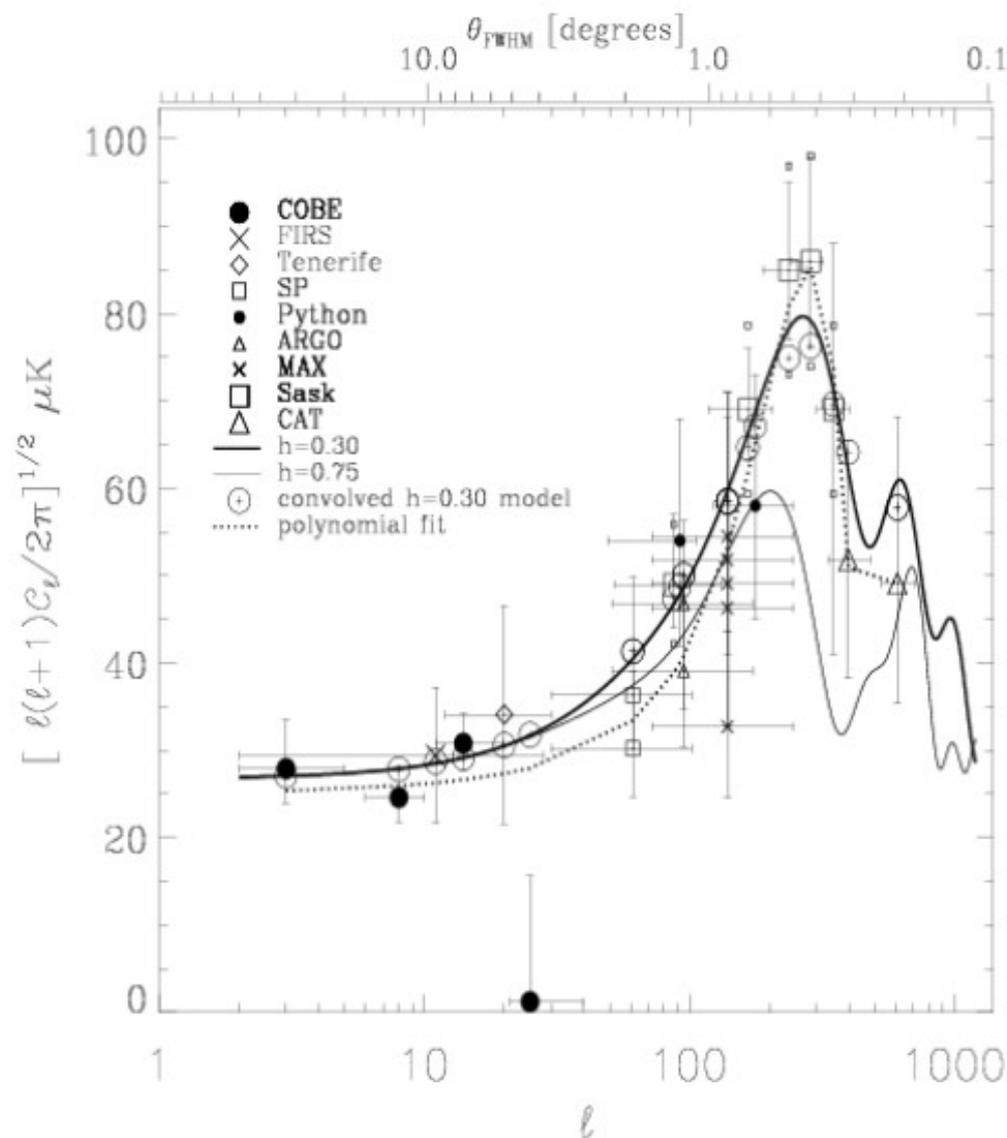
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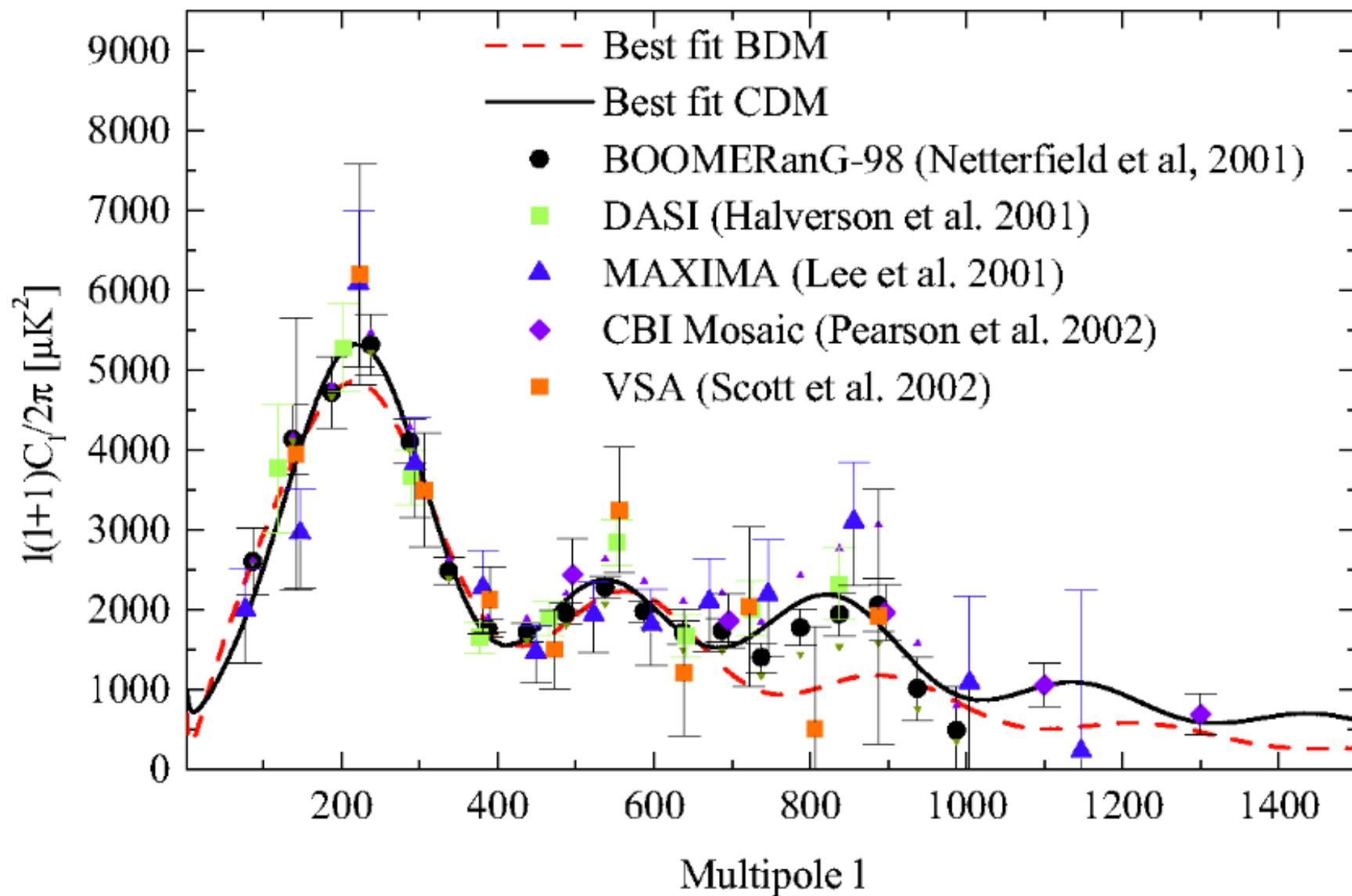
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A Brief History of the CMB Anisotropies Angular Spectrum (Experimental Data)



Collection of CMB anisotropy data from C. Lineweaver et al., 1996

CMB anisotropies pre-WMAP (January 2003)

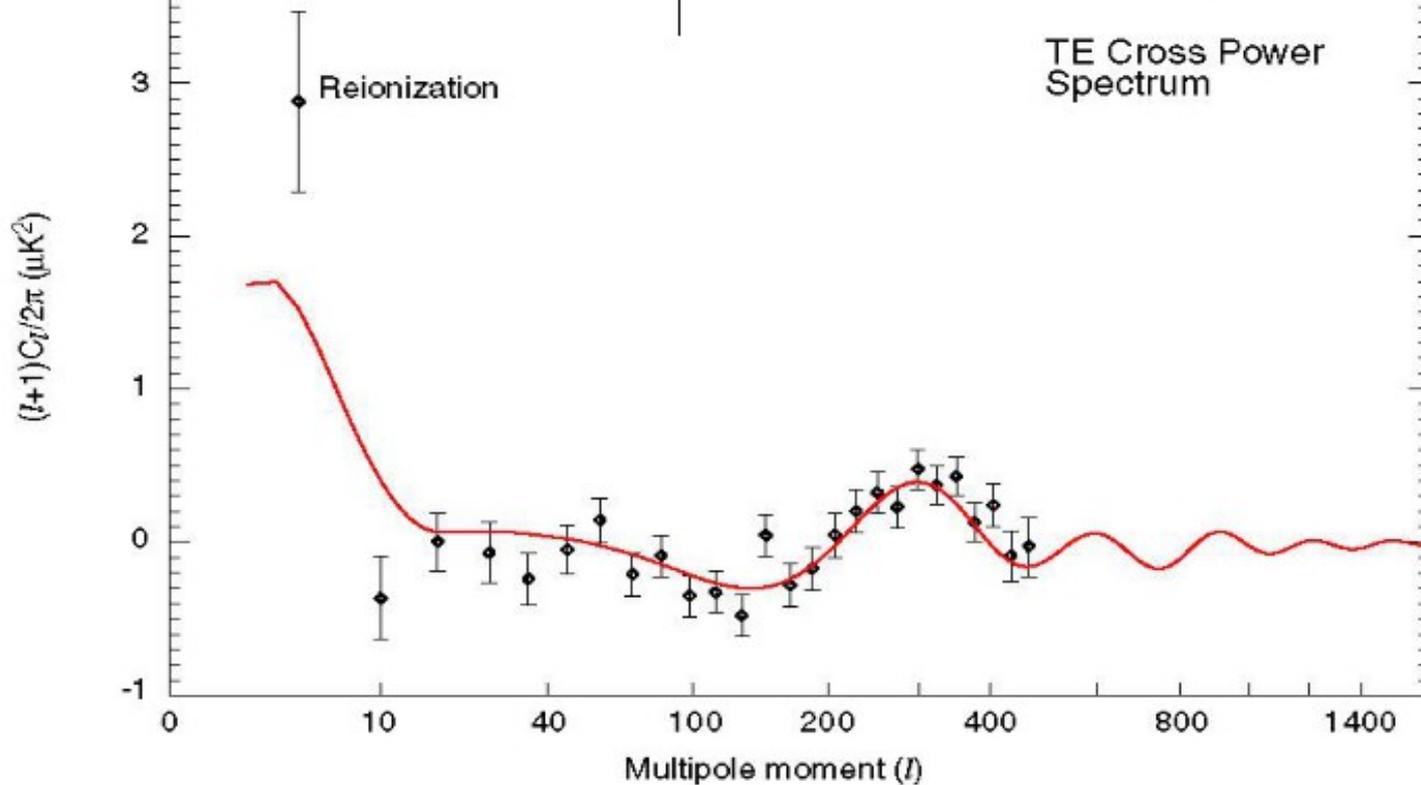
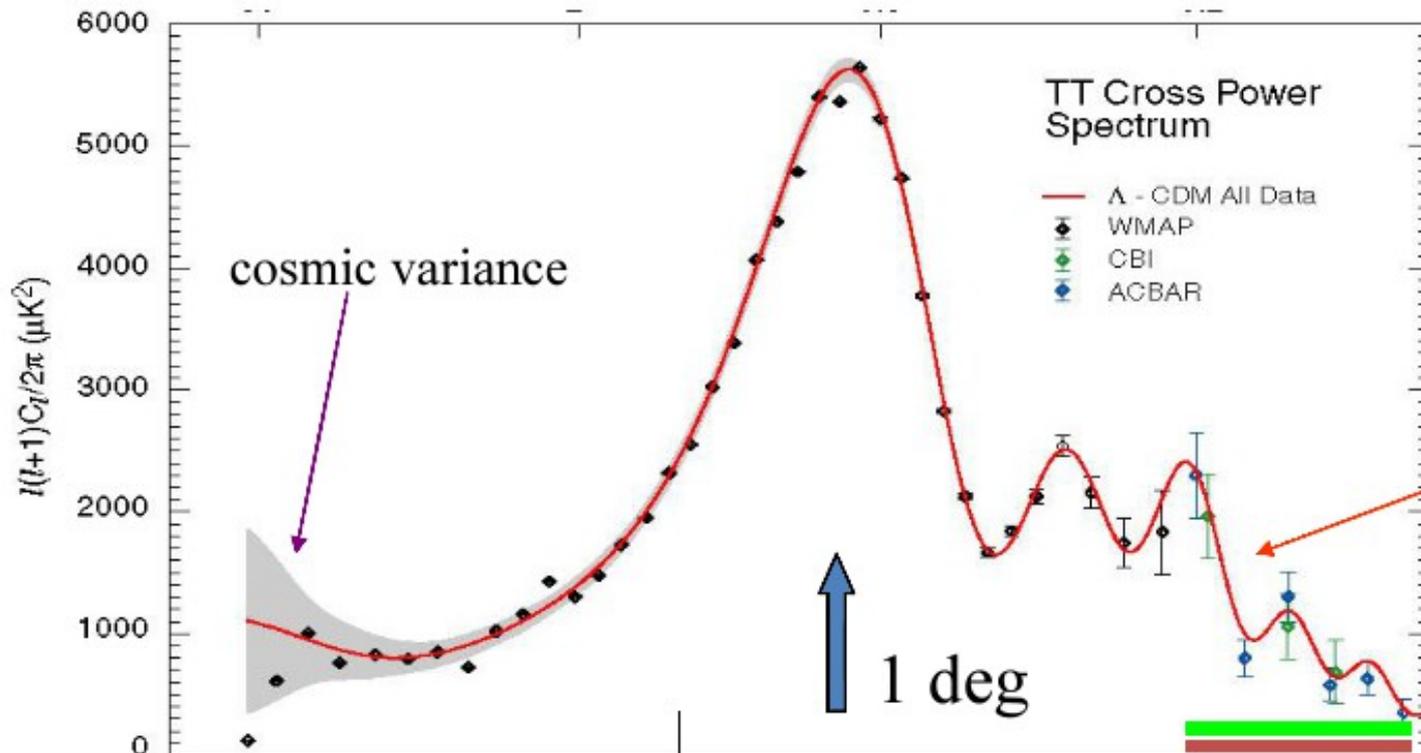


