Automated Virtual MEs for HH Production

Stephen Jones

Work With:
Borowka, Di Vita, Greiner, Heinrich, Kerner, Luisoni, Mastrolia, Peraro, Schlenk, Schubert, Stoyanov, Yundin, Zirke

Overview

1. Introduction

   Motivation for Studying Double Higgs Production

   Overview of Production Channels

   Gluon Fusion $gg \to HH$

2. Project - NLO Virtual MEs for Gluon Fusion

   Form Factor Decomposition

   Integral Reduction

   Master Integrals: Analytic + Numerical Integration

3. Conclusion

Motivation

Multi-Higgs production probes the self-coupling

(n-1) Higgs production probes H^n terms

$$\mathcal{L} \supset -\frac{m_H^2}{2}H^2 - \frac{m_H^2}{2v}H^3 - \frac{m_H^2}{8v^2}H^4$$

Consistent with observed excess (LHC)

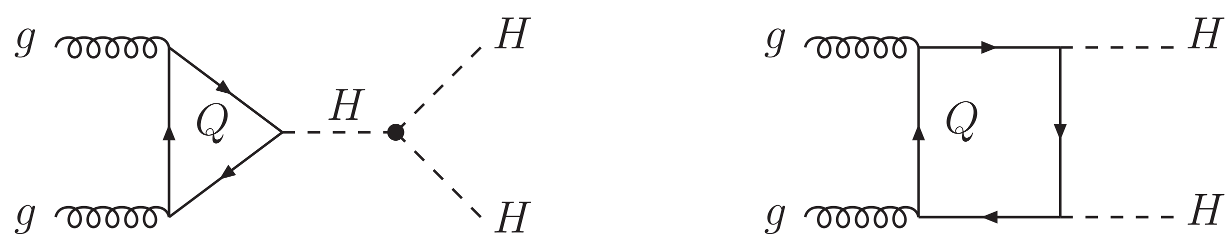
Our focus (x/LHC)

Tiny cross-section (probably not x/LHC)

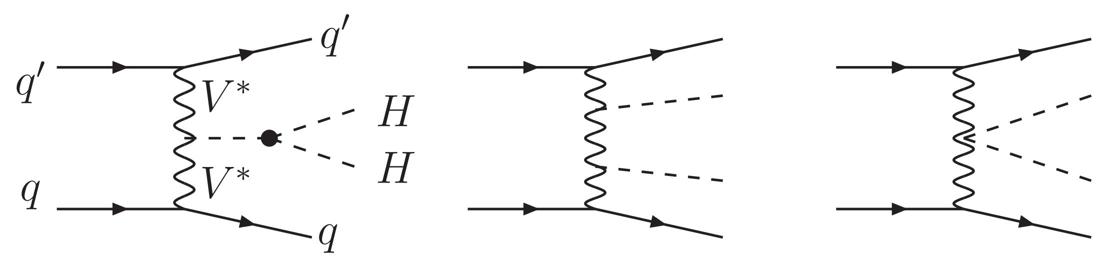
Production Channels

$\sigma(pp \to HH + X)$ @ 13TeV

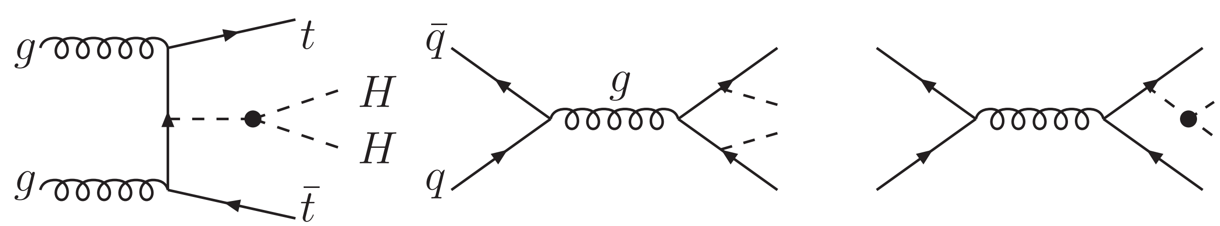
Gluon Fusion



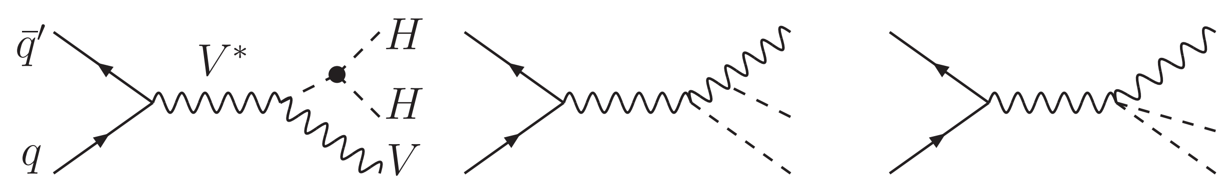
Vector Boson Fusion (VBF)



