Modified teleparallelism: an alternative approach of modified gravity

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Windows on Quantum Gravity
28-30 October, 2015, Madrid

The so-called f(T) theories of gravity are a generalization of the well-known teleparallel equivalent of General Relativity (TEGR) in which:

- **the dynamical relevant object is the tetrad instead of the metric tensor and**
- **the degrees of freedom are encoded in the torsion rather than in the curvature.**

Outline

- Basic definitions
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- Lorentz invariance
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- Summary

Motivation

Modified gravity

Several models of modified gravity have been proposed in order to tackle the shortcomings of General Relativity...

Dark matter and Dark energy (deformations of GR at large scales)
$\hookrightarrow$ there exist any geometrical explanantion for the phenomena associated to the dark sector?

Singularities (deformations of GR at small scales)
$\hookrightarrow$ it is possible to smooth or even to avoid them?

Quantum gravity...

Teleparallel gravity

$\overbrace{\text{tetrad} \leftrightarrow \text{metric}}$

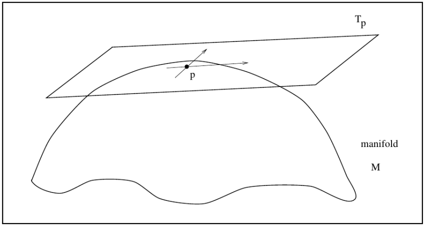
torsion (Weitzenböck connection) $\leftrightarrow$ curvature (Levi-Civita connection)

Teleparallel Equivalent of General Relativity (TEGR)

A $f(T)$ theory is a deformation of TEGR

(likewise $f(R)$ theory is a deformation of GR)

Tetrad



The field of tetrad is

- a set of four orthonormal vectors defining a local frame at every point $\mathbf{e}_a(x^\mu)$
- which constitutes a basis of the tangent space $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$
- whose components are represented as e_a^μ
- and are related by the metric tensor through the orthonormal condition

Torsion

- It could be represented as the exterior derivative of the tetrad: $\mathbf{T}^a = \mathbf{d}\mathbf{e}^a$
- It is related to the antisymmetric part of any connection
- The so-called torsion scalar (or Weitzenböck invariant) (like the curvature escalar R) is obtained by the contraction

$$T = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}$$

where $T^\rho{}_{\mu\nu}$ are the components of the torsion tensor

$\star\ T^\mu{}_{\nu\rho} \;=\; e^\mu_a\,(\partial_\nu e^a_\rho - \partial_\rho e^a_\nu)$

and $S_\rho{}^{\mu\nu}$ are the components of the so-called superpotential

$\star\ 2S_\rho{}^{\mu\nu} = K^{\mu\nu}{}_\rho + T_\lambda{}^{\lambda\mu}\,\delta^\nu_\rho - T_\lambda{}^{\lambda\nu}\,\delta^\mu_\rho$

$\hookrightarrow$ **contorsion** $K^\lambda{}_{\mu\nu} = \overset{W}{\Gamma}{}^\lambda_{\mu\nu} - \overset{LC}{\Gamma}{}^\lambda_{\mu\nu}$ is the difference between Weitzenböck and Levi-Civita connection

$\hookrightarrow$ **Weitzenböck connection**

Weitzenböck connection

$$\overset{\mathrm{W}}{\Gamma}{}^{\mu}{}_{\rho\nu} = e_a^{\mu}\, \partial_\nu e_\rho^a = -e_\rho^a\, \partial_\nu e_a^\mu$$

- It is curvaturaless connection with torsion
- The Weitzenböck spacetime is provided by this connection
- The covariant derivative is obtained through the connection

$$\nabla_\nu V^\mu = \partial_\nu V^\mu + \overset{\mathrm{W}}{\Gamma}{}^{\mu}{}_{\rho\nu}\, V^\rho = \partial_\nu (V^a e_a^\mu) - e_\rho^a\, \partial_\nu e_a^\mu = e_a^\mu \partial_\nu V^a$$

Weitzenböck connection (TEGR - $f(T)$)
$\hookrightarrow$ it has torsion but not curvature

$$\overset{\text{W}}{T}{}^{\rho}{}_{\mu\nu} = \overset{\text{W}}{\Gamma}{}^{\rho}{}_{\mu\nu} - \overset{\text{W}}{\Gamma}{}^{\rho}{}_{\nu\mu} \neq 0$$