Temperature

85% of sky

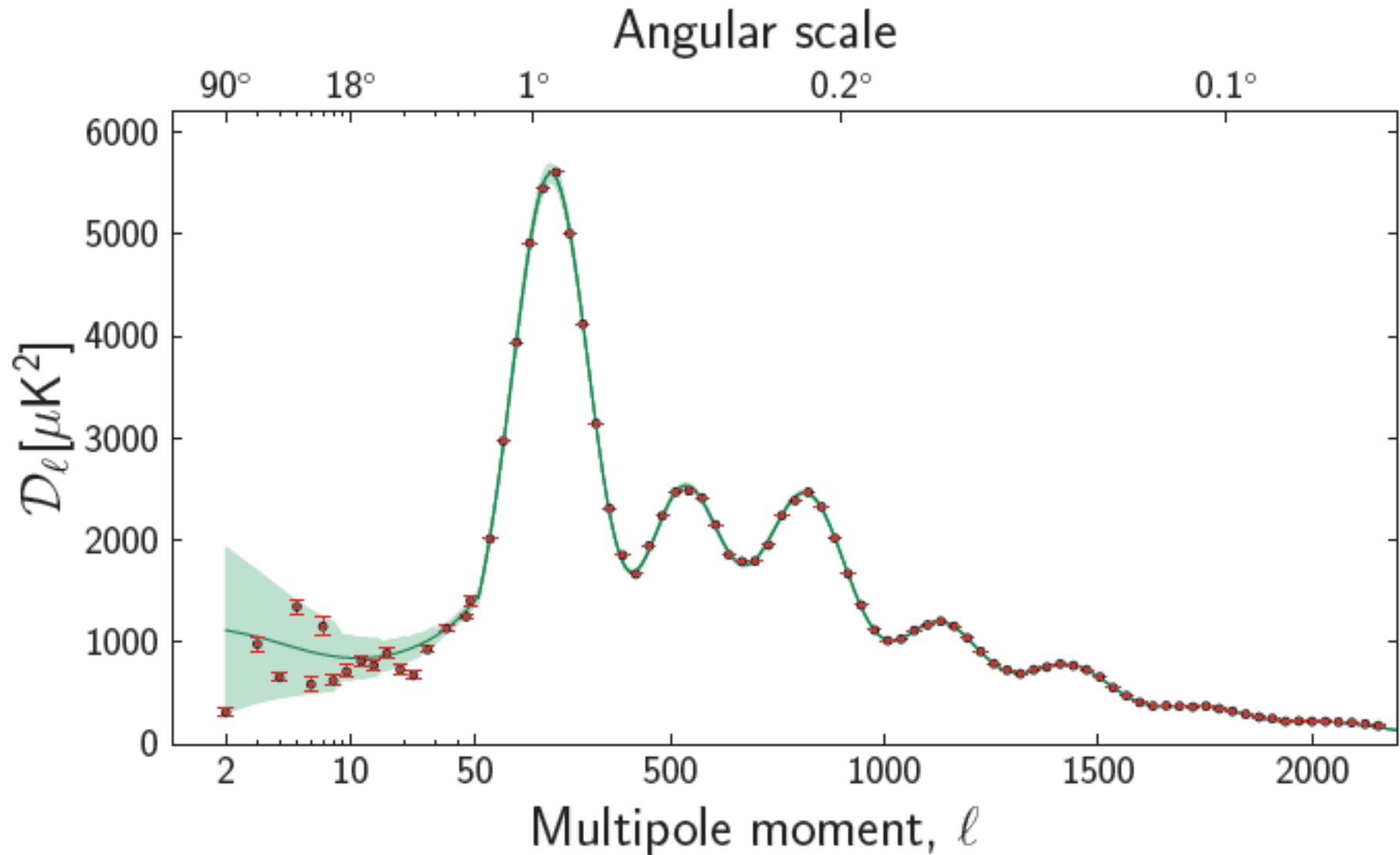
Best fit model

Spergel et al, 2003



Temperature-polarization

Planck 2013 TT angular spectrum



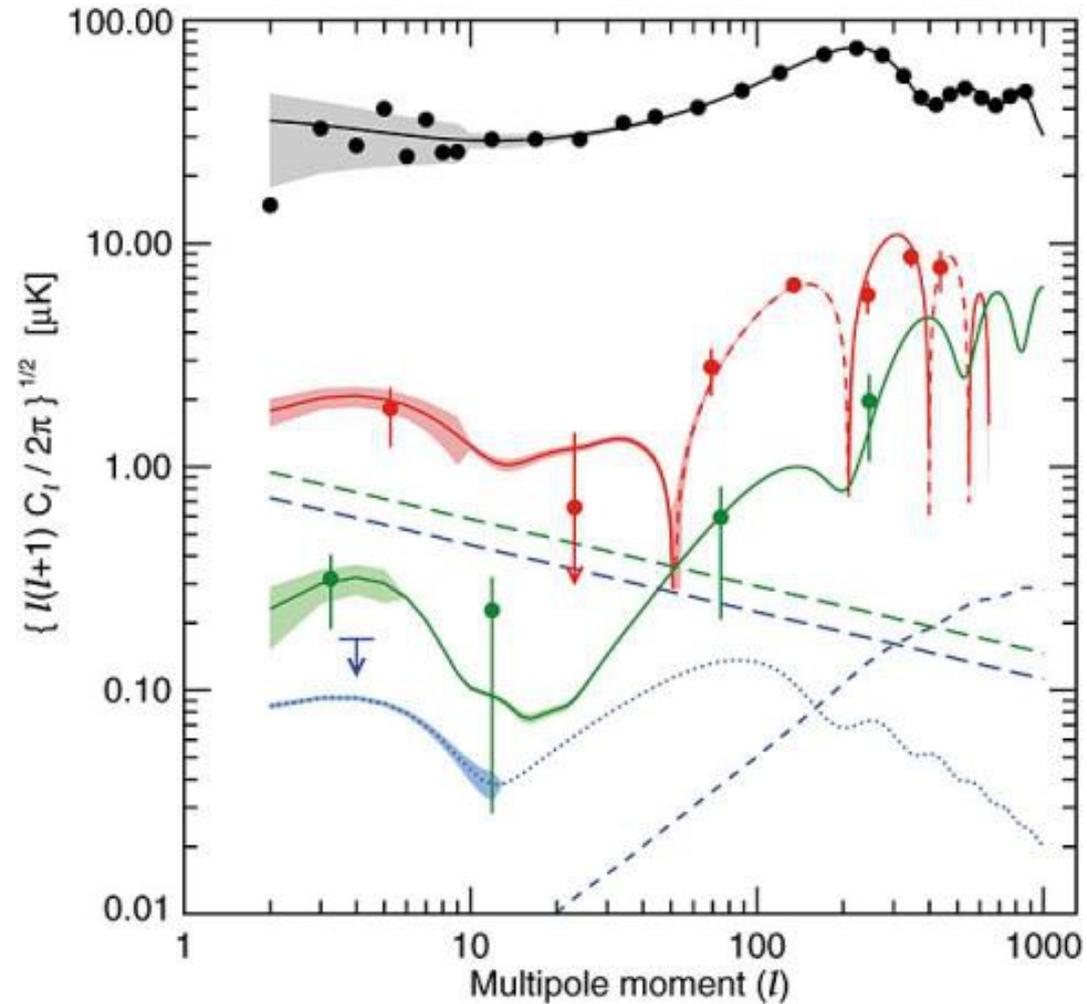
The CMB Angular Power Spectrum

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$$\langle (\Delta T / T)^2 \rangle_{rms} = \sum_l \frac{(2l+1)}{4\pi} C_l \approx \int \frac{l(l+1)}{2\pi} C_l d \ln l$$

We can extract 4 independent angular spectra from the CMB:

- Temperature
- Cross Temperature Polarization
- Polarization type E (density fluctuations)
- Polarization type B (gravity waves)



CMB Anisotropies

Four mechanisms produce primary CMB anisotropies:

- Gravity (Sachs-Wolfe effect)
- Intrinsic (Adiabatic) Fluctuations
- Doppler effect
- Time-Varying Potentials (Integrated Sachs-Wolfe Effect)

$$\frac{\Delta T}{T}(\vec{n}) \doteq \int_0^{\infty} \left[g(z) (\Psi + \Theta_0 + \vec{n} \cdot \vec{v}_b) + e^{-\tau} H^{-1} \dot{\Psi} \right] dz$$

Gravity

Adiabatic

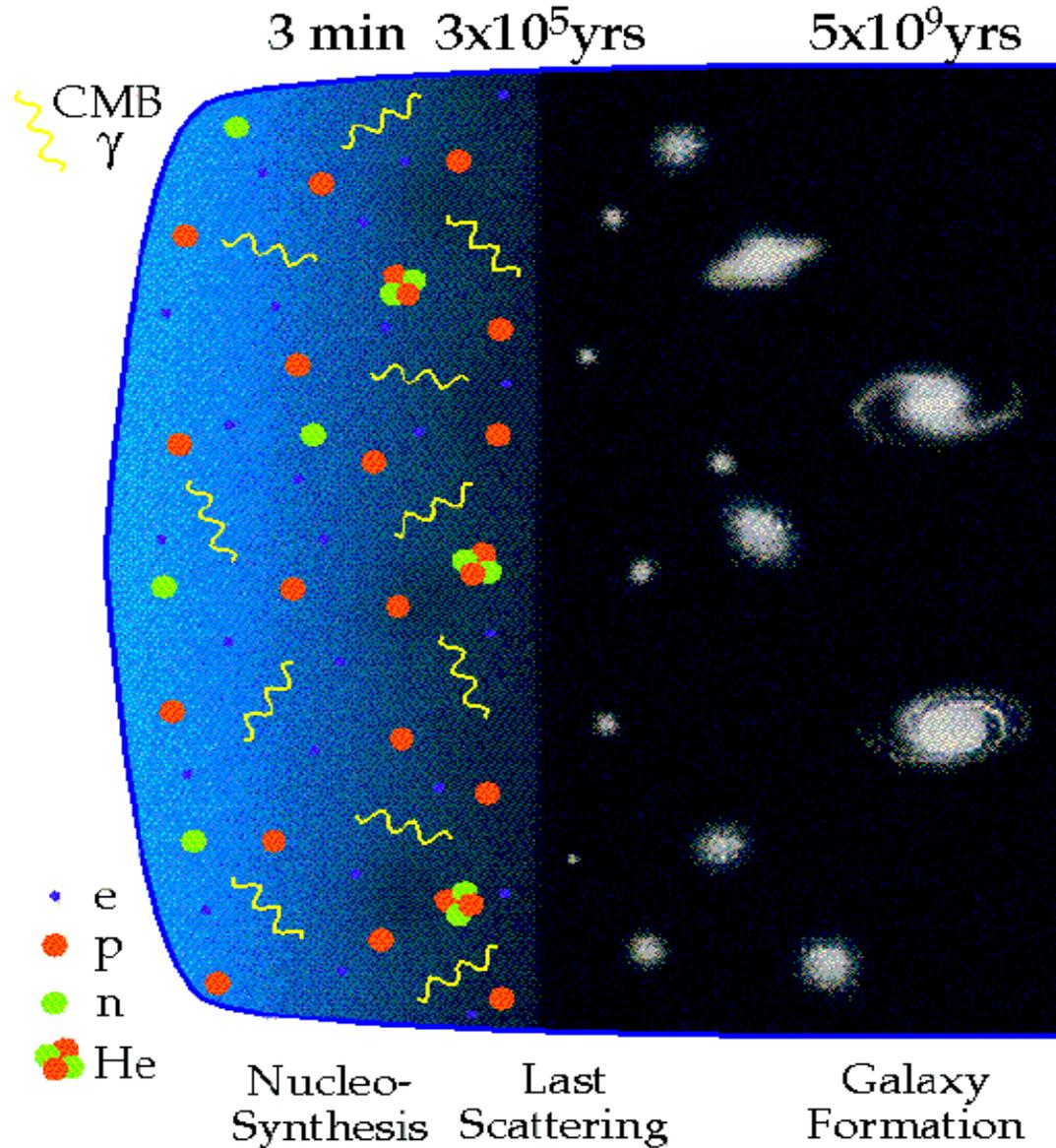
Doppler

ISW

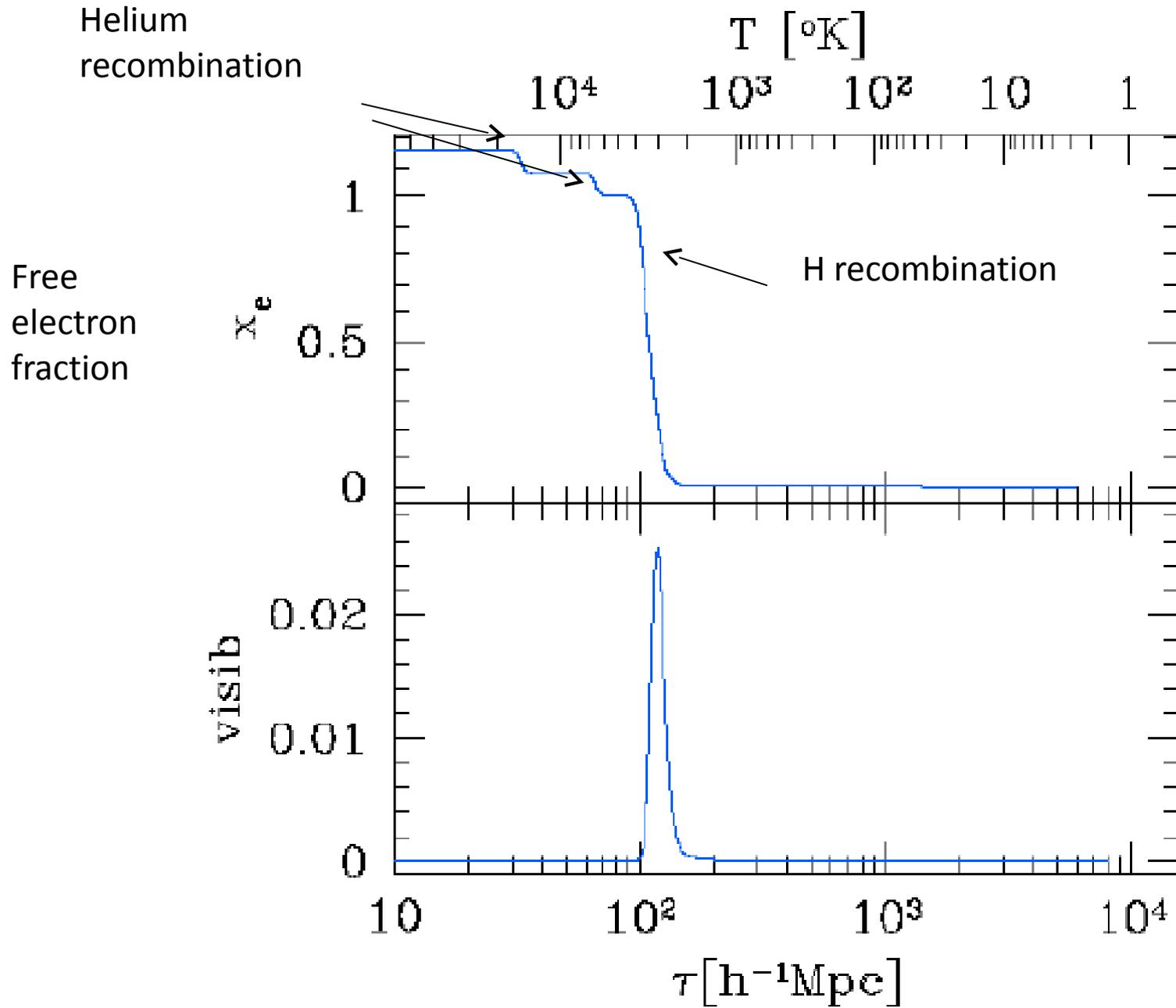
(Very) Brief History

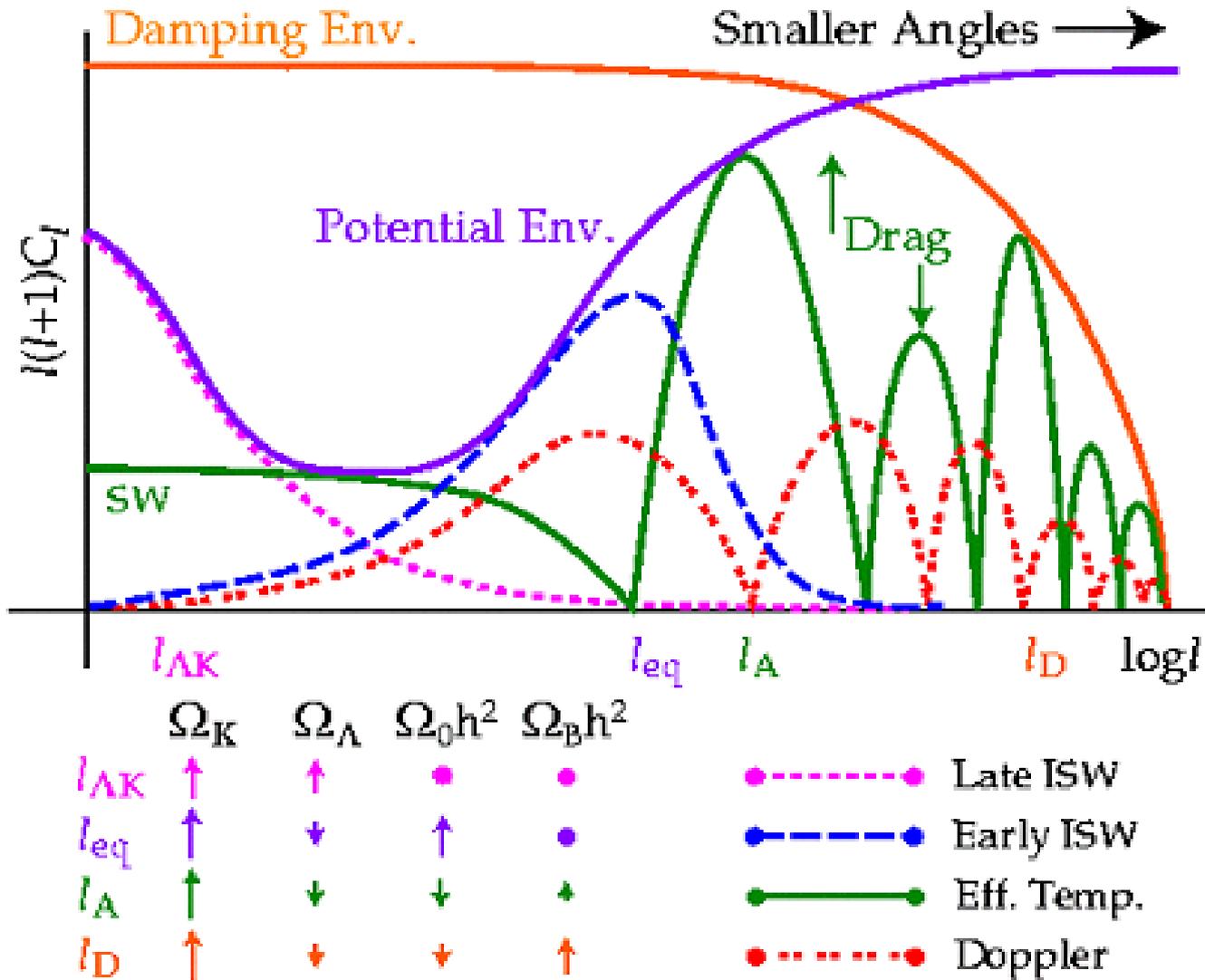
When $T > 3000$ K

plenty of free electrons.
Photons are «trapped» in the primordial plasma.



