mc2hessian

Common tools to estimate PDF uncertainties

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OUTLINE OF THE TALK

INTRODUCTION

PDF UNCERTAINTIES FROM DATA

PDFs have to be estimated from experimental data.

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  - Need tools to combine, compare, benchmark.
  - Need to distribute in a way useful for the community.

PDF PARAMETIZATIONS

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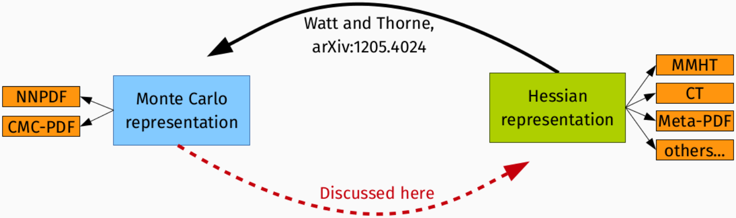
Monte Carlo approach Perform a Monte Carlo simulation sampling from the distribution of replicas.

$$\mathcal{O} \sim \mathcal{O}(f)$$

SUMMARY



Problem addressed here:

⇒ Determine an **unbiased Hessian representation** for **MC** PDFs.

Hessian approach:

- Linear error propagation.

Monte Carlo approach:

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Monte Carlo approach:

- Arbitrary error propagation.
- Easy combination of multiple PDF sets.
- Much less *functional bias.*

THE `mc2hessian` ALGORITHM

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3. Fit each replica to a “Hessian like” functional form.
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- Introduces functional bias.

Given a Monte Carlo prior set of PDFs

$$\{f_\alpha^{(k)}\}_{k=1,\ldots,N_{\rm rep}}\,, \quad \alpha = \{g, u, d, s, \ldots\}\,,$$

use a subset of replicas as parameters of linear expansion:

$$f_\alpha^{(k)} \approx f_{H,\alpha}^{(k)} \equiv f_\alpha^{(0)} + \sum_{i=1}^{N_{\rm eig}} a_i^{(k)} (\eta_\alpha^{(i)} - f_\alpha^{(0)})\,, \quad k = 1, \ldots, N_{\rm rep}$$

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- approximate each replica of the original MC ensemble $f_\alpha^{(k)}$
- by the linear combination $f_{H,\alpha}^{(k)}$
- with coefficients $a_i^{(k)}$
- of deviations from the central value $f_\alpha^{(0)}$
- expanded in the basis of a subset of replicas $\{\eta_\alpha^{(i)}\}_{i=1,\ldots,N_{\rm eig}} \subset \{f_\alpha^{(k)}\}$

DESCRIPTION OF THE METHOD

- We want to go from $N_{rep} = 1000$ MC replicas to N_{eig} eigenvectors.

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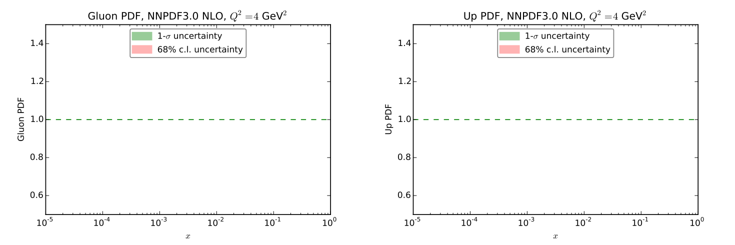
$$\epsilon = \left| \frac{\sigma - (68\%\,\text{c.l})}{\sigma} \right|$$



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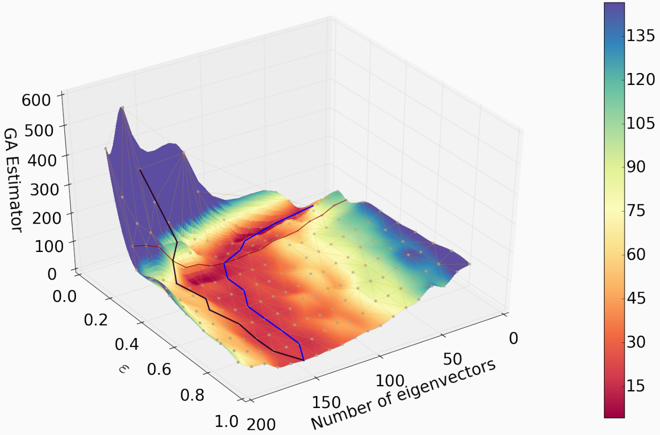
$$\epsilon = \left| \frac{\sigma - (68\%\,\text{c.l})}{\sigma} \right|$$

