Low and High Energy Modeling in Geant4

Hadronic Shower Simulation Workshop
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(on behalf of Geant4 Collaboration)

Overview

- Quark-Gluon String Model
- Bertini Cascade
- Binary Cascade
- CHIPS

Origin of the QGS (Quark-Gluon String) Model

- Author: H-P. Wellisch, M. Komagorov
- Most code unique to Geant4
  - guidance from Dubna QGS model (N.S. Amelin)
  - fragmentation code based on pre-existing FORTRAN

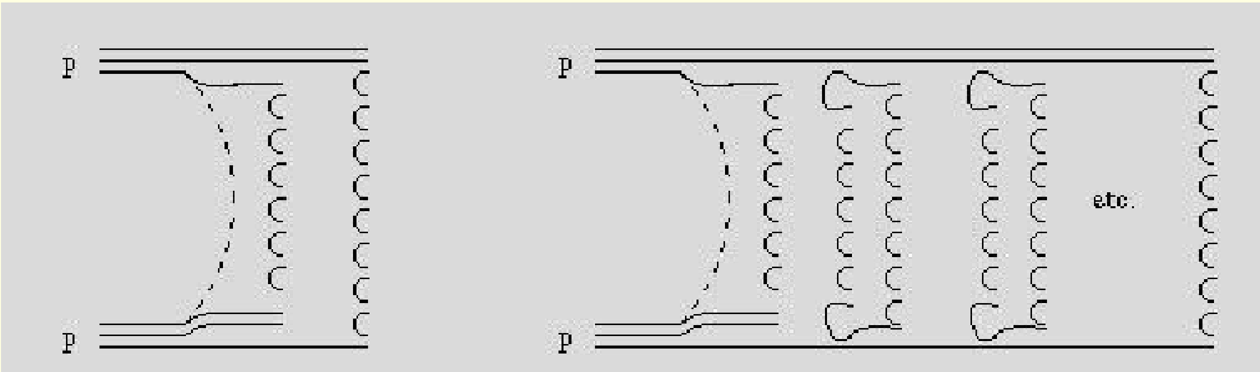
Applicability

- Incident p, n, π, K
- Also for high energy γ when CHIPS model is connected
- ~20 GeV < E < 50 TeV

- Model handles:
  - Selection of collision partners
  - Splitting of nucleons into quarks and diquarks
  - Formation and excitation of quark-gluon string
  - String hadronization
- Damaged nucleus remains. Another Geant4 model must be added for nuclear fragmentation and de-excitation
  - pre-compound model, or CHIPS for nuclear fragmentation

Quark Gluon String Model

- Two or more strings may be stretched between partons within hadrons
  - Strings from cut cylindrical Pomerons
- Parton interaction leads to color coupling of valence quarks
  - sea quarks included too
- Partons connected by quark gluon strings, which hadronize



Quark Gluon String Model Algorithm

- Build up 3-dimensional model of nucleus
- Large γ-factor collapses nucleus to 2 dimensions
- Calculate impact parameter with all nucleons
- Calculate hadron-nucleon collision probabilities
  - based on quasi-eikonal model, using Gaussian density distributions for hadrons and nucleons
- Sample number of strings exchanged in each collision
- Unitarity cut, string formation and decay

The Nuclear Model

- Nucleon momenta are sampled assuming Fermi gas model
- Nuclear density
  - harmonic oscillator shape for $A < 17$
  - Woods-Saxon for others
- Sampling is done in a correlated manner:
  - local phase-space densities are constrained by Pauli principle
  - sum of all nucleon momenta must equal zero

Collision Criterion

- In the Regge-Gribov approach, the probability of an inelastic collision with nucleon i can be written as

$$p_i(b_i, s) = (1/c)(1 - \exp[-2u(b_i, s)]) = \sum_{n=1}^{\infty} p_{i^{(n)}}(b_i, s)$$

- where

$$p_{i^{(n)}}(b_i, s) = (1/c)\exp[-2u(b_i, s)]\frac{[2u(b_i, s)]^n}{n!}$$

is the probability of finding n cut pomerons in the collision

$$u(b_i, s) = \frac{z(s)}{2}\exp(b_i^2/4L(s))$$

is the eikonal amplitude for hadron-nucleon elastic scattering with pomeron exchange

Pomeron Parameters

- The functions z(s) and L(s) contain the pomeron parameters:
  - fitted to N-N, π–N, K-N collision data (elastic, total, single diffraction cross sections)
  - pomeron trajectory: $\alpha_P' = 0.25\ GeV^{-2}$, $\alpha_P(0) = 1.0808$ for π, K, 0.9808 for N

- Other parameters:
  - energy scale $s_0 = 3.0\ GeV^2$ for N, $1.5\ GeV^2$ for π, $2.3\ GeV^2$ for K
  - Pomeron-hadron vertex parameters also included:
    - coupling: $\gamma_P^N = 6.56\ GeV^{-2}$
    - radius of interaction: $R^{2N}{}_P = 3.56\ GeV^{-2}$

Diffractive Dissociation

- Need to sample the probability of diffraction
  - get it from difference of total and inelastic collision probabilities

$$p_{ij}^{diff}(b_{ij},s)=\frac{\mathrm{c}-1}{\mathrm{c}}(p_{ij}^{tot}(b_{ij},s)-p_{ij}(b_{ij},s))$$

  - where c is the “shower enhancement” coefficient
  - c = 1.4 for nucleons, 1.8 for pions
- Splitting off diffraction probabilities with parameter c follows method of Baker 1976

String Formation

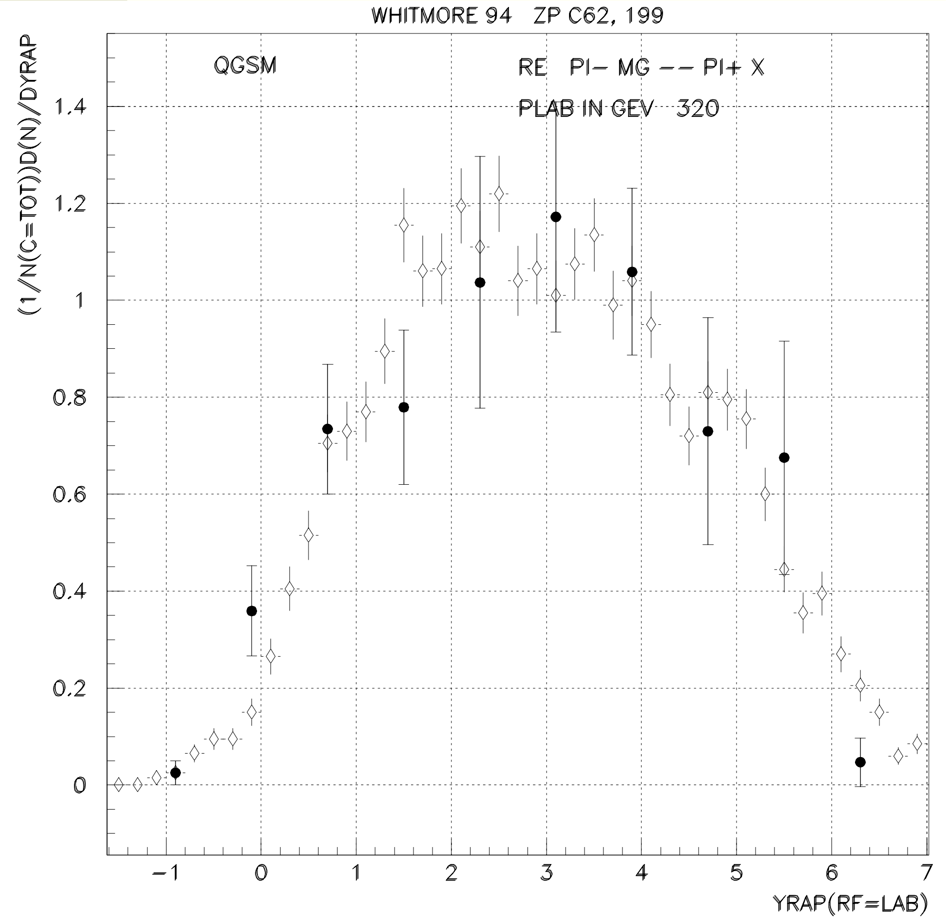
- Cutting the pomeron yields two strings
- String formation is done by parton exchange (Capella 94, Kaidalov 82)
  - for each participating hadron, parton densities are sampled
  - requires quark structure function of hadron
  - parton pairs combined into color singlets
  - sea quarks included with u:d:s = 1: 1: 0.27