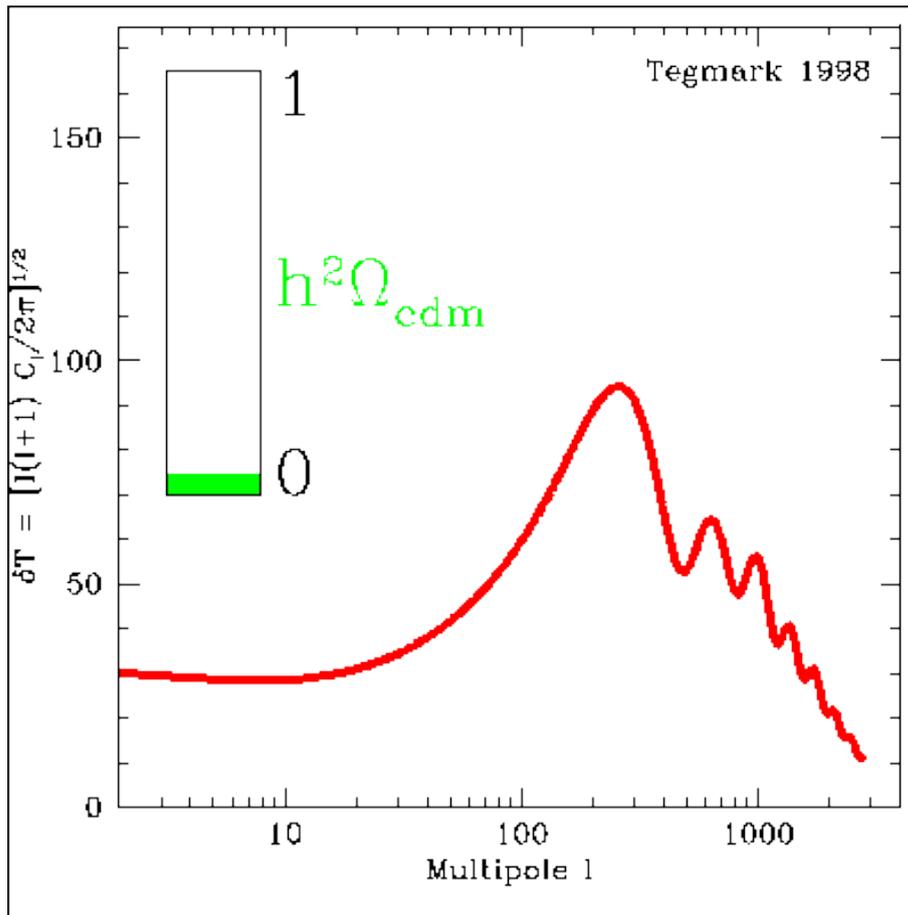
Recombination



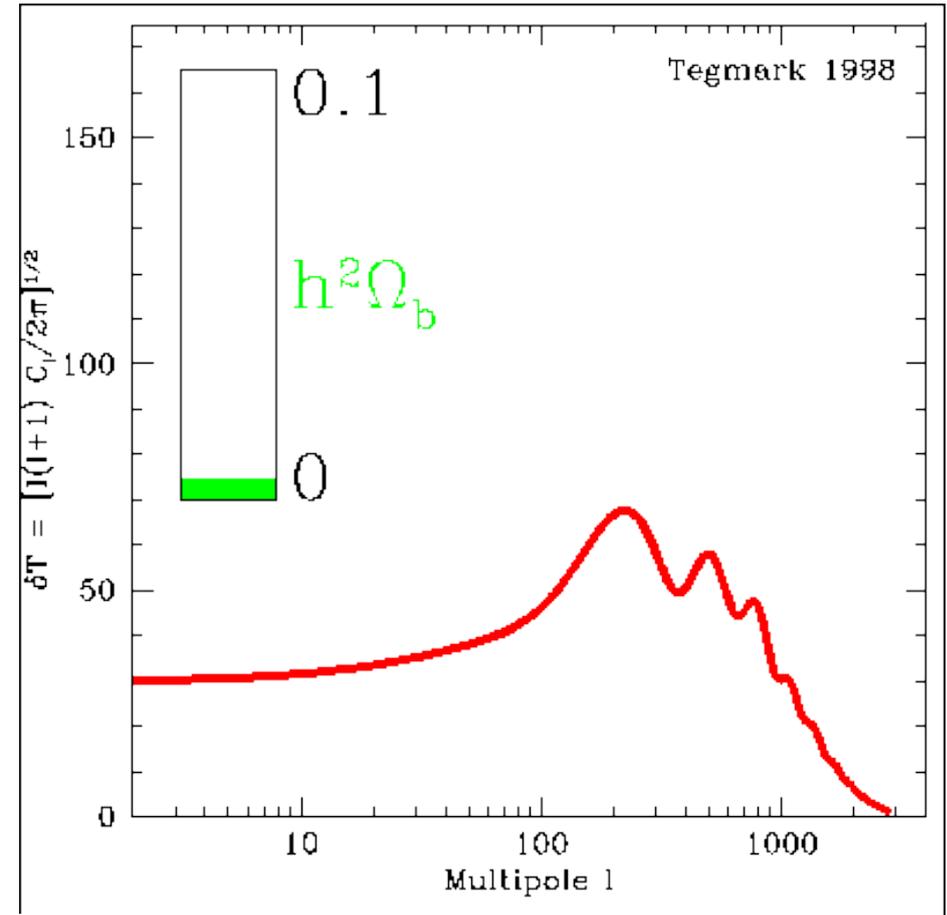


Constraining Cosmological Parameters with CMB

Cold Dark Matter Density

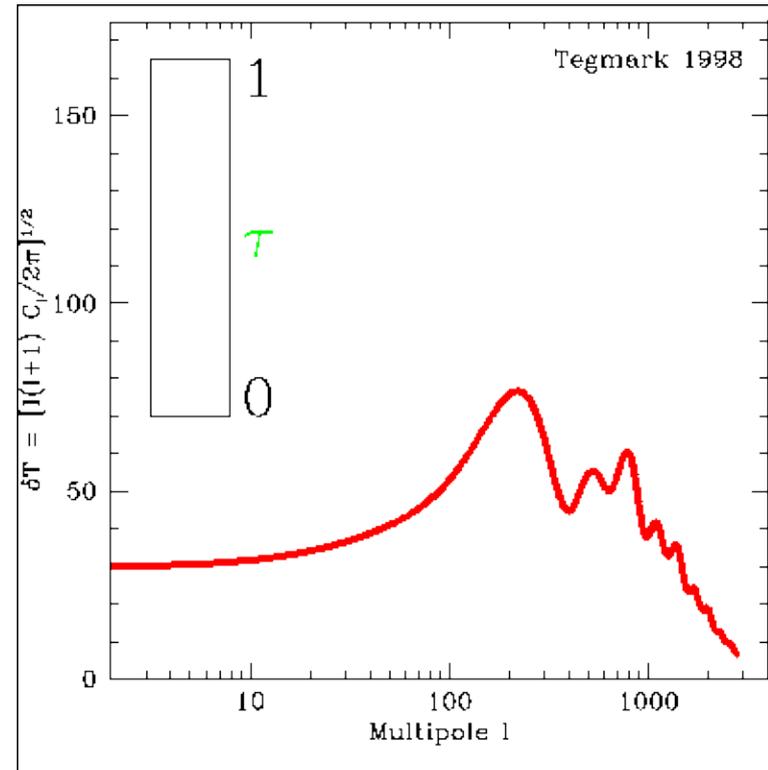
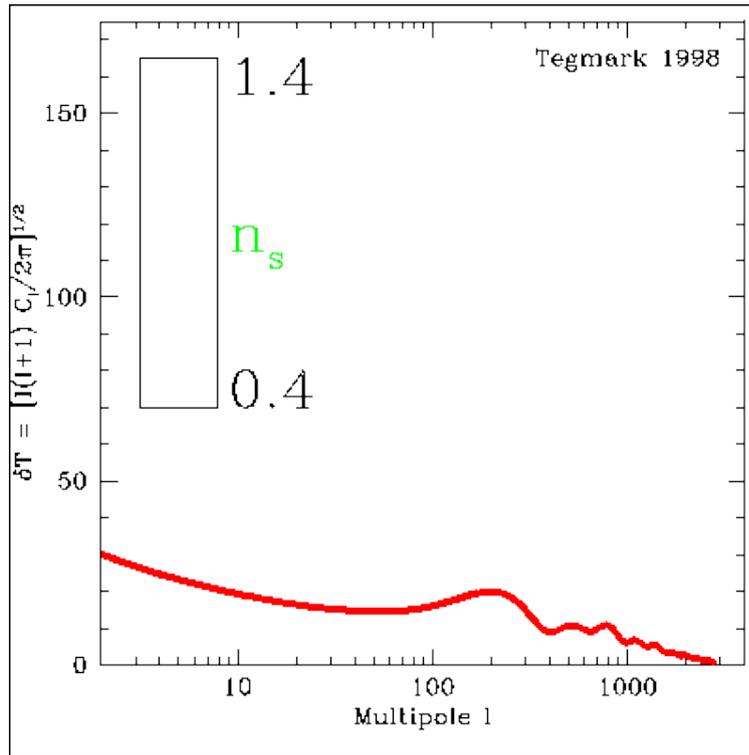


Baryon Density

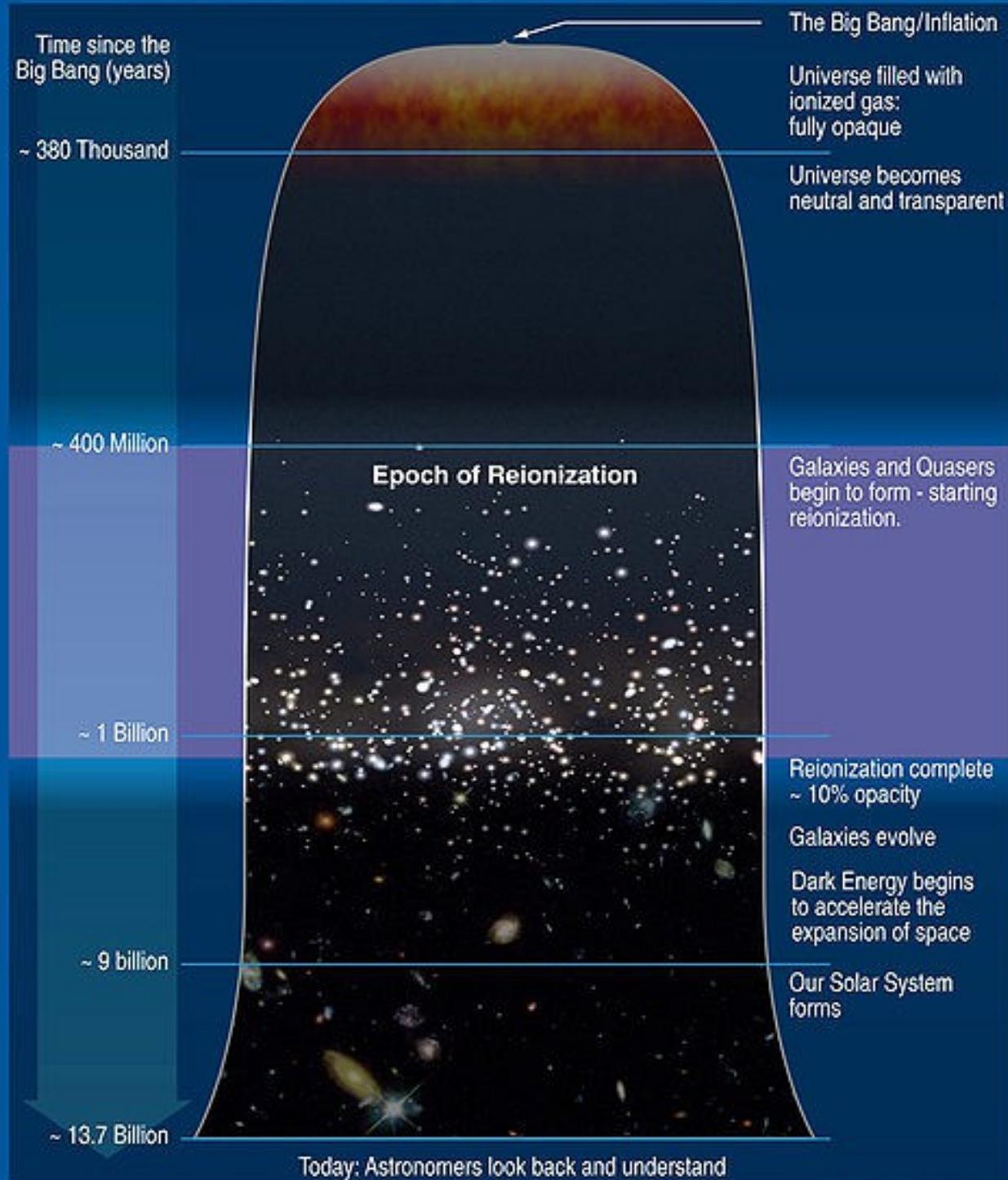


Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015

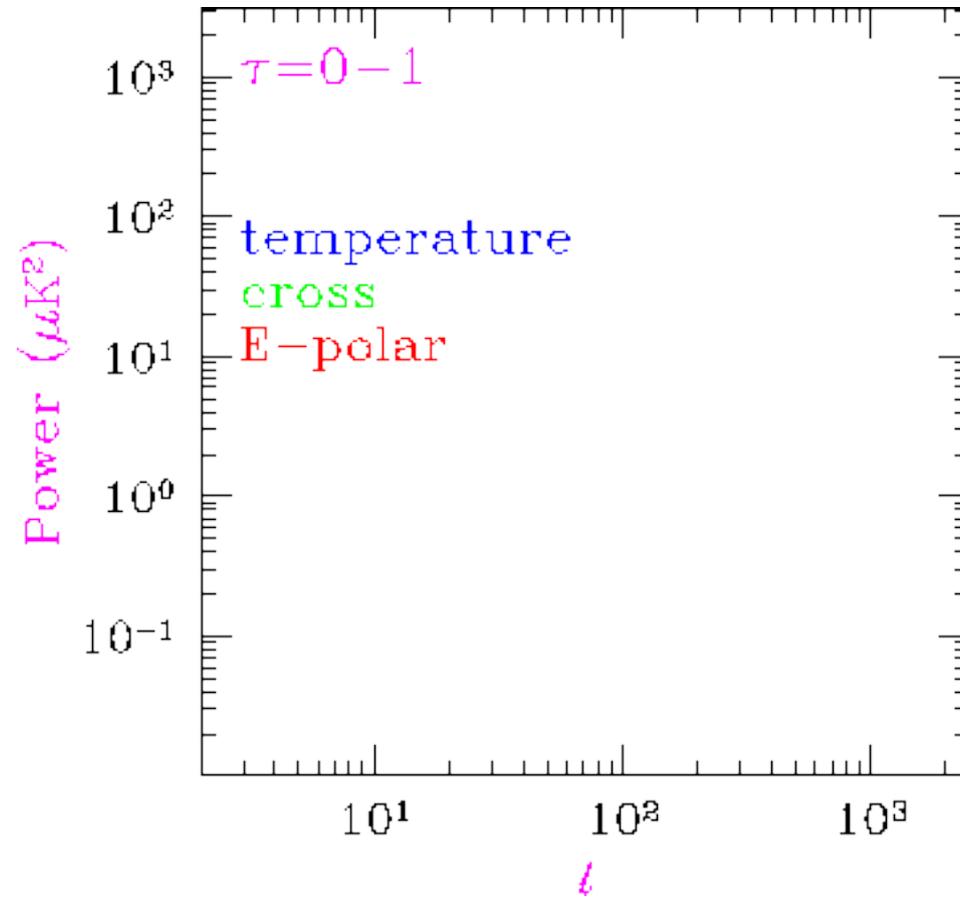
Constraining Cosmological Parameters with CMB



First Stars and Reionization Era



Measuring large scale CMB polarization breaks this degeneracy.