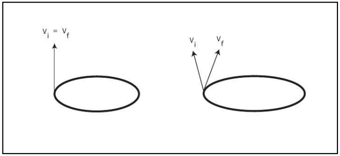
$$\overset{\text{W}}{R}{}^{\lambda\rho}{}_{\nu\mu} = \partial_{\nu}\overset{\text{W}}{\Gamma}{}^{\rho}{}_{\lambda\mu} - \partial_{\mu}\overset{\text{W}}{\Gamma}{}^{\rho}{}_{\lambda\nu} + \overset{\text{W}}{\Gamma}{}^{\rho}{}_{\eta\nu}\overset{\text{W}}{\Gamma}{}^{\eta}{}_{\lambda\mu} - \overset{\text{W}}{\Gamma}{}^{\rho}{}_{\eta\mu}\overset{\text{W}}{\Gamma}{}^{\eta}{}_{\lambda\nu} = 0$$

Levi-Civita connection (GR - $f(R)$)
$\hookrightarrow$ it has curvature but not torsion

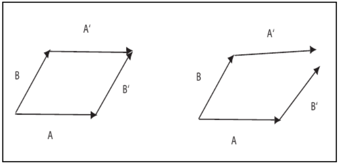
$$\overset{\text{LC}}{T}{}^{\rho}{}_{\mu\nu} = \overset{\text{LC}}{\Gamma}{}^{\rho}{}_{\mu\nu} - \overset{\text{LC}}{\Gamma}{}^{\rho}{}_{\nu\mu} = 0$$

$$\overset{\text{LC}}{R}{}^{\lambda\rho}{}_{\nu\mu} = \partial_{\nu}\overset{\text{LC}}{\Gamma}{}^{\rho}{}_{\lambda\mu} - \partial_{\mu}\overset{\text{LC}}{\Gamma}{}^{\rho}{}_{\lambda\nu} + \overset{\text{LC}}{\Gamma}{}^{\rho}{}_{\eta\nu}\overset{\text{LC}}{\Gamma}{}^{\eta}{}_{\lambda\mu} - \overset{\text{LC}}{\Gamma}{}^{\rho}{}_{\eta\mu}\overset{\text{LC}}{\Gamma}{}^{\eta}{}_{\lambda\nu} \neq 0$$

Torsion and curvature

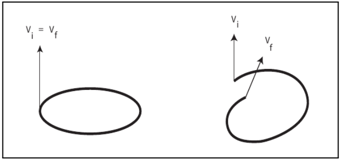


GR $\Rightarrow$ **curvature** $\Rightarrow$ **angular deficit**



TG $\Rightarrow$ **torsion** $\Rightarrow$ **length deficit**

$\hookrightarrow$ **well-known in condensed matter**



EC $\Rightarrow$ **curvature + torsion**

TEGR

[Einstein, Sitzungsber.Preuss.Akad.Wiss.Phys.Math. Kl. 217 (1928); 401 (1930); Math.Ann. 102, 685 (1930)]

Reformulation of GR

Tetrad or *vierbein*

- $\{\mathbf{e}_a(x)\} \rightarrow$ basis of tangent space
- $\{\mathbf{e}^a(x)\} \rightarrow$ co-basis of co-tangent (dual) space

$\hookrightarrow e^a_\mu e^\mu_b = \delta^a_b \quad e^a_\mu e^\nu_a = \delta^\nu_\mu$
$\hookrightarrow \mathbf{e}^a = e^a_\mu dx^\mu \quad \mathbf{e}_a = e^\mu_a \partial x^\mu$

Greek (latin) indices correspond to spacetime (tangent space)

There is a link between the tetrad and the metric through the orthonormal condition

$$\eta_{\hat{a}\hat{b}} = g_{\mu\nu} e^\mu_{\hat{a}} e^\nu_{\hat{b}} \ , \ \eta^{\hat{a}\hat{b}} = g^{\mu\nu} e^{\hat{a}}_\mu e^{\hat{b}}_\nu$$

$$g_{\mu\nu} = \eta_{\hat{a}\hat{b}} e^{\hat{a}}_\mu e^{\hat{b}}_\nu \ , \ g^{\mu\nu} = \eta^{\hat{a}\hat{b}} e^\mu_{\hat{a}} e^\nu_{\hat{b}}$$

