Universality classes for models of inflation

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Windows on Quantum Gravity: Season 2,
Madrid, Spain.

Overview

Simplest inflationary realization.

Homogeneus scalar field ϕ in a homogeneous and isotropic universe:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2\kappa^2}R + \frac{\dot{\phi}^2}{2} - V(\phi)\right), \qquad (ds^2 = dt^2 - a^2(t)d\vec{x}^2) \quad (1)$$

The evolution of the system is described by Friedmann equations ($\kappa^2 = 1$):

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{\rho}{3}, \qquad -2\dot{H} = p + \rho \quad (2)$$

Inflation $\Longleftrightarrow$ Early phase of exponential expansion

$$\left(\frac{\dot{a}}{a}\right)^2 \sim const \iff \rho \sim -p \sim const \iff \left|\frac{\dot{\phi}^2}{2}\right| \ll |V| \sim const \quad (3)$$

dS Spacetime $\Longleftrightarrow$ $H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = constant$ $\Longleftrightarrow$ **Eternally inflating universe**

Nearly dS space $\Longleftrightarrow$ Inflation

Good inflationary models $\Rightarrow$ Slow departure from $1 + p/\rho \sim 0$.

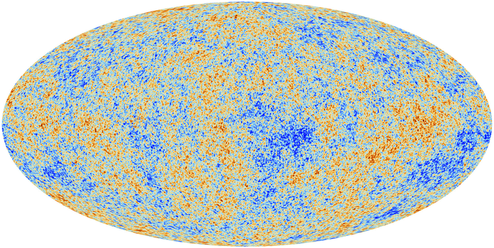
Observables.

Cosmological perturbations $\Longrightarrow$ Scalar and tensor power spectra:

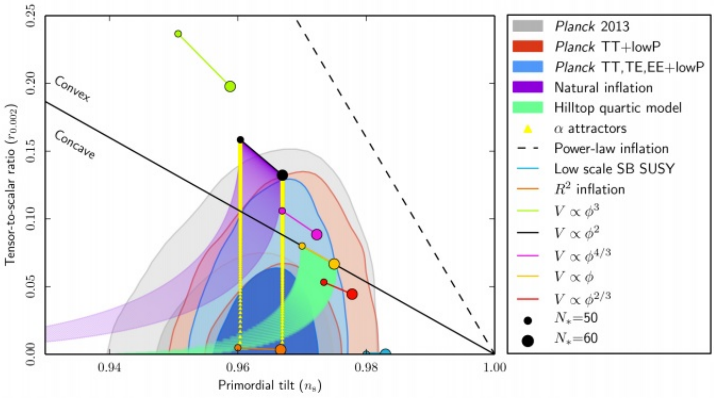
$$\mathcal{P}_s = \frac{1}{4\pi^2}\left.\frac{H^4}{\dot{\phi}^2}\right|_{k=aH} \qquad \mathcal{P}_t = 8\left.\left(\frac{H}{2\pi}\right)^2\right|_{k=aH} \tag{4}$$

Tensor-to-scalar ratio and the spectral index:

$$r \equiv \left.\frac{\mathcal{P}_t}{\mathcal{P}_s}\right|_{k=aH} \qquad n_s - 1 \equiv \left.\frac{d\ln \mathcal{P}_s(k)}{d\ln k}\right|_{k=aH} \tag{5}$$



Model classification.



A wide range of inflationary models has been proposed: Starobinsky inflation, Chaotic inflation, Higgs inflation, . . .

- Do we really need new models?
- Similar results for different models.

$\Longrightarrow$ Systematic classification of inflationary models.

Overview

β-function formalism.

Resonable assumption: $\phi(t)$ **piecewise monotonic**.

$$\text{We invert } \phi(t) \text{ to get } t(\phi) \Longrightarrow H(\phi) \equiv -\frac{1}{2}W(\phi). \qquad (6)$$

Via Hamilton-Jacobi formalism Friedmann equations can be expressed as:

$$W_{,\phi} = \dot{\phi} \qquad\qquad 2V = \frac{3}{2}W^2 - W_{,\phi}^2 \qquad (7)$$

In analogy with QTF we define a **β-function** as:

$$\frac{dg}{d\ln\mu} \equiv \beta(g) \iff \beta(\phi) \equiv \frac{d\phi}{d\ln a} = -2\frac{W_{,\phi}}{W}. \qquad (8)$$

The equation of state can be expressed in terms of the β-function as:

$$1 + \frac{p}{\rho} = \frac{4}{3}\left(\frac{W_{,\phi}}{W}\right)^2 = \frac{\beta^2(\phi)}{3}. \qquad (9)$$

Inflation $\iff$ **Slow departure from** $\beta(\phi) \sim 0$.

Some useful formulas.