The standard cosmological model

- Assumes General Relativity, Inflation, Adiabatic and Scalar Perturbations, flat universe.
- Friedmann-Robertson-Walker (or Friedmann-Lemaitre) metric. Hubble Constant (+1)

$$H_0 = 100h \text{ km/s/Mpc}$$

- 3 Energy components: Baryons, Cold Dark Matter, Cosmological Constant (+3). Flat Universe (-1).

$$\omega_b = \Omega_b h^2 \quad \omega_{CDM} = \Omega_{CDM} h^2$$

- Initial conditions for perturbations given by Inflation: Adiabatic, nearly scale invariant initial power spectrum, only scalar perturbations. Two free parameters (+2): Amplitude and Spectral index.

$$P(k) \approx A_s \left(\frac{k}{k_0} \right)^{n_s}$$

Pivot scale is usually fixed to:

$$k_0 = 0.002 \text{ hMpc}^{-1}$$

- Late universe reionization characterized with a single parameter(+1) : optical depth τ or reionization redshift z_r .

Total: 1+3-1+2+1= 6 parameters.

Probing the Neutrino Number with CMB

If we change the neutrino number we change the Hubble rate at recombination. This will change two important scales: the sound horizon and the damping scale:

$$r_s = \int_0^{t_*} c_s dt/a = \int_0^{a_*} \frac{c_s da}{a^2 H} \quad r_d^2 = (2\pi)^2 \int_0^{a_*} \frac{da}{a^3 \sigma_T n_e H} \left[\frac{R^2 + \frac{16}{15} (1 + R)}{6(1 + R^2)} \right]$$

If we measure the first peaks in the angular spectrum we fix the angular scale of the sound horizon given by:

$$\theta_s = \frac{r_s}{d_a(z_{dec})}$$

We can therefore write the angular damping scale as:

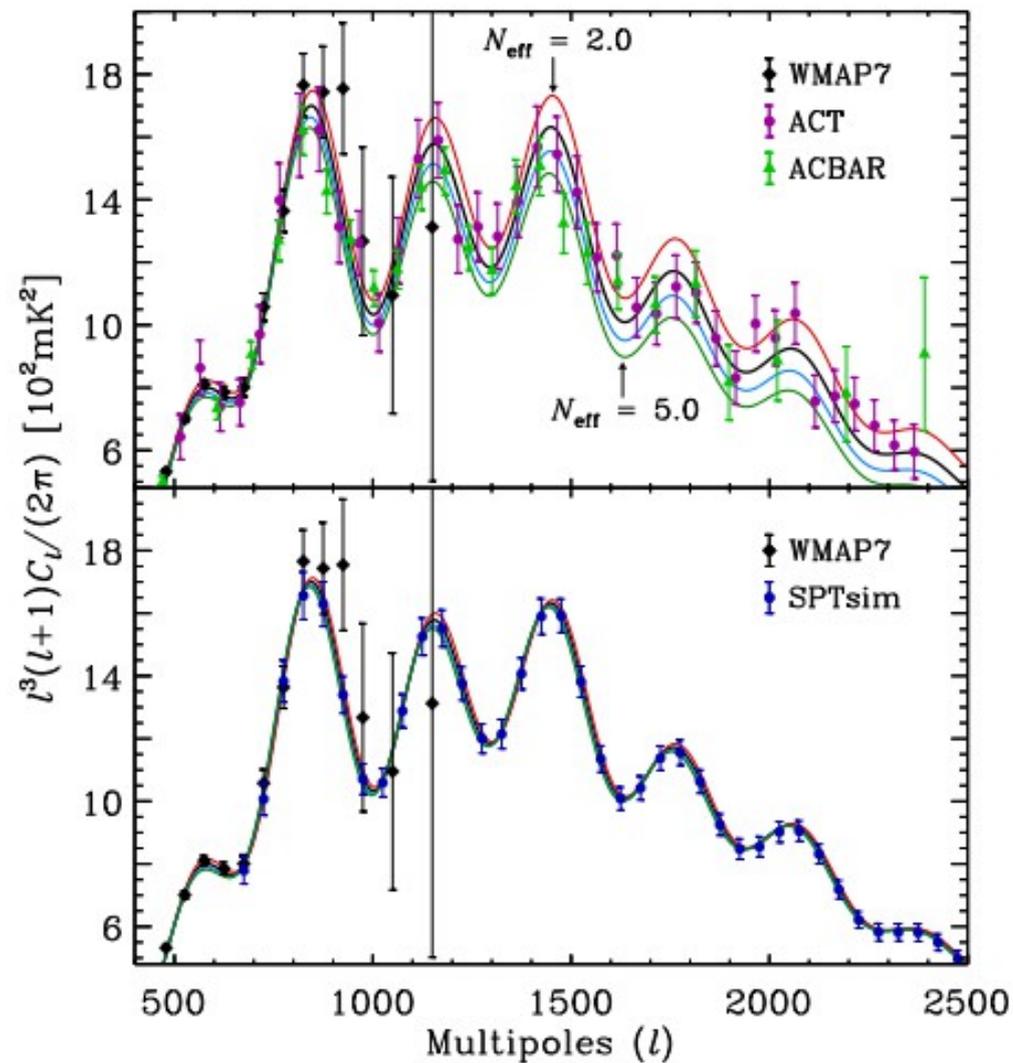
$$\theta_d = \frac{r_d}{d_a(z_{dec})} = \frac{r_d}{r_s} \theta_s \sim \sqrt{H}$$

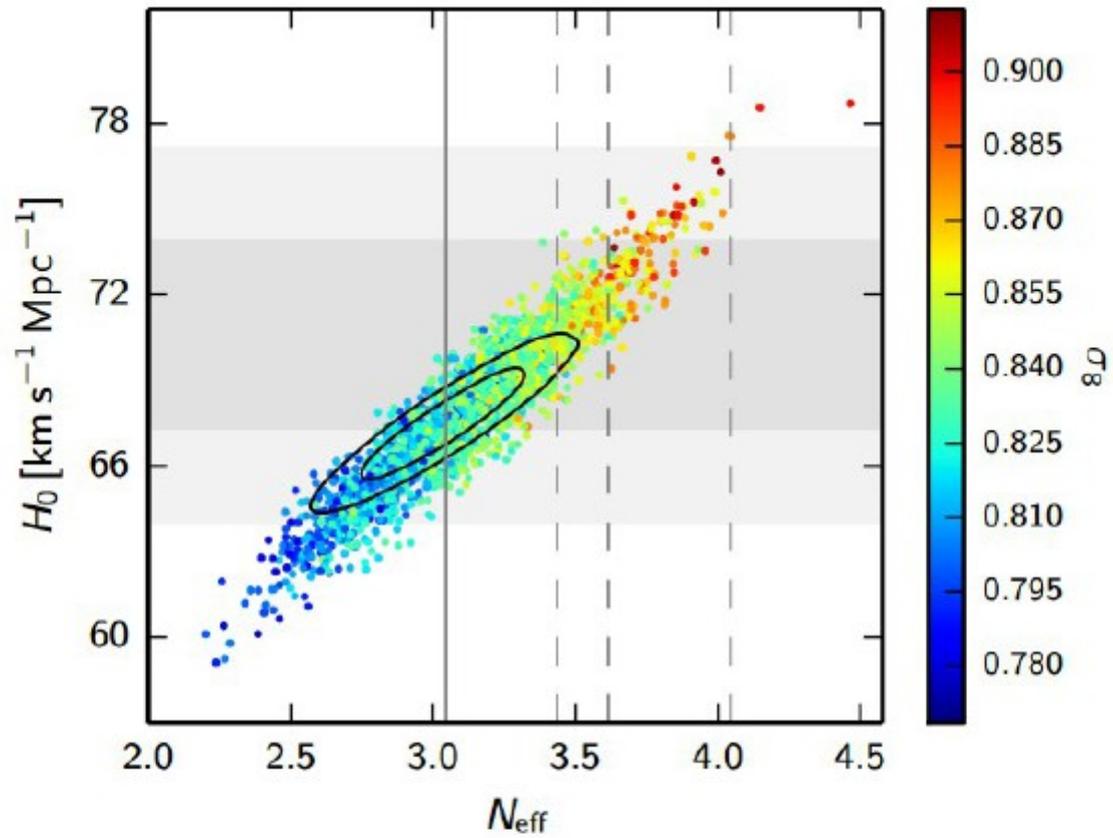
Therefore increasing the neutrino number increases the damping scale !

Probing the Neutrino Number with CMB

Larger Neutrino Number

Larger Damping of CMB anisotropies





$$N_{\text{eff}} = 3.13 \pm 0.32 \quad \text{Planck TT+lowP};$$

$$N_{\text{eff}} = 3.15 \pm 0.23 \quad \text{Planck TT+lowP+BAO};$$

$$N_{\text{eff}} = 2.99 \pm 0.20 \quad \text{Planck TT, TE, EE+lowP};$$

$$N_{\text{eff}} = 3.04 \pm 0.18 \quad \text{Planck TT, TE, EE+lowP+BAO}.$$

Probing the Neutrino Number with CMB

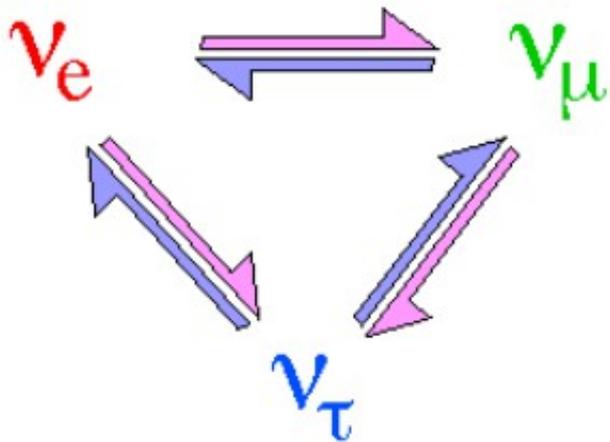
Parameter uncertainty	Planck	Planck+ACTPol	CMBPol
$\sigma(\Omega_b h^2)$	0.00020	0.00013 (1.5)	0.000048 (4.1)
$\sigma(\Omega_c h^2)$	0.0025	0.0015 (1.7)	0.00058 (4.3)
$\sigma(\theta_s)$	0.00044	0.00024 (1.8)	0.000075 (5.9)
$\sigma(\tau)$	0.0043	0.0035 (1.2)	0.0023 (1.9)
$\sigma(n_s)$	0.0073	0.0049 (1.5)	0.0026 (2.8)
$\sigma(\log[10^{10} A_s])$	0.019	0.013 (1.5)	0.0078 (2.4)
$\sigma(N_{eff})$	0.18	0.11 (1.6)	0.044 (4.1)

TABLE IV. 68% c.l. errors on cosmological parameters in the case of extra background of relativistic particles N_{eff} . The numbers in brackets show the improvement factor σ_{Planck}/σ respect to the Planck experiment.

Neutrino Masses

We know that flavour neutrino oscillations exists

From present evidences of oscillations from experiments measuring atmospheric, solar, reactor and accelerator neutrinos

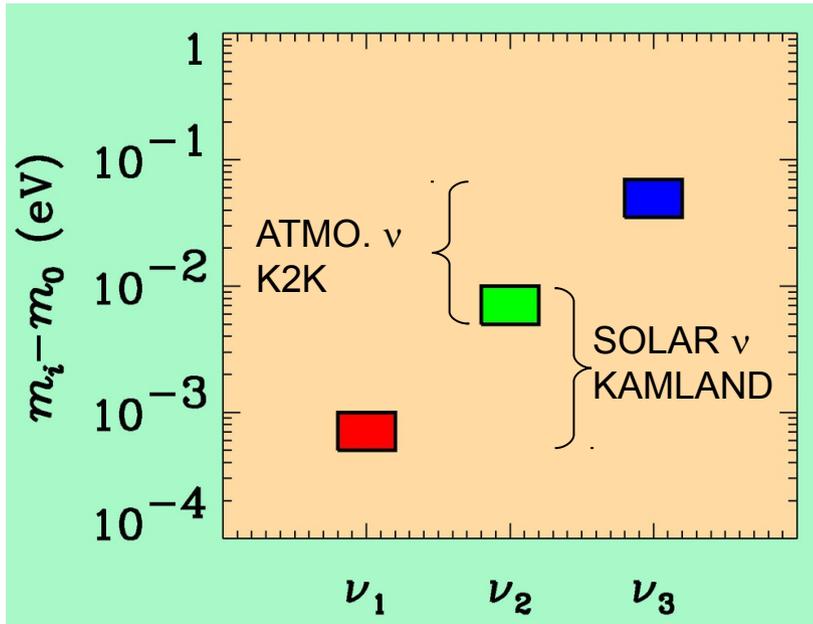


$$(e, \mu, \tau) \leftrightarrow (\nu_1, \nu_2, \nu_3)$$

$$\nu_{\alpha L} = \sum_{i=1}^3 U_{\alpha i} \nu_{iL}, \quad (\alpha = e, \mu, \tau)$$

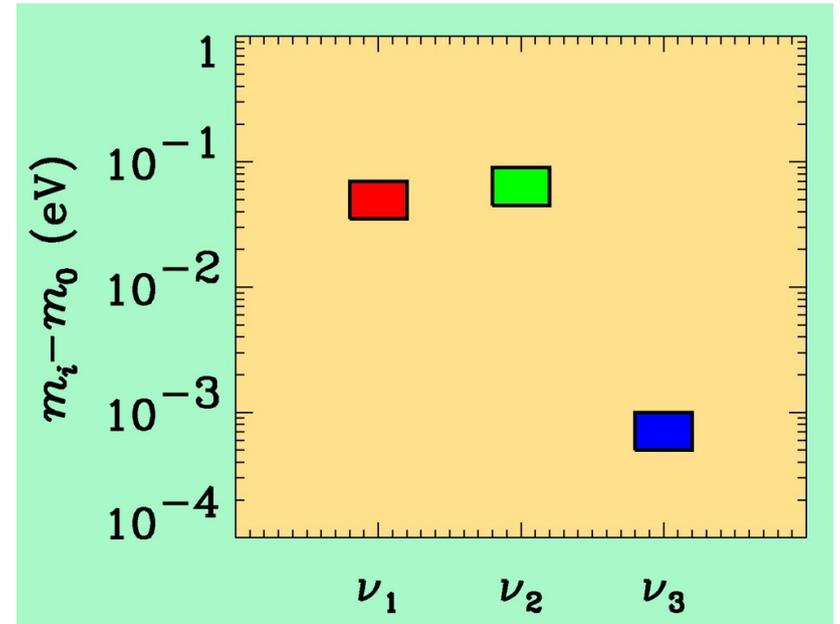
$$\begin{array}{c}
 \nu_e \\
 \nu_\mu \\
 \nu_\tau
 \end{array}
 \begin{bmatrix}
 & \nu_1 & \nu_2 & \nu_3 \\
 & c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
 -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
 s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
 \end{bmatrix}
 \times \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) .$$

If neutrino masses are hierarchical then oscillation experiments do not give information on the absolute value of neutrino masses



