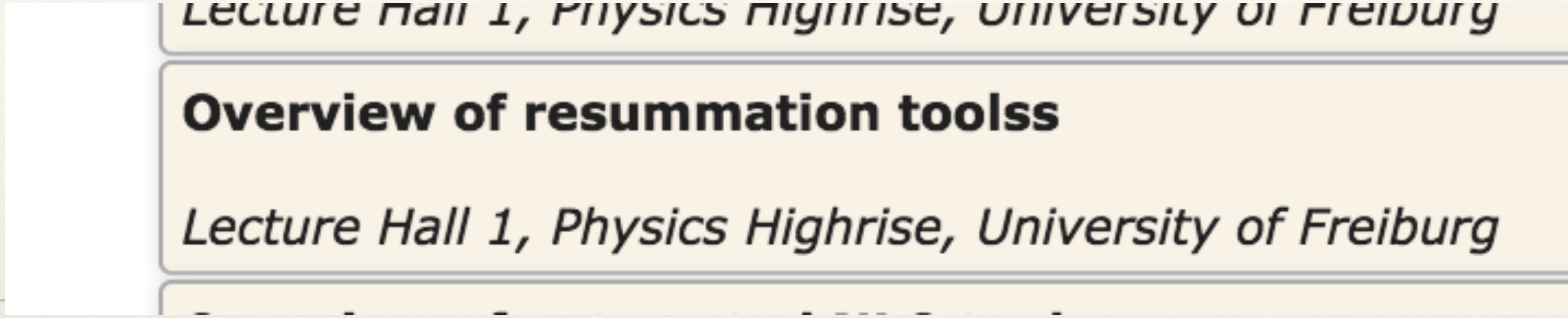
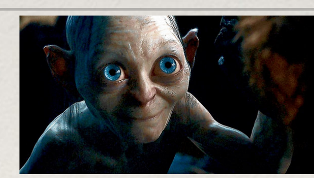


Overview of resummation "toolsses"

HiggsTools First Annual Meeting, Freiburg



Eric Laenen

Universiteit van Amsterdam
NIKHEF
University of Utrecht

Perturbative series in QFT

- Typical perturbative behavior of observable

$$\hat{O}_2 = 1 + \alpha(L^2 + L + 1) + \alpha^2(L^4 + L^3 + L^2 + L + 1) + \ldots$$

  - α is the coupling of the theory (QCD, QED, ..)
  - L is some numerically large logarithm
  - “1” = π^2, ln2, anything no
  - Notice: *effective* expansion parameter is αL^2. Problem occurs if is this >1!!
  - Possible fix: reorganize/resum terms such that

$$\begin{aligned}\hat{O} &= 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \ldots \\ &= \exp\left(\underbrace{\underbrace{Lg_1(\alpha_s L)}_{LL} + g_2(\alpha_s L)}_{NLL} + \alpha_s g_3(\alpha_s L) + \ldots\right)\underbrace{C(\alpha_s)}_{\text{constants}} \\ &\quad + \text{ suppressed terms}\end{aligned}$$

- Notice the definition of LL, NLL, etc

LL, NLL,.. and matching to fixed order

- Leading-log, next-to-leading log, etc
  - Schematic overview

$$O = \alpha_s^p \left(\underbrace{\underbrace{C_0}_{\text{LL,NLL}} + C_1\alpha_s}_{\text{NNLL}} + \dots \right) \exp\left[\underbrace{\underbrace{\underbrace{(\sum_{n=1} \alpha_s^n L^{n+1} c_n)}_{\text{LL}} + (\sum_{n=1} \alpha_s^n L^n d_n)}_{\text{NLL}} + (\sum_{n=1} \alpha_s^n L^{n-1} e_n)}_{\text{NNLL}} + \dots \right]$$

  - Systematic expansion in α_s in the exponent
    - ✓ If we can find the coefficients c_n, d_n, e_n, C_0, C_1 etc
  - It is directly clear how to combine this with an exact NLO or NNLO calculation
    - ✓ Expand the resummed version to the next order in α_s . Add the NLO and resummed, but subtract the order α_s - expanded resummed result, to avoid double counting.

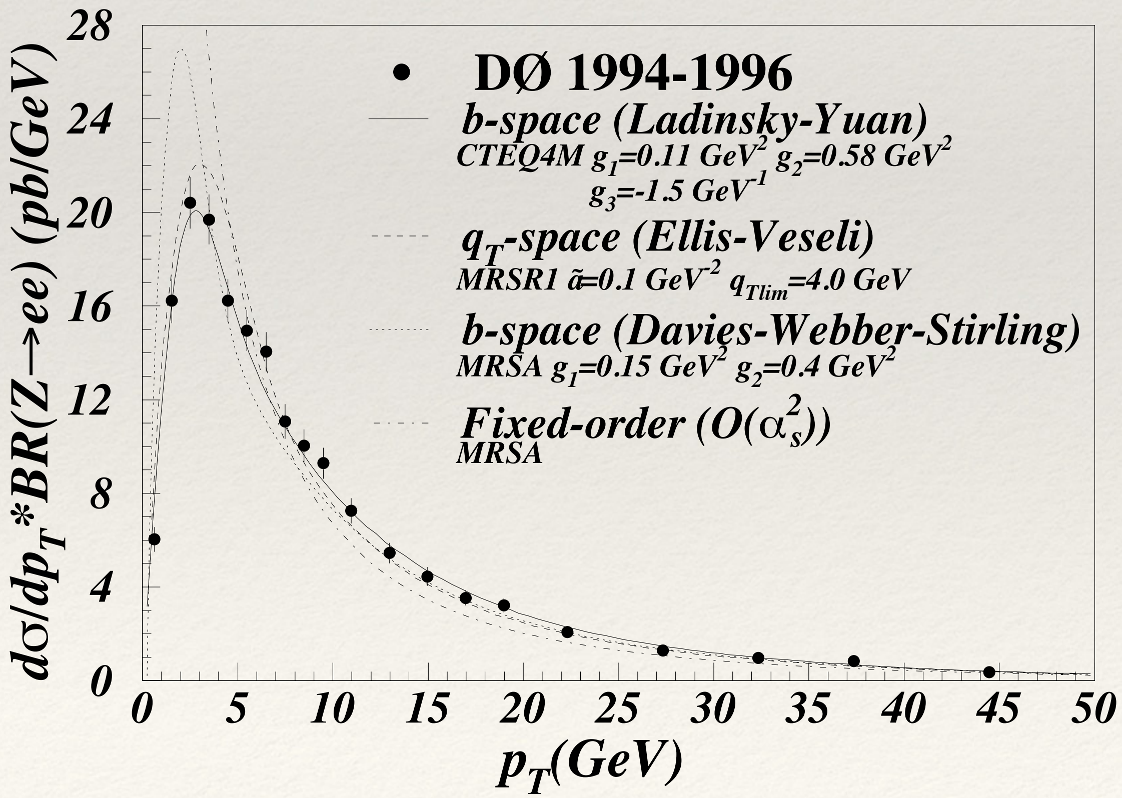
$$O_{\text{NLO matched}} = O_{\text{NLO}} + O_{\text{resummed}} - (O_{\text{resummed}})\big|_{\text{expanded to } \mathcal{O}(\alpha_s)}$$