TEGR

[Hayashi & Shirafuji, PRD 19, 3524 (1979)]

$$S = \frac{1}{2\kappa} \int d^4x \;\; e \;\; S_\rho{}^{\mu\nu} \, T^\rho{}_{\mu\nu} = \frac{1}{2\kappa} \int d^4x \;\; e \;\; \mathbb{S} \cdot \mathbb{T} = \frac{1}{2\kappa} \int d^4x \;\; e \;\; T$$

$$\kappa = 8\pi G; \; c = \hbar = 1; \; e = \det[e_\mu^{\hat{a}}] = \sqrt{-g}$$

T is the torsion scalar (or Weitzenböck scalar)

Equivalence

$$\boxed{R = -T + 2e^{-1}\partial_\rho(e\, T_\mu^{\mu\rho})}$$

$\Rightarrow$ the Lagrangians are equal up to a 4-divergence!

Modified teleparallel gravity: $f(T)$ theories

[Ferraro & Fiorini, PRD 75, 084031 (2007), Ferraro, AIP Conf. Proc., 1471, 103 (2012)]

A generalization of TEGR...

$$S = \frac{1}{2\kappa} \int d^4x \, e \, f(T)$$

differents f produce differents theories $f(T)$

- $f_{\mathrm{UV}}(T) \to$ UV deformations (small scales)
- $f_{\mathrm{IR}}(T) \to$ IR deformations (large scales)

Equations of motion

$$-2\kappa e_a^\lambda T_\lambda^\nu = - \; e_a^\nu \; f(T) + \\ + 4\left[e^{-1}\partial_\mu(e \; S_a^{\mu\nu}) + e_a^\lambda T^\rho_{\mu\lambda} \; S_\rho^{\mu\nu}\right] \; f'(T) + \\ + 4 \; S_a{}^{\mu\nu} \; \partial_\mu(T) f''(T)$$

- ⋆ T_λ^ν is the usual energy-momentum tensor
- ⋆ 2nd order eqns for the tetrad (the torsion is built from 1st derivatives of the tetrad) $\to f(T) = T \Rightarrow$ TEGR
- ⋆ 16 independent components $\Rightarrow$ extra dof [Li, Miao & Miao, JHEP 07, 108 (2011)]

A lot of work was done in the context of $f(T)$ gravity (just for mention the first references...)

- alternative explanation for the acceleration of the universe [Bengochea & Ferraro, PRD 79, 124019 (2009); Linder, PRD 81, 127301 (2010); Wu & Yu, PLB 692, 176 (2010); 693, 415 (2010); Bamba, Geng, Lee & Luo, JCAP. 03 (2011); Dent, Dutta & Saridakis (2010), PRD 83, 023508 (2011); Wu & Yu, EPJC 71, 1552 (2011)]
- successful attempts in order to smooth or even to avoid the singularities present in LFRW cosmologies [Ferraro & Fiorini, PRD 75, 084031 (2007); Ferraro & Fiorini PLB 702, 75 (2011); Tamanini & Böhmer PRD 86, 044009 (2012)]

  and black holes geometries like Schwarzschild [Ferraro & Fiorini PRD, 84, 083518 (2011)] and Kerr spacetimes [Bejarano, Ferraro & Guzmán, EPJC, 75, 77 (2015)]

$f(R)$ vs $f(T)$...

modified GR

- Gravity is encoded in the curvature through the LC connection $\overset{\text{LC}}{\Gamma}{}^{\lambda}_{\mu\nu}$ (without torsion)
- **GR**
$$S_{GR} = \frac{1}{2\kappa}\int d^4x\,\sqrt{-g}\,R$$
- $f(R)$
$$S_{f(R)} = \frac{1}{2\kappa}\int d^4x\,\sqrt{-g}\,f(R)$$
- R is the curvature scalar (Ricci scalar)
- 4th order eqns: R contains 2nd derivatives of the metric (metric f.)
- extra dof: 1
  $\hookrightarrow$ scalar field

[Buchdahl, MNRAS 150, 1 (1970)]

modified TERG

- Gravity is encoded in the torsion through the W connection $\overset{\text{W}}{\Gamma}{}^{\lambda}_{\mu\nu}$ (without curvature)
- **TEGR**
$$S_{TEGR} = \frac{1}{2\kappa}\int d^4x\;e\;T$$
- $f(T)$
$$S_{f(T)} = \frac{1}{2\kappa}\int d^4x\;e\;f(T)$$
- T is the torsion scalar (Weitzenböck scalar)
- 2nd order eqns: T contains 1st derivatives of the tetrad
- extra dof: 3
  $\hookrightarrow$ massive vector field
  $\hookrightarrow$ no-massive vector field $+$ scalar field

