Vector boson pair production at NNLO

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Vector boson pair production

- Different processes: ($\gamma\gamma$, $W^{\pm}\gamma$, $Z^0\gamma$, Z^0Z^0, $W^{\pm}W^{\pm}$, $W^{\pm}Z^0$)
- Testing structure of the Standard Model
  - Gauge boson self-couplings
  - Anomalous couplings
- Higgs boson production
  - Background
  - Signal-background interference
  - Off-shell production
- Beyond Standard Model
  - Similar final state signatures: background

Vector boson pair production: status

- NLO QCD
  - On-shell production (B. Mele, P. Nason, G. Ridolfi; S. Frixione)
  - Including leptonic decays (L. Dixon, Z. Kunszt, A. Signer; MCFM)
- Gluon fusion
  - Leading order: one loop (E.W.N. Glover, J. van der Bij)
- NLO electroweak
  - Ongoing work (E. Accomando, A. Denner, C. Meier; A. Bierweiler, T. Kasprzik, J. Kühn; M. Billoni, S. Dittmaier, B. Jäger, C. Speckner; A. Denner, S. Dittmaier, M. Hecht, S. Pasold; J. Baglio, L.D. Ninh and M. M. Weber)
- NNLO QCD
  - Including decays ($Z^0\gamma, W^{\pm}\gamma$) (M. Grazzini, D. Rathlev, S. Kallweit)
  - On-shell (Z^0Z^0, W^+W^-) (F. Cascioli et al.)

NNLO QCD corrections

- Two-loop matrix elements
  - Explicit poles from loop integration
- One-loop matrix elements with one extra parton
  - Explicit poles from loop integration
  - Implicit poles from phase space integration
  - Computed with automated methods
    (e.g. OpenLoops: F. Cascioli, P. Maierhöfer, S. Pozzorini)
- Tree-level matrix elements with two extra partons
  - Implicit poles from phase space integration
- Method to cancel infrared singularities between channels

Kinematics

- Vector boson pair production

$$q(p_1) + \bar{q}'(p_2) \longrightarrow V_1(p_3) + V_2(p_4)$$

$$p_1^2 = p_2^2 = 0\,, \qquad p_3^2 \neq 0\,, \qquad p_4^2 \neq 0$$

$$s = (p_1 + p_2)^2\,, \qquad t = (p_1 - p_3)^2\,, \qquad u = (p_2 - p_3)^2$$

  - Non-linear kinematical boundary

$$s \geq \left(\sqrt{p_3^2} + \sqrt{p_4^2}\right)^2, \qquad \frac{1}{2}(p_3^2 + p_4^2 - s - \kappa) \leq t \leq \frac{1}{2}(p_3^2 + p_4^2 - s + \kappa)$$

$$\kappa\left(s, p_3^2, p_4^2\right) = \sqrt{s^2 + p_3^4 + p_4^4 - 2(s\, p_3^2 + p_3^2\, p_4^2 + p_4^2\, s)}$$

Two-loop corrections

- Helicity amplitude for four-lepton production

$$q(p_1) + \bar{q}'(p_2) \to V_1(p_3) + V_2(p_4) \to l_5(p_5) + \bar{l}_6(p_6) + l_7(p_7) + \bar{l}_8(p_8)$$

- Two basic amplitudes

$$M_{LLL}(p_1, p_2; p_5, p_6, p_7, p_8) = S_L^{\mu\nu}(p_1^-, p_2^+, p_3)\, L_{L\mu}(p_5^-, p_6^+) L_{L\nu}(p_7^-, p_8^+)\,,$$
$$M_{RLL}(p_1, p_2; p_5, p_6, p_7, p_8) = S_R^{\mu\nu}(p_1^+, p_2^-, p_3)\, L_{L\mu}(p_5^-, p_6^+) L_{L\nu}(p_7^-, p_8^+)\,,$$

- Lorentz and gauge invariance
  - Expressed in terms of 10 basic tensor structures
  - Schouten identity in 4 dimensions: 9 independent structures