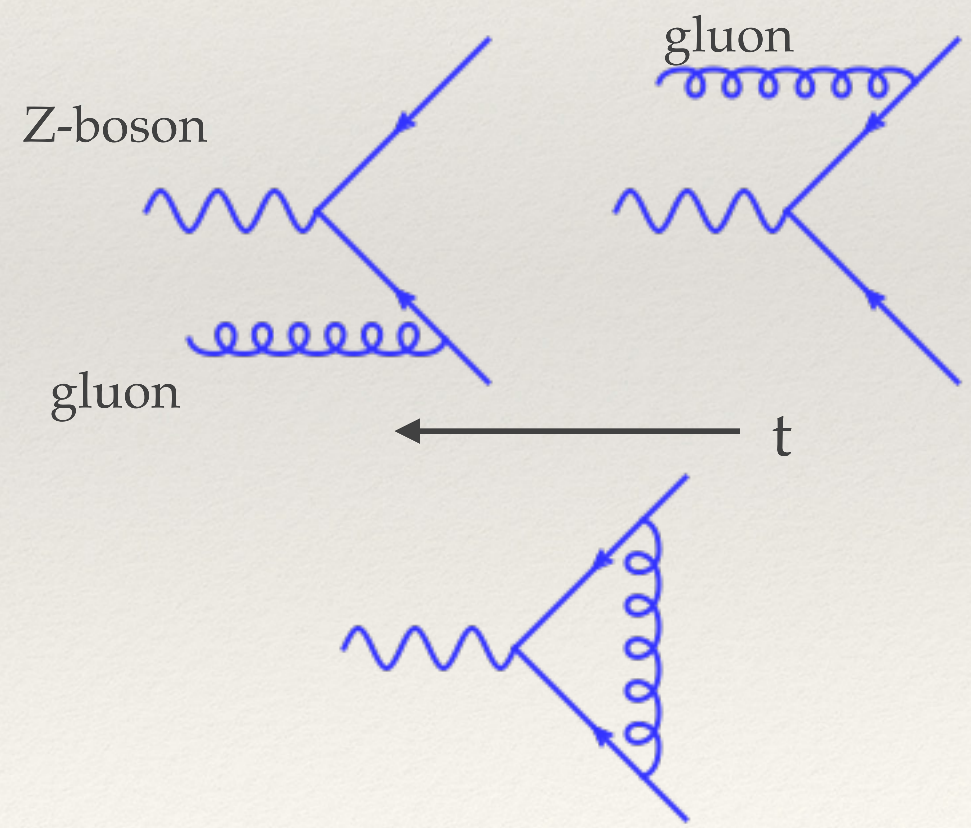
      - generalization to NNLO is "obvious"

- Various examples of logs

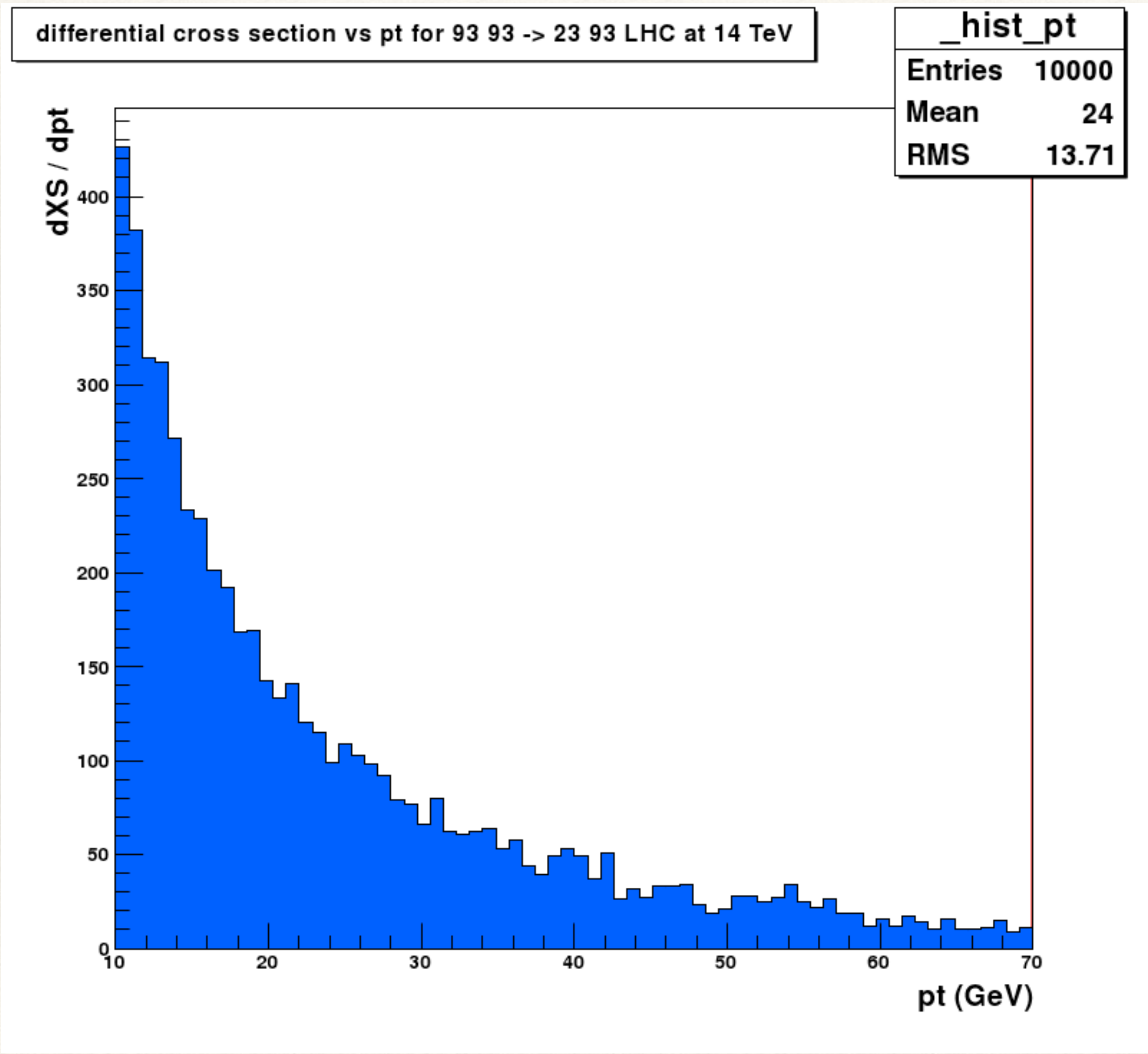
Example of double log: recoil logs

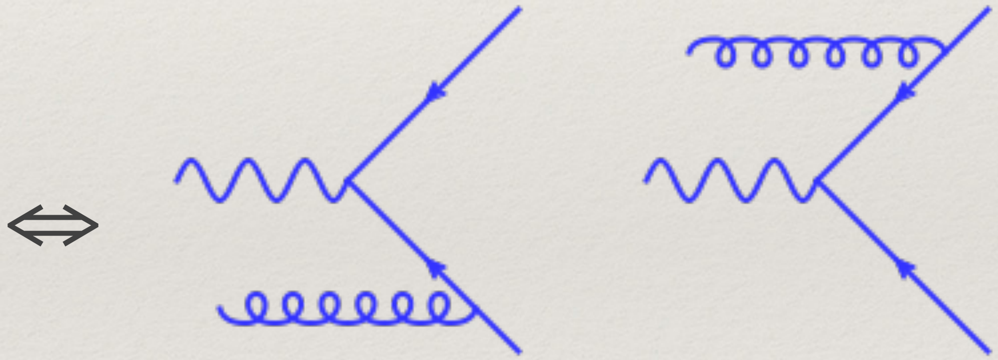
- Eg. pT of Z-bosons produced in hadron collisions
  - Z-boson gets p_T from recoil agains (soft) gluons
  - Visible logs (argument made of measured quantities)
    - 1 emission: with gluon very soft: divergent
      - virtual: large negative bin at pT=0
  - The turn-over at pT around 5 GeV is only explained by resummation, not by finite order calculations