[Ferraro & Fiorini, PRD 75, 084031 (2007)]

Local Lorentz invariance

[Li, Sotiriou & Barrow, PRD 83, 064035 (2011)]

- $f(T)$ **gravity is global invariant under Lorentz transformations**

$$\boxed{T = -R + 2e^{-1}\partial_\nu(eT_\sigma^{\sigma\nu}) \longrightarrow f(\,-R + 2e^{-1}\partial_\nu(eT_\sigma^{\sigma\nu})\,)}$$

$\hookrightarrow$ The surface term remains **encapsulate** insde the function f breaking the local Lorentz invariance.

- Of course, if $f(T) = T$ (i.e. TEGR), the local Lorentz invariance is preserved

Torsion in teleparallel gravity

- The torsion $\mathbf{T}^a$ could be descompose as a linear combination of three irreducible parts (tensorial$^{(1)}$, vectorial$^{(2)}$, axial$^{(3)}$)

$$T^a = a_1 \, {}^{(1)}T^a + a_2 \, {}^{(2)}T^a + a_3 \, {}^{(3)}T^a$$

$${}^{(1)}T_{\lambda\mu\nu} = \frac{1}{2}(T_{\lambda\mu\nu} + T_{\mu\lambda\nu}) + \frac{1}{6}(g_{\nu\lambda} \, {}^{(2)} + g_{\nu\mu} T_\lambda \, {}^{(2)}T_\nu) - \frac{1}{3} g_{\lambda\mu} \, {}^{(2)}T_\nu$$

$${}^{(2)}T_\mu = T^\lambda_{\lambda\mu}$$

$${}^{(3)}T_\mu = \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}$$

which is in general non-local invariant under Lorentz transformation.

- By a particular choice of the coefficients $a_2 = -2a_1$ y $a_3 = -a_1/2$ (fine tuning), it is obtained a Lagrangian which turns out to be local Lorentz invariant (up to a boundary term) $\rightarrow$ **TEGR**

  $\hookrightarrow$ the dynamical context of the theory does not differ from GR: the equation of motion are the same!

So far...

- $f(T)$ theories of gravity are not invariant under local Lorentz transformation
- wiht the exception, of course, of $f(T) = \alpha + \beta T$ which corresponds to TEGR
- a non-invariant theory under local Lorentz transformation is sensitive to the tetrad selected
- although local Lorentz transformations do not affect the metric, they do change the $f(T)$ field equations since different tetrads (even though providing the same metric) could represent different physical theories

[Liberati, CQG 30, 133001 (2013)]

Lorentz invariance has to be tested at all scales; in particular, at small scales since in the context of generalized teleparallelism, Lorentz violations are expected to occurs at extremely small scales.

Suitable tetrads in $f(T)$ theory

[Ferraro & Fiorini Phys. Lett. B 702, 75 (2011); Tamanini & Böhmer Phys. Rev. D 86, 044009 (2012)]

▶ due to the lack of local Lorentz invariance non-trivial frames could arise
▶ such is the case of **Schwarzschild** and **Kerr** frames for which the naive frames straightforward obtained from the line element of the metric are not the appropiate frame solution (the same happens with close **LFRW** spacetimes)

$\Downarrow$

different tetrads could lead to the same metric

but they are not necessarily good tetrads

- solve the dynamical equations
- without restrictions on the functional form of f
- general relativity limit has to be recovered (if $f(T) = T$)

$\Downarrow$

Finding the suitable tetrad could be a very hard task...

Schwarzschild trivial frame

[Ferraro & Fiorini PRD, 84, 083518 (2011)]

The trivial diagonal frame from the usual line element of the Schwarzschild geometry is not a consistent solution of $f(T)$ gravity

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2\, d\Omega^2$$

$$\begin{aligned} e^0 &= \left(1 - \frac{2M}{r}\right)^{1/2} dt \\ e^1 &= \left(1 - \frac{2M}{r}\right)^{-1/2} dr \\ e^2 &= r\, d\theta \\ e^3 &= r\, sin\,\theta\, d\varphi \end{aligned}$$