- We construct a *figure of merit* and optimize with a *genetic algorithm*:

$$\mathrm{ERF}_{\sigma} = \sum_{i=1}^{N_x} \sum_{\alpha=1}^{N_f} \left| \frac{\sigma^{\mathrm{PDF}}_{H,\alpha}(x_i, Q_0^2) - \sigma^{\mathrm{PDF}}_{\alpha}(x_i, Q_0^2)}{\sigma^{\mathrm{PDF}}_{\alpha}(x_i, Q_0^2)} \right|$$

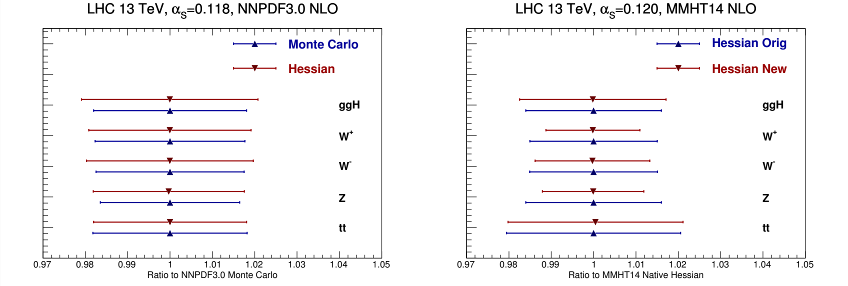
SELECTING THE OPTIMAL BASIS



- **Surface:** GA minimum for estimator in function of ϵ and N_{eig}.
- **Blue curve:** surface minimum; **black curve:** estimator with large ϵ.

PHENOMENOLOGY

LHC inclusive cross-sections @ 13 TeV



- **Good agreement** for LHC inclusive cross-sections, below 10%.
- **Also** for a large number of differential distributions at the LHC 7 TeV.

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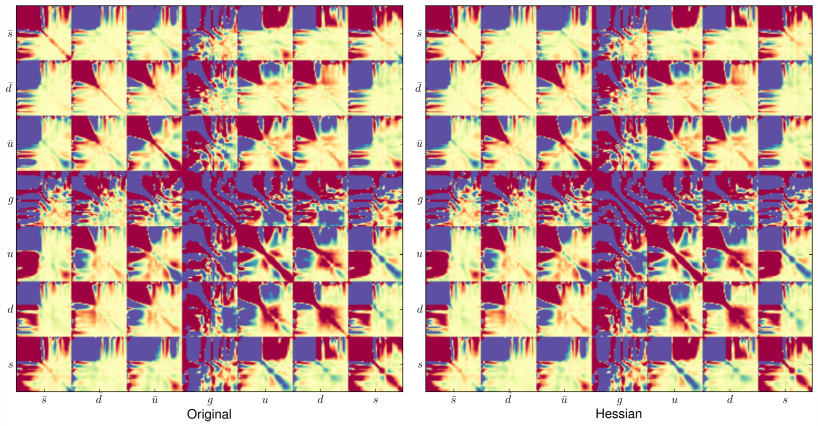
- Maximize *"agreement"* of Hessian and MC covariance matrices.
- It can be reduced to a lineal algebra problem!
  - (pick linear combinations of replicas corresponding to the dominant eigenvalues, using singular value decomposition).

Results for 100-eigenvector Hessian.



DELIVERY

- The `mc2hessian` program is public available at

  `github.com/scarrazza/mc2hessian`

- Further **optimizations** in progress before final release.
- NNPDF3.0 Hessian version available in `LHAPDF6` soon:
  - `NNPDF30_nlo_as_0118_hessian`
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- Any other MC set can be converted using directly the public code.

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OUTLOOK

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Hopefully `mc2hessian` will be used to deliver the Standard PDFs for tasks like Higgs cross section measurements.

Questions?