Divergence near p_T=0





Physics near small p_T

- At finite order

$$\frac{d\sigma}{dp_T} = c_0\delta(p_T) + \alpha_s\left(c_2^1\frac{\ln p_T}{p_T} + c_1^1\frac{1}{p_T} + c_0^1\delta(p_T)\right) + \ldots$$

  - hence the real divergence toward p_T near zero

- Resummed

$$\frac{d\sigma}{dp_T} = c_0 \exp\left[-c_2^1\alpha_s \ln^2(p_T) + \ldots\right]$$

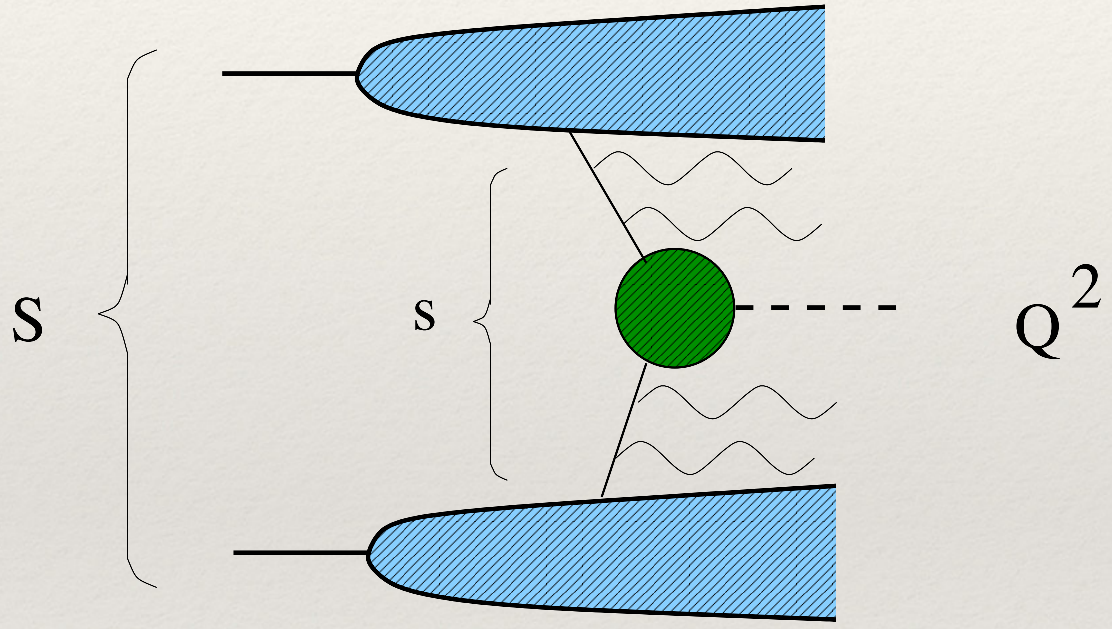
  - ✓ this is also the effective behaviour of the parton shower there

- Notice:
  - finite order oscillates wildly near small p_T, and may be negative
  - resummed is positive, and it tracks the data well

- Physics of resummed answer:
  - probability of the process **not** to emit at small p_T is vanishingly small
    - ✓ There is violent acceleration of color charges after all..

Another example: threshold logs

- Logarithm of "energy above threshold Q^2" $\ln^2(1 - Q^2/s)$
  - "Invisible" logs": argument made up of integration variables
  - Typical effect: enhancement of cross section



$$S \geq s \geq Q^2$$

Resummation 101

- Cross section for n extra gluons

Phase space measure **Squared matrix element**

$$\sigma(n) = \frac{1}{2s}\int d\Phi_{n+1}(P, k_1, \ldots, k_n) \times |\mathcal{M}(P, k_1, \ldots, k_n)|^2$$

- When emissions are soft, can factorize phase space measure and matrix element [eikonal approximation]

$$d\Phi_{n+1}(P, k_1, \ldots, k_n) \longrightarrow d\Phi(P) \times \Big(d\Phi_1(k)\Big)^n \frac{1}{n!}$$

- Sum over all orders

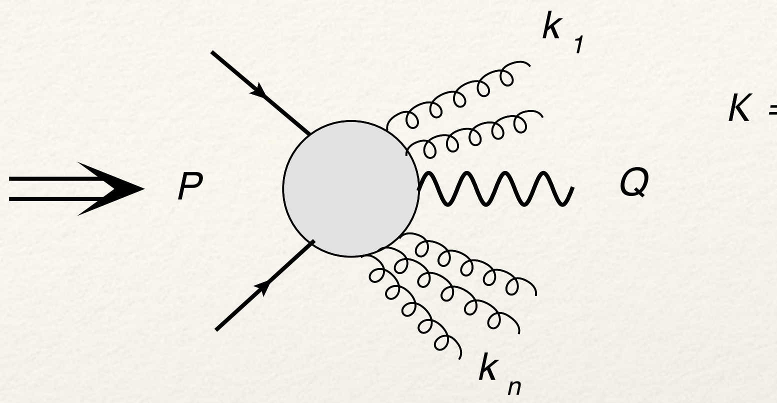
$$|\mathcal{M}(P, k_1, \ldots, k_n)|^2 \longrightarrow |\mathcal{M}(P)|^2 \times \Big(|\mathcal{M}_{1\text{ emission}}(k)|^2\Big)^n$$

$$\sum_n \sigma(n) = \sigma(0) \times \exp\left[\int d\Phi_1(k)|\mathcal{M}_{1\text{ emission}}(k)|^2\right]$$

- Incorporate Theta or Delta functions in space space
  - but these must factorize similarly, or they cannot go into exponent

Phase space in resummation

- Kinematic condition expresses "z" in terms of gluon energies



$$s = Q^2 - 2P\cdot K - K^2 \qquad \delta\Big(1 - \frac{Q^2}{s} - \sum_i \frac{2k_i^0}{\sqrt{s}}\Big)$$

  - or conservation of transverse momentum $\delta^2(Q_T - \sum_i p_T^i)$

- Transform (e.g. Laplace or Fourier) factorizes the phase space

$$\int_0^\infty dw\, e^{-w\,N}\delta\Big(w - \sum_i w_i\Big) = \prod_i \exp(-w_i N) \qquad \int d^2Q_T\, e^{ib\cdot Q_T}\,\delta^2(Q_T - \sum_i p_T^i) = \prod_i e^{ib\cdot p_T^i}$$

- So can go into exponent

$$\sum_n \sigma(n) = \sigma(0) \times \exp\Big[\int d\Phi_1(k)|\mathcal{M}_{1\ \mathrm{emission}}(k)|^2(\exp(-w\,N) - 1)\Big]$$

  - Large logs: ln(N) or ln(bQ)

Resummation and factorization

- Very generically, if a quantity factorizes, one can resum it
  - Renormalization; factorizes UV modes into Z-factor

$$G_B(g_B, \Lambda, p) = Z\Big(\frac{\Lambda}{\mu}, g_R(\mu)\Big) \times G_R\Big(g_R(\mu), \frac{p}{\mu}\Big)$$

  - Evolution equation (here RG equation)

$$\mu\frac{d}{d\mu}\ln G_R\Big(g_R(\mu), \frac{p}{\mu}\Big) = -\mu\frac{d}{d\mu}\ln Z\Big(\frac{\Lambda}{\mu}, g_R(\mu)\Big) = \gamma(g_R(\mu))$$

  - Solving = resumming

$$G_R\left(\frac{p}{\mu}, g_R(\mu)\right) = G_R\left(1, g_R(p)\right) \underbrace{\exp\left[\int_p^\mu \frac{d\lambda}{\lambda}\gamma(g_R(\lambda))\right]}_{\text{resummed}}$$

Resummation and factorization

- Type of factorization dictates resummation
  - small x [ln(x)] ⟶ k_T factorization
    - ✓ Regge, High-Energy,..
  - large x [ln(1-x)] ⟶ near-threshold factorization
    - ✓ Threshold, Sudakov
- Factorization is essentially separating degrees of freedom
  - Systematic approach in Soft Collinear Effective Theory

Soft Collinear Effective Theory

Bauer, Fleming, Pirjol, Stewart,...