Associated top pair



Double Higgs-strahlung

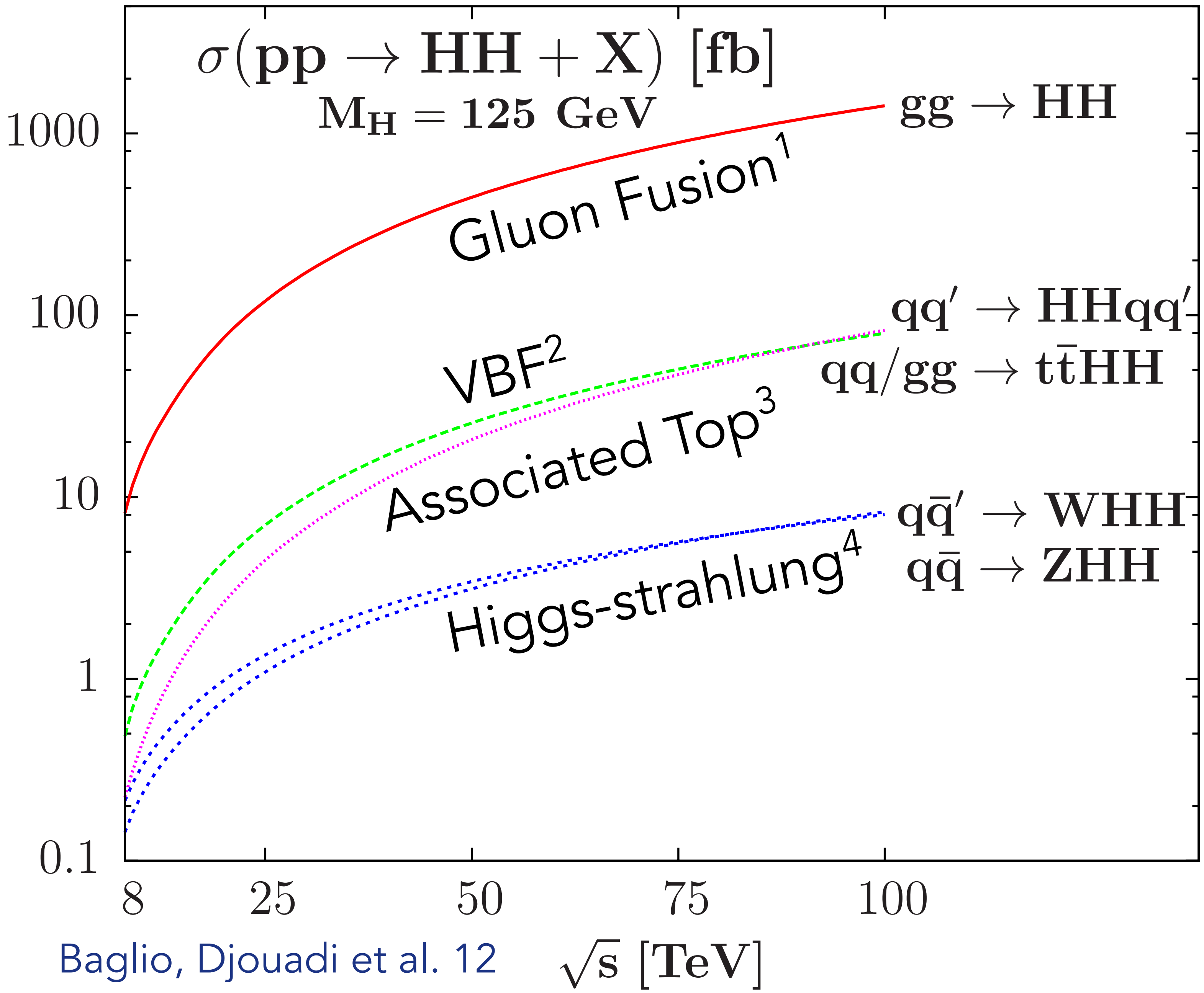


…

Baglio, Djouadi et al. 12

Production Channels

$$\sigma(pp \to HH + X) \sim \frac{1}{1000}\sigma(pp \to H + X)$$



Baglio, Djouadi et al. 12

[1] NLO QCD HEFT, HPAIR
Plehn, Spira, Zerwas 96, 98; Dawson et al. 98

[2] NLO QCD, VBFNLO
Baglio, Djouadi et al. 12

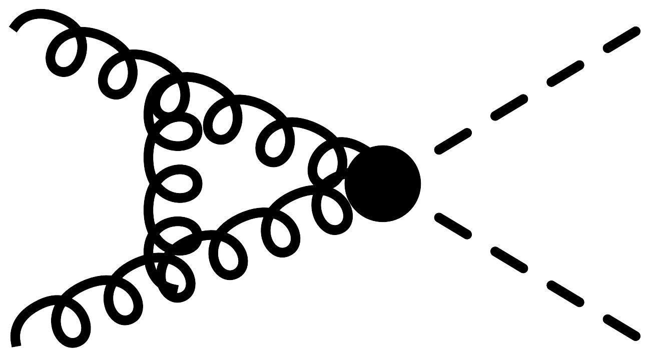
[3] LO QCD (NLO, aMC@NLO)
Frederix, Frixione et al. 14

[4] NNLO QCD
Baglio, Djouadi et al. 12

Gluon Fusion

1. LO (1-loop), Dominated by $Q = t, b$ (b contributes 1%)
   Glover, van der Bij 88

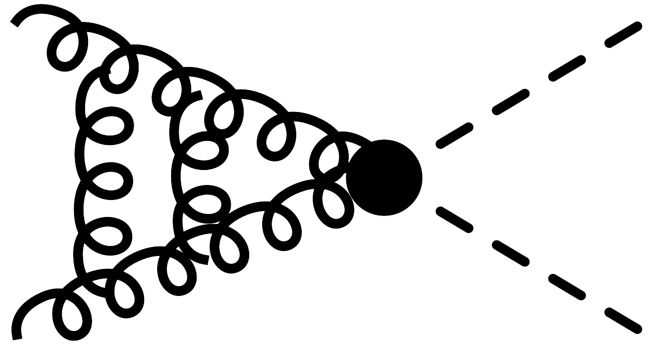
2. Born Improved NLO H(iggs)EFT $K \approx 2$
   Plehn, Spira, Zerwas 96, 98; Dawson, Dittmaier, Spira 98



   A. Including m_T in Real radiation
      Maltoni, Vryonidou, Zaro 14

   B. Including $\mathcal{O}(1/m_T^{12})$ terms in Virtual MEs
      Grigo, Hoff, Melnikov, Steinhauser 13; Grigo, Hoff 14

3. Born Improved NNLO HEFT +20%
   De Florian, Mazzitelli 13



   Including matching coefficients
   Grigo, Melnikov, Steinhauser 14

   NNLL Soft Gluon Resummation +30%
   Shao, Li, Li, Wang 13

Gluon Fusion (NLO HEFT)

Tension between corrections to HEFT!

Born Improved NLO QCD HEFT

$$\mathrm{d}\sigma^V_{\mathrm{NLO}}(m_T) \approx \mathrm{d}\bar{\sigma}^V_{\mathrm{NLO}}(m_T) = \frac{\sigma^V_{\mathrm{NLO}}(m_T \to \infty)}{\sigma^V_{\mathrm{LO}}(m_T \to \infty)} \sigma^V_{\mathrm{LO}}(m_T)$$

$$\mathrm{d}\sigma^R(m_T \to \infty)$$

$$K \approx 2$$

A. Maltoni et al.14

$$\mathrm{d}\bar{\sigma}^V(m_T)$$

$$\mathrm{d}\sigma^R(m_T)$$

$$-10\%$$

B. Grigo, Hoff 14

$$\mathrm{d}\sigma^V_{\mathrm{NLO}}(m_T) \approx \mathrm{d}\bar{\sigma}^V_0 + \mathrm{d}\bar{\sigma}^V_0 \frac{\lambda^2}{m_T^2} + \ldots + \mathrm{d}\bar{\sigma}^V_6 \frac{\lambda^{12}}{m_T^{12}}$$

$$\mathrm{d}\sigma^R(M_t \to \infty)$$

$$+10\%$$

Real-virtual Cancellations Spoilt?

Stable?

Gluon Fusion NLO

Goal: Compute $pp \to HH + X$ via Gluon Fusion $gg \to HH$

NLO (2-loop) with full top mass dependence

Neglect b contribution

Tools: `GoSam` (uses `QGRAF` and `FORM`) + `SecDec`

Cullen et al. 14 Nogueira 93 Vermaseren et al. 12 Borowka et al. 15

`GoSam (https://gosam.hepforge.org)`

Automated calculation of 1-loop amplitudes

SM/BSM

Interface to BLHA1/BLHA2 for real radiation

Shopping List

Virtual MEs (HH):

Channels:

$gg \to HH$ $\quad$ ~~$q\bar{q} \to HH$~~

Contributes at NNLO

	Diagrams
Tree	0
1-loop	8
2-loop	122

Real Radiation (HH + j...):

Huge simplification!