Two-loop corrections

- In spinor helicity notation

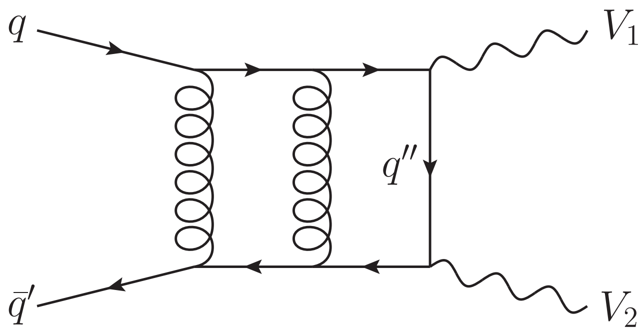
$$
\begin{aligned}
&M_{LLL}(p_1,p_2;p_5,p_6,p_7,p_8) \\
&= [1\,p_3\,2\rangle \Big\{ E_1\,\langle 15\rangle\langle 17\rangle[16][18] \\
&+ E_2\,\langle 15\rangle\langle 27\rangle[16][28] + E_3\,\langle 25\rangle\langle 17\rangle[26][18] \\
&+ E_4\,\langle 25\rangle\langle 27\rangle[26][28] \; + E_5\langle 57\rangle[68] \Big\} \\
&+ E_6\,\langle 15\rangle\langle 27\rangle[16][18] + E_7\,\langle 25\rangle\langle 27\rangle[26][18] \\
&+ E_8\,\langle 25\rangle\langle 17\rangle[16][18] + E_9\,\langle 25\rangle\langle 27\rangle[16][28]
\end{aligned}
$$

- Compute coefficients using projectors
  - Two independent calculations of coefficients (F. Caola, J. Henn, K. Melnikov, A. Smirnov, V. Smirnov; A. von Manteuffel, L. Tancredi, TG)

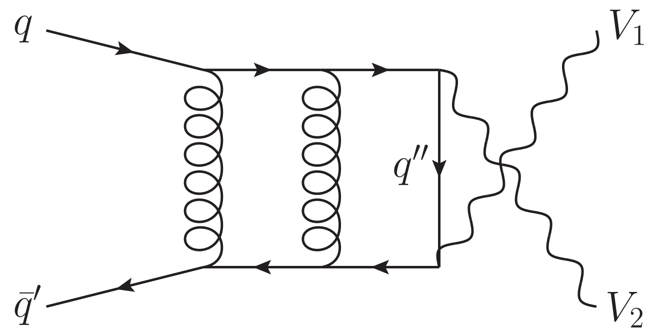
Two-loop corrections

- Classes of diagrams

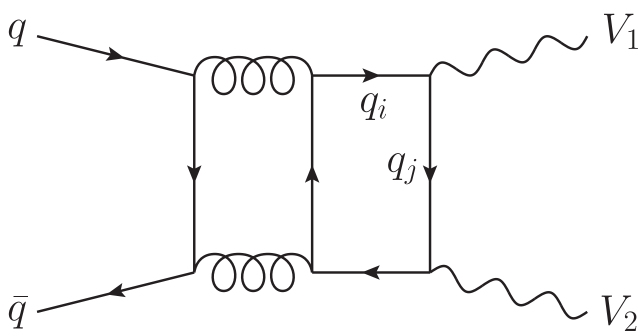
$[A]$



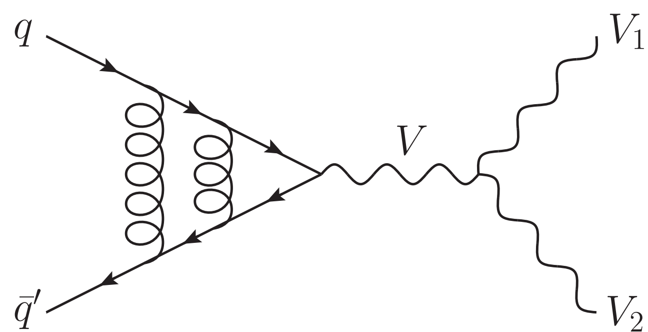
$[B]$



$[C]$



$[F_V]$



Two-loop corrections

- Reduction to master integrals
  - 84 independent integrals
  - 111 integrals including crossings
- Kinematical variables: linearize kinematics

$$s = \bar{m}^2(1+\bar{x})^2, \qquad p_3^2 = \bar{m}^2\bar{x}^2(1-\bar{y}^2),$$
$$t = -\bar{m}^2\bar{x}((1+\bar{y})(1+\bar{x}\bar{y}) - 2\bar{z}\bar{y}(1+\bar{x})), \qquad p_4^2 = \bar{m}^2(1-\bar{x}^2\bar{y}^2)$$