- Effective field theory approach: SCET
  - Distinguish separate **fields** for soft, collinear, hard partons, and ultrasoft gluons

$$\mathcal{L}_{SCET,qq} = \bar{\xi}_n (in \cdot D + i\not{D}_{c,\perp} \frac{1}{i\bar{n} \cdot D_c} i\not{D}_{c,\perp}) \frac{\not{\bar{n}}}{2} \xi_n - \frac{1}{4} \mathrm{Tr}\{G^c_{\mu\nu} G^{c,\mu\nu}\}$$

    - ✓ Powerful power counting. Using +,-,T notation

$$p_h \sim Q(1,1,1) \quad p_c \sim Q(\lambda, 1, \sqrt{\lambda}) \quad p_s \sim Q(\lambda, \lambda, \lambda)$$

    - ✓ Fields scale similarly:

$$\xi_n \sim \lambda \quad \xi_{\bar{n}} \sim \lambda^2 \quad A_s \sim \lambda \quad \bar{n} \cdot A_c \sim \lambda^0$$

  - 2 gauge transformations, collinear and ultrasoft
    - ✓ and two types of Wilson lines:

$$W_c(x) \qquad S_n(x)$$

Soft Collinear Effective Theory

Bauer, Fleming, Pirjol, Stewart,...

- Decouple soft gluons from collinear via field redefinition $\xi_n(x) \to S_n(x)\xi_n^{(0)}(x)$

$$\bar{\xi}\frac{\bar{n}\!\!\!/}{2}n\cdot D\,\xi \to \bar{\xi}^{(0)}\frac{\bar{n}\!\!\!/}{2}n\cdot D_c\,\xi^{(0)}$$

  - Soft gluons do not of course fully disappear from every observable
  - Can form soft functions (matrix elements of soft Wilson lines)
- Resummation: match and run
  - Write observable (e.g. σ_{DY}) as

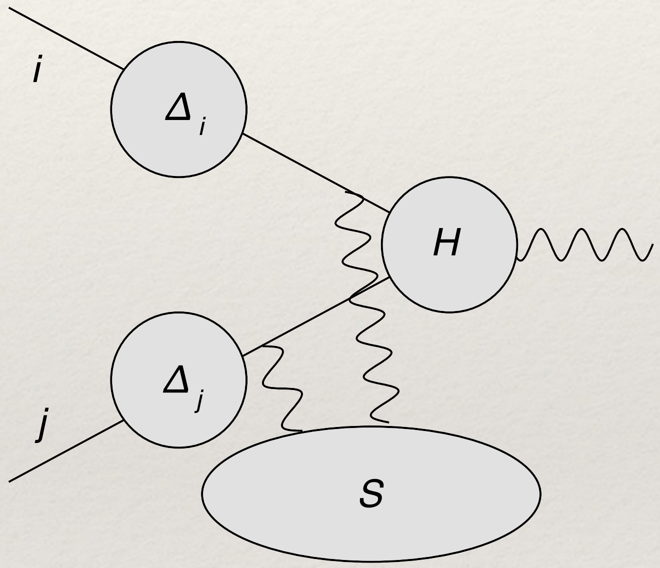
$$\langle O_{QCD}\rangle \qquad \text{and as} \qquad \prod_i \langle O^i_{SCET}\rangle \times C^i_{\text{match}}$$

  - Solve RG equations for O^i_{SCET}
  - Find C by 1-loop (or 2-loop) calculations on both sides
- Powerful method

Factorization for threshold resummation

- $\Delta_i(N)$: initial state soft+collinear radiation effects
  - real+virtual
  - $\alpha_s^n \ln^{2n} N$
- $S_{ij}(N)$: soft, non-collinear radiation effects
  - $\alpha_s^n \ln^n N$
- H: hard function, no soft and collinear effects

$$\sigma(N) = \sum_{ij} \phi_i(N)\phi_j(N) \times \underbrace{\left[\Delta_i(N)\Delta_j(N)S_{ij}(N)\, H_{ij}\right]}_{\hat{\sigma}_{ij}(N)}$$



$$\begin{aligned}\Delta_i(N) &= \exp\left[\ln N \frac{C_F}{2\pi b_0 \lambda}\{2\lambda + (1-2\lambda)\ln(1-2\lambda)\} + ..\right] \\ &= \exp\left[\frac{2\alpha_s C_F}{\pi}\ln^2 N + ..\right]\end{aligned}$$

Resummed cross sections

Sterman; Catani, Trentadue

Threshold-resummed Drell-Yan cross section

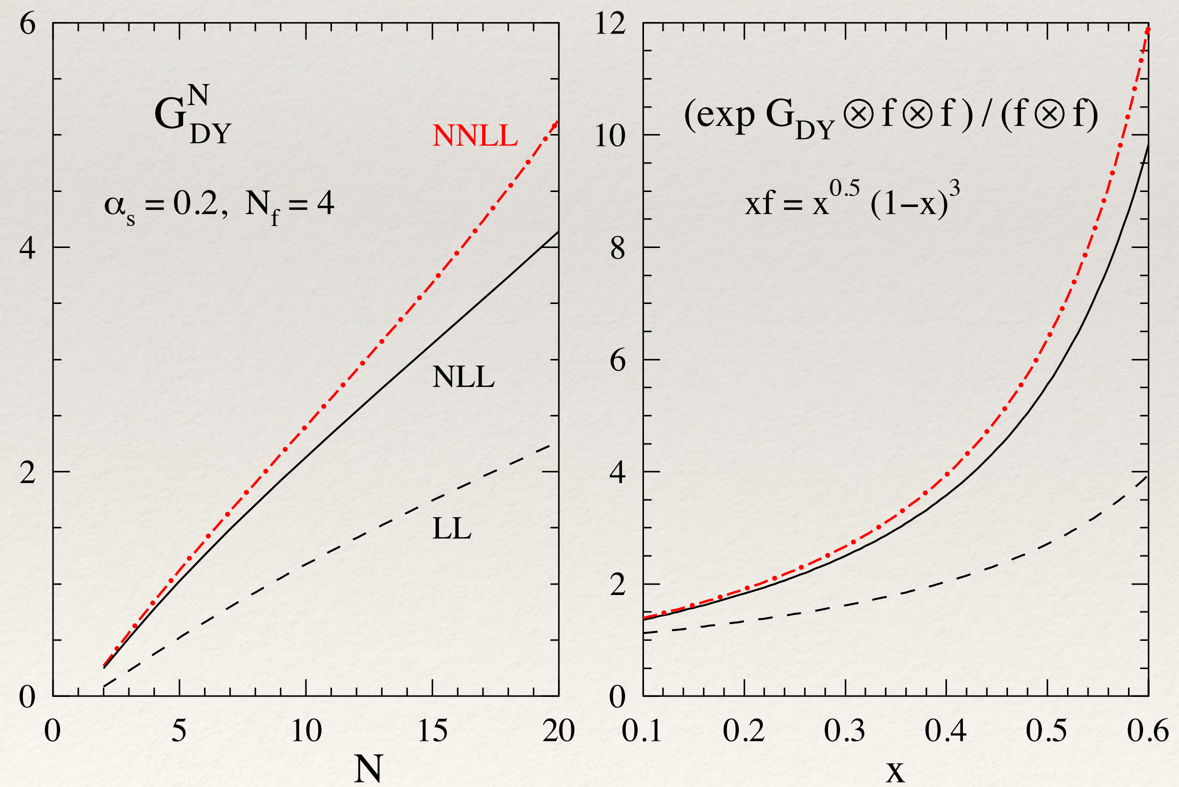
Functions in exponent depend only on coupling