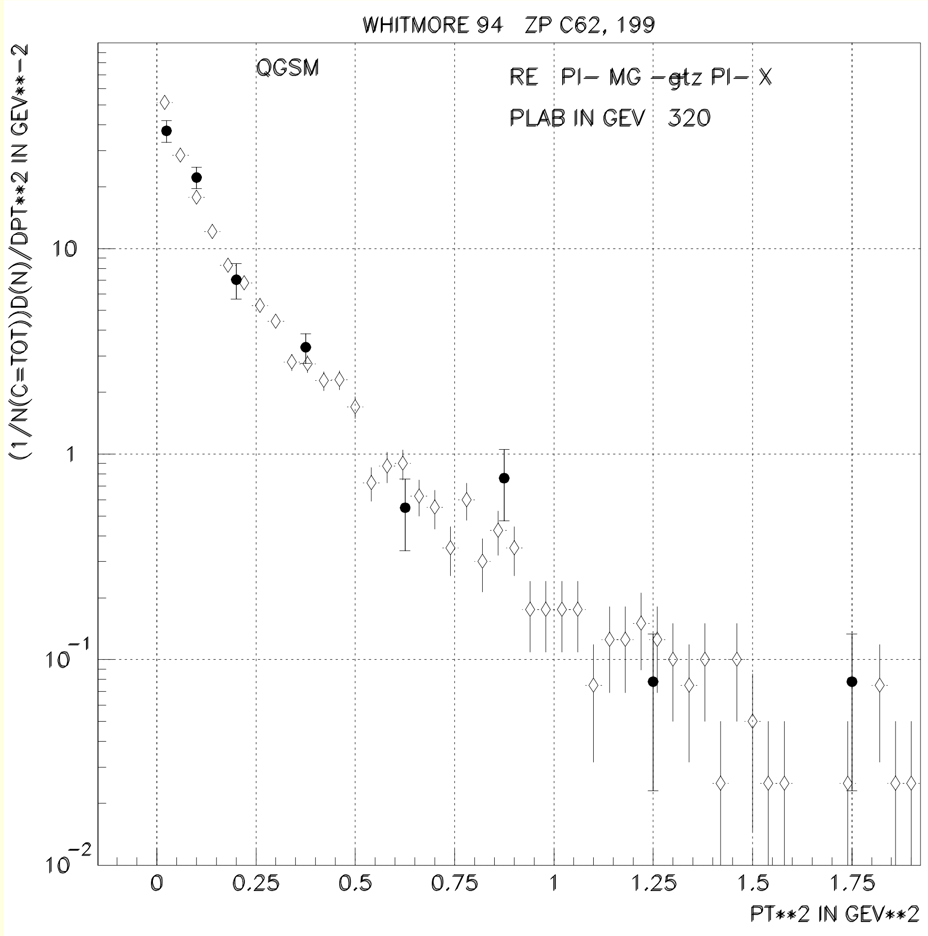
Longitudinal String Fragmentation

- String extends between constituents
- Break string by inserting q-qbar pair according to
  - u : d : s : qq = 1 : 1 : 0.27 : 0.1
- At break -> new string + hadron
- Gaussian P_t , $\langle P_t^2 \rangle$ = 0.5 GeV
- Created hadron gets longitudinal momentum from sampling QGSM fragmentation functions
  - Lund functions also available

QGSM - Results

pi- Mg → pi+ X , Plab 320 GeV/c

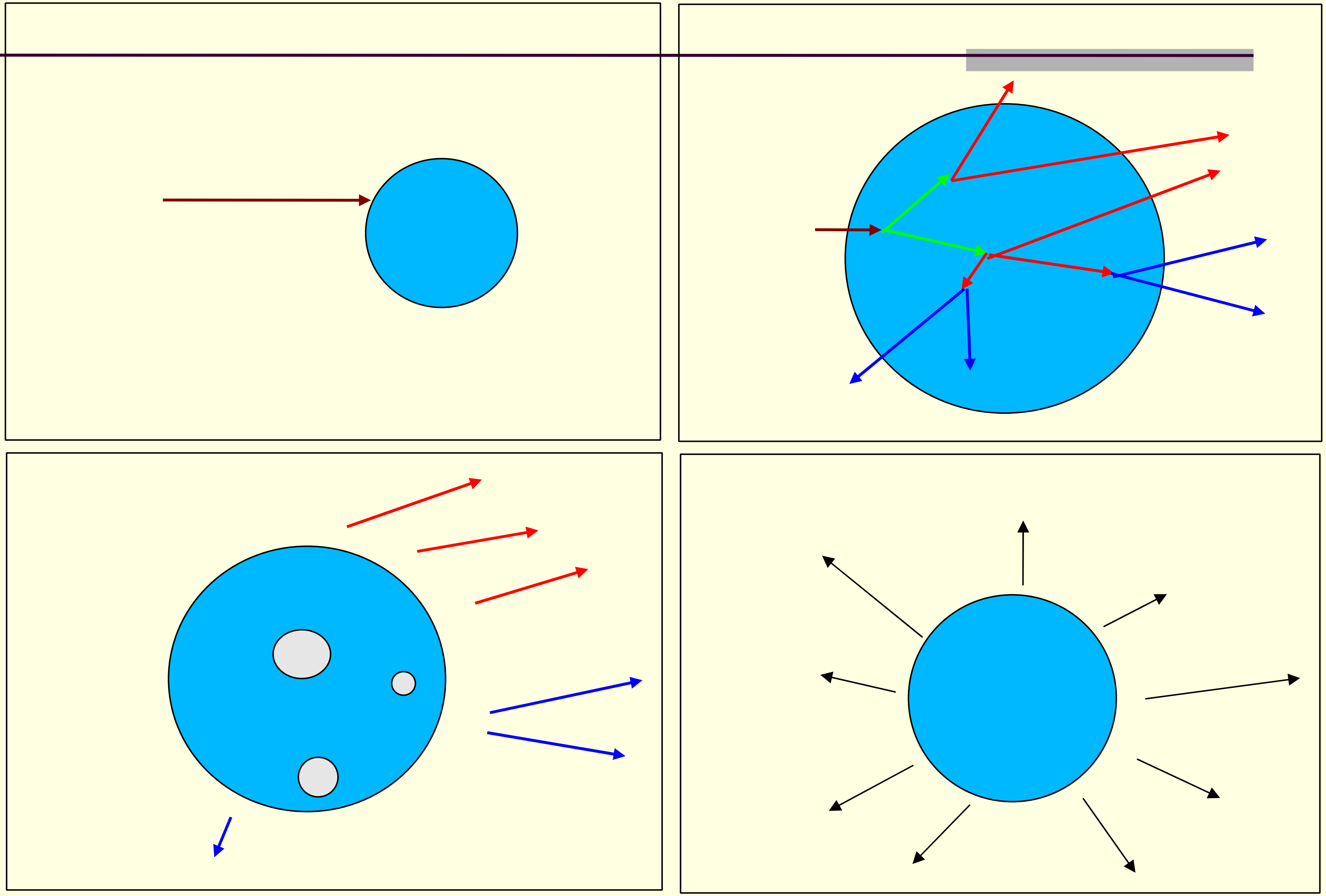




Rapidity $\eta = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$

P_t^2 [GeV^2]

Cascade Modeling Concept



Geant4 Bertini Cascade: Origin

- A re-engineered version of the INUCL code of N. Stepanov (ITEP)
- Employs many of the standard INC methods developed by Bertini (1968)
  - using free particle-particle collisions within cascade
  - step-like nuclear density
- Similar methods used in many different intra-nuclear transport codes

Applicability of the Bertini Cascade

- inelastic scattering of p, n, π, K, Λ, Σ, Ξ
- incident energies: $0 < E < 10$ GeV
  - upper limit determined by lack of partial final state cross sections and the end of the cascade validity region
  - lower limit due to inclusion of internal nuclear de-excitation models
- in principle, can be extended to:
  - anti-baryons
  - ion-ion collisions

Bertini Cascade Model

- The Bertini model is a classical cascade:
  - it is a solution to the Boltzmann equation on average
  - no scattering matrix calculated
- Core code:
  - elementary particle collider: uses free cross sections to generate secondaries
  - cascade in nuclear medium
  - pre-equilibrium and equilibrium decay of residual nucleus
  - nucleus modelled as three concentric spheres of different densities; density constant within sphere

Bertini Cascade Modeling Sequence (1)

- Nuclear entry point sampled over projected area of nucleus
- Incident particle is transported in nuclear medium
  - mean free path from total particle-particle cross sections
  - nucleus modeled as 3 concentric, constant-density shells
  - nucleons have Fermi gas momentum distribution
  - Pauli exclusion invoked
- Projectile interacts with a single nucleon
  - hadron-nucleon interactions based on free cross sections and angular distributions
  - pions can be absorbed on quasi-deuterons