- Derive differential equations in dimensionless variables
  - Transform to canonical basis (J. Henn)
    - Iterative removal of rational factors and ε -terms (S. Caron-Huot, J. Henn; A. von Manteuffel, L. Tancredi, TG)

Two-loop corrections

- Differential equations for vector of master integrals

$$d\vec{M}(\epsilon;\bar{x},\bar{y},\bar{z}) = \epsilon \sum_{k=1}^{20} A_k \, d\ln(\bar{l}_k)\, \vec{M}(\epsilon;\bar{x},\bar{y},\bar{z})$$

- With alphabet

$$\begin{aligned}\{\bar{l}_1,\ldots,\bar{l}_{20}\} = \{&2, \bar{x}, 1+\bar{x}, 1-\bar{y}, \bar{y}, 1+\bar{y}, 1-\bar{x}\bar{y}, 1+\bar{x}\bar{y}, 1-\bar{z}, \bar{z},\\
&1+\bar{y}-2\bar{y}\bar{z}, 1-\bar{y}+2\bar{y}\bar{z}, 1+\bar{x}\bar{y}-2\bar{x}\bar{y}\bar{z}, 1-\bar{x}\bar{y}+2\bar{x}\bar{y}\bar{z},\\
&1+\bar{y}+\bar{x}\bar{y}+\bar{x}\bar{y}^2-2\bar{y}\bar{z}-2\bar{x}\bar{y}\bar{z}, 1+\bar{y}-\bar{x}\bar{y}-\bar{x}\bar{y}^2-2\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z},\\
&1-\bar{y}-\bar{x}\bar{y}+\bar{x}\bar{y}^2+2\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z}, 1-\bar{y}+\bar{x}\bar{y}-\bar{x}\bar{y}^2+2\bar{y}\bar{z}-2\bar{x}\bar{y}\bar{z},\\
&1-2\bar{y}-\bar{x}\bar{y}+\bar{y}^2+2\bar{x}\bar{y}^2-\bar{x}\bar{y}^3+4\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z}+2\bar{x}\bar{y}^3\bar{z},\\
&1-\bar{y}-2\bar{x}\bar{y}+2\bar{x}\bar{y}^2+\bar{x}^2\bar{y}^2-\bar{x}^2\bar{y}^3+2\bar{y}\bar{z}+4\bar{x}\bar{y}\bar{z}+2\bar{x}^2\bar{y}^3\bar{z}\}\end{aligned}$$

Two-loop corrections

- **Integration of differential equation**
  - Alphabet linear in *z*: integration yields GHPLs

$$G(w_1, \ldots, w_n; z) = \int_0^z \frac{dt}{t - w_1} G(w_2, \ldots, w_n; t)$$

  - Boundary condition: function of *(x,y)* at fixed *z*
  - Again from differential equations, first in *x*, then in *y*
- **Solution contains many instabilities**
  - Individual GHPLs non-analytic at each zero of the alphabet
  - Equal-mass limit can not be taken smoothly

Two-loop corrections

- Simplify master integrals: use reparametrization

$$s = m^2(1+x)(1+xy), \qquad p_3^2 = m^2,$$
$$t = -m^2 xz, \qquad p_4^2 = m^2 x^2 y$$

- New alphabet

$$\begin{aligned}\{l_1,\ldots,l_{19}\} = \{&x, 1+x, y, 1-y, z, 1-z, -y+z, 1+y-z, 1+xy, 1+xz, xy+z,\\ &1+y+xy-z, 1+x+xy-xz, 1+y+2xy-z+x^2yz,\\ &2xy+x^2y+x^2y^2+z-x^2yz, 1+x+y+xy+xy^2-z-xz-xyz,\\ &1+y+xy+y^2+xy^2-z-yz-xyz,\\ &-xy+z+xz+xyz, -y+z+yz+xyz\}\end{aligned}$$