$$\frac{d\sigma^{\rm resum}}{dQ^2}(z) = \int_C \frac{dN}{2\pi\, i}\, z^{-N}\, \hat\sigma(N)$$

$$\sigma(N) = \exp\left[-\int_0^1 dx\, \frac{x^{N-1}-1}{1-x}\left\{\int_{Q^2}^{Q^2(1-x)^2} \frac{d\mu}{\mu}\, A(\alpha_s(\mu)) + D(\alpha_s((1-x)Q))\right\}\right] \times \left(1+\alpha_s(Q^2)\frac{C_F}{\pi}+\ldots\right)$$

$$\hat\sigma_{DY}(N,Q^2) = g_0(Q^2)\, \exp\left[G^N_{DY}(Q^2)\right]$$
$$G^N_{DY} = \ln N g_1(\lambda) + g_2(\lambda) + \alpha_s g_3(\lambda) + \ldots, \qquad \lambda = \beta_0 \alpha_s \ln N$$

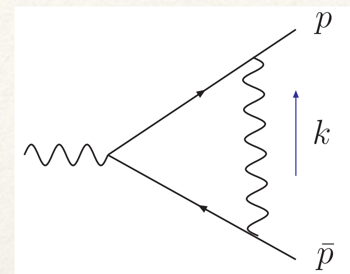
A. Vogt



Good convergence in exponent

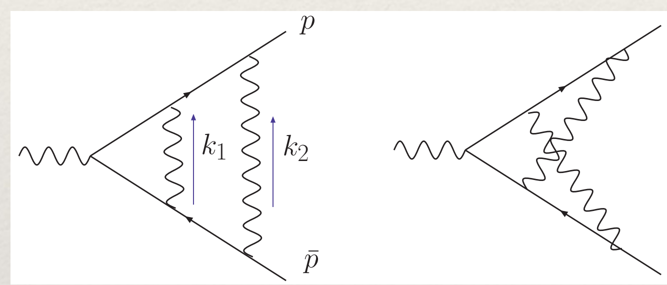
Eikonal exponentiation

One loop vertex correction, in eikonal approximation



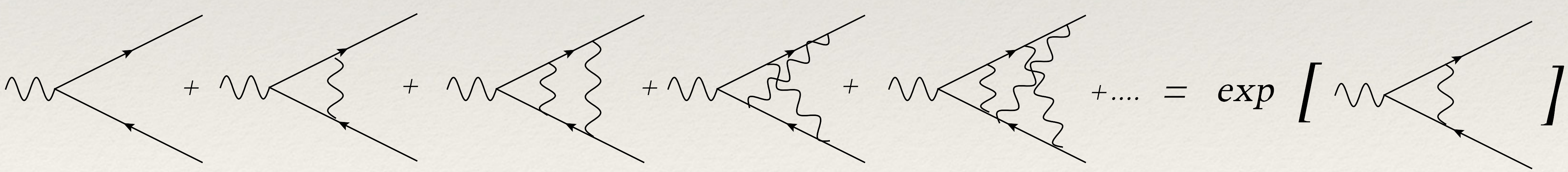
$$\mathcal{A}_0 \int d^n k \frac{1}{k^2} \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)}$$

Two loop vertex correction, in eikonal approximation



$$\mathcal{A}_0 \frac{1}{2} \left(\int d^n k \frac{1}{k^2} \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)} \right)^2$$

Exponential series



Yennie, Frautschi, Suura

Resummation using path integrals

EL, Stavenga, White

Use textbook result

$$\text{Sum of all diagrams} = \exp\Big(\text{Connected diagrams}\Big)$$

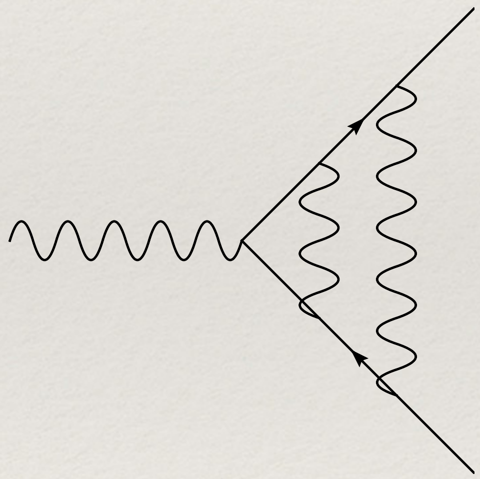
$$f = e^{i\int dt(\frac{1}{2}\dot{x}^2 + p\cdot A + ..)}$$

Write scattering amplitude as first-quantized path integral

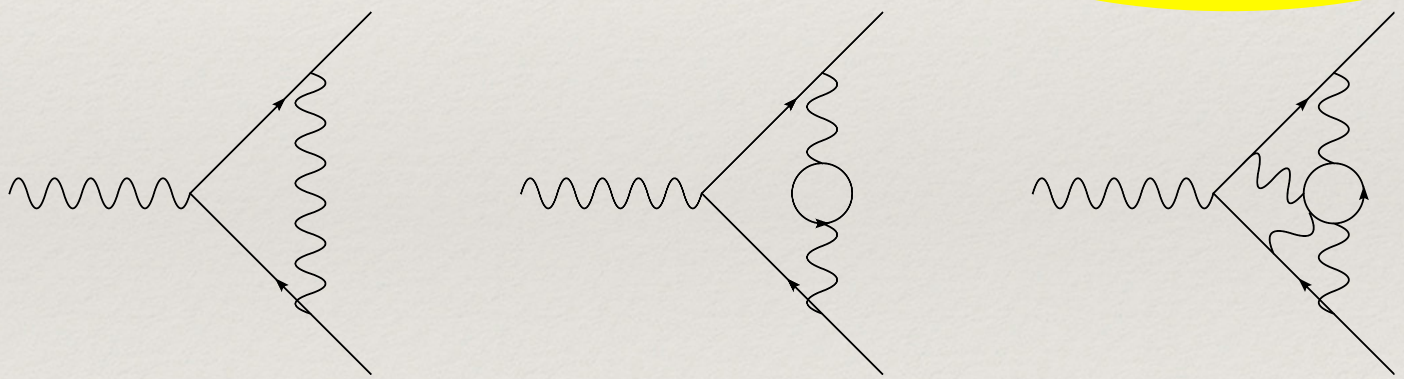
$$M(p_1, p_2, \{k\}) = \int \mathcal{D}A_s\, \mathcal{D}x(t)\, H[x]\, f_1[A_s, x(t)]\, f_2[A_s, x(t)]\, e^{iS[A_s]}$$

