Effective Field Theories in the Quest for BSM Physics

Raquel Gómez Ambrosio
Università & INFN @Torino & CMS @CERN

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Outline

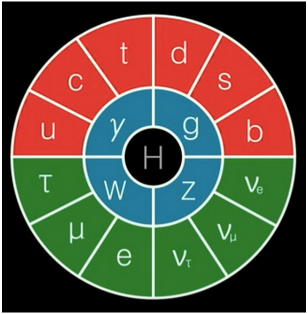
Introduction

The Standard Model Today

- (Higgs) particle with $J^{CP} = 0^{++}$, $M_H = 125.09 \pm 0.24$ GeV found in 2012.
- No new physics found in the experiment (deviations, if any, are very tiny)

To-Do List

- Neutrino masses
- Dark Matter
- Graviton



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Search for BSM physics: Shifting the couplings

- Search deviations in the Higgs couplings, *aka* “kappa framework”
- Problem 1: All measurements seem consistent with no deviations:
  1. Need NNLO and beyond (signal + background)
  2. Need more precise experiments (more resolution)
- Problem 2: The κ's don't have a direct physical interpretation

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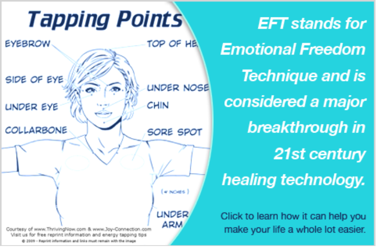
What is Effective Field Theory

Definition:

An Effective field theory (EFT) is a field theory, designed to reproduce the behaviour of some underlying (in general, unknown) physical theory in some limited regime. It focuses on the degrees of freedom relevant to that regime, simplifying the problem though letting aside some important physics.



Don't confuse it with "Emotional Freedom Technique" . . .

What is EFT. Some examples

- Landau-Ginzburg theory for superconductivity, the (non)-linear sigma model for (anti)-ferromagnetism, Fermi theory for β-decay ...
- Example: Interactions between nucleons and pions. Promote the chiral transformation for the *nucleon field* to a non linear one:

$$\psi(x) \to e^{2i\gamma_5 \vec{\tau}\cdot\vec{\phi}(x)}\psi(x) \qquad \begin{cases} \psi(x) & \text{Nucleon field} \\ \phi(x) & \text{Pion field.} \end{cases}$$

- **Caveat:** The same effective phenomena can be achieved through many different original scenarios. You need educated guesses, good luck, and reliable experimental results in order to find the *"real"* one.

Why EFT?

- Large scale physics, as we know it, is made of effective field theories: fluid dynamics, solid state physics, condensed matter physics . . . Why shouldn't particle physics be one too?

- Newton's theory of gravity is an effective low-energy theory of general relativity, which is itself some low-energy effective theory of a quantum theory of gravity.

- As a hint: it doesn't seem reasonable to use the renormalization-group flow *ad infinitum* to reach arbitrarily short distance scales (Wilsonian interpretation of the SM)

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- Maybe not ...

Two basic things the EFT apprentice has to know:

1) Top-down Vs. Bottom-up approach

- In the **Top-down** approach,
  - Start from a complete high energy theory
  - Study its behaviour in its infrared regime
  - Assuming heavy modes decouple, calculations become simpler.
- In the **Bottom-up** approach,
  - Start from an low-energy known theory (the SM).
  - Study its high energy behaviour.
  - Add operators consistent with the symmetries.
  - Match the unknown with experimental observations.

Two basic things the EFT apprentice has to know:

2) Relevant, Marginal and Irrelevant operators

- Introduced by K. Wilson, in the context of renormalization
- Check mass dimension Vs. spacetime dimensions $\begin{cases} [\mathcal{O}] < d & \text{relevant} \\ [\mathcal{O}] = d & \text{marginal} \\ [\mathcal{O}] > d & \text{irrelevant} \end{cases}$
- Relevant and marginal operators, like the ones in $\mathcal{L}_{SM}$, become increasingly large at higher energies
- Irrelevant operators, decrease at higher energies, and are not *"influential"*

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{dim=5} + \mathcal{L}_{dim=6} + \cdots =$$
$$= (\sim \Lambda^0) + (\sim \Lambda^{-1}) + (\sim \Lambda^{-2}) + \ldots$$

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SM EFT (bottom-up approach)

$$\mathcal{L}_{eff} = \underbrace{\mathcal{L}_{SM}}_{\text{dim 4}} + \underbrace{\sum_i \frac{c_i\, \mathcal{O}_i}{\Lambda^2}}_{\text{dim 6}} + \underbrace{\cdots}_{\text{higher dim. operators}}$$

- c_i are Wilson coefficients
- One can build 80 dim-6 operators (compatible with $SU(2) \times SU(3) \times U(1)$ and lepton/baryon conservation)
- Eqs. of motion, reduce this set to a 59-operator basis *(for one generation of particles! for three → 2499 operators)*

Hands on EFT: HowTo

From the UV theory to the EFT (Top-down approach)

- Integrate out the heavy fields of the UV theory

$$e^{iS_{eff}[\phi](\mu)} = \int \mathcal{D}\Phi \, e^{iS_{UV}[\phi,\Phi](\mu)}$$

- Where μ is the scale where the UV theory matches the EFT ($\mu \sim m$), don't mistake it for Λ_{UV},Λ_{Planck}.