- Basis functions with arguments: ratios of letters

$$\mathrm{Li}_n(\xi) = -G(0_{n-1}, 1; \xi), \qquad \mathrm{Li}_{2,2}(\xi_1,\xi_2) = G\left(0, \frac{1}{\xi_1}, 0, \frac{1}{\xi_1\xi_2}; 1\right)$$

Two-loop corrections

- Simplify master integrals
  - Compute symbol of master integrals and of new basis functions (A. Goncharov, M. Spradlin, A. Volovich, C. Vergu, C. Duhr)
  - Match coefficients of basis functions
    - Coproduct calculus (A. Goncharov; C. Duhr; L. Tancredi, E. Weihs, TG)
    - Huge pattern matching problem
    - Use pre-computed combinations to reduce complexity (A. von Manteuffel, R. Schabinger, H.X. Zhu)

Two-loop corrections

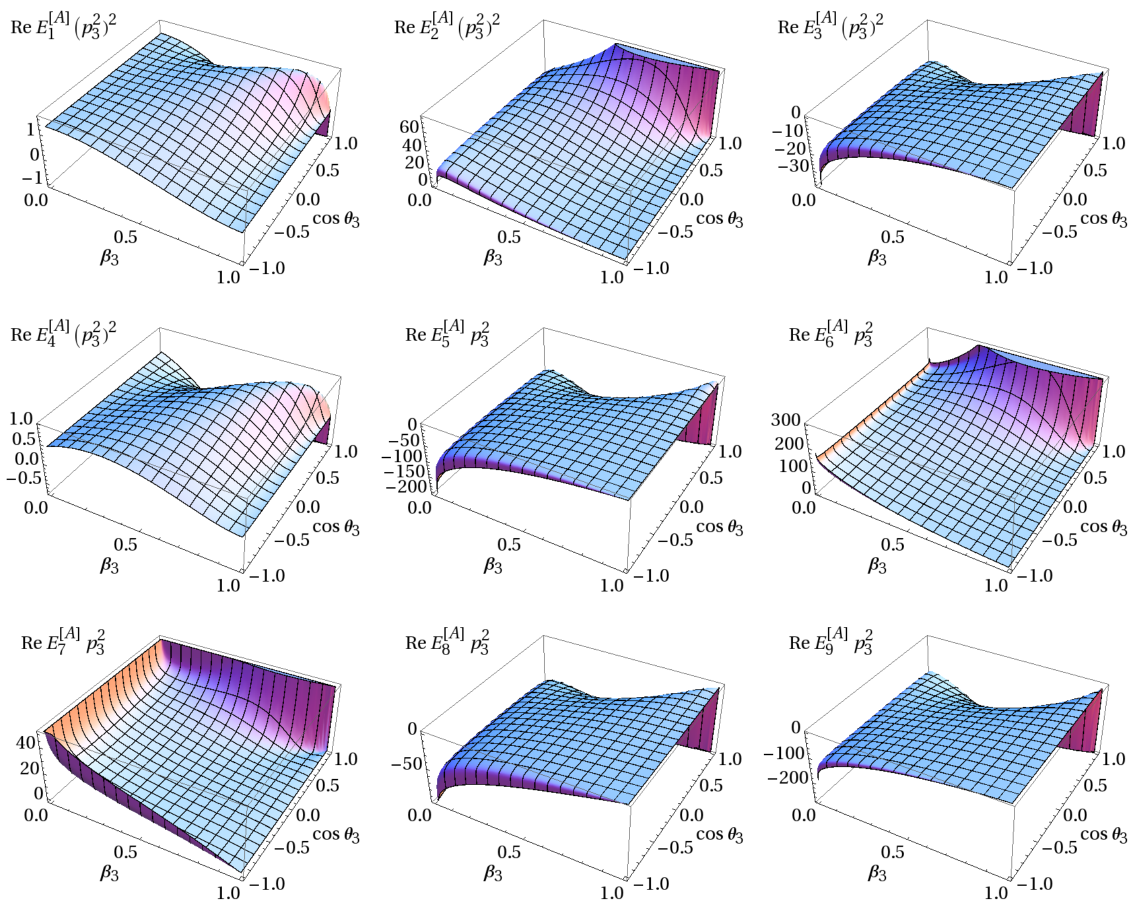
- **Infrared structure factorizes**
  (S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini)