x(t): path of charged particle

Eikonal vertices are sources for gauge bosons along line

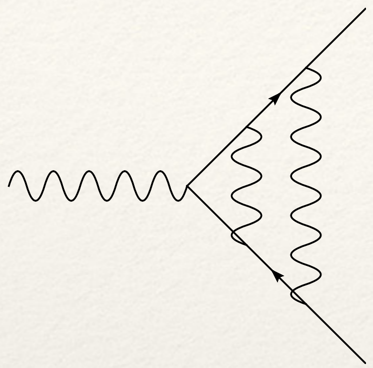


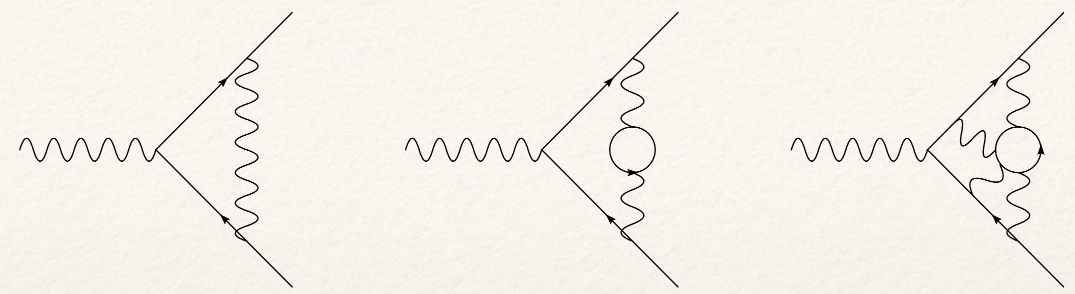
Disconnected



Connected

Non-abelian exponentiation: webs





- Not immediately obvious how this could work (the path integral must be a real exponential), since
  - Source terms have non-abelian charges, so don't commute
  - External line factors are path-ordered exponentials
  - Nevertheless

$$\sum_D \mathcal{F}_D C_D = \exp\left[\sum_i \bar{C}_i w_i\right]$$

Gatheral; Frenkel, Taylor; Sterman

- Proven using replica trick (from statistical physics)

EL, Stavenga, White

- Generalized to multiple colored external lines

Gardi, EL, Stavenga, White

Mitov, Sterman, Sung

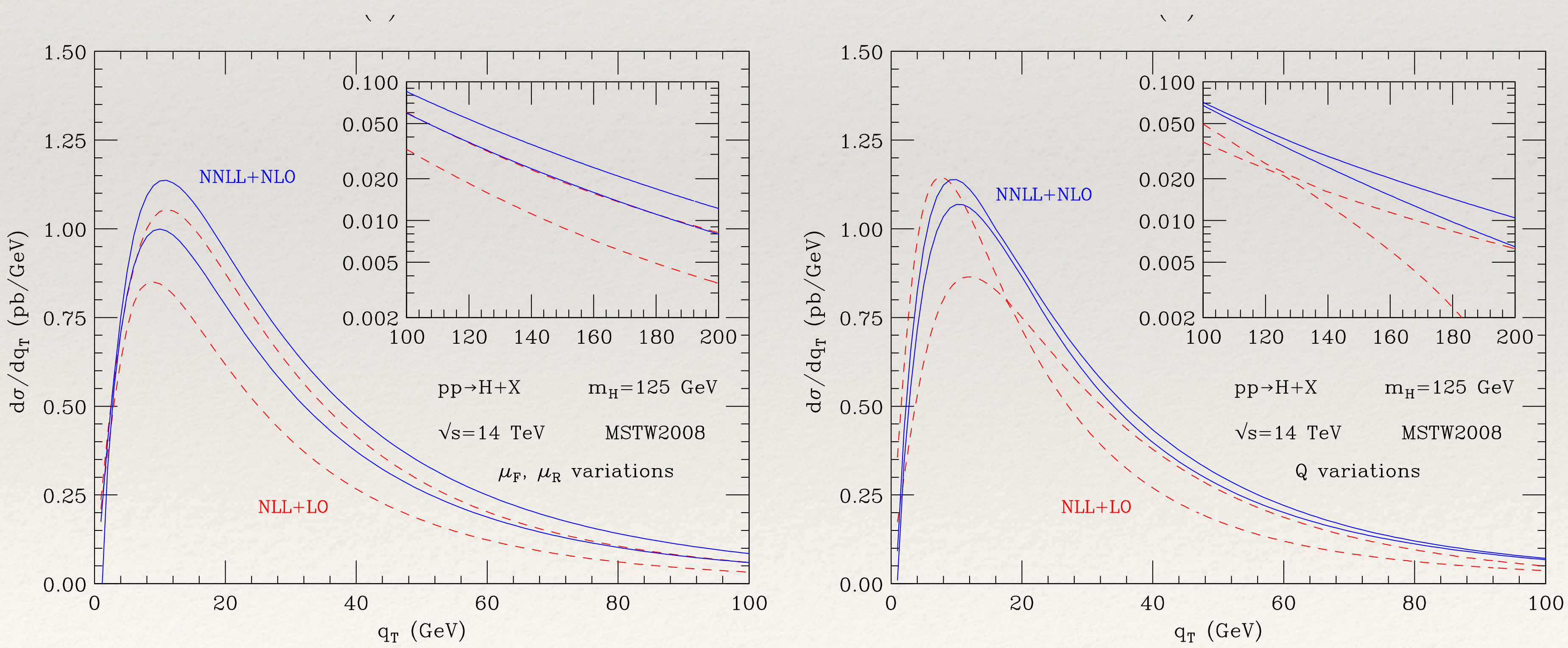
Tranverse momentum resummation

de Florian, Ferrara, Grazzini, Tommasini

- Method: b-space resummation
  - Code: `HqT`
- Key part of resummation formula

$$S_c(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_c(\alpha_S(q^2)) \ln\frac{M^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$

  - first 3 orders of A_c, and 2 of B_c are known. NNLL+NNLO accuracy.



Higgs + (no) jets

- Large logs due to exclusive binning: $\ln(p_T^{veto}/m)$. Resummations using SCET
- Accuracies have gone up
  - NNLL'+NNLO — Stewart, Tackmann, Walsh, Zuberi
  - N3LL'+NNLO — Becher, Neubert, Rothen
    - ✓ and include careful assessment of uncertainties
- Resummation also here enters also the realm where once only Monte Carlo roamed..
  - how to combine resummed Higgs cross sections in different exclusive jet bins
    - ✓ "resummed theory covariance matrix" — Boughezal, Xiaohui, Petriello, Tackmann, Walsh
    - ✓ reduction of factor 2 in theoretical uncertainties!
  - automation @ NNLO+NLO! — Becher, Frederix, Neubert, Rothen

N^3LO Higgs from small and large x

Ball, Bonvini, Forte, Marzani, Ridolfi

- So far, focus on large N
- Interesting idea: use analyticity structure in complex N space
  - From large N (large x) and N=1 (small x) resummation
    - ✓ Sudakov $\ln^i N$, BFKL $1/(N-1)^i$
  - Switch to $(1-z)^2/z$ — EL, Magnea, Stavenga
  - Leads in Mellin space to

$$\ln N \to \psi_0(N)$$

    - ✓ Removes branchpoint at N=0
- Corrections beyond NNLO about 17% (m=125 GeV at 8 TeV)