Normal hierarchy

$$m_3 > m_2 > m_1$$



Inverted hierarchy

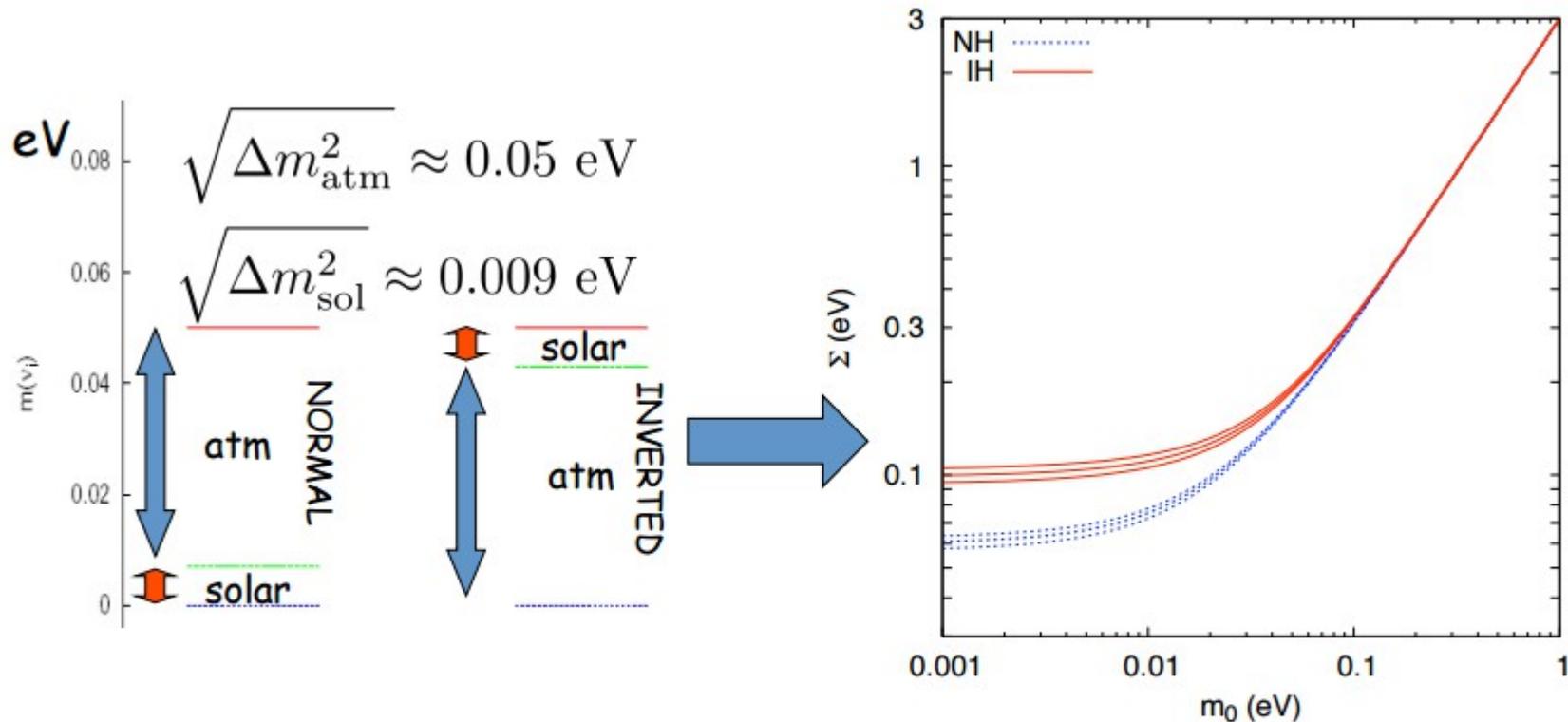
$$m_2 > m_1 > m_3$$

Moreover neutrino masses can also be degenerate

$$m_1, m_2, m_3 \gg \delta m_{\text{atmospheric}}$$

Neutrino Absolute Mass Scale

Oscillation data do not fix the absolute mass scale.



$$0.06(0.1) \text{ eV} \lesssim \sum_i m_i \lesssim 6 \text{ eV}$$

A large range of values (about 2 orders of magnitude) is still allowed by current experiments !

Energy density from neutrinos (after decoupling)

If they are relativistic:

$$\rho_\nu = \frac{7\pi^2}{120} N_{eff} \left(\frac{4}{11}\right)^{4/3} T_\gamma^4$$

$$\Omega_\nu h^2 = \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{eff} \Omega_\gamma h^2$$

$$\Omega_\nu h^2 = 0.58 \times 10^{-5} N_{eff}$$

($T_\gamma = 2.726 \text{ K}$)

If they are not relativistic:

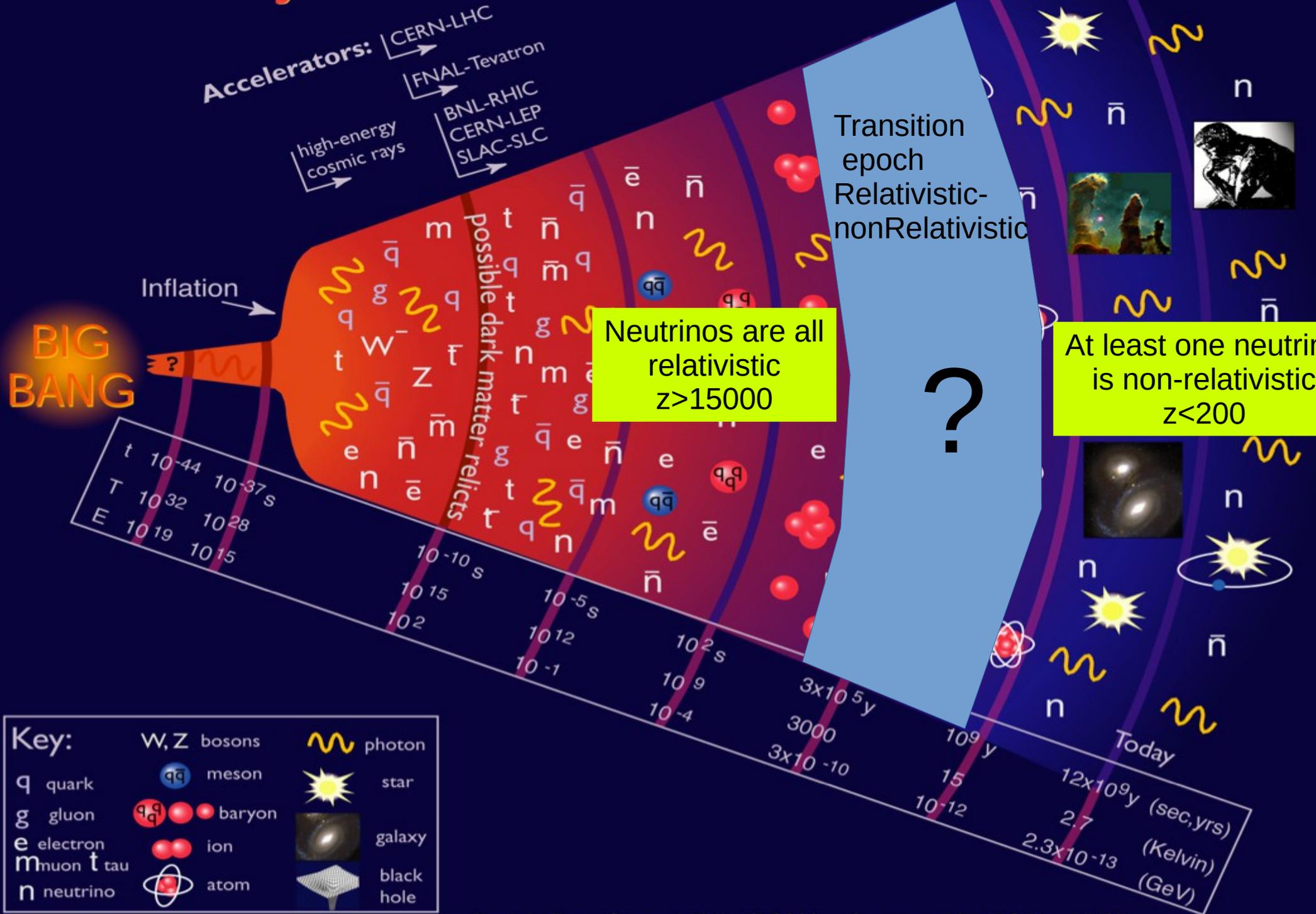
$$\rho_\nu = \sum n_i m_i$$

$$n_i = \frac{6\zeta(3)}{11\pi^2} T_\gamma^3$$

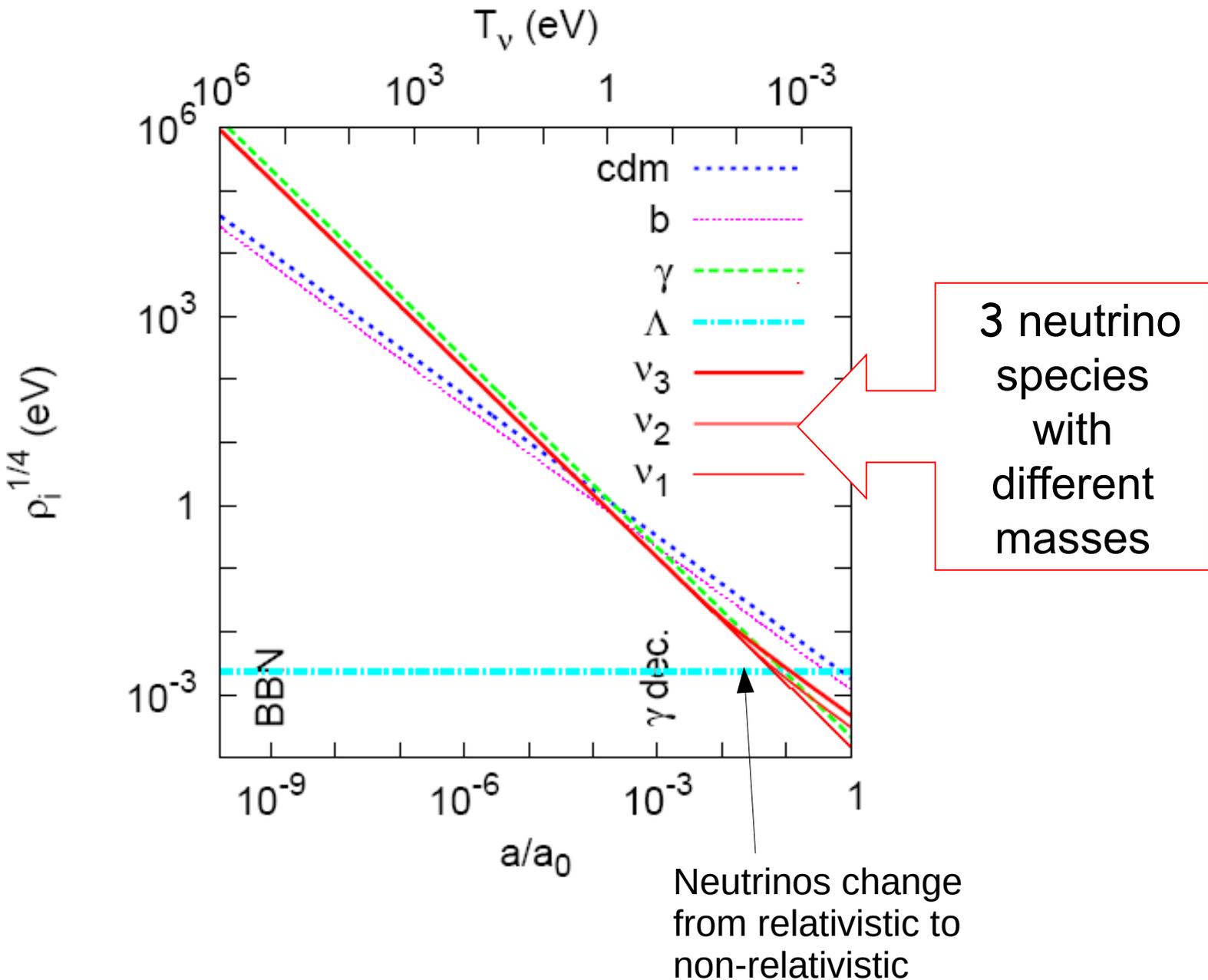
$$\Omega_\nu h^2 = \frac{\sum m_i}{93.2 \text{ eV}}$$

($T_\gamma = 2.726 \text{ K}$)

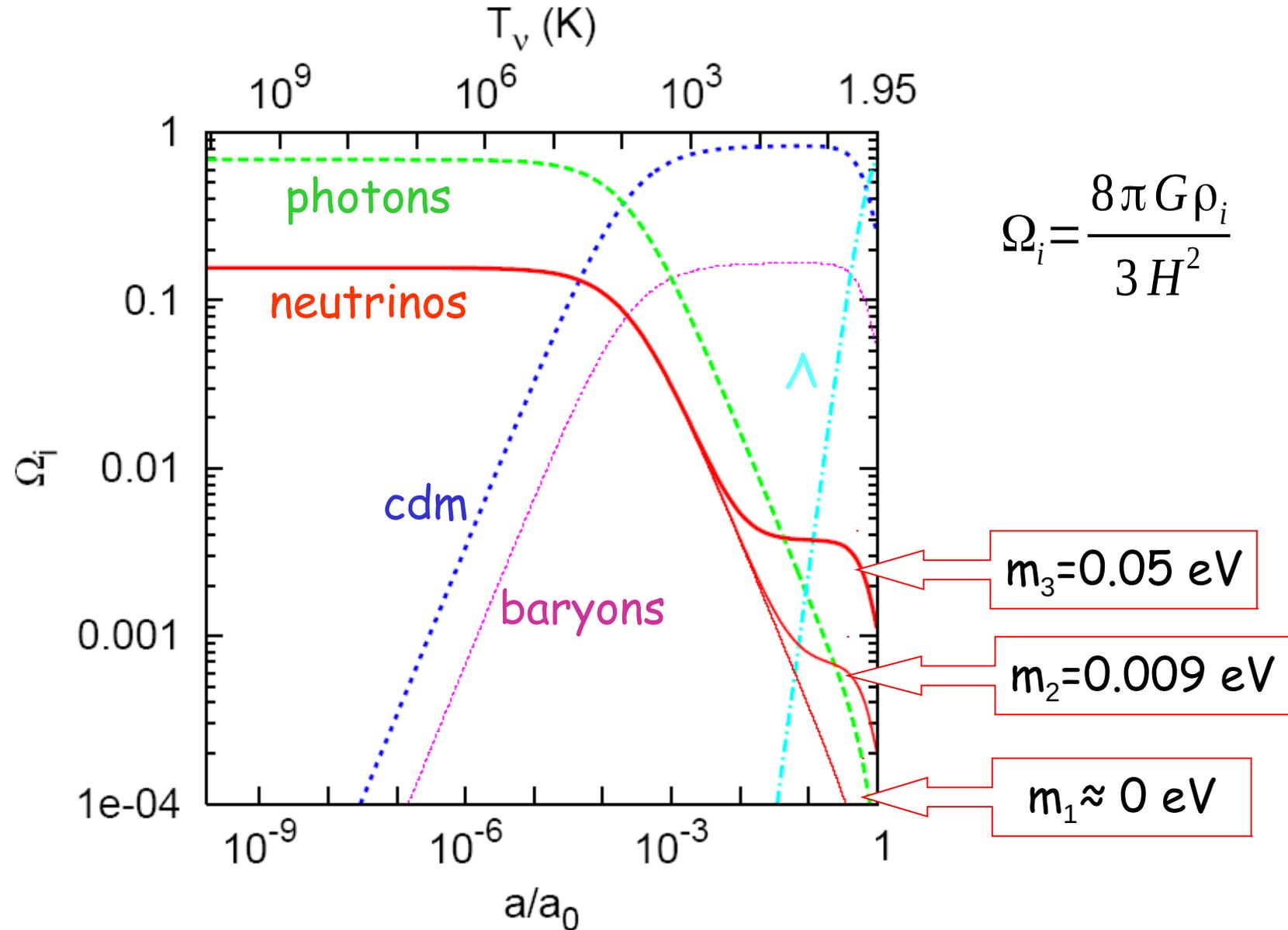
History of the Universe



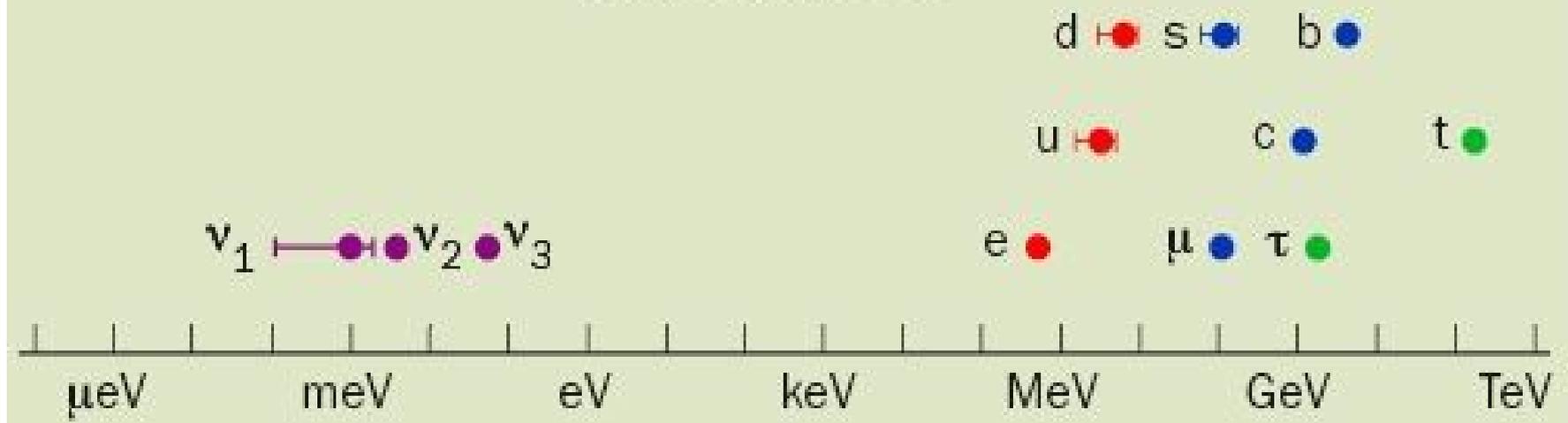
Evolution of the background densities: 1 MeV \rightarrow now