1-j Channels:

$gg \to HH + g$

$gq \to HH + q$ $\quad$ $g\bar{q} \to HH + \bar{q}$

$q\bar{q} \to HH + g$

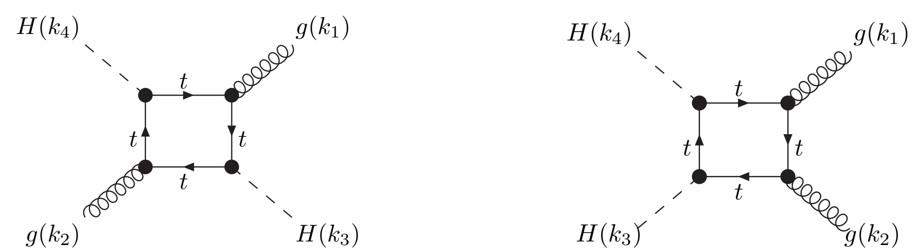
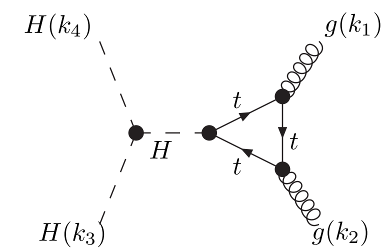
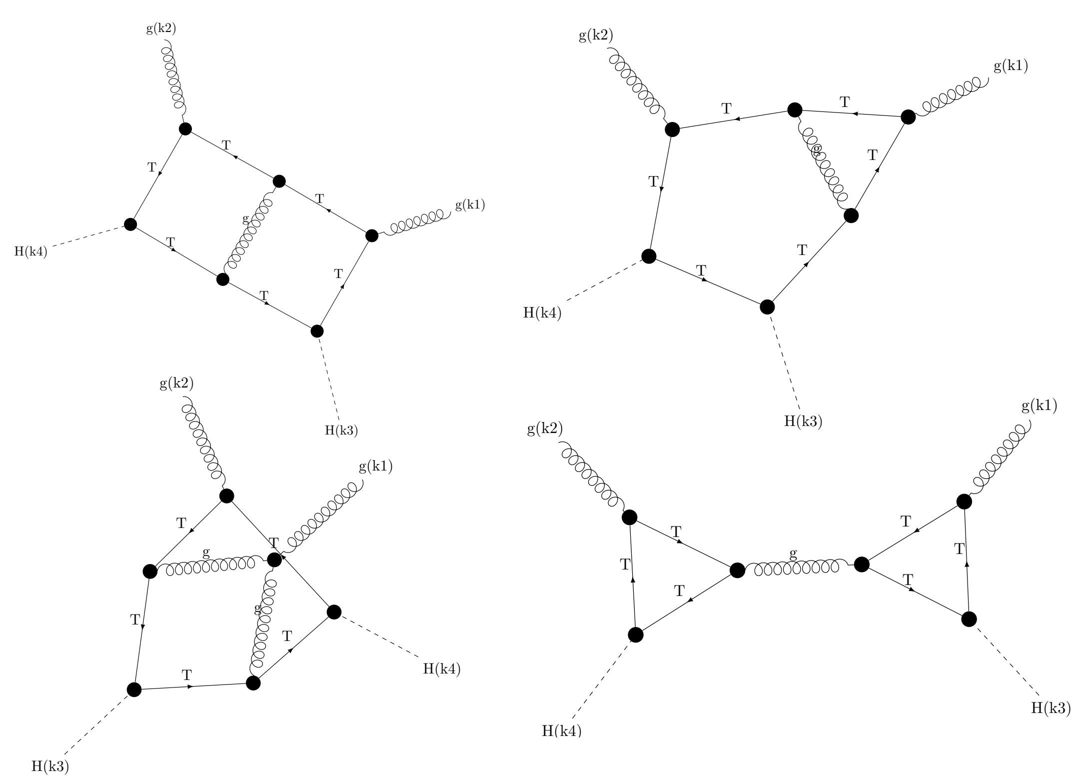
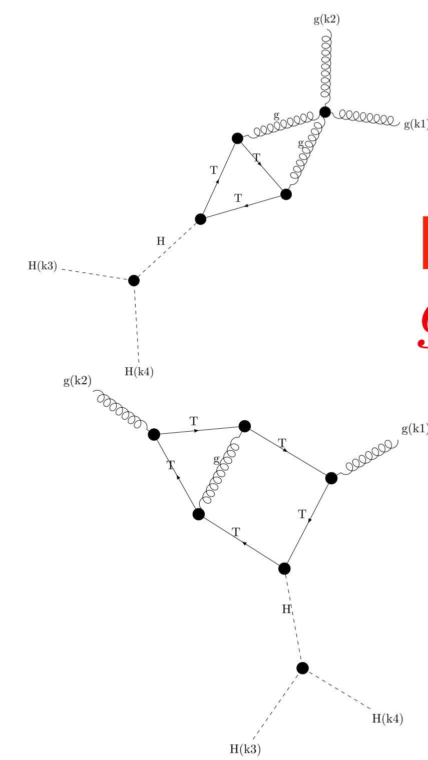
	Diagrams
Tree ⊗ Double	0
1-loop ⊗ Single	54+8+8+8