N^3LL resummation

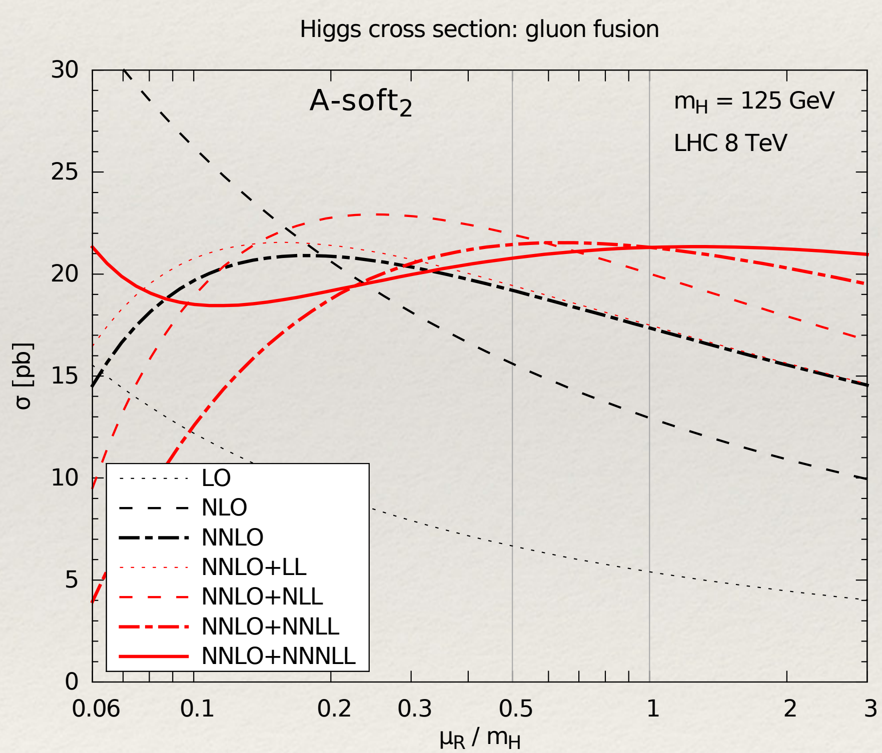
- Mellin space analysis

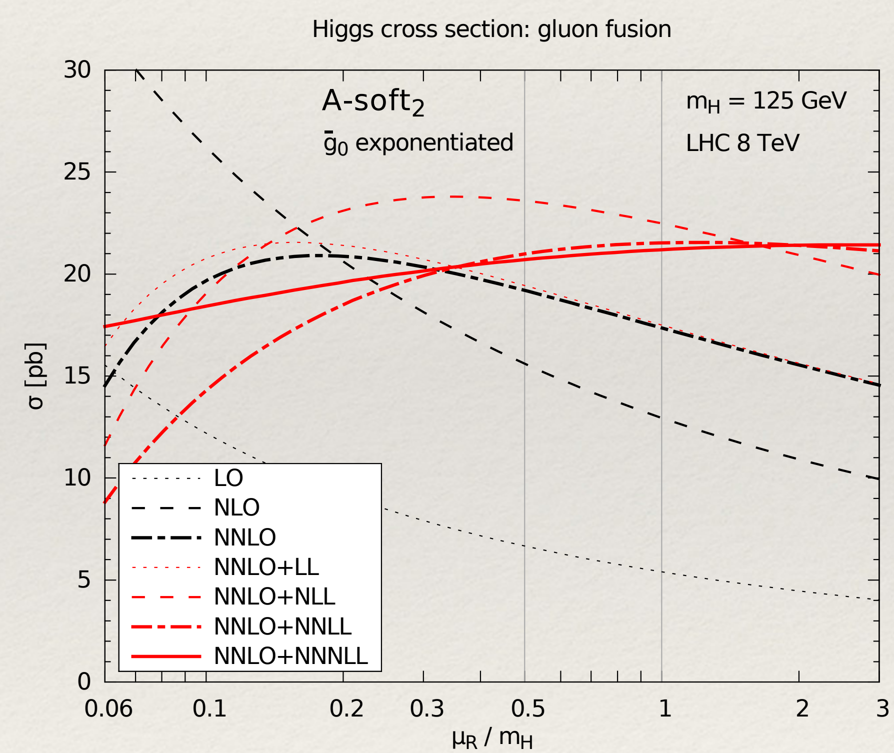
Catani, Cieri, de Florian, Ferrera, Grazzini

Bonvini, Marzani

  - Include information from N=1 pole (~ next-to-soft terms)
  - For inverse Mellin transform, employ both Minimal Prescription and Borel prescription
    - ✓ but not much difference
  - Nice progression, especially with exponentiated constants

Eynck, EL, Magnea





  - Code: ResHiggs and ggHigs

N^{3}LO Higgs and subleading powers

- Large logs in N^{3}LO are present in resummed expression

Anasthasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

$$\Delta_{c,N} = \exp\left\{\int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[2\int_{M^2}^{(1-z)^2M^2}\frac{dq^2}{q^2}A_c(\alpha_s(q^2) + D_c(\alpha_s((1-z)^2M^2)\right]\right\}$$

  - A and D known to 3rd order
- Earlier result added also ln(1-z) logs
- Full N^{3}LO result [Falko Dulat's tallk] realized full power of (1-z) threshold expanion

Generic large x behavior

- For DY, DIS, Higgs, singular behavior when x⟶ 1

$$\delta(1-x) \qquad \left[\frac{\ln^i(1-x)}{1-x}\right]_+ \qquad \ln^i(1-x)$$

  - delta-function: pure virtuals
  - plus distributions: resummable to all orders (N3LL for Higgs production now)
  - NLP logarithms, systematics are beginning to emerge
- Method of regions allows their computation

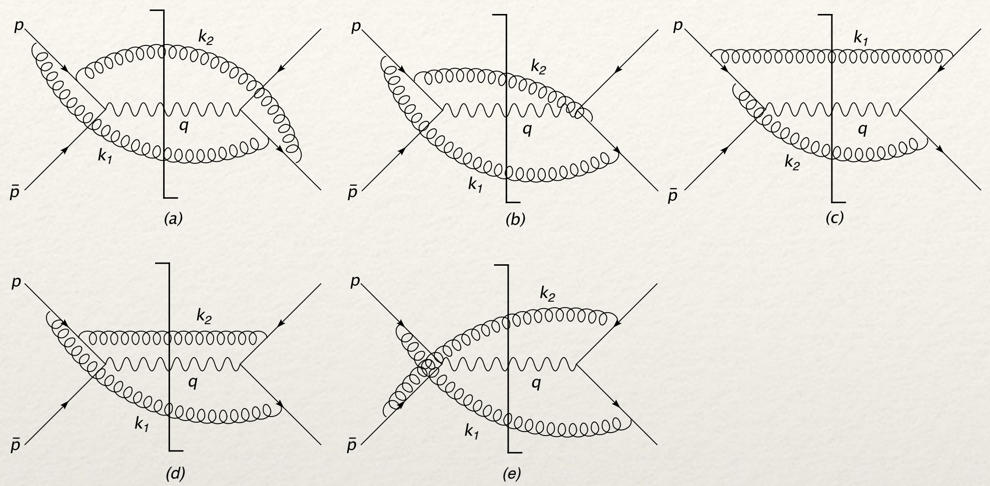
$$(1-x)^p \ln^q(1-x)$$

    - ✓ at least to p=37
- Can they be predicted?

NLP logs in Drell-Yan at NNLO

EL, Magnea, Stavenga, White

- Check NLP Feynman rules for NNLO Drell-Yan *double real* emission (only C_F^2 terms)



  - Result at NE level, agrees with equivalent exact result

$$K^{(2)}_{\rm NE}(z) = \left(\frac{\alpha_s}{4\pi}C_F\right)^2 \Bigg[-\frac{32}{\epsilon^3}\mathcal{D}_0(z) + \frac{128}{\epsilon^2}\mathcal{D}_1(z) - \frac{128}{\epsilon^2}\log(1-z) - \frac{256}{\epsilon}\mathcal{D}_2(z) + \frac{256}{\epsilon}\log^2(1-z) - \frac{320}{\epsilon}\log(1-z) + \frac{1024}{3}\mathcal{D}_3(z) - \frac{1024}{3}\log^3(1-z) + 640\log^2(1-z) \Bigg],$$

$$\mathcal{D}_i = \left[\frac{\log^i(1-z)}{1-z}\right]_+$$

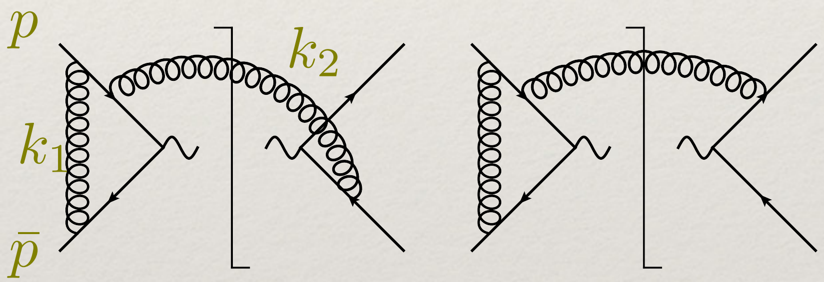
- Next, 1 Real- 1 Virtual (only C_F^2 terms)
  - ✓ virtual gluon not necessarily soft
  - ✓ we redid exact calculation again, for comparison