Schwarzschild non-trivial frame: the good frame

The appropiate non-trivial frame is found by applying a radial boost to the naive frame (squared root) corresponding to the metric written in isotropic coordinates

$$ds^2 = A(\rho)^2\, dt^2 - B(\rho)^2\, \left(dx^2 + dy^2 + dz^2\right)$$

$$\rho = \sqrt{x^2 + y^2 + z^2} \qquad \sqrt{r^2 - 2Mr} + r - M = 2\rho \qquad A(\rho) = \frac{2\rho - M}{2\rho + M} \qquad B(\rho) = \left(1 + \frac{M}{2\rho}\right)^2$$

$$\boxed{\gamma(\rho) = \left(1 - \beta^2(\rho)\right)^{-\frac{1}{2}} \qquad \beta(\rho) = v(\rho)/c}$$

$$\begin{aligned}
\bar{e}^0 &= A(\rho)\gamma(\rho)\, dt - \frac{B(\rho)}{\rho}\sqrt{\gamma^2(\rho) - 1}\,[x\, dx + y\, dy + z\, dz] \\
\bar{e}^1 &= -\frac{A(\rho)}{\rho}\sqrt{\gamma^2(\rho) - 1}\, x\, dt + B(\rho)\Big[(1 + \frac{\gamma(\rho) - 1}{\rho^2}x^2)\, dx + \frac{\gamma(\rho) - 1}{\rho^2}\, x\, y\, dy + \frac{\gamma(\rho) - 1}{\rho^2}\, x\, z\, dz\Big] \\
\bar{e}^2 &= -\frac{A(\rho)}{\rho}\sqrt{\gamma^2(\rho) - 1}\, y\, dt + B(\rho)\Big[\frac{\gamma(\rho) - 1}{\rho^2}\, x\, y\, dx + (1 + \frac{\gamma(\rho) - 1}{\rho^2}y^2)\, dy + \frac{\gamma(\rho) - 1}{\rho^2}\, y\, z\, dz\Big] \\
\bar{e}^3 &= -\frac{A(\rho)}{\rho}\sqrt{\gamma^2(\rho) - 1}\, z\, dt + B(\rho)\Big[\frac{\gamma(\rho) - 1}{\rho^2}\, x\, z\, dx + \frac{\gamma(\rho) - 1}{\rho^2}\, y\, z\, dy + (1 + \frac{\gamma(\rho) - 1}{\rho^2}z^2)\, dz\Big]
\end{aligned}$$

Null tetrad

Given an orthonormal tetrad $\{\mathbf{e}^{\hat{a}}\}$, it is possible to define the corresponding null tetrad $\{\mathbf{n}^a\} = \{\mathbf{l}, \mathbf{n}, \mathbf{m}, \overline{\mathbf{m}}\}$ like

$$\mathbf{l} = \frac{(\mathbf{e}^{\hat{0}} + \mathbf{e}^{\hat{1}})}{\sqrt{2}} \quad \mathbf{n} = \frac{(\mathbf{e}^{\hat{0}} - \mathbf{e}^{\hat{1}})}{\sqrt{2}} \quad \mathbf{m} = \frac{(\mathbf{e}^{\hat{2}} + i\mathbf{e}^{\hat{3}})}{\sqrt{2}} \quad \overline{\mathbf{m}} = \frac{(\mathbf{e}^{\hat{2}} - i\mathbf{e}^{\hat{3}})}{\sqrt{2}}$$

$[\mathbf{l} \cdot \mathbf{l} = 0,\ \mathbf{n} \cdot \mathbf{n} = 0,\ \mathbf{m} \cdot \mathbf{m} = 0,\ \overline{\mathbf{m}} \cdot \overline{\mathbf{m}} = 0]$

In this new basis, the Minkowski metric is

$$g_{\mu\nu} = \eta_{ab} n^a_\mu n^b_\nu \quad \text{con} \quad \eta_{ab} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Then,

$$\boxed{\mathrm{g} = \mathrm{n} \otimes \mathbf{l} + \mathbf{l} \otimes \mathbf{n} - \mathbf{m} \otimes \overline{\mathbf{m}} - \overline{\mathbf{m}} \otimes \mathbf{m}}$$

Searching for solutions...

Teleparallelism with null tetrads

The transformation between orthonormal and null tetrads is linear and constant $\Rightarrow$ the teleparallel formulation is valid either in the orthonormal or null approach.