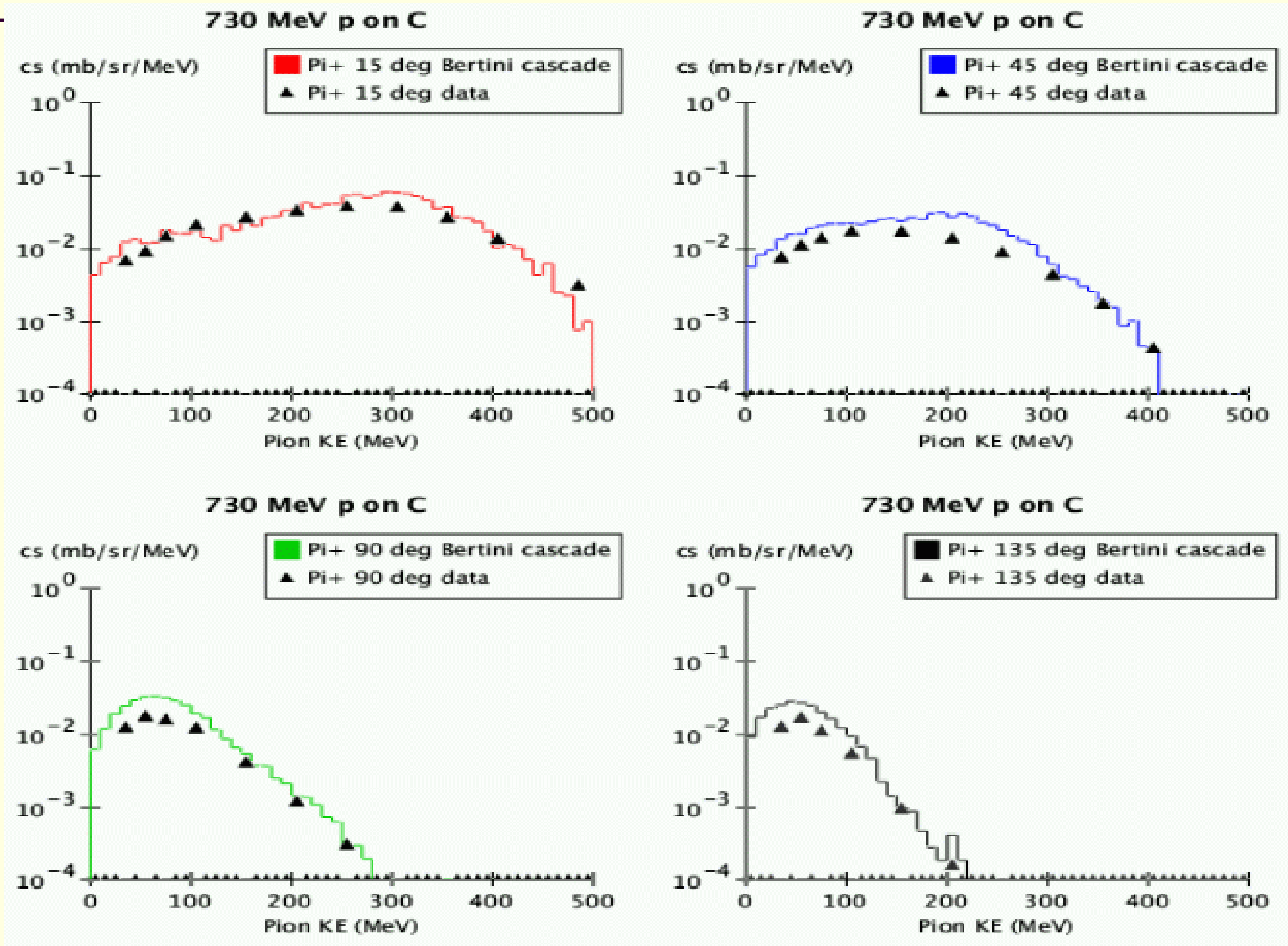
Bertini Cascade Modeling Sequence (2)

- Each secondary from initial interaction is propagated in nuclear potential until it interacts or leaves nucleus
  - can have reflection from density shell boundaries
  - currently no Coulomb barrier
- As cascade collisions occur, exciton states are built up, leading to equilibrated nucleus
  - selection rules for p-h state formation: $\Delta p = 0, +/1,$
    $\Delta h = 0, +/-1, \quad \Delta n = 0, +/-2$
- Model uses its own exciton routine based on that of Griffin
  - Kalbach matrix elements used
  - level densities parametrized vs. Z and A

Bertini Cascade Modeling Sequence (3)

- Cascade ends and exciton model takes over when secondary KE drops below 20% of its original value or 7 X nuclear binding energy
- Nuclear evaporation follows for most nuclei
  - emission continues as long as excitation is large enough to remove a neutron or α
  - γ emission below 0.1 MeV
- For light, highly excited nuclei, Fermi breakup
- Fission also possible

Validation of the Bertini Cascade



Origin and Applicability of the Binary Cascade

- H.P. Wellisch and G. Folger (CERN)
- Based in part on Amelin's kinetic model
- Incident p, n
  - $0 < E < \sim 3$ GeV
- light ions
  - $0 < E < \sim 3$ GeV/A
- π
  - $0 < E < \sim 1.5$ GeV

Binary Cascade

- Hybrid between classical cascade and full QMD model
- Detailed model of nucleus
  - nucleons placed in space according to nuclear density
  - nucleon momentum according to Fermi gas model
- Nucleon momentum taken into account when evaluating cross sections, collision probability
- Collective effect of nucleus on participant nucleons described by optical potential
  - numerically integrate equation of motion

Binary Cascade Modeling (1)

- Nucleon-nucleon scattering (t-channel) resonance excitation cross-sections are derived from p-p scattering using isospin invariance, and the corresponding Clebsch-Gordan coefficients
  - elastic N-N scattering included
- Meson-nucleon inelastic (except true absorption) scattering modelled as s-channel resonance excitation. Breit-Wigner form used for cross section.
- Resonances may interact or decay
  - nominal PDG branching ratios used for resonance decay
  - masses sampled from Breit-Wigner form

Binary Cascade Modeling (2)

- Calculate imaginary part of the R-matrix using free 2-body cross-sections from experimental data and parametrizations
- For resonance re-scattering, the solution of an in-medium BUU equation is used.
  - The Binary Cascade at present takes the following strong resonances into account:
    - The delta resonances with masses 1232, 1600, 1620, 1700, 1900, 1905, 1910, 1920, 1930, and 1950 MeV
    - Excited nucleons with masses 1440, 1520, 1535, 1650, 1675, 1680, 1700, 1710, 1720, 1900, 1990, 2090, 2190, 2220, and 2250 MeV

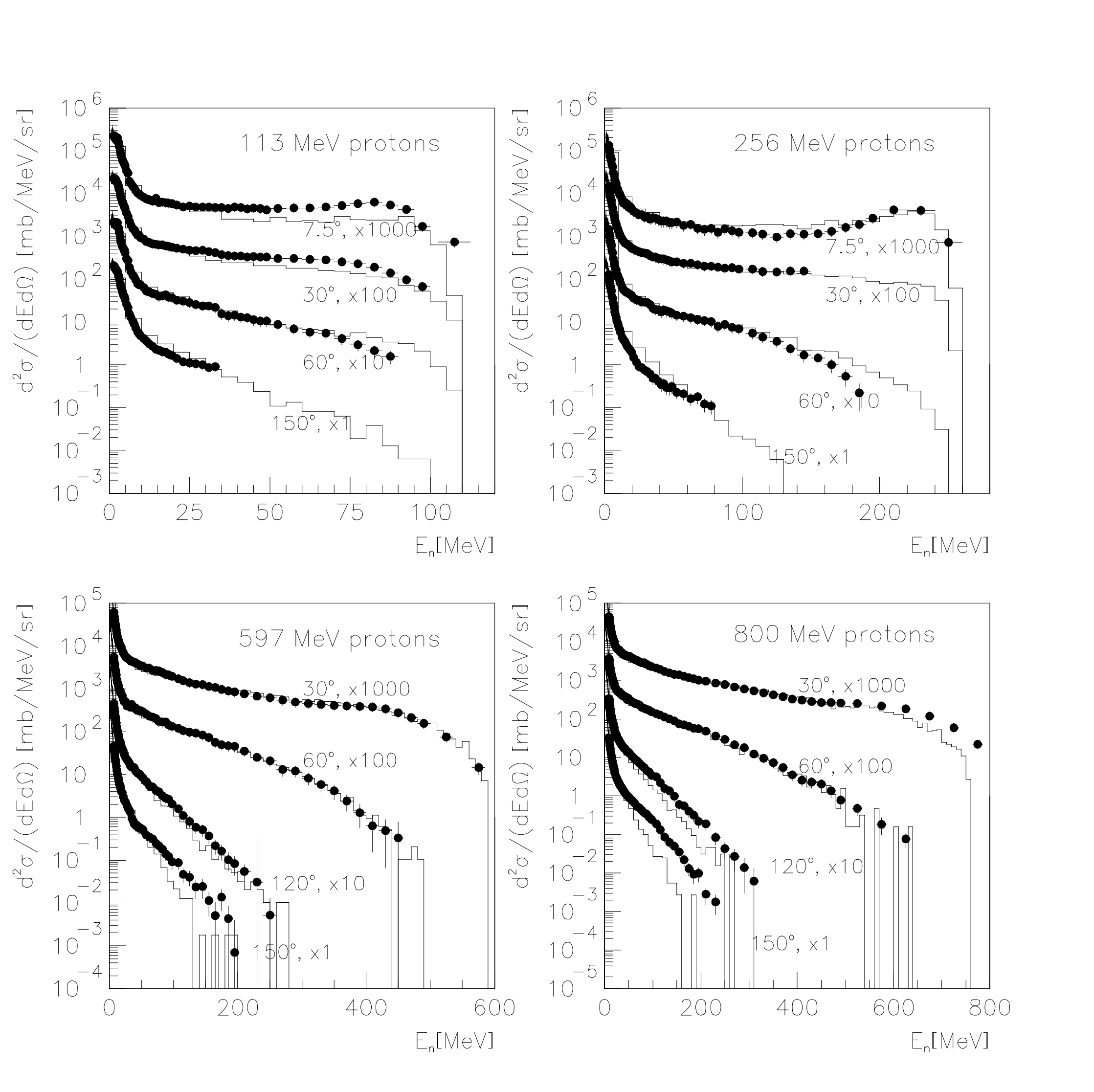
Binary Cascade Modeling (3)