`GoSam` for MEs + Catani-Seymour Dipole Subtraction

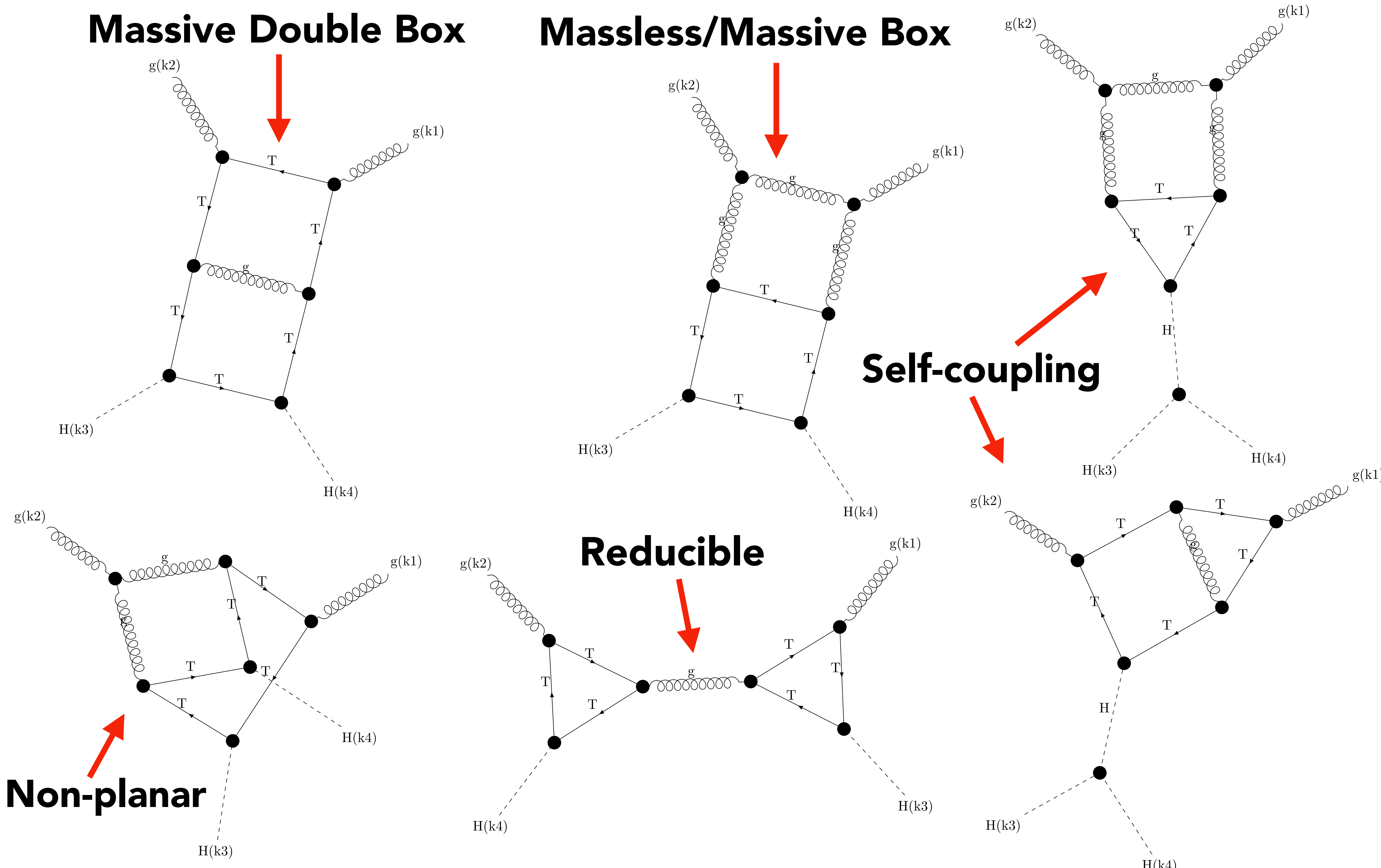
Catani, Seymour 96

Virtual MEs

Boxes & Triangles

	Yukawa only (≤ 4-point)	Self-coupling (3-point)
LO	 6 Diagrams	 2 Diagrams
NLO	 101 Diagrams	 **Known** $gg \to H$ 21 Diagrams

Diagrams


Massive Double Box
Massless/Massive Box
Self-coupling
Reducible
Non-planar
g(k2)
g(k1)
H(k3)
H(k4)
T
g
H

Form Factor Decomposition

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Decomposition:

Form Factors (Contain integrals)

$$\mathcal{M}^{\mu\nu} \propto A_1(s,t,m_H^2,m_T^2,d)T_1^{\mu\nu} + A_2(s,t,m_H^2,m_T^2,d)T_2^{\mu\nu}$$

(Tensor) Basis

Choose: $\mathcal{M}^{++} = \mathcal{M}^{--} = -A_1$

$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -A_2$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_2^\mu p_1^\nu}{p_1 \cdot p_2} \qquad p_T^2 = \frac{ut - m_H^4}{s}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_2^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2p_2 \cdot p_3 p_3^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\mu p_3^\nu}{p_T^2}$$

Glover, van der Bij 88

Form Factor Decomposition

Construct Projectors:

No Integrals

$$P_j^{\mu\nu} = \sum_{i=1}^{2} B_{ji}(s,t,m_H^2,d)T_i^{\mu\nu}$$

Same Basis as amplitude

Such that:

$$P_{1\,\mu\nu}\mathcal{M}^{\mu\nu} = A_1$$

$$P_{2\,\mu\nu}\mathcal{M}^{\mu\nu} = A_2$$

Explicitly; separately calculate the contraction of each projector with $\mathcal{M}^{\mu\nu}$

Recall:

$$\mathcal{M}^{++} = \mathcal{M}^{--} = -A_1$$

$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -A_2$$

- Self-coupling diagrams are 1PR by cutting a scalar propagator
- By angular momentum conservation they contribute only to A_1

Current Status: Projectors constructed/ input by hand

Integral Reduction

Integral Reduction (dramatically) reduces the number of integrals!

Integrals	1-loop	2-loop
Direct	63	9865
+ Symmetries	21	1601
+ IBPs	8	~300-400

3 Finite Boxes, 4 Finite Triangles + (d-4) x Bubble!

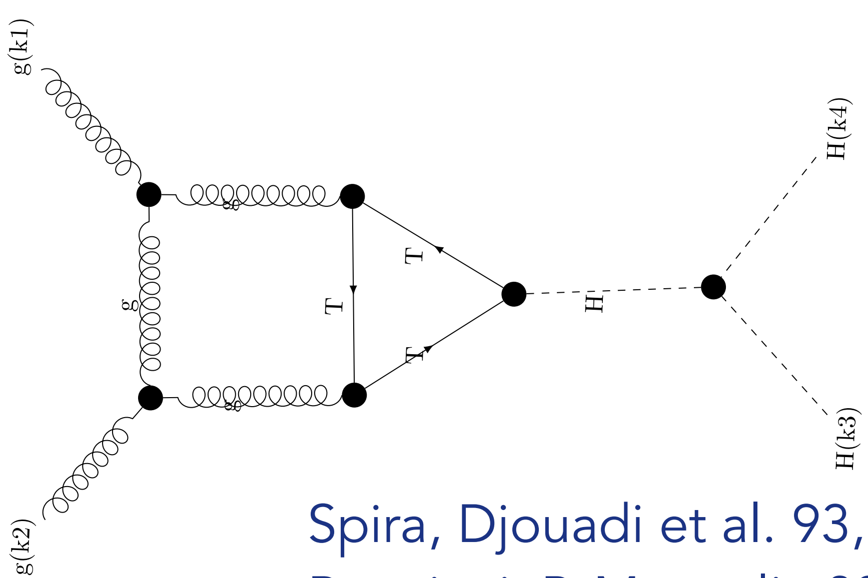
Current Status: Writing `GoSam` interface to existing Integral Reduction tools: `Reduze`, `LiteRed`, `FIRE`

Manteuffel, Studerus 12; Lee 13; Smirnov, Smirnov 13

Master Integrals

Double Higgs Production Master Integrals are tough!