Evolution of the background densities: 1 MeV \rightarrow now



fermion masses

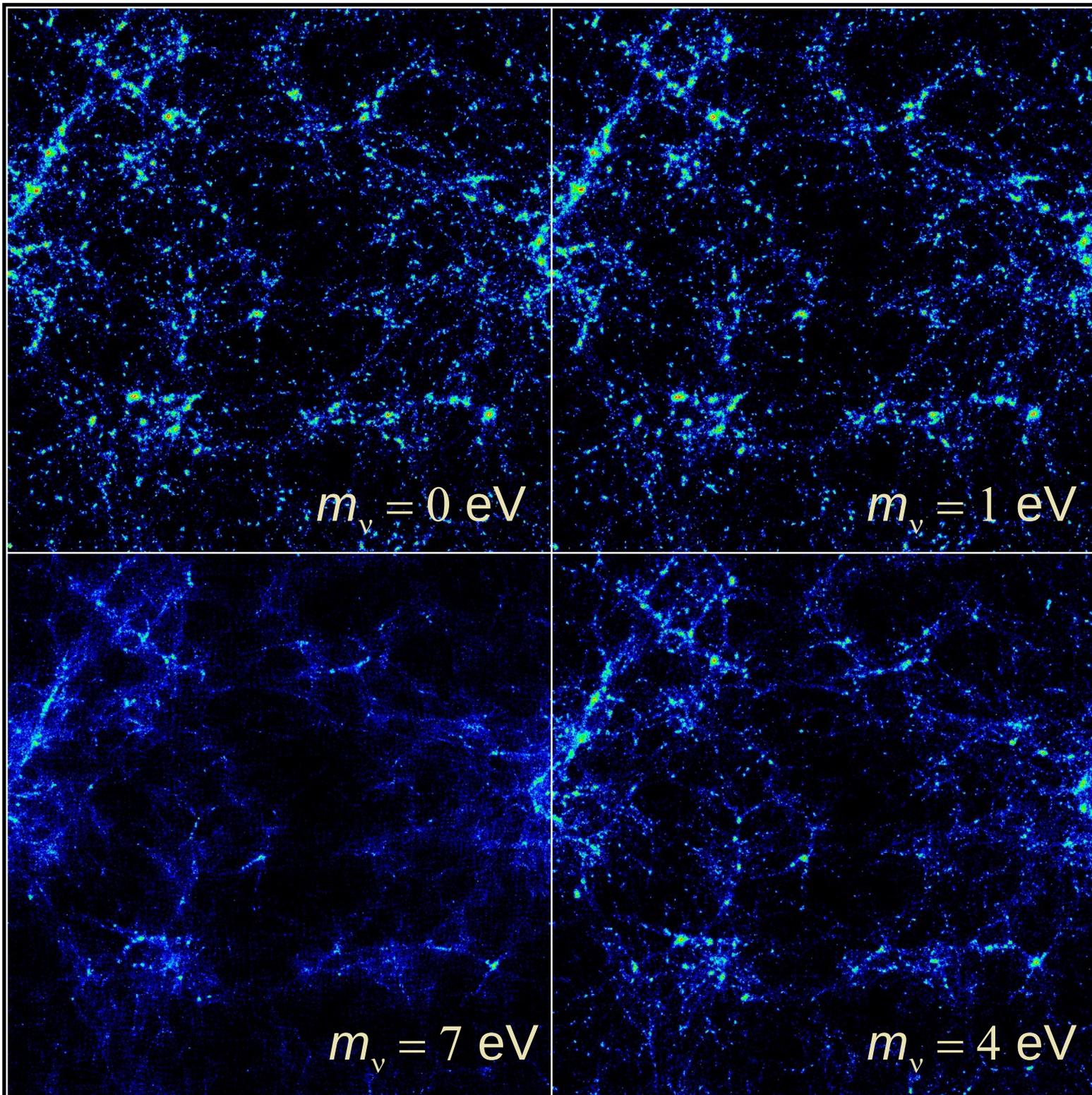


Massive Neutrinos affect large scale structure !

- ◆ We can relate the neutrino abundance in the universe to the total mass:

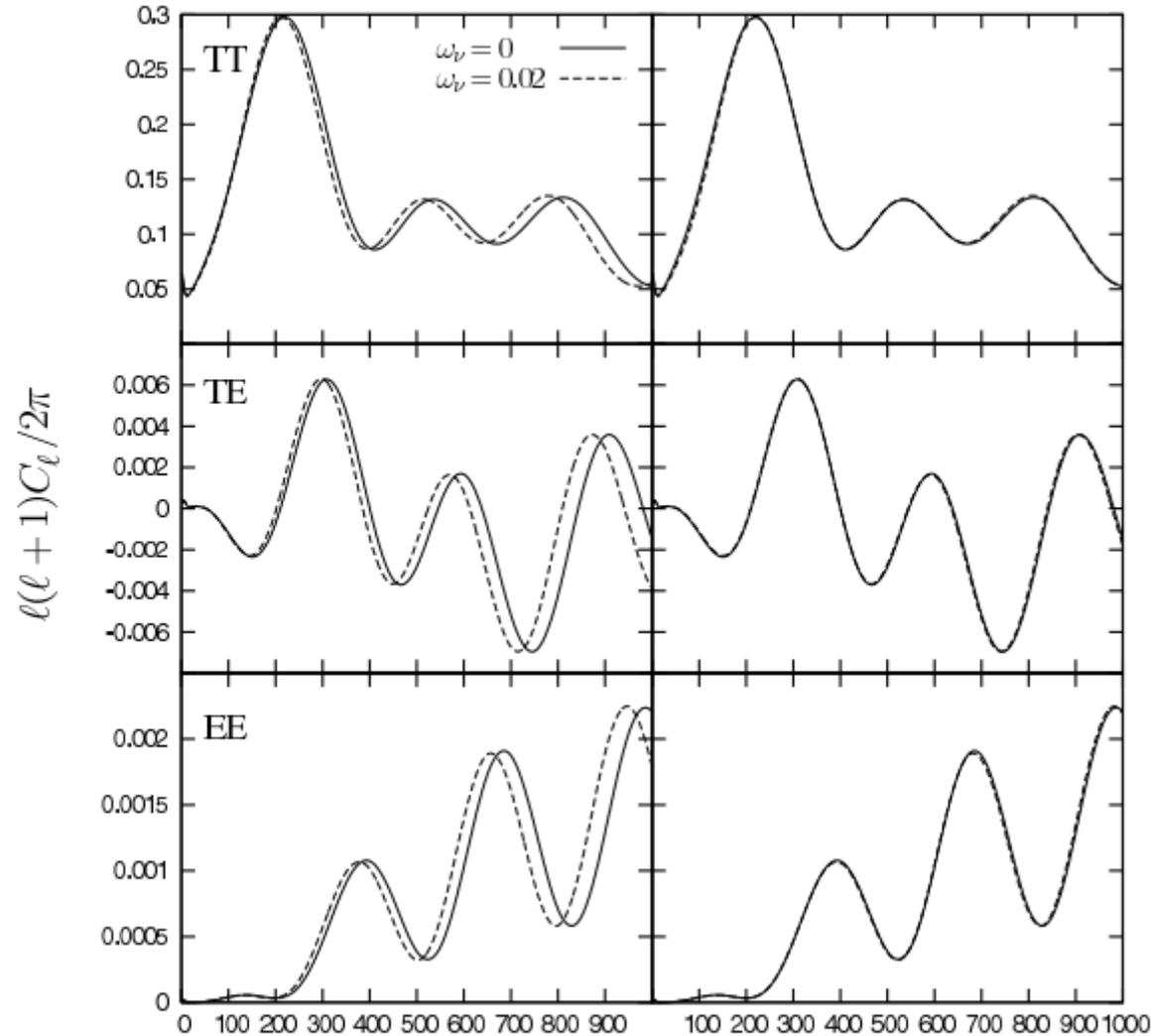
$$\Omega_{\nu} h^2 = \frac{\Sigma m_{\nu}}{93.6 eV}$$

- ◆ Less clustering in universe with massive neutrinos.



Ma '96

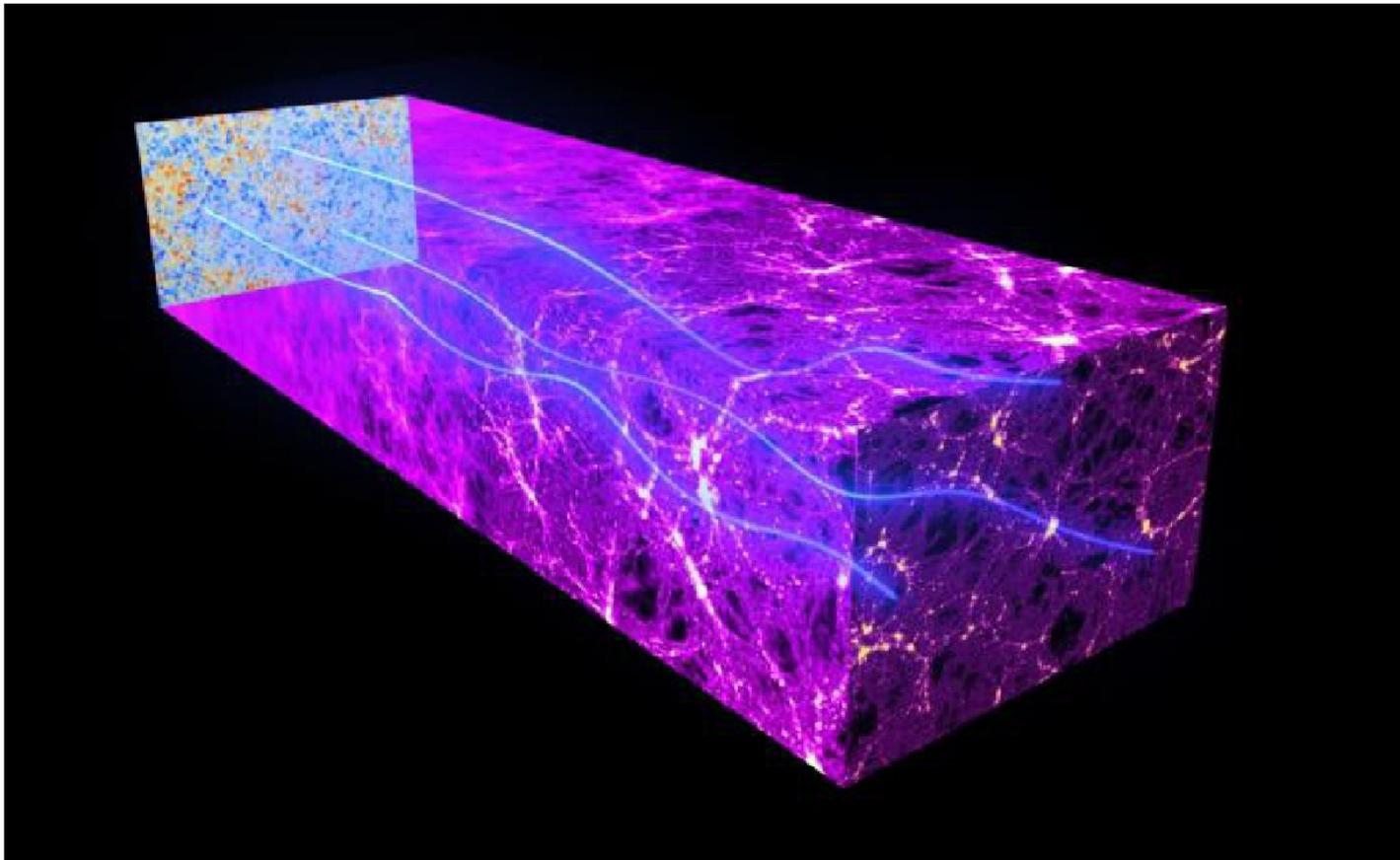
CMB bounds on neutrino masses



If neutrinos have a mass larger than $m \sim 0.3$ eV they are non-relativistic at recombination. At this point, increasing their mass increases the dark matter component at recombination. The change in the gravitational potential at early times with massive neutrinos leads to a slight changes in Cl below the first acoustic peak (see Ma & Bertshchinger 1998, Fukugita et al, 2006). In principle, CMB should constrain neutrino masses to $m < 2$ eV but constraints are actually much better because of lensing.

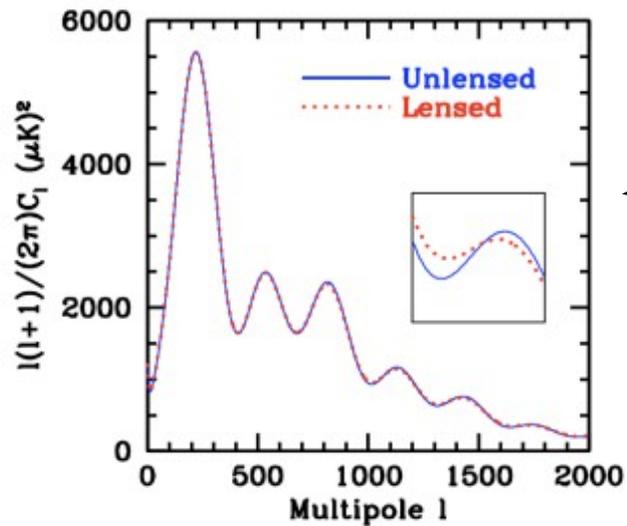
Gravitational Lensing

The gravitational effects of intervening matter bend the path of CMB light on its way from the early universe to the Planck telescope. This “gravitational lensing” distorts our image of the CMB



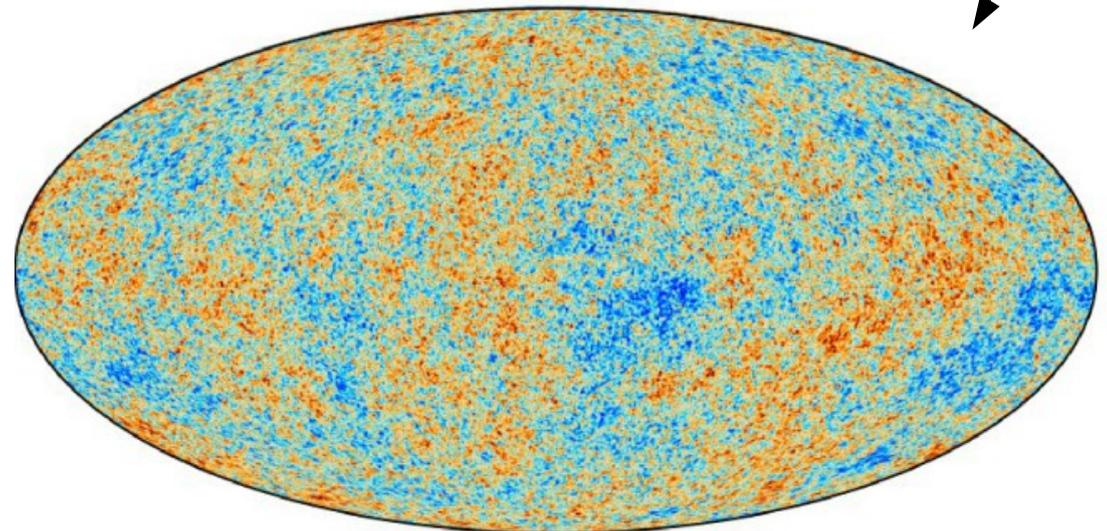
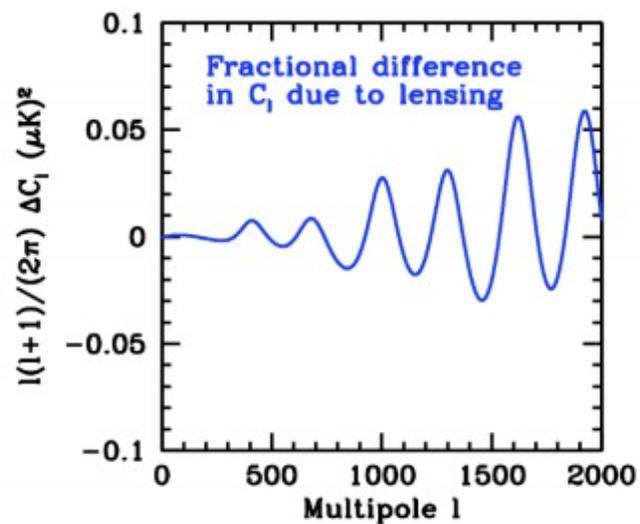
CMB anisotropy do not probe just recombination epoch !
CMB lensing probes structure formation around $z \sim 2$ (broadly)