- Nucleon-nucleon elastic scattering angular distributions taken from Arndt phase shift analysis of experimental data
- Pauli blocking implemented in its classical form
  - finals state nucleons occupy only states above Fermi momentum
- True pion absorption is modeled as s-wave absorption on quasi-deuterons
- Coulomb barrier taken into account for charged hadrons

Binary Cascade Modeling (4)

- Cascade stops when mean energy of all scattered particles is below A-dependent cut
  - varies from 18 to 9 MeV
  - if primary below 45 MeV, no cascade, just precompound
- When cascade stops, the properties of the residual exciton system and nucleus are evaluated, and passed to a pre-equilibrium decay code for nuclear de-excitation

Binary Cascade - results



p Pb -> n X

Chiral Invariant Phase Space (CHIPS)

- Origin: M.V. Kosov (CERN, ITEP)
  - Manual for the CHIPS event generator, KEK internal report 2000-17, Feb. 2001 H/R.
- Use:
  - capture of negatively charged hadrons at rest
  - anti-baryon nuclear interactions
  - gamma- and lepto-nuclear reactions
  - back end (nuclear fragmentation part) of QGSC model

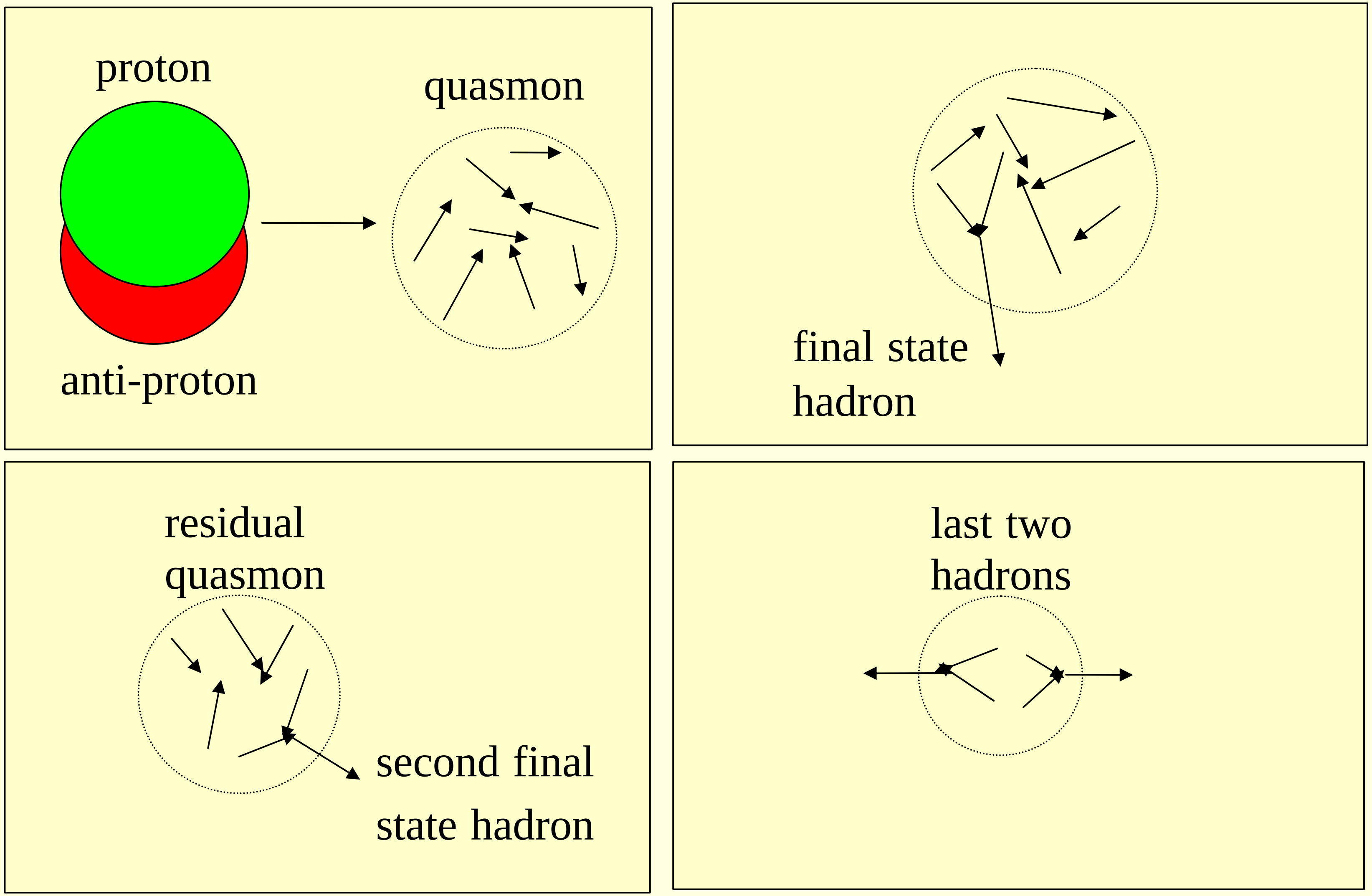
CHIPS Fundamental Concepts

- Quasmon: an ensemble of massless partons uniformly distributed in invariant phase space
  - a 3D bubble of quark-parton plasma
  - can be any excited hadron system or ground state hadron
- Critical temperature T_C : model parameter which relates the quasmon mass to the number of its partons:
  - $M^2_Q = 4n(n-1)T^2_C$ => $M_Q \sim 2nT_C$
  - T_C = 180 – 200 MeV
- Quark fusion hadronization: two quark-partons may combine to form an on-mass-shell hadron
- Quark exchange hadronization: quarks from quasmon and neighbouring nucleon may trade places

CHIPS Applications

- u,d,s quarks treated symmetrically (all massless)
  - model can produce kaons, but s suppression parameter is needed, η suppression parameter also required
  - real s-quark mass is taken into account by using masses of strange hadrons
- CHIPS is a universal method for fragmentation of excited nuclei (containing quasmons).
- Unique, initial interactions were developed for:
  - interactions at rest such as π^- capture, pbar annihilation
  - gamma- and lepto-nuclear reactions
  - hadron-nuclear interaction in-flight are in progress
- Anti-proton annihilation on p and π^- capture at rest in a nucleus illustrate two CHIPS modelling sequences

Modeling Sequence for Proton – antiproton Annihilation (1)



Modeling Sequence for Proton - antiproton Annihilation (2)

- anti-proton and proton form a quasmon in vacuum
  - no quark exchange with neighboring nucleons
  - $n = M/2T_C$ quark-partons uniformly distributed over phase space with spectrum $dW/kdk \propto (1 - 2k/M)^{n-3}$
- quark fusion occurs
  - calculate probability of two quark-partons in the quasmon to combine to produce effective mass of outgoing hadron:
    - sample k in 3 dimensions
    - second quark momentum q from spectrum of n-1 quarks
    - integrate over vector q with mass shell constraint for outgoing hadron