- Massive propagators
- Off-shell legs

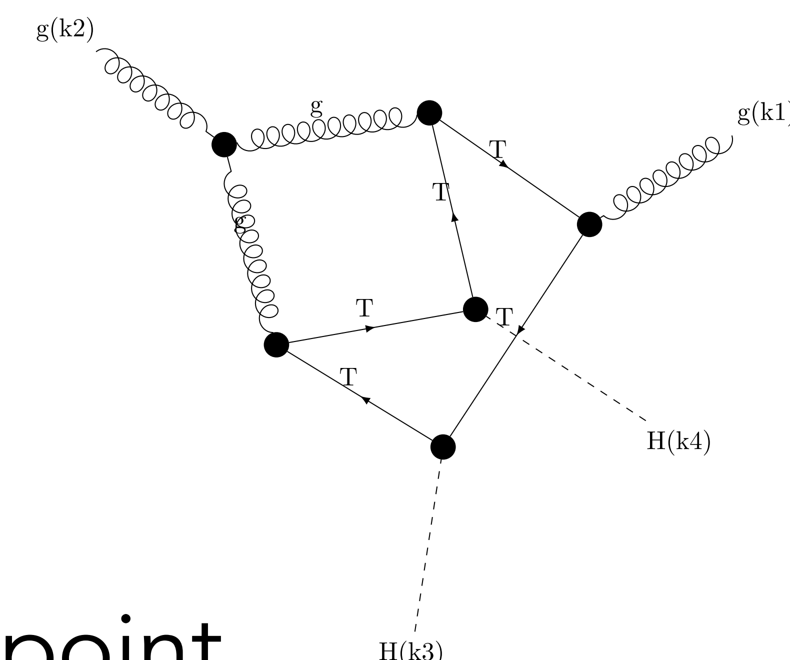


Spira, Djouadi et al. 93, 95;
Bonciani, P. Mastrolia 03,04;
Anastasiou, Beerli et al. 06;
...

3-point

2 scales s, m_T^2

Known analytically



4-point

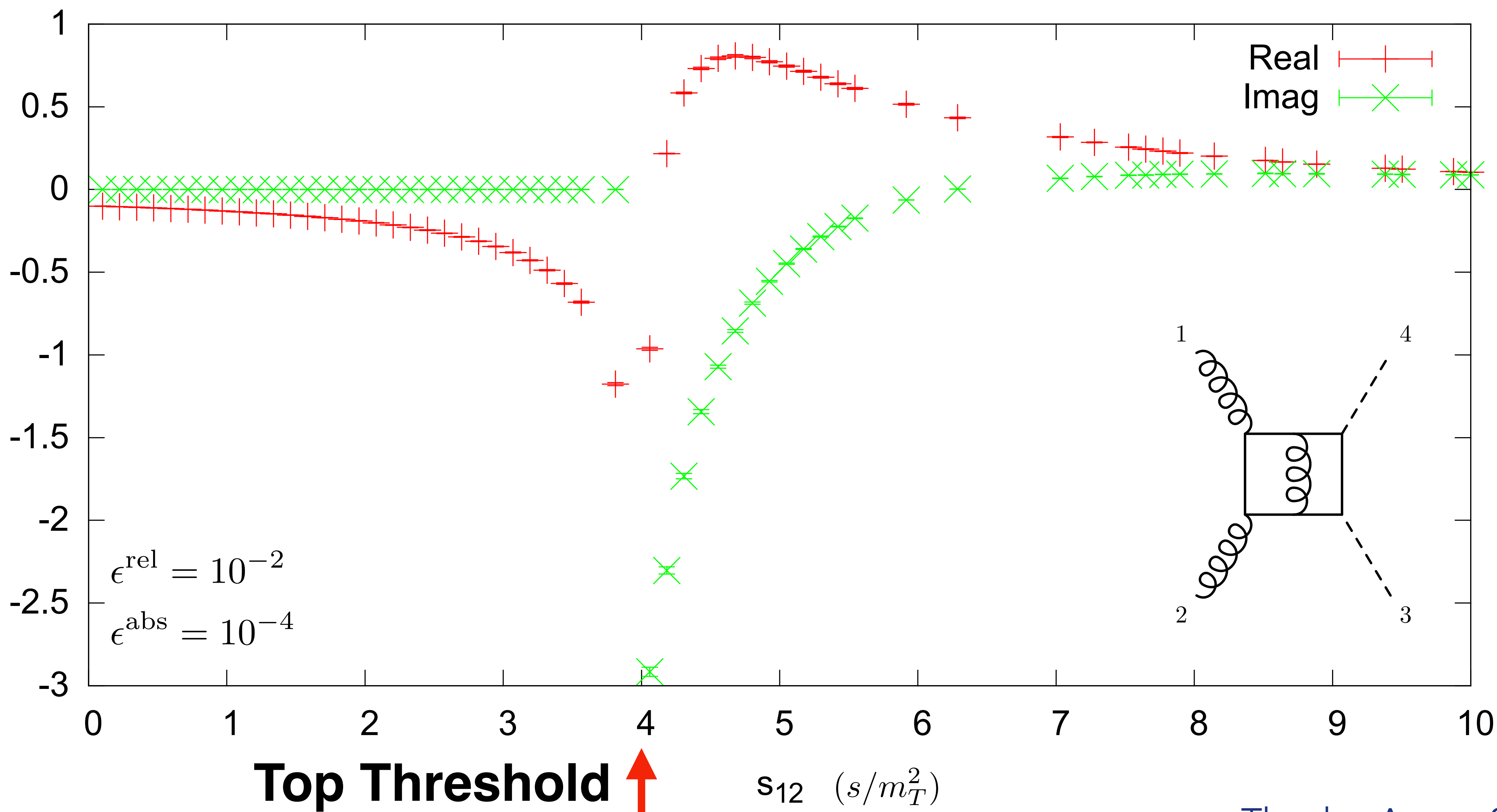
4 scales s, t, m_T^2, m_H^2

Numerical (`SecDec`)

Master Integrals (Numerical)