- Use saddle point approximation:

$$S_{eff} \approx \underbrace{S[\Phi_C]}_{\text{tree-level}} + \underbrace{\frac{i}{2}\mathrm{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi_C}\right)}_{\text{one-loop}}$$

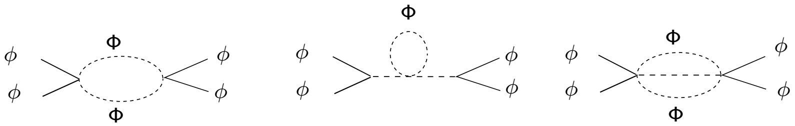
Diagrammatical interpretation

- Clear form the path integral point of view: Only Φ is "dynamical" (ϕ is fixed)

$$\int \mathcal{D}\Phi \; e^{iS_{UV}[\phi,\Phi](\mu)} \neq \int \mathcal{D}\Phi\mathcal{D}\phi \; e^{iS_{UV}[\phi,\Phi](\mu)}$$

- Recall the background field method: $\phi \to \phi + \tilde{\phi}$

$$\mathcal{L}(\phi) \to \mathcal{L}(\phi_i + \tilde{\phi}_i) = \mathcal{L}(\phi_i) + \underbrace{\mathcal{L}_1}_{=0} + \underbrace{\frac{1}{2}\frac{\delta^2\mathcal{L}}{\delta\phi_i\phi_j}\bigg|_{\tilde{\phi}=\phi}\phi_i\phi_j}_{\text{1-loop}} + \dots$$



Covariant Derivative Expansion

- First proposed by M. Gaillard (1986), see also Murayama (1412.1837)
- Elegant method to evaluate the one loop effective action:

$$S_{eff} \approx \underbrace{S\left[\Phi_C\right]}_{\text{tree-level}} + \underbrace{\frac{i}{2}\mathrm{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi_C}\right)}_{\text{one-loop}}$$

- **Bonus:** Respect gauge invariance at each step!
- **Bonus 2:** Covariant expansion of S_{eff} leads directly to the $\mathcal{O}_i$'s

Underlying idea

- $D_\mu = \partial_\mu - iA_\mu \quad \rightarrow \quad D_\mu \mathcal{O} = [D, \mathcal{O}]$
- Make a expansion on D^2, instead of the *usual* $(\partial^2 - m^2)$
- All terms in the expansion are proportional to D^2 or $[D, \mathcal{O}]$ (i.e. gauge invariant/covariant)

CDE. A simple example: Scalar $\mathcal{L}$

$$\mathcal{L}(\Phi,\phi) = \left[\Phi^\dagger B(\phi) + B^\dagger(\phi)\Phi\right] + \Phi^\dagger\left((iD)^2 - m^2 - U(\phi)\right)\Phi + \dots$$

- Find Φ_c:

$$\left.\frac{\delta S}{\delta \Phi}\right|_{\Phi=\Phi_c} = 0 \quad \rightarrow \quad \Phi_c = \frac{-B(\phi)}{(iD)^2 - m^2 - U(\phi)}$$

- Expand:

$$\Phi_c = \left[\frac{1}{1 - \frac{1}{m^2}(-D^2 - U)}\right]\frac{1}{m^2}B \sim \frac{1}{m^2}B + \frac{1}{m^2}(-D^2-U)\frac{1}{m^2}B + \\ + \frac{1}{m^2}(-D^2-U)\frac{1}{m^2}(-D^2-U)\frac{1}{m^2}B + \dots$$

- And replace:

$$\mathcal{L}_{eff,tree} = B^\dagger\frac{1}{m^2}B + B^\dagger \underbrace{\frac{1}{m^2}(-D^2-U)\frac{1}{m^2}}_{\text{dim-6 operator}} B + \text{higher dim. ops}$$

Once you know Φ_c you can evaluate:

$$S_{eff} \approx \underbrace{S\left[\Phi_C\right]}_{\text{tree-level}} + \underbrace{\frac{i}{2}\operatorname{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta\Phi^2}\right|_{\Phi_C}\right)}_{\text{one-loop}}$$

For the previous example:

$$\frac{i}{2}\ \operatorname{Tr}\ \log\left(\left.\frac{\delta^2 S}{\delta\phi^2}\right|_{\Phi_C}\right) = (D)^2 - m^2 - U(\phi)$$

Or, in the general case:

$$\Delta S_{eff,1loop} \propto i\, c_s \operatorname{Tr}\log\left[D^2 + m^2 + U(\phi_c(x))\right] \qquad \begin{cases} c_s = 1/2 & \text{Real scalar} \\ c_s = 1 & \text{Complex scalar} \\ c_s = -1/2 & \text{Fermion} \end{cases}$$