$$\Omega^{(1),\text{finite}} = \Omega^{(1)} - I_1(\epsilon)\,\Omega^{(0)}\,,$$
$$\Omega^{(2),\text{finite}} = \Omega^{(2)} - I_1(\epsilon)\,\Omega^{(1)} - I_2(\epsilon)\,\Omega^{(0)}$$

- **Checks**
  - Pole structure
  - Vanishing of Furry-type contributions
  - High-energy limit (G. Chachamis, M. Czakon, D. Eiras)
  - Agreement with independent calculation
    (F. Caola, J. Henn, K. Melnikov, A. Smirnov, V. Smirnov)

Two-loop corrections

- **Coefficients of tensor structures**



Two-loop corrections

- Results for the amplitudes

`http://vvamp.hepforge.org`

  - $q\bar{q} \to V_1 V_2$
    (A. von Manteuffel, L. Tancredi, TG)
  - $gg \to V_1 V_2$
    (A. von Manteuffel, L. Tancredi)
  - $q\bar{q} \to V\gamma$
    (L. Tancredi, TG)
  - $gg \to V\gamma$
    (L. Tancredi, E. Weihs, TG)

Real radiation at NNLO

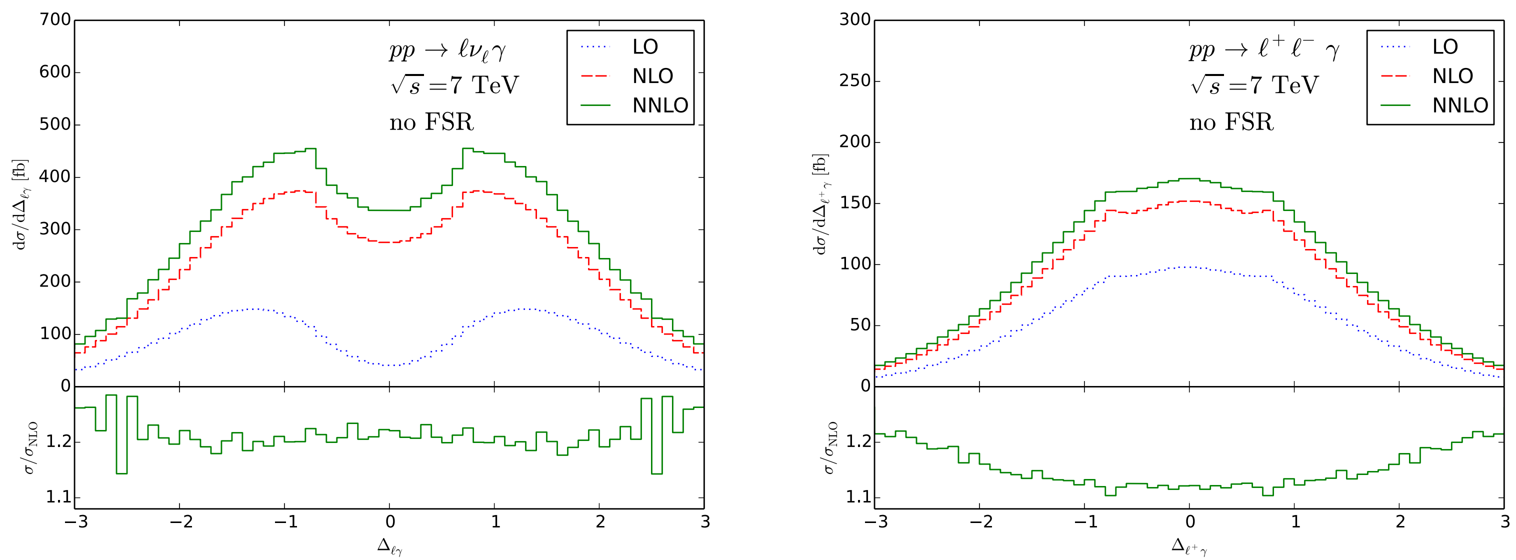
- q_T-subtraction (S. Catani, M. Grazzini)
  - Production of colourless final states at hadron colliders
    - Universal behaviour in the limit of small transverse momentum, known from resummation
    - Use small-q_T limit to construct subtraction term

$$d\sigma^F_{NNLO} = \mathcal{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\mathrm{jet}}_{NLO} - d\sigma^{CT}_{NLO} \right]$$