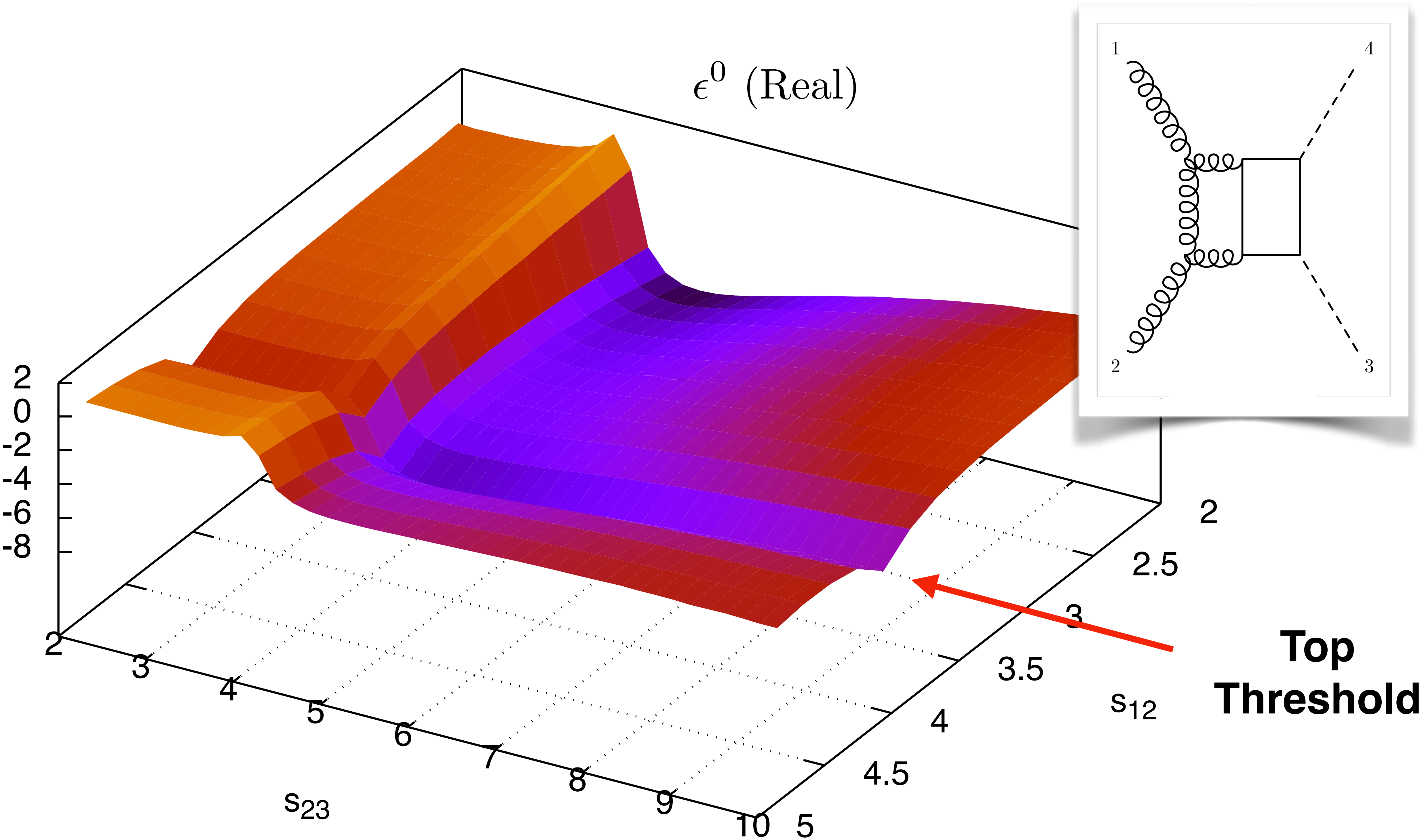
SecDec (https://secdec.hepforge.org)

Evaluate Dimensionally regulated parameter integrals numerically



Thanks: Anton Stoyanov

Master Integrals (Numerical)



Thanks: Anton Stoyanov

Conclusion

HH Production

- Key measurement for probing the self coupling (HL-LHC era)
- HEFT implies that the NLO K factor for gluon fusion is large

  Tension between corrections to HEFT

  Full NLO corrections important

Ongoing/Future

- Extend available tools to allow computation of (some) 2-loop virtual MEs
- Fully exploit analytically known master integrals: Loopedia (?)
- Implement interface to numerical tools for unknown master integrals

Please stay tuned!

Backup

Gluon Fusion NLO

Strategy: Extend `GoSam` to handle 2-loop virtual MEs

Now: Just $gg \to HH$

Future: Other 2-loop processes

- At 1-loop `GoSam` utilises existing efficient numerical implementations for:

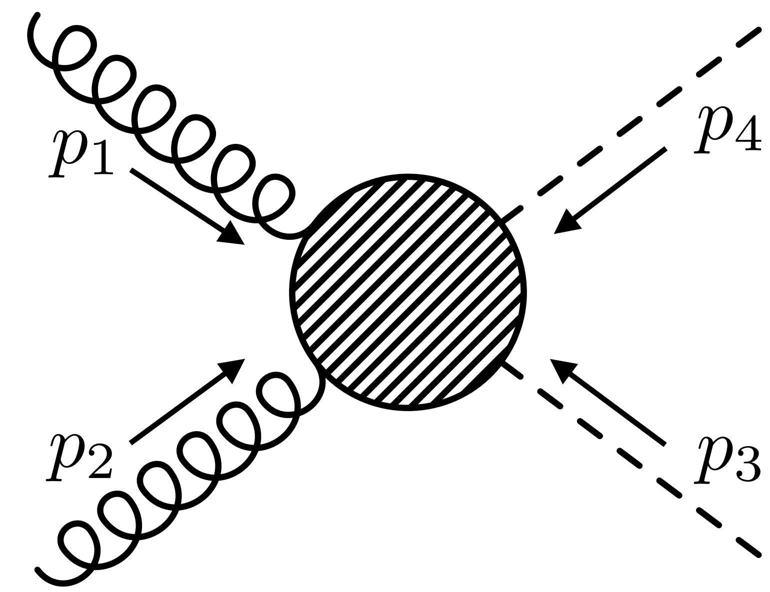
  Tensor/Integrand Reduction, `Golem95`/ `Ninja`/ `Samurai`
  Guillet et al. 13; Peraro et al. 14; Mastrolia et al. 10

  Master integrals, `OneLOop`/ `QCDLoop`/ `LoopTools`
  van Hameren 10; Ellis, Zanderighi 07; Hahn, Perez-Victoria 98

- Not available beyond 1-loop, need to make significant changes to `GoSam`

Form Factor Decomposition

$g(p_1)g(p_2) \to H(-p_3)H(-p_4) \quad \sum_{i=1}^{4} p_i = 0$



CDR (Dim = d)