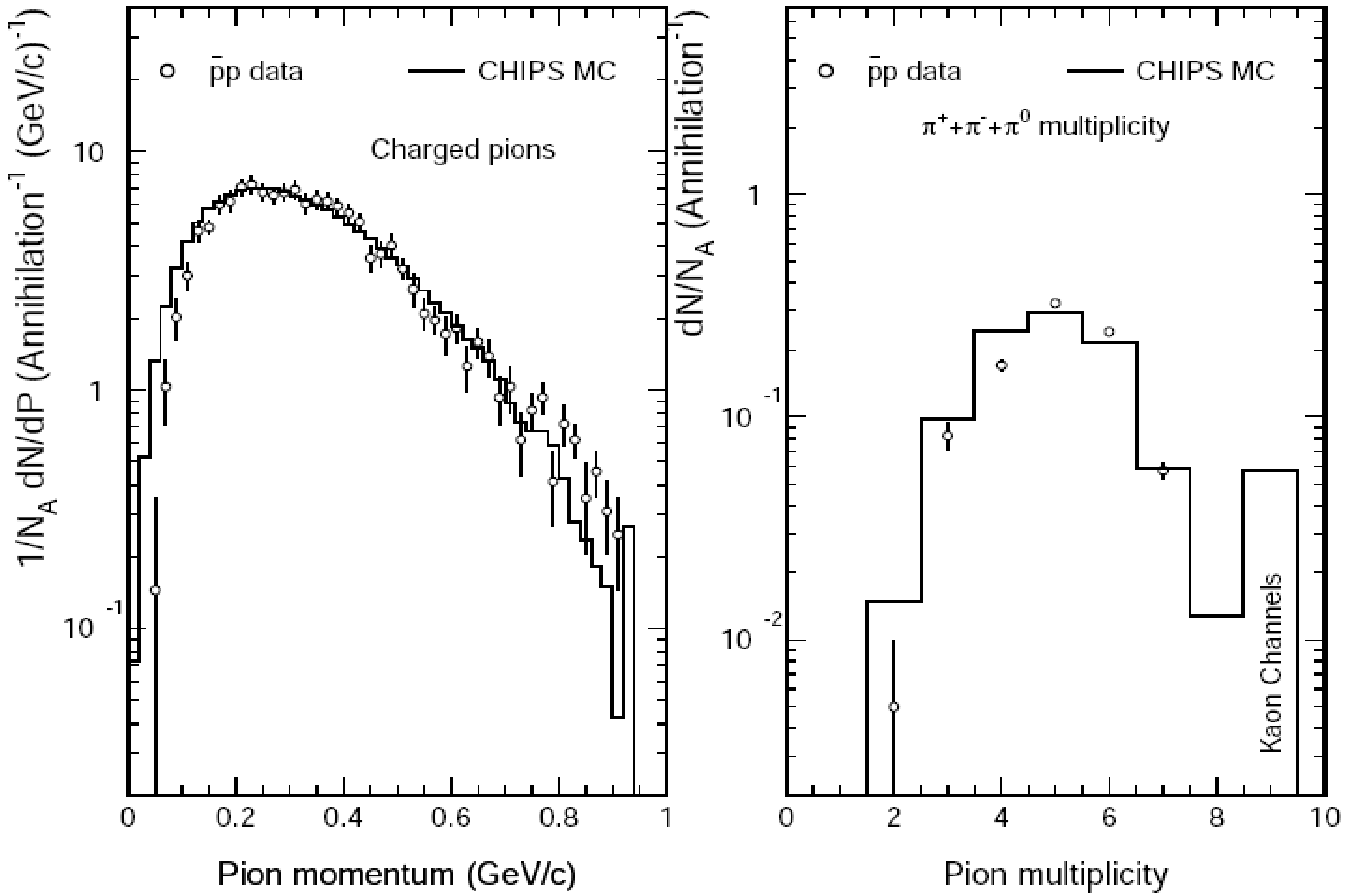
Modeling Sequence for Proton - antiproton Annihilation (3)

- determine type of final state hadron to be produced
  - probability that hadron of given spin and quark content is produced: $P = (2s_h + 1)\, z^{N-3}\, C_Q$
  - C_Q is the number of ways a hadron h can be made from the choice of quarks in the quasmon
  - z^{N-3} is a kinematic factor from the previous momentum selection
- first hadron is produced, escapes quasmon
- randomly sample residual quasmon mass, based on original mass M and emitted hadron mass

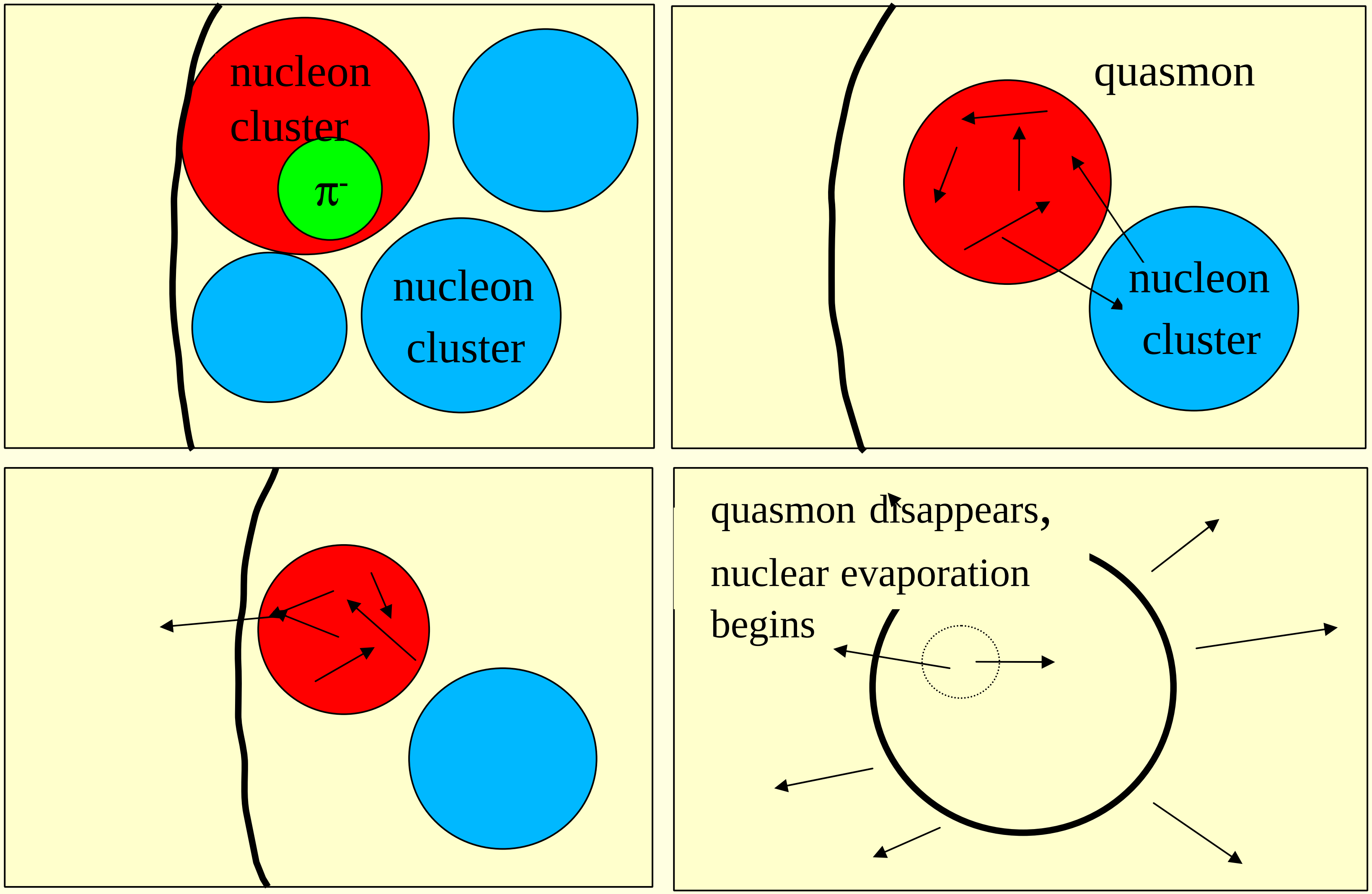
Modeling Sequence for Proton - antiproton Annihilation (4)

- Repeat quark fusion with reduced quasmon mass and quark-parton content
- hadronization process ends when minimum quasmon mass m_{min} is reached
  - m_{min} is determined by quasmon quark content at final step
  - depending on quark content, final quasmon decays to two hadrons or a hadron and a resonance
  - kaon multiplicity regulated by the s-suppression parameter (s/u = 0.1)
  - η/η' suppression regulated by η–suppression parameter (0.3)

Validation of CHIPS for Proton Anti-Proton Annhilation



Modeling Sequence for π^- Capture at Rest in a Nucleus (1)



Modeling Sequence for π^- Capture at Rest in a Nucleus (2)

- pion captures on a subset or cluster of nucleons
  - resulting quasmon has a large mass, many partons
  - capture probability is proportional to number of clusters in nucleus
  - 3 clusterization parameters determine number of clusters
- both quark exchange and quark fusion occurs
  - only quarks and diquarks can fuse
  - mesons cannot be produced, so quark-anti-quark cannot fuse as in vacuum case (p-pbar)
  - because q-qbar fusion is suppressed, quarks in quasmon exchange with neighboring nucleon or cluster
    - produces correlation of final state hadrons

Modeling Sequence for π^- Capture at Rest in a Nucleus (3)

- some final state hadrons escape nucleus, others are stopped by Coulomb barrier or by over-barrier reflection
- as in vacuum, hadronization continues until quasmon mass reaches lower limit m_{min}
  - in nuclear matter, at this point nuclear evaporation begins
  - if residual nucleus is far from stability, a fast emission of p, n, α is made to avoid short-lived isotopes

Known Problems and Improvements (1)

- QGS:
  - gaussian sampling of p_T too simple => incorrect diffraction, not enough π^- suppression in p scattering
  - internal cross sections being improved
- Medium energy (~10 GeV - 60 GeV):
  - too low for QGS, HEP models
  - too high for cascade, LEP models
  - improved parametrized model being developed
- Cascades:
  - no Coulomb barrier in Bertini

Known Problems and Improvements (2)

- CHIPS:
  - originally designed only as final state generator, not intended for projectile interaction with nucleus
  - extension planned for inelastic scattering
  - neutrino scattering recently added