Extracting Lensing Signal from the CMB

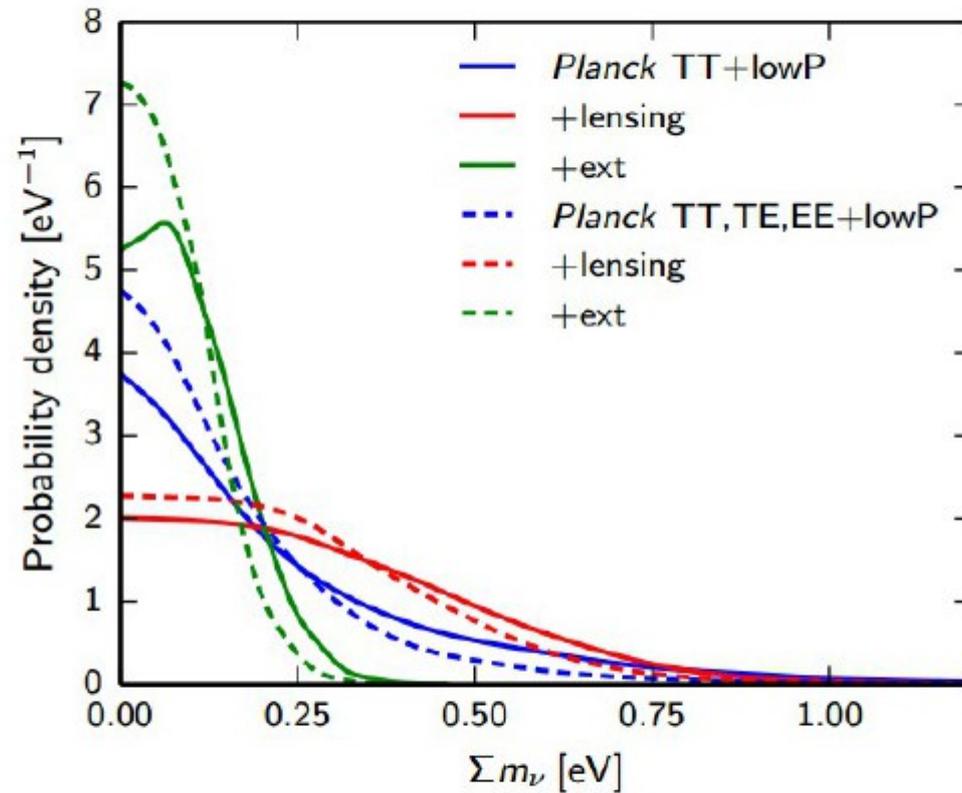


Extraction at Angular Spectrum Level
Effects on CMB angular spectrum.
More massive neutrinos \rightarrow less lensing.

Extraction at map level.
Lensing introduce a correlation between C_l 's.
This creates a non-zero trispectrum (4-point c.f.).



3 standard Massive Neutrinos



$$\left. \begin{array}{l} \Sigma m_\nu < 0.23 \text{ eV} \\ \Omega_\nu h^2 < 0.0025 \end{array} \right\} 95\%, \text{ Planck TT+lowP+lensing+ext.}$$

Constraining Neutrino Masses with CMB only (forecasts)

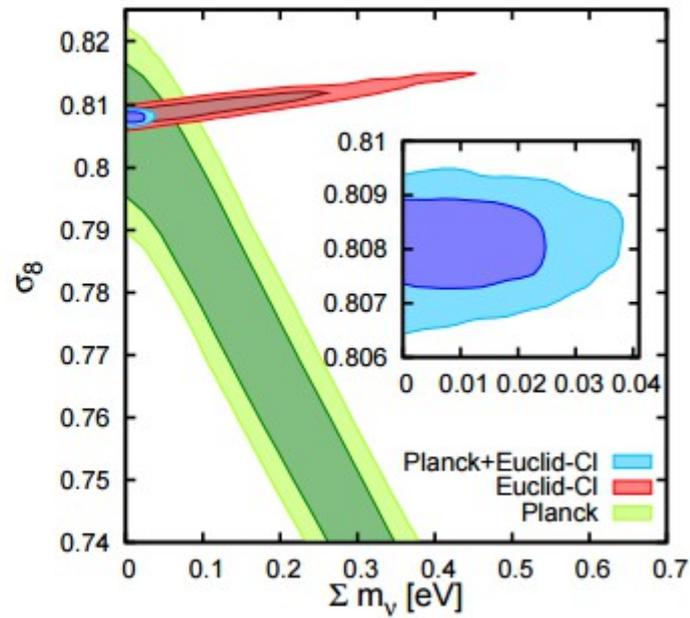
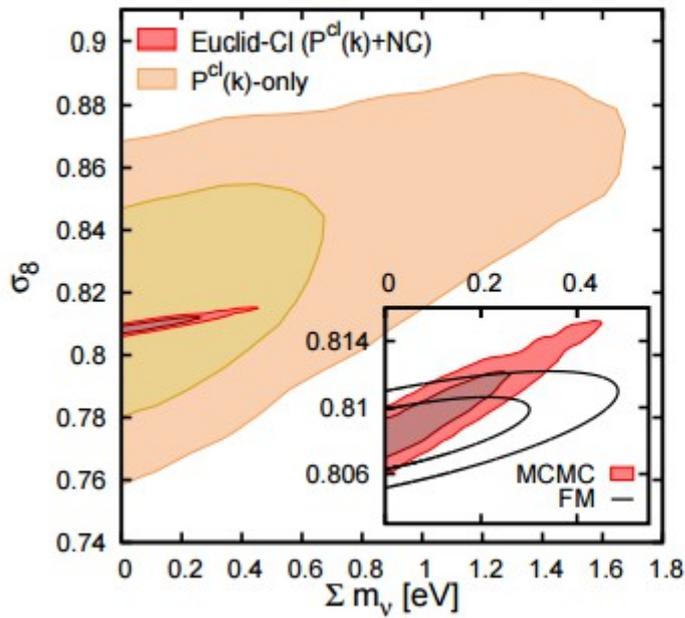
Parameter uncertainty	Planck	Planck+ACTPol	CMBPol
$\sigma(\Omega_b h^2)$	0.00014	0.000081 (1.7)	0.000033 (4.2)
$\sigma(\Omega_c h^2)$	0.0017	0.0010 (1.7)	0.00071 (2.4)
$\sigma(\theta_S)$	0.00028	0.00016 (1.7)	0.000062 (4.5)
$\sigma(\tau)$	0.0042	0.0034 (1.2)	0.0023 (1.8)
$\sigma(n_S)$	0.0034	0.0022 (1.5)	0.0016 (2.1)
$\sigma(\log[10^{10} A_S])$	0.013	0.0094 (1.4)	0.0065 (2.0)
$\sigma(\sum m_\nu)$	< 0.16	< 0.08 (2.0)	< 0.05 (3.2)

(eV)

TABLE III. 68% c.l. errors on cosmological parameters in the case of massive neutrinos. The numbers in brackets show the improvement factor σ_{Planck}/σ respect to the Planck experiment.

Galli et al, Phys.Rev.D82:123504,2010

Thanks to lensing we can constrain neutrino masses using just CMB data !



Model		Λ CDM+ m_ν			
Data		Planck	P^{cl} -only	Euclid-CI	Euclid-CI+Planck
$\sum m_\nu$ [eV]	68% CL	< 0.41	< 0.41	< 0.17	< 0.017
	95% CL	< 0.74	< 1.20	< 0.35	< 0.031

Backup Slides

Caveats

- Current measurements of the neutron lifetime based on two different methods are in disagreement by about ~ 8 s. Systematic errors for Helium production could be underestimated (see e.g. Salvati et al., arXiv:1507.07243).

$$\tau_n^{\text{bottle}} = (879.6 \pm 0.8) \text{ [s]} \quad \tau_n^{\text{beam}} = (888.0 \pm 2.1) \text{ [s]}$$

- $d(p,\gamma)^3\text{He}$ rate that enters in the calculation of D is not well measured (with most recent measurements dating 1998) and is about 10% smaller than what ab initio computations suggest. Again, systematics could be underestimated (see Di Valentino et al., Phys. Rev. D 90, 023543 (2014)).

Reaction	Rate Symbol	$\sigma_{2\text{H}/\text{H}} \cdot 10^5$
$p(n, \gamma)^2\text{H}$	R_1	± 0.002
$d(p, \gamma)^3\text{He}$	R_2	± 0.062
$d(d, n)^3\text{He}$	R_3	± 0.020
$d(d, p)^3\text{H}$	R_4	± 0.013

2 pages Explanation

A classic result is that if all the matter contributing to the Cosmic density is able to cluster, the fluctuations grow as the Cosmic scale factor:

$$\delta \approx a$$

If only a fraction Ω_* can cluster the the equation is generalized to

$$\delta \approx a^p \quad p \approx \Omega_*^{3/5}$$

In the radiation dominated era $p=0$ and so we don't have clustering.
In the recent Λ -dominated epoch again, $p=0$. Fluctuations grow only in the matter dominated epoch with a net growth of

$$\left(\frac{a_{\Lambda D}}{a_{MD}} \right)^p \approx 4700^p$$

Massive non relativistic neutrinos are unable to cluster on small scales because of their high velocities. Between matter domination and dark energy domination they constitute a roughly constant fraction of the matter density:

$$f_\nu = 1 - \Omega_\nu$$

Since the neutrino number density is determined by standard model neutrino Freezeout, the fraction is a function of the sum of the 3 neutrino masses:

$$f_\nu \approx \frac{M_\nu}{\Omega_* h^2 \times 92.5 \text{eV}}$$

The net fluctuation growth factor is therefore given by:

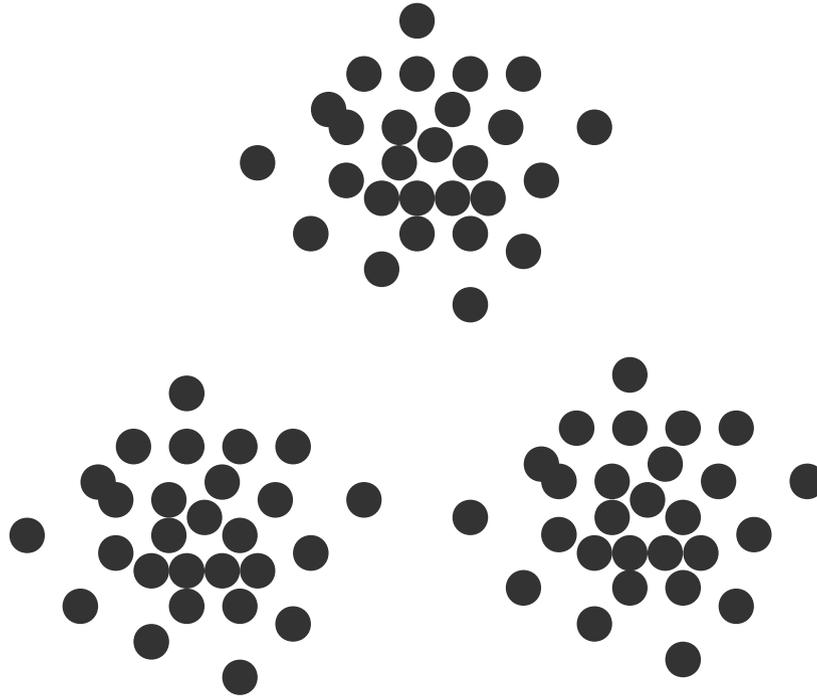
$$\left(\frac{a_{\Lambda D}}{a_{MD}} \right)^p \approx 4700^p \approx 4700^{(1-f_\nu)^{3/5}} \approx 4700 e^{-4 f_\nu}$$

The power spectrum is the variance of fluctuations in Fourier space, so Massive neutrinos suppress it by a factor:

$$P(k, f_\nu) \cong e^{-8 f_\nu} P(k, 0)$$

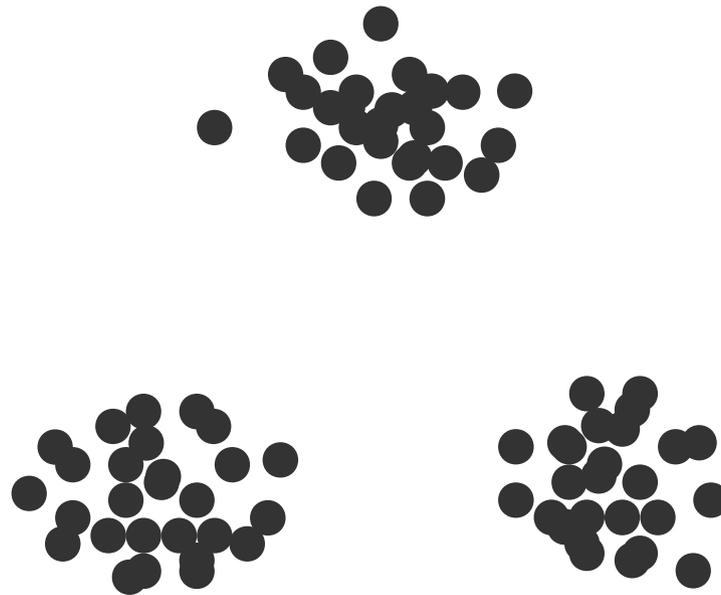
Structure formation after equality

baryons and CDM
experience
gravitational
clustering



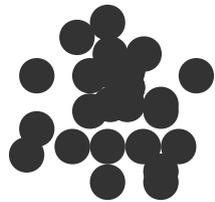
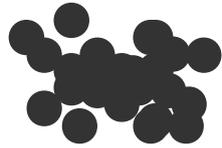
Structure formation after equality

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Structure formation after equality