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

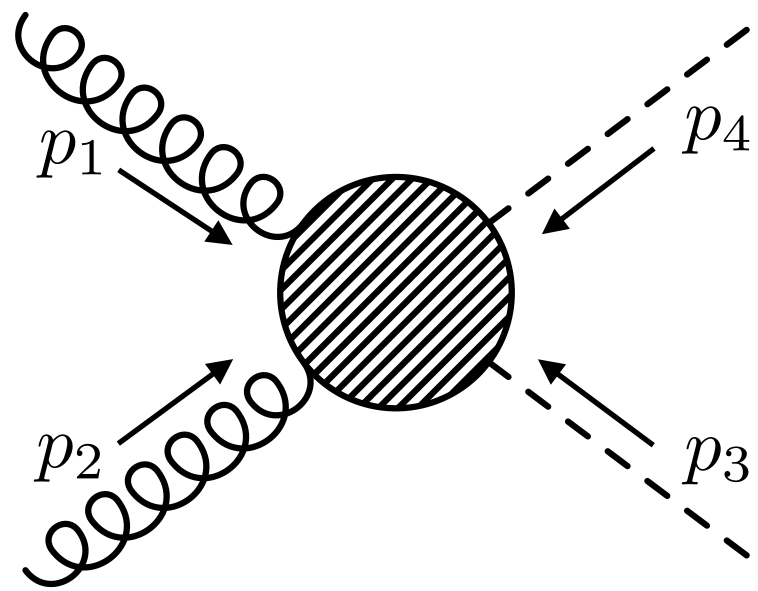
Decompose:
$$\begin{aligned}\mathcal{M}^{\mu\nu} = a_{00} g^{\mu\nu} &+ a_{11} p_1^\mu p_1^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu \\ &+ a_{21} p_2^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{23} p_2^\mu p_3^\nu \\ &+ a_{31} p_3^\mu p_1^\nu + a_{32} p_3^\mu p_2^\nu + a_{33} p_3^\mu p_3^\nu\end{aligned}$$

a_{ij} **functions of Mandelstams +** d

p_i **linearly indep.**

Form Factor Decomposition

$$g(p_1)g(p_2) \to H(-p_3)H(-p_4) \quad \sum_{i=1}^{4} p_i = 0$$



CDR (Dim = d)

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Decompose: $\mathcal{M}^{\mu\nu} = a_{00} g^{\mu\nu} + \cancel{a_{11} p_1^\mu p_1^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu}$

$$+ a_{21} p_2^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{23} p_2^\mu p_3^\nu$$

$$+ a_{31} p_3^\mu p_1^\nu + a_{32} p_3^\mu p_2^\nu + a_{33} p_3^\mu p_3^\nu$$

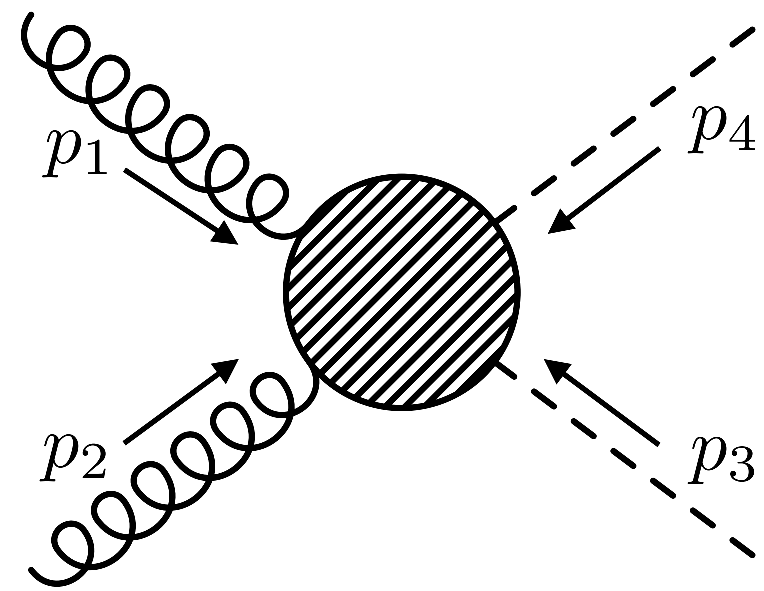
a_{ij} **functions of Mandelstams +** d

p_i **linearly indep.**

Transversity: $g(p_1): \ \epsilon^1_\mu p_1^\mu = 0$

Form Factor Decomposition

$g(p_1)g(p_2) \to H(-p_3)H(-p_4) \quad \sum_{i=1}^{4} p_i = 0$



CDR (Dim = d)

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Decompose:
$$\mathcal{M}^{\mu\nu} = a_{00} g^{\mu\nu} + \cancel{a_{11} p_1^\mu p_1^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu}$$
$$+ a_{21} p_2^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{23} p_2^\mu p_3^\nu$$
$$+ a_{31} p_3^\mu p_1^\nu + a_{32} p_3^\mu p_2^\nu + a_{33} p_3^\mu p_3^\nu$$

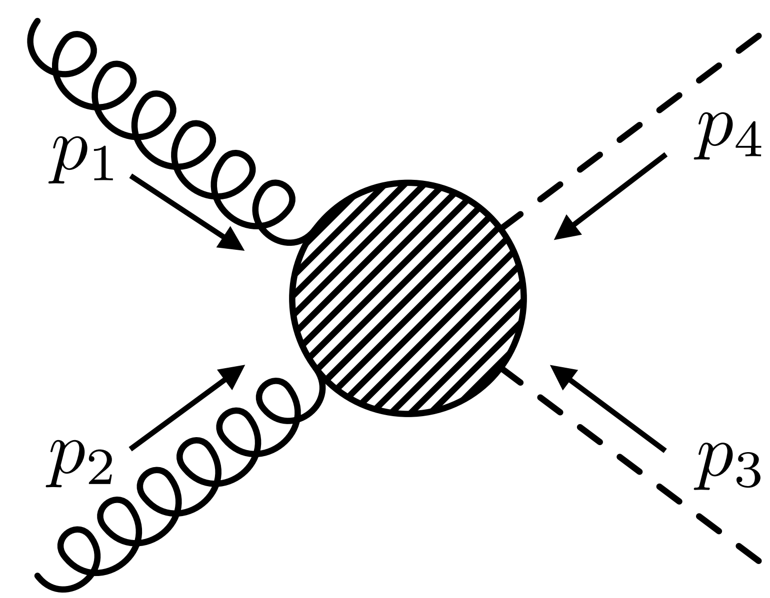
a_{ij} **functions of Mandelstams +** d

p_i **linearly indep.**

Transversity: $g(p_1): \ \epsilon^1_\mu p_1^\mu = 0$ $\quad g(p_2): \ \epsilon^2_\nu p_2^\nu = 0$

Form Factor Decomposition

$g(p_1)g(p_2) \to H(-p_3)H(-p_4) \quad \sum_{i=1}^{4} p_i = 0$



CDR (Dim = d)

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Decompose: $\mathcal{M}^{\mu\nu} = a_{00} g^{\mu\nu} + a_{11} p_1^\mu p_1^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu$
$+ a_{21} p_2^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{23} p_2^\mu p_3^\nu$
$+ a_{31} p_3^\mu p_1^\nu + a_{32} p_3^\mu p_2^\nu + a_{33} p_3^\mu p_3^\nu$

a_{ij} **functions of Mandelstams +** d

p_i **linearly indep.**

Transversity: $g(p_1): \; \epsilon^1_\mu p_1^\mu = 0$ $\quad g(p_2): \; \epsilon^2_\nu p_2^\nu = 0$