    - Subtraction defined after integration over all but one variables (non-local)
  - Implementation based on NLO calculation for F+jet

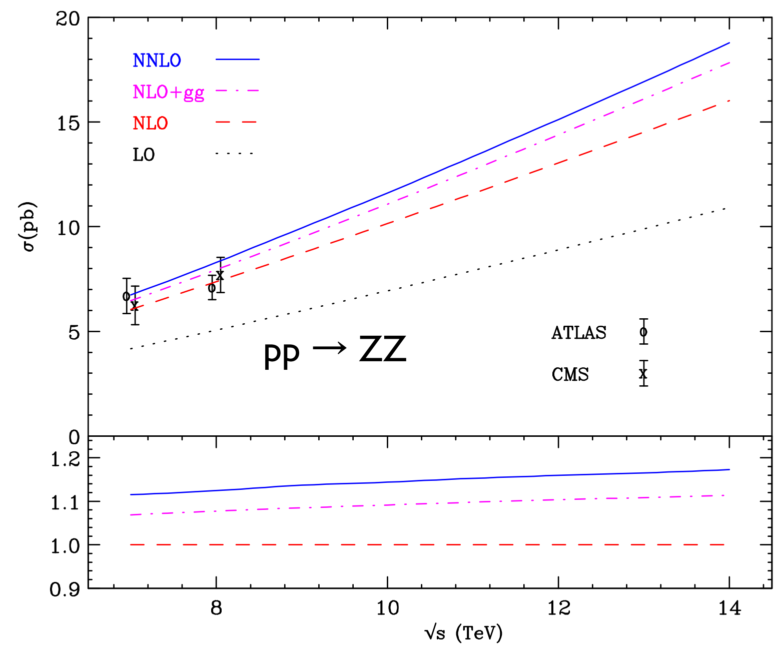
NNLO QCD for $Z^0\gamma$ and $W^{\pm}\gamma$

- Calculational setup including leptonic decays
  - q_T subtraction
  - Tree-level and one-loop matrix elements: OpenLoops
  - Integrator and infrastructure: MUNICH (S. Kallweit)
- Results (M. Grazzini, S. Kallweit, D. Rathlev)



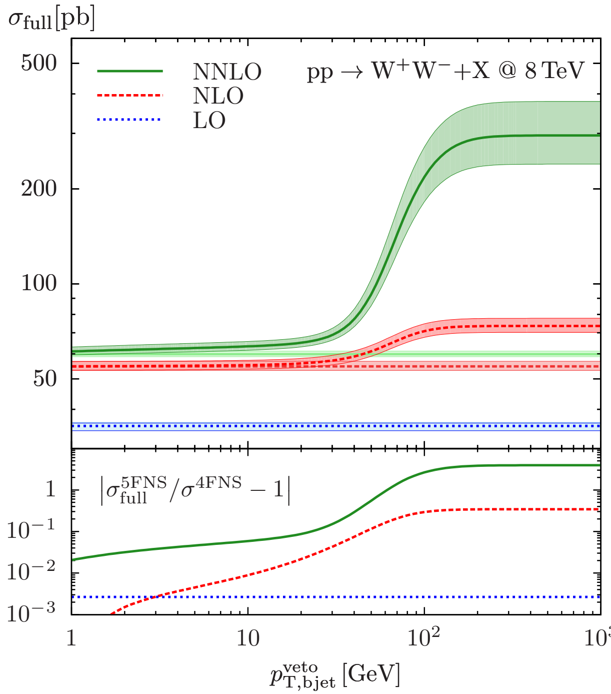
NNLO QCD for Z^0Z^0

- On-shell production: total cross section
  - q_T-subtraction
  - Tree-level and one-loop matrix elements: OpenLoops
- Results (F. Cascioli, M. Grazzini, S. Kallweit, P. Maierhöfer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi, E. Weihs, TG)
  - Moderate NNLO corrections
  - about half of the NNLO correction due to leading order $gg \to ZZ$
  - No reduction of scale uncertainty NLO $\to$ NNLO