Strategy

$$\begin{cases} \star\ \mathbf{l} \longrightarrow e^{\lambda(x)}\ \mathbf{l} \\ \star\ \mathbf{n} \longrightarrow e^{-\lambda(x)}\ \mathbf{n} \end{cases} \Rightarrow \boxed{\lambda(x) \text{ such that } T = 0}$$

Vacuum solutions in $f(T)$ gravity

A tetrad with $T = 0$ which corresponds to a vacuum solution of TEGR $\Rightarrow$ is also a solution in any $f(T)$ theory (for f sufficiently smooth)

If $T = 0$ the equations of motion (in vacuum) reads

$$0=4\, e^{-1}\partial_\mu(e\, S_a{}^{\mu\nu}) + 4\, e_a^\lambda\, T^\rho{}_{\mu\lambda}\, S_\rho{}^{\mu\nu}$$

$\Downarrow$

Vacuum solutions of TEGR with cosmological constant

$\hookrightarrow$ Schwarzschild $\mapsto$ radial boost **[Ferraro & Fiorini PRD, 84, 083518 (2011)]**
$\hookrightarrow$ Kerr $\mapsto$ null tetrad **[Bejarano, Ferraro & Guzmán, EPJC, 75, 77 (2015)]**

are also vacuum solutions (without deformations) for any $f(T)$ theory

Kerr geometry

[Bejarano, Ferraro & Guzmán, EPJC, 75, 77 (2015)]

Null tetrad asociated to Kerr spacetime ($a = J/M$ and $\Sigma = r^2 + a^2 c^2\theta$)

$$n^a_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{\lambda(t,r,\theta)}(1-\frac{2Mr}{\Sigma}) & e^{\lambda(t,r,\theta)}(1+\frac{2Mr}{\Sigma}) & 0 & e^{\lambda(t,r,\theta)} a\, s^2\theta \left(1+\frac{2Mr}{\Sigma}\right) \\ e^{-\lambda(t,r,\theta)} & -e^{-\lambda(t,r,\theta)} & 0 & -e^{-\lambda(t,r,\theta)} a\, s^2\theta \\ 0 & 0 & r + ia\, c\theta & i\, s\theta(r + ia\, c\theta) \\ 0 & 0 & r - ia\, c\theta & -i\, s\theta(r - ia\, c\theta) \end{pmatrix}$$

Torsion invariant

$$T = \frac{2}{\Sigma^3}\left(\Sigma^2 - 4\,a^2\cos^2\theta\,(\Sigma + m\,r) - 2\,r\,\Sigma^2\,\partial_t\lambda\right)$$

By setting $T = 0$, it possible to solve the equations obtaining the function

$$\lambda(t,r,\theta) = \frac{t}{2\,r}\left(1 - 4\,a^2\cos^2\theta\,\frac{\Sigma + m\,r}{\Sigma^2}\right) + \lambda_1(r,\theta)$$

$\hookrightarrow$**There exist a (null) tetrad, solution of the dynamical equations, such that $T = 0$, therefore this is also a rotating vacuum solution in $f(T)$ gravity**

Boyer-Lindquist coordinates: $x^\mu = (\tilde{t}, r, \theta, \varphi)$ $\begin{cases} d\varphi = d\phi + \frac{a}{r^2+a^2-2Mr}\,dr \\ d\tilde{t} = dt + \frac{2Mr}{r^2+a^2-2Mr}\,dr \end{cases}$

Recap

In the teleparallel picture,

- the gravitational field is described by torsion
- the dynamical variable is the tetrad
- the simplest model of teleparallelism lead to an equivalent dynamical formulation of GR $\rightarrow$ TEGR

  $\hookrightarrow$ the degree of freedom are encoded in the torsion

  $\hookrightarrow$ the torsion represents an alternative agent of gravitational interaction
- $f(T)$ is a generalization of TEGR (a particular teleparallel theory of gravity)
- The advantage of 2nd order field equations is paid by the lack of local Lorentz invariance
- It is worth mentioning that teleparallel approach allows to formulate gravity as a gauge theory (Yang-Mills type). In this sense, teleparallelism and its generalizations have achieved a great interest

- By introducing the null tetrad formalism, it is possible to obtain an easy way to find a dynamical solution such that the torsion invariant T vanishes which means not only that this tetrad is a solution of TEGR but also it is a solution of any $f(T)$ theory suffiently smooth

  ↪ Although modified theories attempt to smooth or even though to avoid the singularities...

  ↪ ...the Kerr metric is not deformable for any $f(T)$ (idem Schwarzschild)

- The null strategy is able for any kind of geometry?
  ↪ Kerr-Schild type metrics?
  ↪ Petrov (classification) type D spacetime?
  (Schwarszchild, Kerr, ...)
  ↪ Cosmological models?

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