Diagnosis: method of regions

Vernazza, Bonocore, EL, Magnea, Melville, White

- Method of region approach, extended to next power — Beneke, Smirnov
  - Allow treatment of (next-to-)soft and (next-to-)collinear on equal footing
- How does it work?
  - Divide up k_1 (=loop-momentum) integral into hard, 2 collinear and a soft region, by appropiate scaling

$$\text{Hard}: \quad k_1 \sim \sqrt{\hat{s}}\,(1,1,1)\,; \qquad \text{Soft}: \quad k_1 \sim \sqrt{\hat{s}}\,(\lambda^2,\lambda^2,\lambda^2)\,;$$
$$\text{Collinear}: \quad k_1 \sim \sqrt{\hat{s}}\,(1,\lambda,\lambda^2)\,; \quad \text{Anticollinear}: \quad k_1 \sim \sqrt{\hat{s}}\,(\lambda^2,\lambda,1)\,.$$



  - expand integrand in λ, to leading and next-to-leading order
  - but then integrate over *all* k_1 anyway!
  - Treat emitted momentum as soft and incoming momenta as hard

$$k_2^\mu = (\lambda^2,\lambda^2,\lambda^2) \qquad p^\mu = \tfrac{1}{2}\sqrt{s}n_+^\mu \qquad \bar{p}^\mu = \tfrac{1}{2}\sqrt{s}n_-^\mu$$

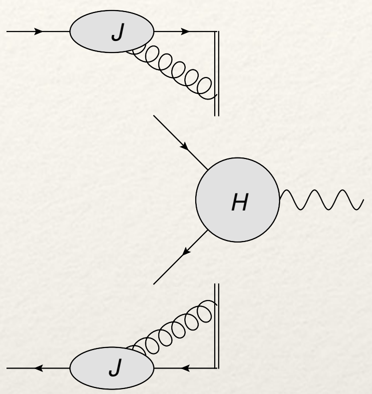
MoR

Vernazza, Bonocore, EL, Magnea, Melville, White

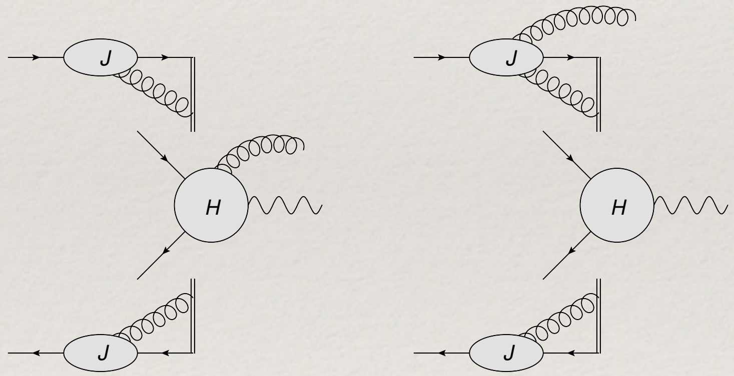
- Findings
  - Hard region (expansion in λ^2)
    - ✓ reproduces already all plus-distributions, and some NLP logarithms
  - Soft region (expansion in λ^2)
    - ✓ all integrals are scale-less, hence all zero in dimensional regularization
  - (anti-)collinear regions (expansion in λ)
    - ✓ only give NLP logarithms, once all diagrams in set are summed
- Result:
  - the full $K^{(1)}_{1r,1v}$ is reproduced, including constants
    - ✓ Collinear regions give only NLP logarithms
    - ✓ Clearly, one must first expand in ε, then in soft momentum
- For predictive power, need factorization

New: a factorization approach

Bonocore, EL, Magnea, Melville, Vernaza, White
arXiv:1503.05156



- Can we *predict* the log(1-z) logarithms?
- Can we *resum* the log(1-z) logarithms to NLL, NNLL etc?
    - For both we need to factorize the cross section, as we did earlier
        - ✓ H contains both the hard and the soft function (non-collinear factors)
        - ✓ J: incoming jet functions
- Next, add one extra soft emission, as in Low's theorem. Let every blob radiate!



Del Duca, 1991

- ✓ Can we compute each new "blob + radiation?", and put it together?
- ✓ New: radiative jet function

$$J_\mu\left(p, n, k, \alpha_s(\mu^2), \epsilon\right) u(p) = \int d^d y \, \mathrm{e}^{-\mathrm{i}(p-k)\cdot y} \, \langle 0 \,|\, \Phi_n(y, \infty)\, \psi(y)\, j_\mu(0) \,|\, p \rangle$$

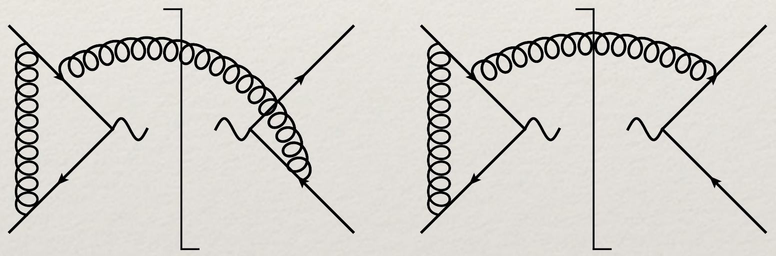
Factorization approach: main formula

Del Duca, 1991

- Upshot: a factorization formula for the emission amplitude

$$\mathcal{A}^{\mu}(p_j,k) = \sum_{i=1}^{2}\left(q_i\,\frac{(2p_i-k)^{\mu}}{2p_i\cdot k-k^2} + q_i\,G_i^{\nu\mu}\,\frac{\partial}{\partial p_i^{\nu}} + G_i^{\nu\mu}\,J_{\nu}(p_i,k)\right)\mathcal{A}(p_i;p_j)$$

- Remarks
  - for logs: to be contracted with cc amplitude
  - only process dependent terms are H and A
  - J_μ is needed at loop level, done
  - In dim.reg.: J is scale-less, so =1



NE logs in factorization approach

Bonocore, EL, Magnea, Melville, Vernaza, White
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- Now put it all together, contract with cc amplitude and on integrate over phase space
  - Can do so in organized fashion

$$d\sigma = d\Phi_{3,\mathrm{LP}} \left(\mathcal{P}_{\mathrm{LP}} + \mathcal{P}_{\mathrm{NLP}}\right) + d\Phi_{3,\mathrm{NLP}} \mathcal{P}_{\mathrm{LP}}$$

- Result:

Find agreement with exact result, including constants:
four powers of logarithms

  - first steps toward resummation of NLP logarithms
  - could generalize “threshold expansion”

Conclusions

- Various resummation tools
  - factorization + resummation
  - straight exponentiation of soft effect (“webs”)
  - systematically improvable, as fixed order
- Benefits:
  - less uncertainty, better physics description
- Progress
  - more exclusive cross sections
  - understanding of analytic structure
  - next-to-soft logarithms