And, after some *simple algebra*, also $\mathcal{L}_{eff,\text{1-loop}}$ ...

$$
\begin{aligned}
\Delta\mathcal{L}_{\text{eff,1-loop}} = \frac{c_s}{(4\pi)^2}\,\text{tr}\Bigg\{ & \\
&+ m^4\left[-\frac{1}{2}\left(\log\frac{m^2}{\mu^2}-\frac{3}{2}\right)\right] \\
&+ m^2\left[-\left(\log\frac{m^2}{\mu^2}-1\right)U\right] \\
&+ m^0\left[-\frac{1}{12}\left(\log\frac{m^2}{\mu^2}-1\right)G'^2_{\mu\nu}-\frac{1}{2}\log\frac{m^2}{\mu^2}\,U^2\right] \\
&+ \frac{1}{m^2}\left[-\frac{1}{60}\left(P_\mu G'_{\mu\nu}\right)^2-\frac{1}{90}G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu}-\frac{1}{12}\left(P_\mu U\right)^2-\frac{1}{6}U^3-\frac{1}{12}UG'_{\mu\nu}G'_{\mu\nu}\right] \\
&+ \frac{1}{m^4}\left[\frac{1}{24}U^4+\frac{1}{12}U\left(P_\mu U\right)^2+\frac{1}{120}\left(P^2U\right)^2+\frac{1}{24}\left(U^2G'_{\mu\nu}G'_{\mu\nu}\right)\right. \\
&\qquad\left.-\frac{1}{120}\left[(P_\mu U),(P_\nu U)\right]G'_{\mu\nu}-\frac{1}{120}\left[U[U,G'_{\mu\nu}]\right]G'_{\mu\nu}\right] \\
&+ \frac{1}{m^6}\left[-\frac{1}{60}U^5-\frac{1}{20}U^2\left(P_\mu U\right)^2-\frac{1}{30}\left(UP_\mu U\right)^2\right] \\
&+ \frac{1}{m^8}\left[\frac{1}{120}U^6\right]\Bigg\}.
\end{aligned}
$$

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Summary & Open Questions

- We presented a method to write down an *operative* EFT from a given UV model
- Integrating out the heavy fields and expanding the result in a CDE we find the $\mathcal{L}_{eff}$ for such models
- After that, we can do physics with the effective model, and match its observables with standard model ones by means of the RG flow equations (next time!)

Open Questions

- What is the range of validity of the effective theory?
- Is there a smart way to combine the bottom-up and top-down approaches of EFT?


THANK YOU FOR YOUR ATTENTION
NO QUESTIONS, PLEASE
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Backup

1-Loop $\mathcal{L}_{eff}$

$$\Delta S_{eff,1loop} \propto i\, c_s\, \mathrm{Tr} \log \left[D^2 + m^2 + U(\phi_c(x)) \right]$$

$$\Delta S_{eff,1loop} = i\, c_s \int d^4x \int \frac{d^4q}{(2\pi)^4}\, \mathrm{tr} \left(e^{iq\cdot x} \log \left[D^2 + m^2 + U(\phi_c(x)) \right] e^{-iq\cdot x} \right)$$

$$\Delta S_{eff,1loop} = i\, c_s \int d^4x \int \frac{d^4q}{(2\pi)^4}\, \mathrm{tr} \left(e^{iD\cdot \frac{\partial}{\partial q}} \log \left[-(iD_\mu - q_\mu)^2 + m^2 + U(\phi_c(x)) \right] e^{-iD\cdot \frac{\partial}{\partial q}} \right)$$

$$
\begin{aligned}
e^{iD_\mu \frac{\partial}{\partial q}}(iD_\mu - q_\mu)e^{-iD_\mu \frac{\partial}{\partial q}} = \\
\sum_{n=0}^{\infty} \frac{1}{n!}\left[P \cdot \frac{\partial}{\partial q}\right]^n (iD_\mu) - \sum_{n=0}^{\infty} \frac{1}{n!}\left[P \cdot \frac{\partial}{\partial q}\right]^n q_\mu = \dots \\
= -(q_\mu + \tilde{G}_{\mu\nu})
\end{aligned}
$$

where,

$$
\tilde{G}_{\mu\nu} = \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}[iD_{\alpha_1}, [iD_{\alpha_2}, [\dots, [P_{\alpha_n}, [D_\mu, D_\nu]]]]] \frac{\partial^n}{\partial q_{\alpha_1} \partial q_{\alpha_2} \dots \partial q_{\alpha_n}}
$$

And,

$$
e^{iD_\mu \frac{\partial}{\partial q}}(U)e^{-iD_\mu \frac{\partial}{\partial q}} = \dots = \tilde{U}
$$

where,

$$
\tilde{U} = \sum_{n=0}^{\infty} \frac{1}{n!}[iD_{\alpha_1}, [iD_{\alpha_2}, [\dots, [P_{\alpha_n}, U]]]] \frac{\partial^n}{\partial q_{\alpha_1} \partial q_{\alpha_2} \dots \partial q_{\alpha_n}}
$$