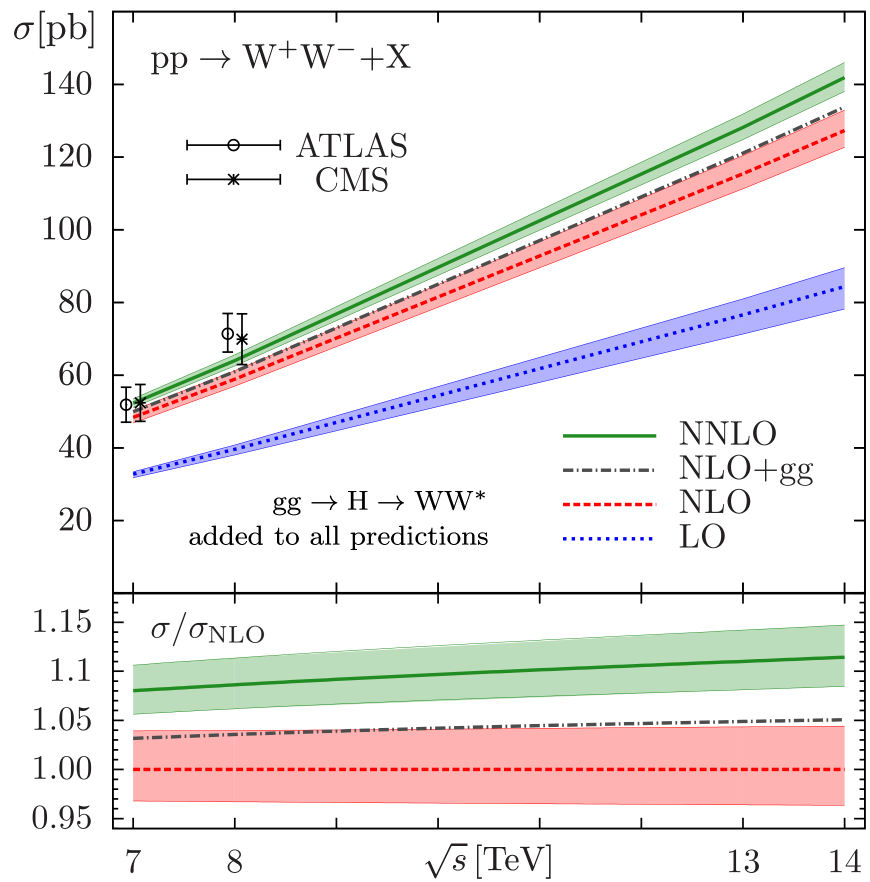
pp $\rightarrow$ W$^+$W$^-$ at NNLO

- **Total on-shell cross section** (M. Grazzini, S. Kallweit, P. Maierhöfer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi, TG)
- **At higher orders: pp $\rightarrow$ W$^+$W$^-$ not well defined**
  - NLO: contribution from gb $\rightarrow$ WWb
  - NNLO: contribution from $q\bar{q}$/gg $\rightarrow$ WW$b\bar{b}$
  - Can not be removed consistently in 5FNS
  - Define 5FNS contribution from scaling behaviour with top quark width
  - Good agreement of 4FNS and 5FNS



$pp \to W^+W^-$ at NNLO

- Total cross section in 4FNS
  - Improved description of data
  - Data based on interpolation from fiducial region
  - Calls for fully differential description, including vector boson decays and off-shell effects



Summary

- Vector boson pair procuction
  - Precision observable for electroweak measurements
- Complete NNLO QCD description emerging
  - Two-loop amplitudes
  - Implementation using q_T subtraction
- On-shell results
  - Moderate NNLO corrections
  - Substantial Born-level gluon fusion contribution
- More to come
  - Full calculation, including off-shell effects and decays
  - Gluon fusion at NLO