Backup Slides

String Formation

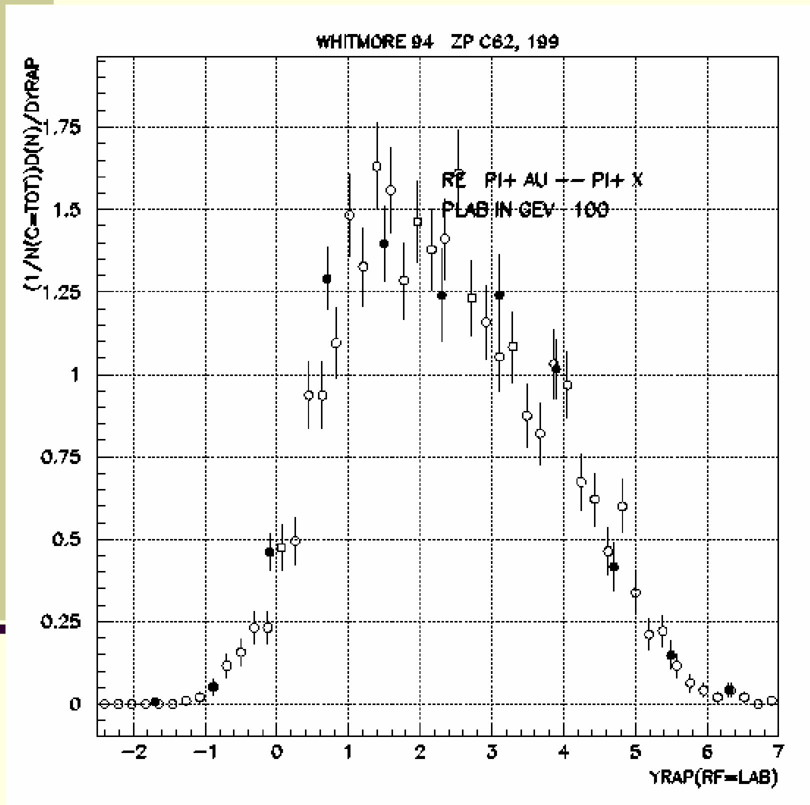
- Cutting the pomeron yields two strings
- String formation is done by parton exchange (Capella 94, Kaidalov 82)
  - for each participating hadron h, parton densities are sampled

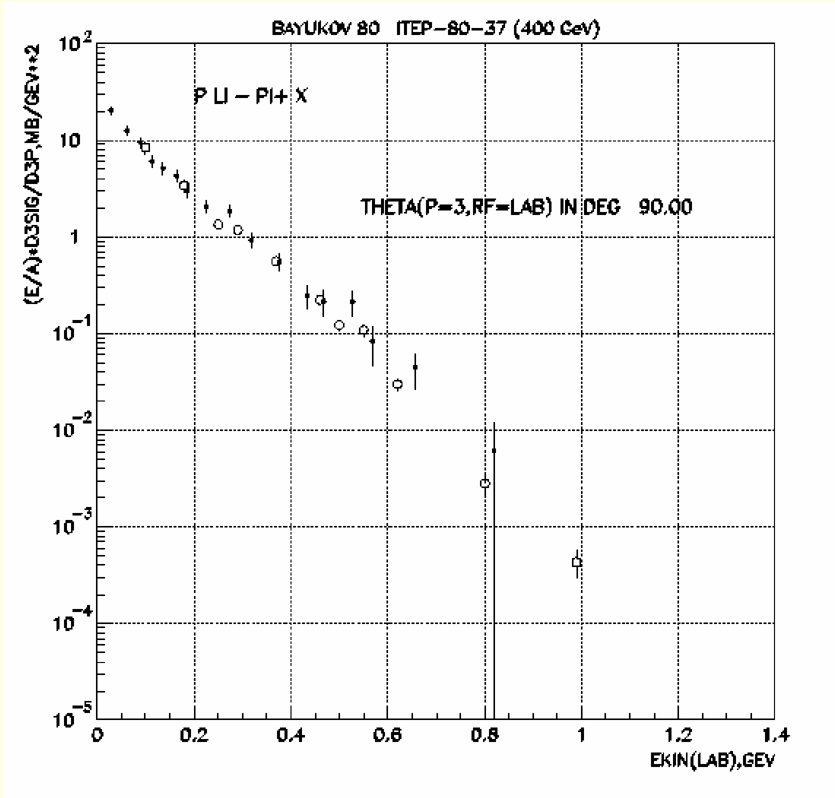
$$f^h(x_1, x_2, \ldots, x_{2n-1}, x_{2n}) = f_0 \prod_{i=1}^{2n} u_{p_i}^h(x_i)\,\delta(1 - \sum_{i=1}^{2n} x_i)$$

  - parton pairs combined to form color singlets
  - u is quark structure function of hadron h
  - sea quarks included with u:d:s = 1: 1: 0.27

QGS Model

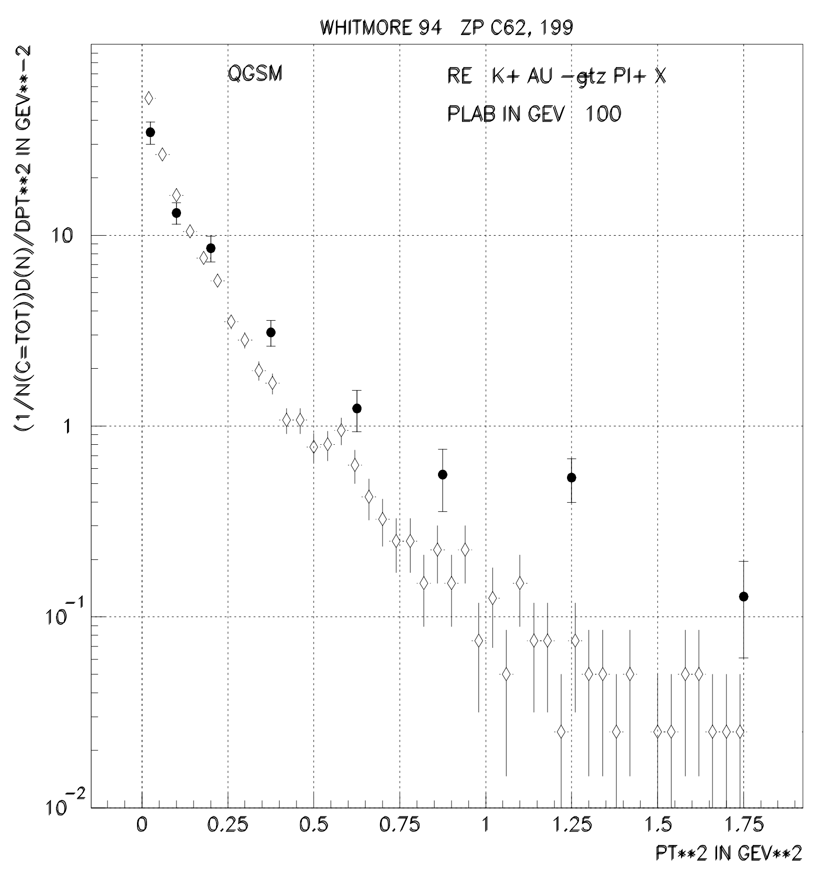
Pion and proton scattering

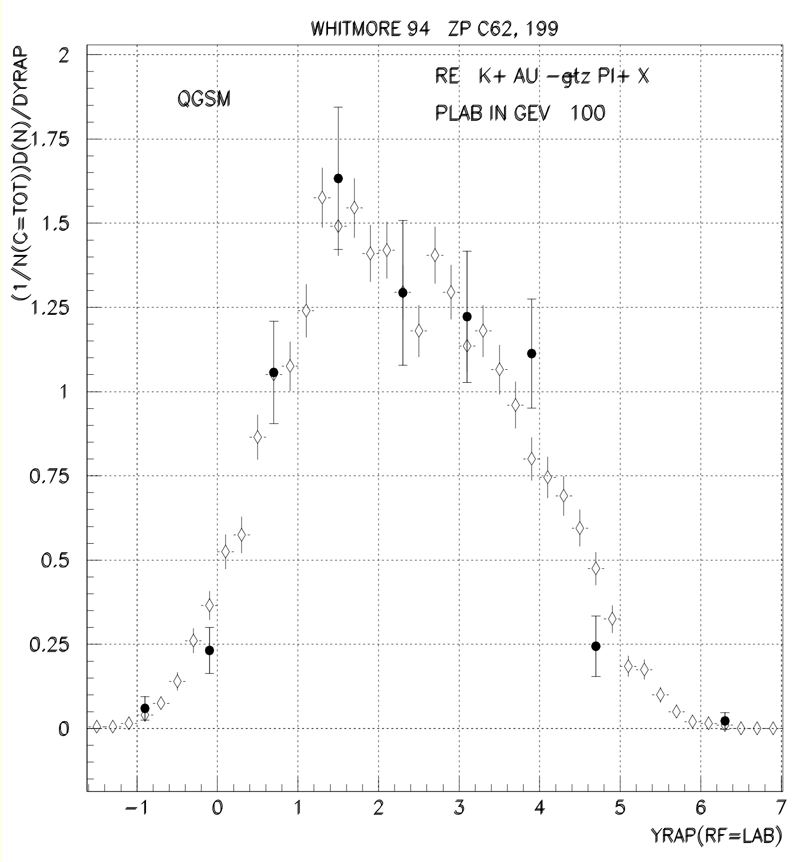




QGS Model

K+ Scattering from Au (π^+ inclusive)