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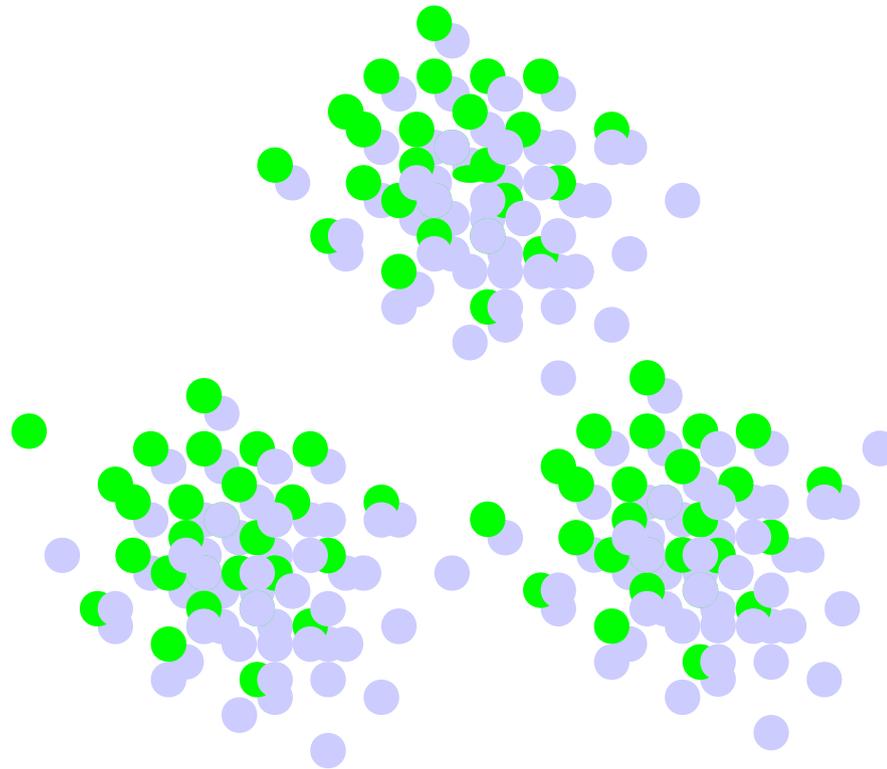


growth of $\delta\rho/\rho(k,t)$ fixed by
« gravity vs. expansion » balance

$$\delta\rho/\rho \propto a$$

Structure formation after equality

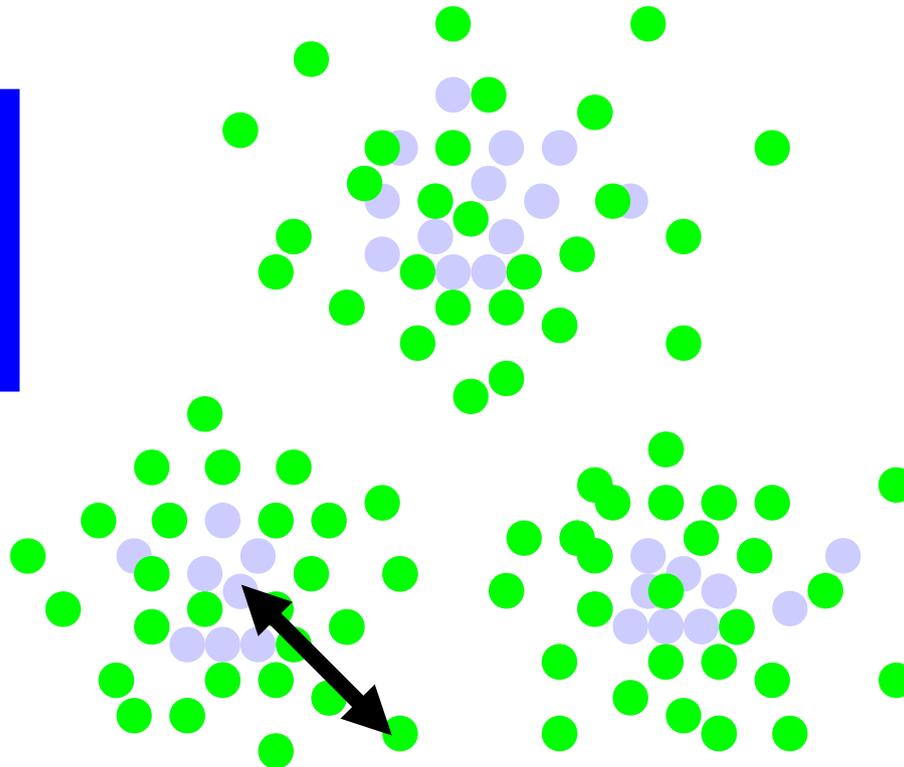
baryons and CDM
experience
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neutrinos
experience
free-streaming
with
 $v = c$ or $\langle p \rangle / m$

Structure formation after equality

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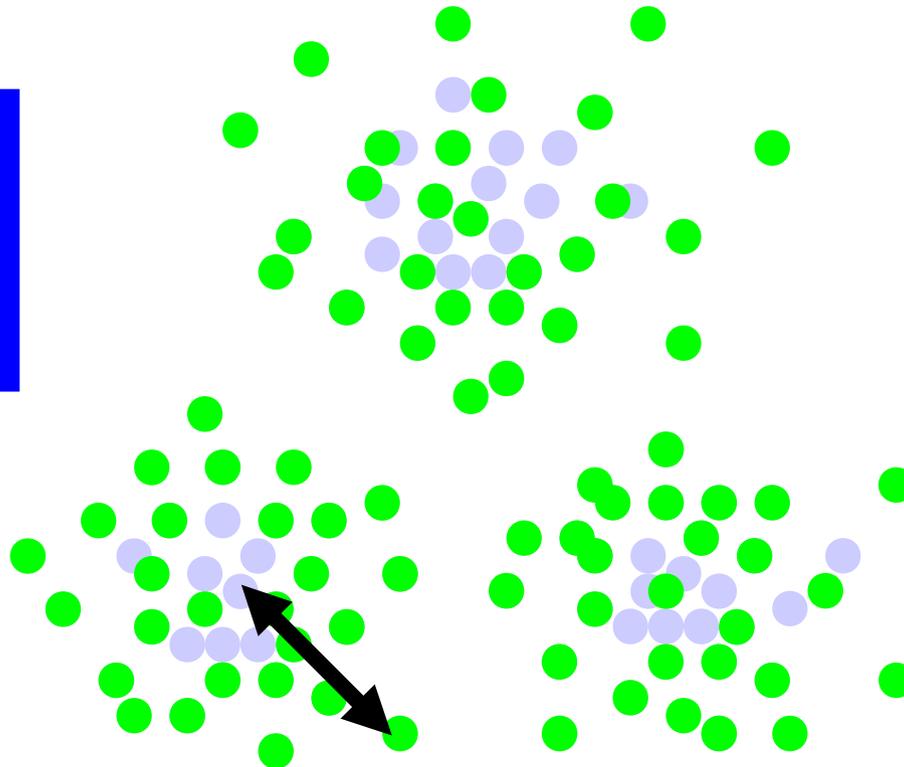


neutrinos
experience
free-streaming
with
 $v = c$ or $\langle p \rangle / m$

neutrinos cannot cluster below a diffusion length

Structure formation after equality

baryons and CDM
experience
gravitational
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neutrinos
experience
free-streaming
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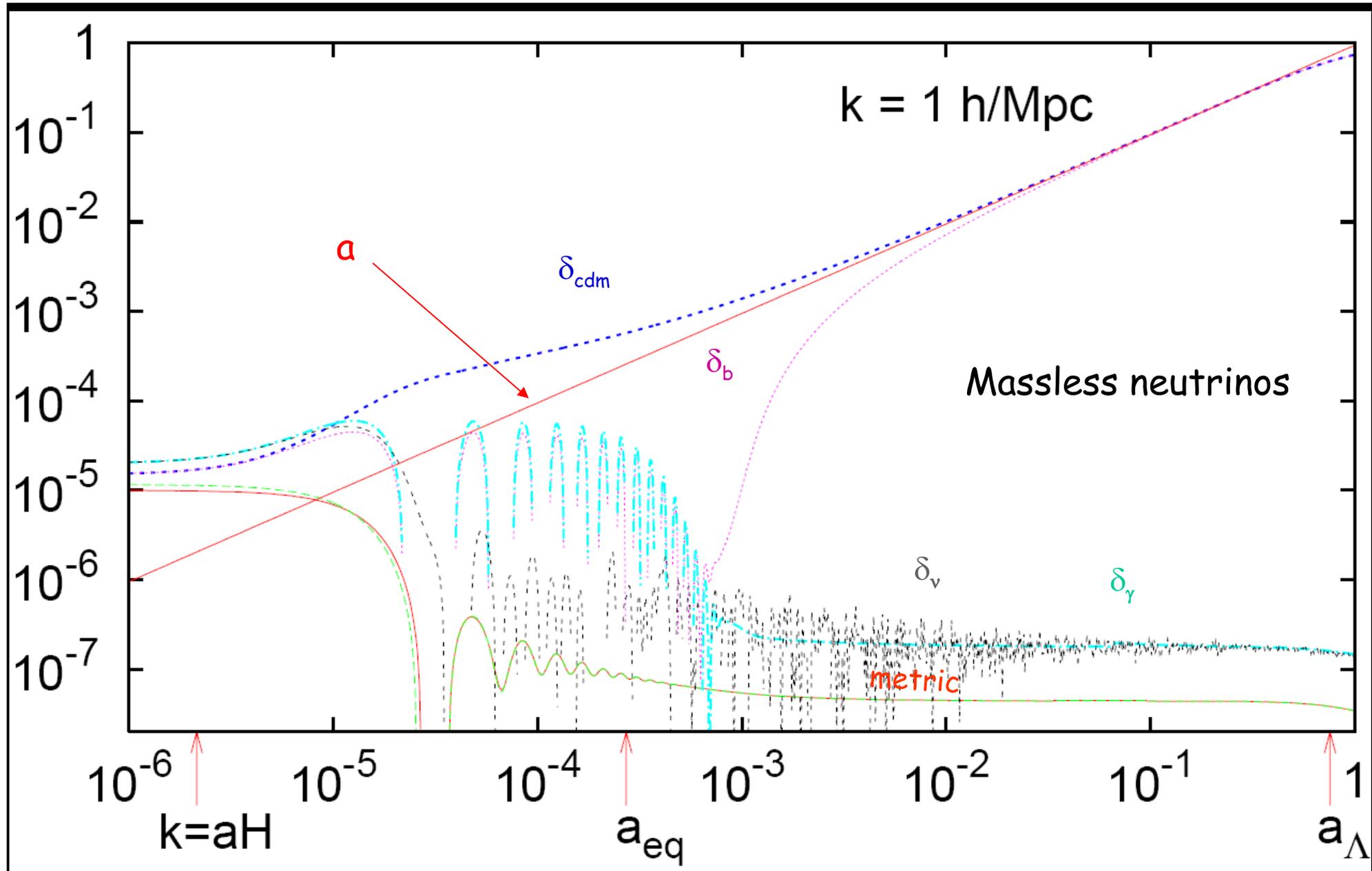
for $(2\pi/k) < \lambda$,

free-streaming suppresses growth of structures during MD

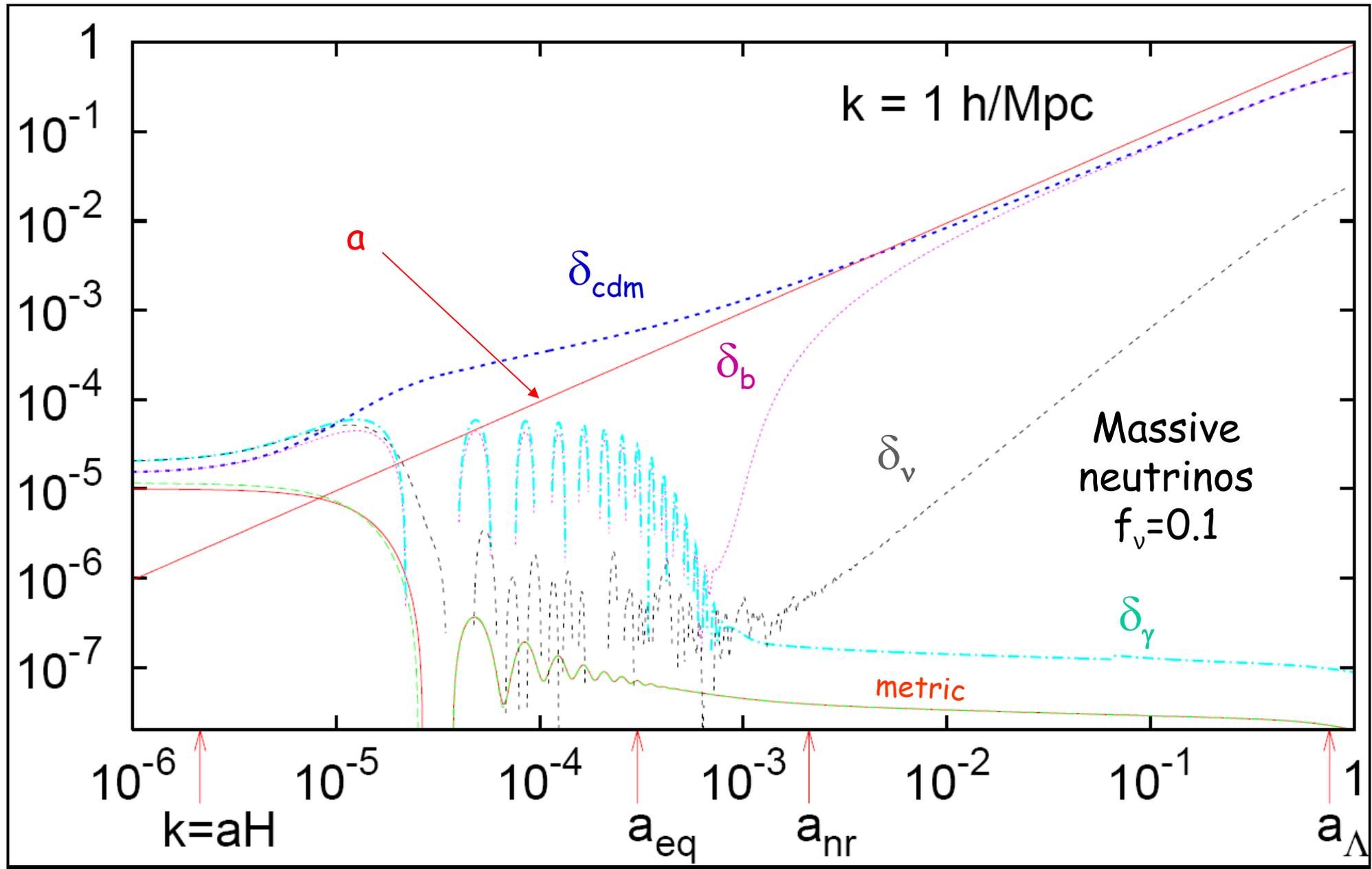
$$\delta\rho/\rho \propto a^{1-3/5 f_v}$$

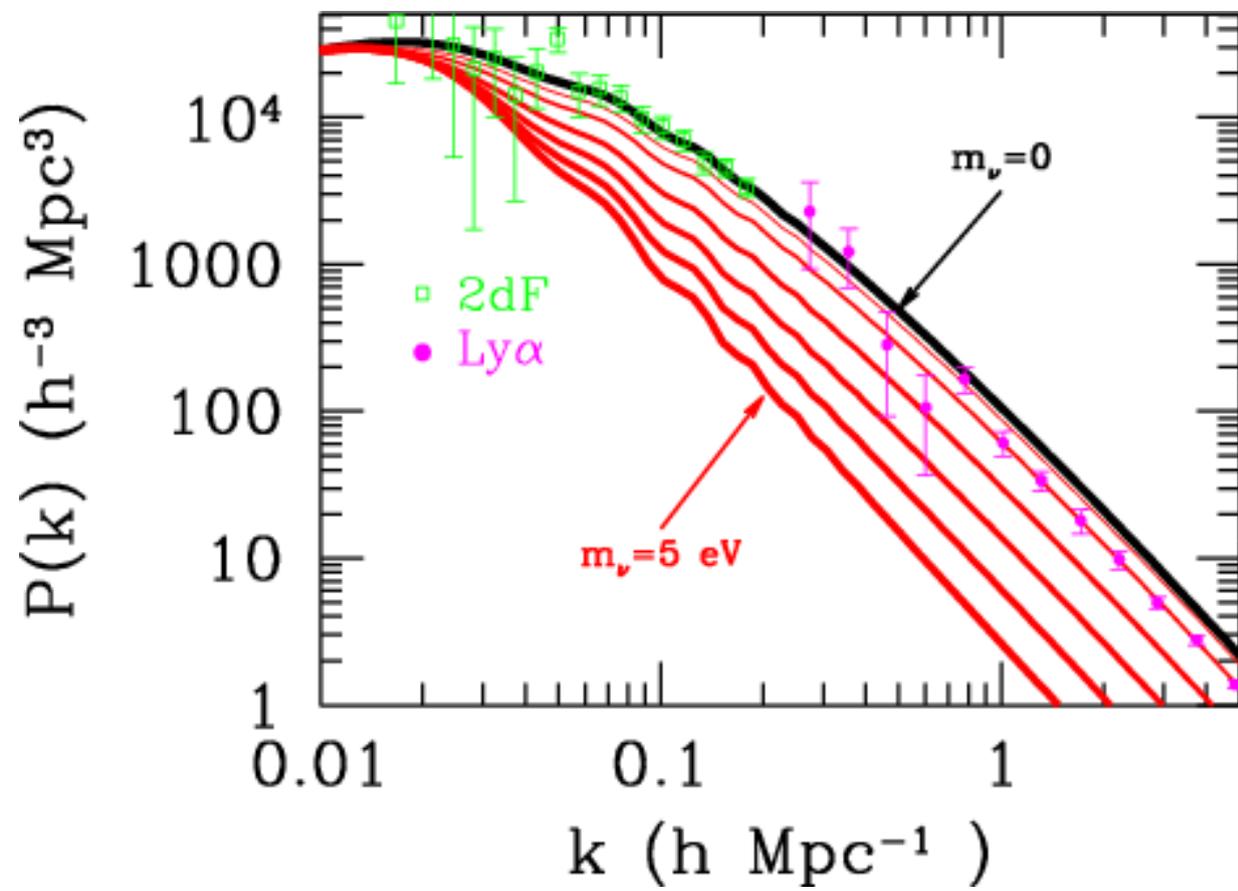
$$\text{with } f_v = \rho_v / \rho_m \approx (\Sigma m_\nu) / (15 \text{ eV})$$

Structure formation after equality



Structure formation after equality





...but we have degeneracies...

- Lowering the matter density suppresses the power spectrum
- This is virtually degenerate with **non-zero neutrino mass**