Ward/Gauge: $p_{1\mu} \mathcal{M}^{\mu\nu} = 0$, $p_{2\nu} \mathcal{M}^{\mu\nu} = 0$ Gives further identities

Integrals

k_i Loop momenta, p_i L.I. External momenta,

$N_i = (q_i^2 - a)$ Propagator-1, $q_i = \sum_{i=1}^{j} b_i k_i + \sum_{i=1}^{m} c_i p_i$

After Dirac algebra (Traces):

$$A_j \supset \int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2 \frac{f(k_1 \cdot k_1, k_1 \cdot k_2, \ldots, k_2 \cdot p_3)}{N_1 \cdots N_7}$$

Number of Scalar products:

$$S = \frac{l(l+1)}{2} + lm$$

$l = 2$ # Loops

$m = 3$ # L.I External momenta

$S = 9$

Integrals

k_i Loop momenta, p_i L.I. External momenta,

$N_i = (q_i^2 - a)$ Propagator^{-1}, $\quad q_i = \sum_{i=1}^{j} b_i k_i + \sum_{i=1}^{m} c_i p_i$

After Dirac algebra (Traces):

$$A_j \supset \int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2 \frac{f(k_1 \cdot k_1, k_1 \cdot k_2, \ldots, k_2 \cdot p_3)}{N_1 \cdots N_7}$$

(Max) 7 Propagators in Diagram

$S >$ **#Propagators: Irreducible Numerators**

Number of Scalar products:

$$S = \frac{l(l+1)}{2} + lm$$

$l = 2$ # Loops

$m = 3$ # L.I External momenta

$S = 9$

Integral Reduction

Integral family: Add propagators s.t. all scalar products can be expressed in terms of (inverse) propagators

$$A_j \supset \int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2 \frac{1}{N_1^{\alpha_1} \cdots N_9^{\alpha_9}} \equiv I(\alpha_1, \dots, \alpha_9)$$

Encode all integrals by their propagator powers

Current Status: Integral families input by hand

Symmetries: $I(\alpha_1, \dots, \alpha_9) = I(\sigma(\alpha_1), \dots, \sigma(\alpha_9))$ ← **For some** $\alpha_i > 0$

Integration-by-parts (IBP) /Lorentz Invariance (LI) Identities

Tkachov 81; Chetyrkin, Tkachov 81 — Gehrmann, Remiddi 99

Laporta/ S-Bases algorithms to automate application of these identities

Laporta 01; Smirnov, Smirnov 06

Uncertainties

Total Cross Section:

Born Improved NLO HEFT

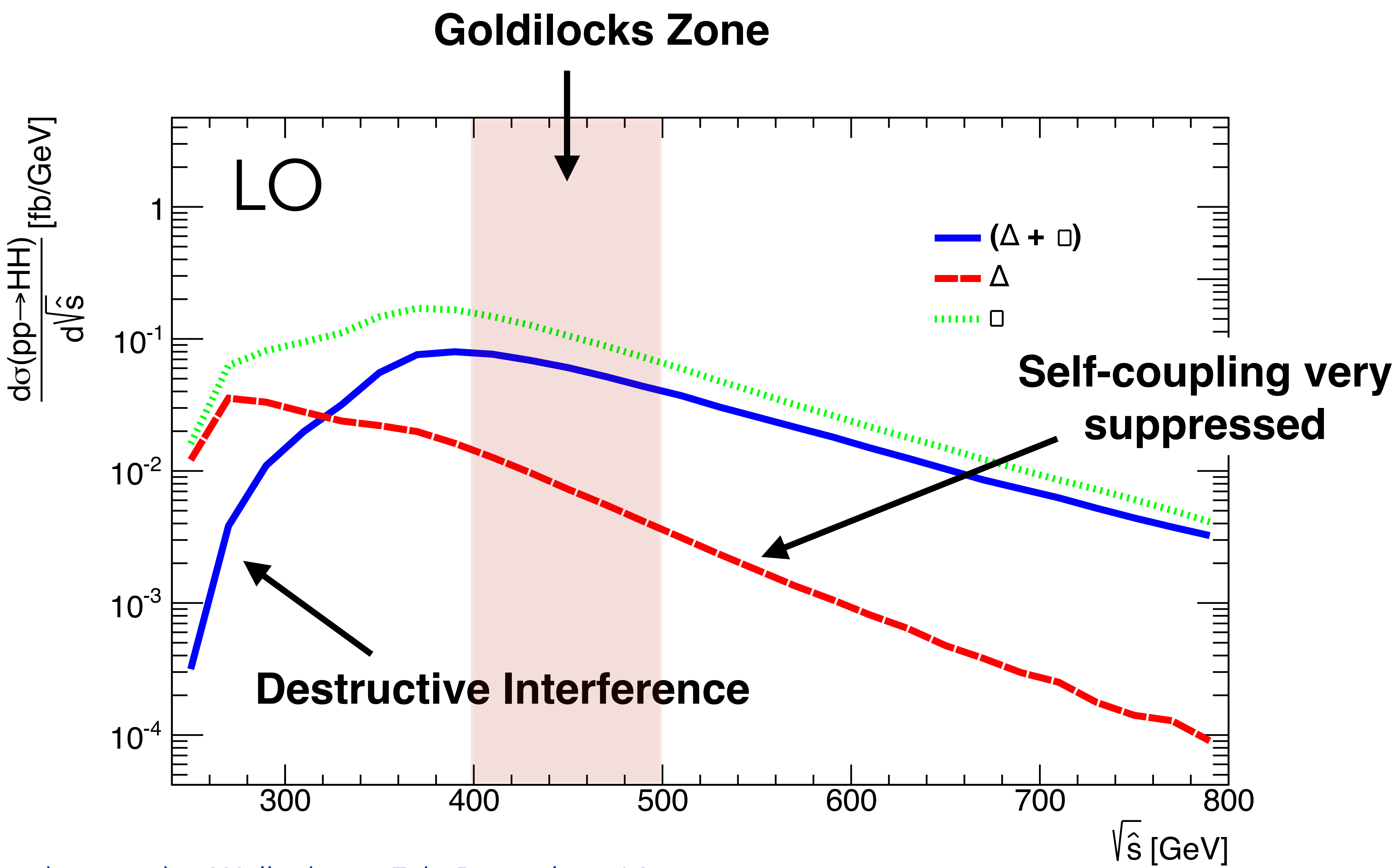
Scale $\mu_0 = \mu_R = \mu_F = M_{HH}$, Variation: $[\frac{\mu_0}{2}, 2\mu_0]$

Some arguments for switching to $\mu_0 = M_{HH}/2$ (account for NNLL?)

Scale	15-20%
PDF + α_s	6-7%
EFT (NLO)	~10%
Total	30-40%

See: Eg… Baglio, Djouadi et al. 12

Self-Coupling Sensitivity



Slawinska, van den Wollenberg, Eijk, Bentvelsen 14