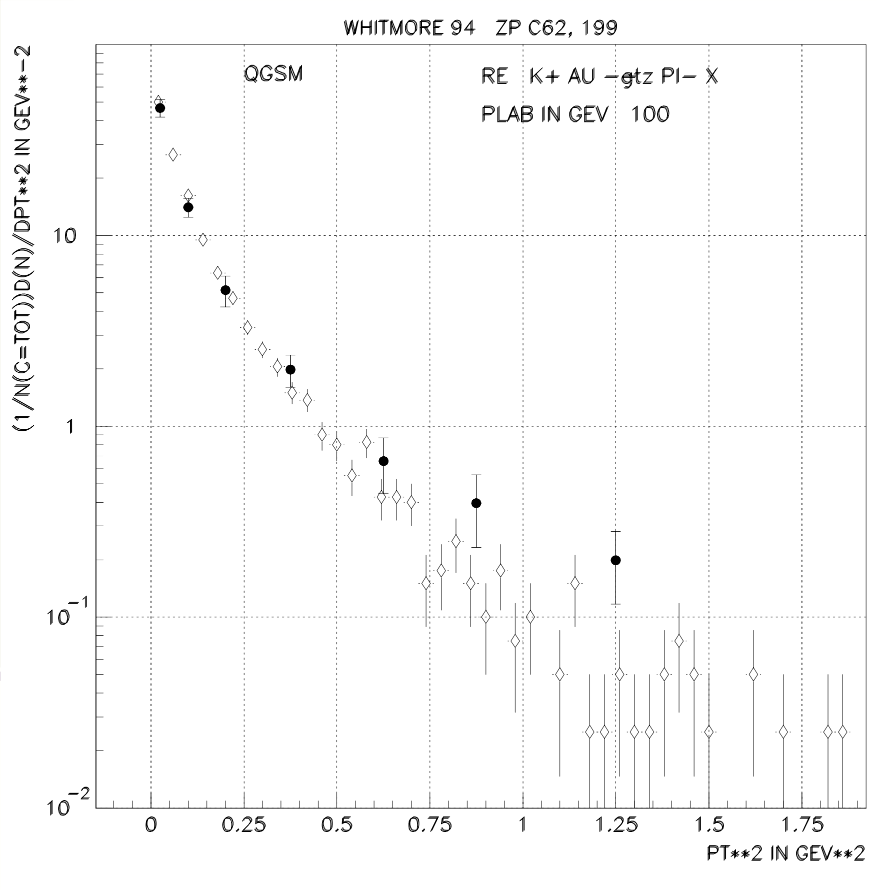


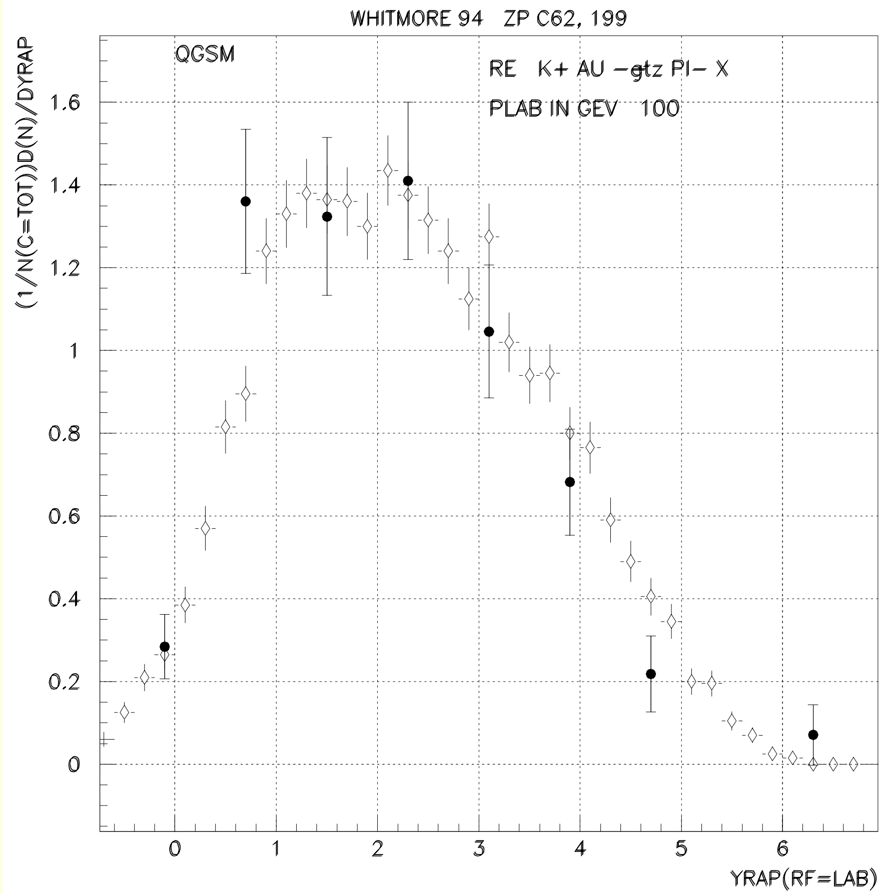


Solid dots: J.J.Whitmore et.al., Z.Phys.C62(1994)199

QGS Model

K+ Scattering from Au (π^- inclusive)





Solid dots: J.J.Whitmore et.al., Z.Phys.C62(1994)199

Chiral Invariant Phase Space (CHIPS)

- Hadron spectra reflect spectra of quark-partons within quasmon
  - 1-D quark exchange:

    $k + M = q + E, \;\; k = p - q \;\; \Rightarrow \;\; k = (E - M + p)/2$

  - 1-D quark fusion:

    $k + q = E, \;\; k - q = p \;\; \Rightarrow \;\; k = (E + p)/2$

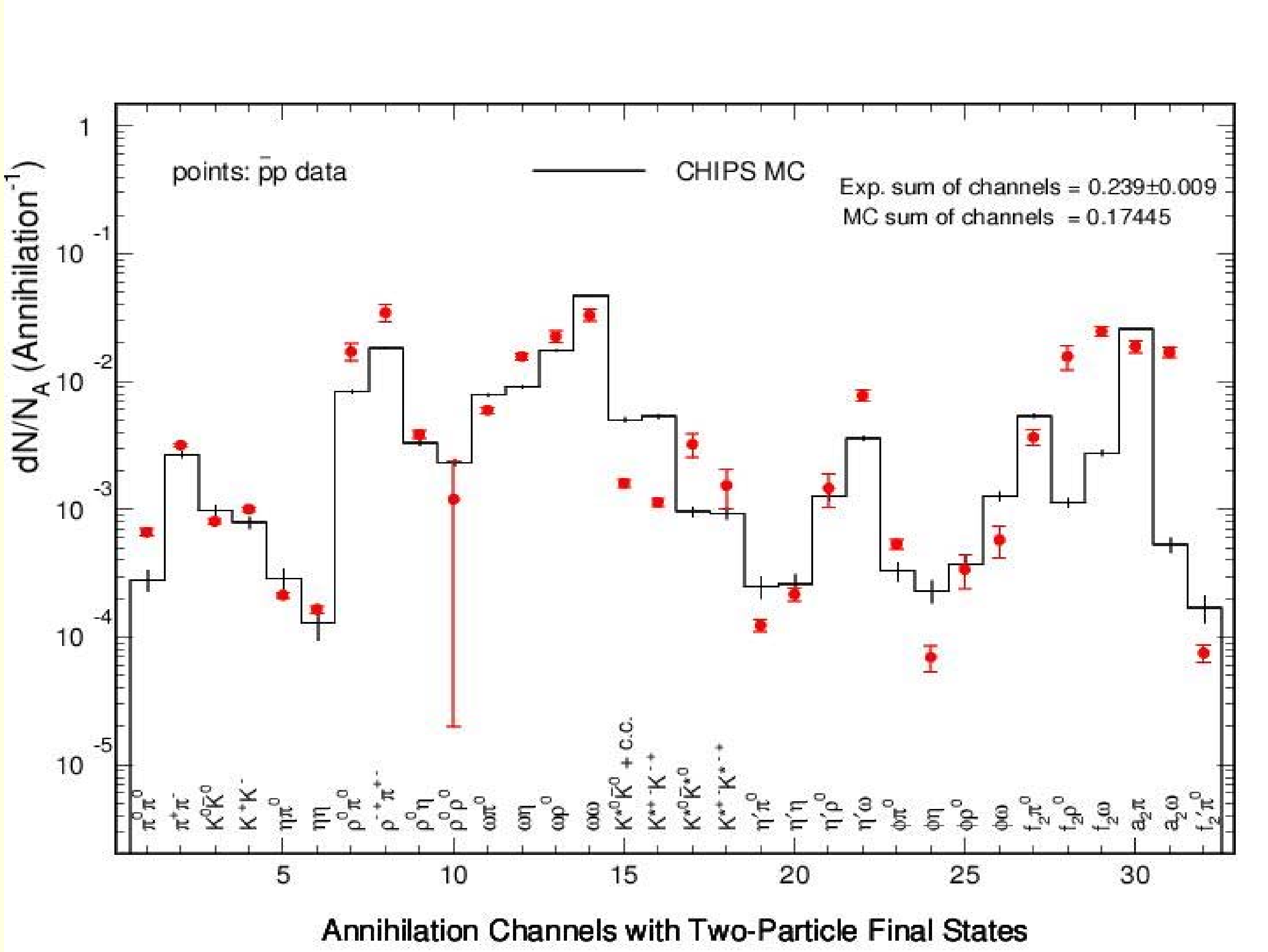
Currently Implemented Mechanisms (1)

- Negative meson captured by nucleon or nucleon cluster:
  - $dE_{\pi} = m_{\pi}$, $dE_K = m_K + m_N - m_{\Lambda}$
- Negative hyperon captured by nucleon or nucleon cluster:
  - $dE_{\Sigma-} = m_{\Sigma-} - m_{\Lambda}$, $dE_{\Xi-} = m_{\Xi-} + m_N - 2m_{\Lambda}$, $dE_{\Omega} = m_{\Omega} + 2m_N - 3m_{\Lambda}$
- Nuclear capture of anti-baryon:
  - annihilation happens on nuclear periphery
  - 4π explosion of mesons irradiates residual nucleus
  - secondary mesons interacting with residual nucleus create more quasmons in nuclear matter
  - large excitation: $dE = m_{antibaryon} + m_N$

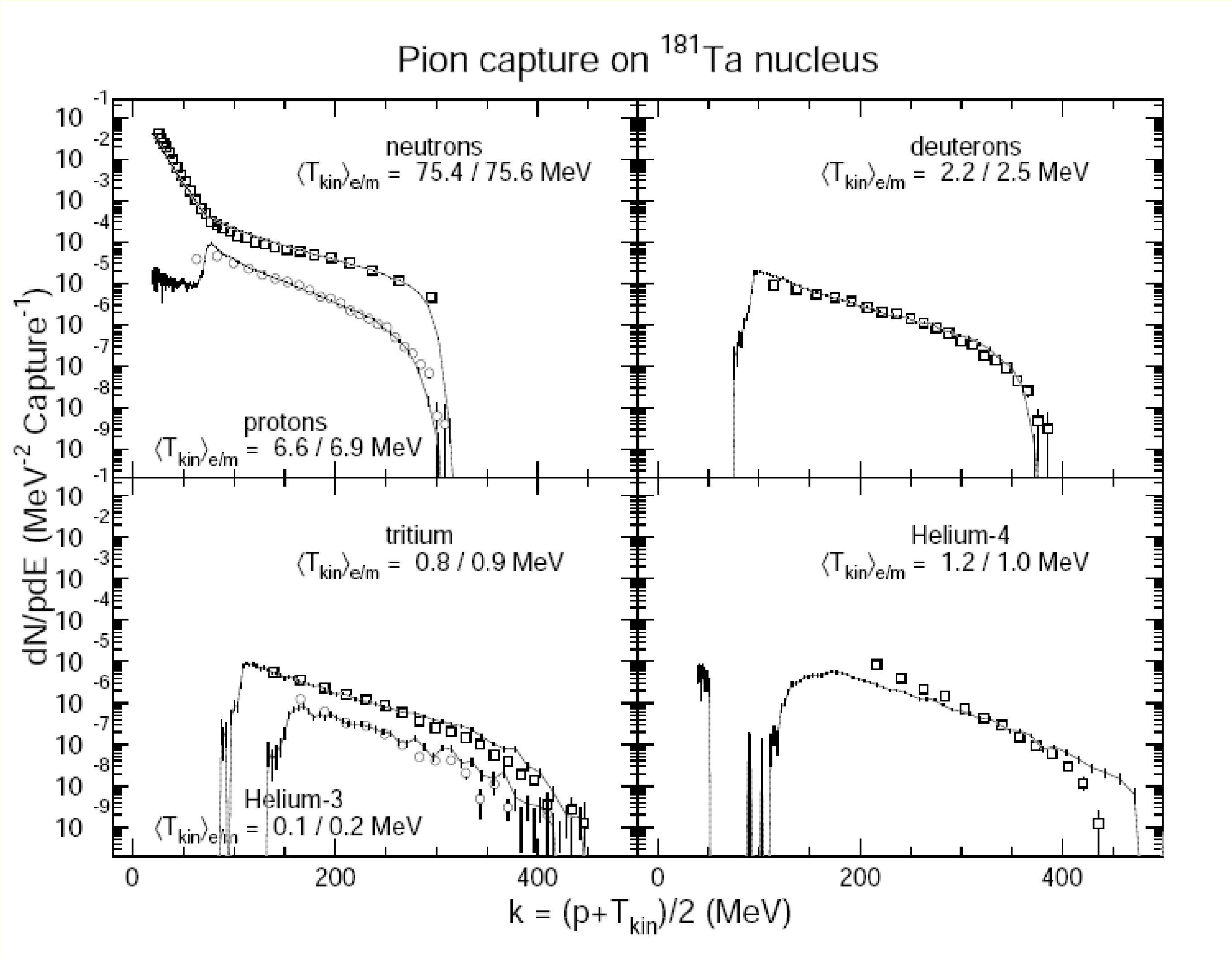
Currently Implemented Mechanisms (2)

- In photo-nuclear reactions γ is absorbed by a quark-parton
  - $dE_\gamma = E_\gamma$
- In back-end of string-hadronization (QGSC model) soft part of string is absorbed:
  - $dE_{QGSC} = 1$ GeV/fm
- lepto-nuclear reactions γ^* , W^* are absorbed by quark-parton:
  - $dE_l = E_{\gamma*}$, $\cos(\theta_k) = (2k/\nu \quad Q^2)/2kq$, $Q^2 = q^2 - \nu^2$
  - with $k < M/2$, if $q - \nu < m_N$, virtual γ cannot be captured by one nucleon

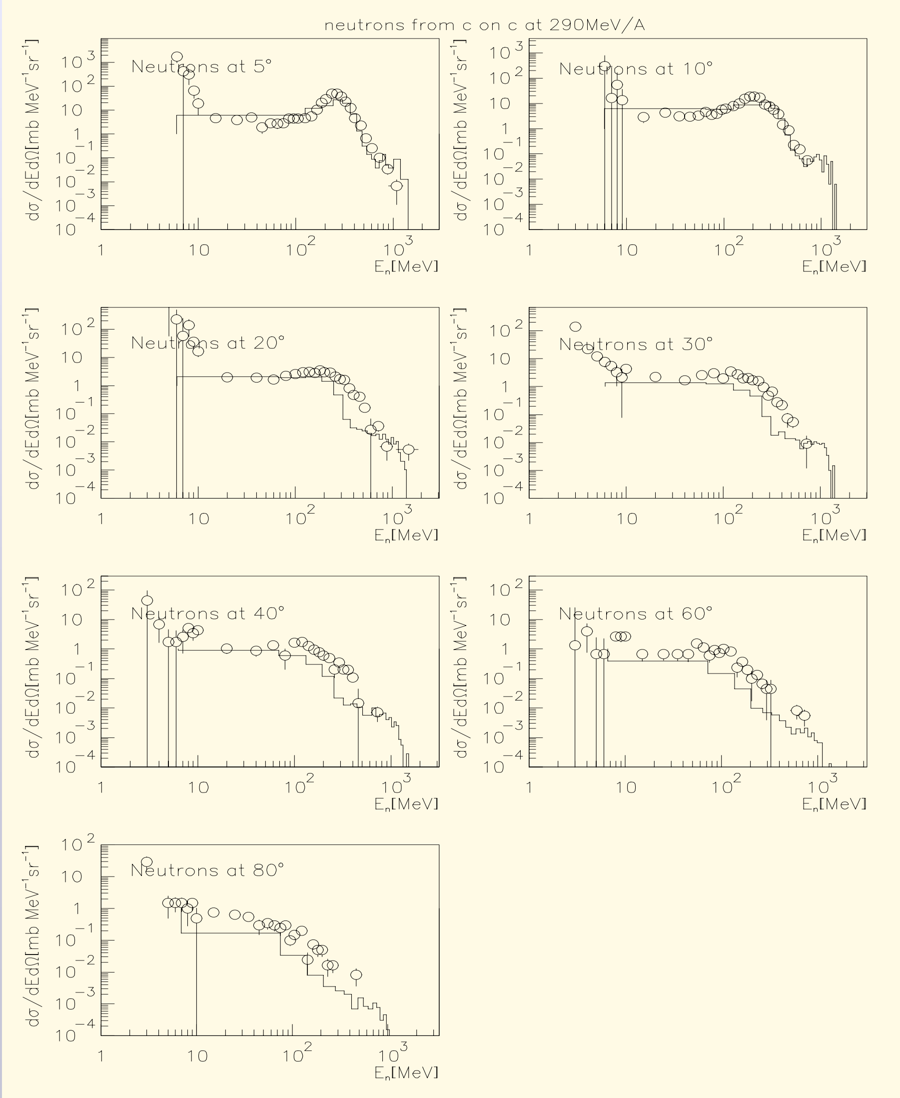
P-pbar Annihilation into Two Body Final States



Validation of CHIPS Model for Pion Capture at Rest on Tantalum



Neutrons from C on C at 290 MeV/c



p+Ta=h+X, 400 GeV, 90°