- **Superpotential**:

$$W(\phi) = W_0 \exp\left\{-\frac{1}{2}\int_{\phi_f}^{\phi} \mathrm{d}\hat{\phi}\,\beta(\hat{\phi})\right\} \tag{10}$$

- **Potential**:

$$V(\phi) = \frac{3}{4}\left(1 - \frac{\beta^2}{6}\right) W^2(\phi) \tag{11}$$

- **Number of e-foldings**:

$$N(\phi) = -\int_{\phi_f}^{\phi} \frac{\mathrm{d}\hat{\phi}}{\beta(\hat{\phi})} \tag{12}$$

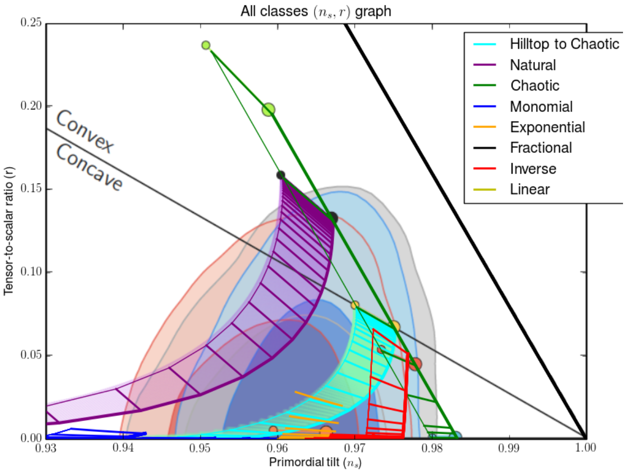
- **Scalar spectral index and its running**:

$$n_s - 1 \simeq -\left[2\beta_{,\phi} + \beta^2\right] \qquad \alpha_s \simeq -2\beta^2\beta_{,\phi} - 2\beta\beta_{,\phi\phi} \tag{13}$$

- **Tensor-to-scalar ratio**:

$$\mathrm{r} = 8\beta^2 \tag{14}$$

Comparison with Planck results on Inflation.



β-functions

$\beta(\phi) = \beta_q(\kappa\phi)^q \qquad 1 < q$

$\beta(\phi) = \beta_1(\kappa\phi)$

$\beta(\phi) = \dfrac{-\beta_p}{(\kappa\phi)^p} \qquad 1 < p$

$\beta(\phi) = \dfrac{-\beta_1}{(\kappa\phi)}$

$\beta(\phi) = \dfrac{-\hat{\beta}_p}{(\kappa\phi)^p} \qquad 0 < p < 1$

$\beta(\phi) = -\beta_\gamma \exp[\gamma\kappa\phi]$

$\beta(\phi) = (\kappa f)^{-1} \tan(\phi/\beta f)$

$\beta(\phi) = \dfrac{\beta_p}{f} \dfrac{p(\phi/f)^{p-1}}{1-(\phi/f)^p}$

P. Binetruy, E. Kiritsis, J. Mabillard, M. P. and C. Rosset, JCAP **1504**, no. 04, 033 (2015) [arXiv:1407.0820 [astro-ph.CO]].

Overview

Mapping dS into AdS.

Exists a correspondence between cosmologies and domain-wall spacetimes!
(P. McFadden and K. Skenderis, J. Phys. Conf. Ser. **222**, 012007 (2010))

Let us express the metric and the scalar field as:

$$\mathrm{d}s^2 = \eta\,\mathrm{d}z^2 + a^2(z)d\vec{x}^2, \qquad \Phi = \phi(z), \tag{15}$$

where:

- $\eta = -1 \rightarrow$ Cosmology, z is the cosmic time.
- $\eta = +1 \rightarrow$ Domain-wall, z is the radial coordinate.

The configuration of eq.(15) solves the field equations for the action:

$$\mathcal{S}_\phi = \eta \frac{m_p^2}{2} \int \mathrm{d}^4x \sqrt{-g} \left(-R + g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi + \frac{2}{m_p^2} V(\Phi) \right). \tag{16}$$

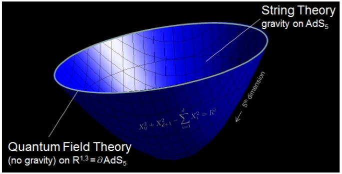
Via HJ formalism we express the equations of motion ($2/m_p^2 = 1$):

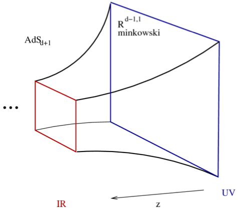
$$\frac{\dot{a}}{a} = -\frac{W(\phi)}{2}, \qquad \dot{\phi} = W_{,\phi}(\phi), \qquad 2\eta V(\phi) = -\frac{3}{2} W^2(\phi) + W_{,\phi}^2(\phi). \tag{17}$$

Every FRW solution of a model with potential $V(\phi)$ corresponds to a domain-wall solution of a model with potential $-V(\phi)$.

(A)dS/CFT and Holografic Inflation.

Asymptotic dS spacetime (Inflation) $\Longleftrightarrow$ Asymptotic AdS domain-walls.





With the language of (A)ds/CFT:

Departure from nearly (A)dS geometry $\Longleftrightarrow$ Deformation of CFT through RG flow

New light on some aspects:

- RG flow away from a fixed point in the past $\sim$ Inflation,
  RG flow towards a fixed point in the future $\sim$ Dark Energy?!
- Fine tuning on the initial conditions for inflation..

Overview

Attractor for inflation at strong coupling.

Let us consider a scalar field with a non minimal coupling with gravity:

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(-\frac{\Omega(\phi)}{2\kappa^2} R + X - V_J(\phi) \right) , \qquad X \equiv \frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi . \quad (18)$$

R. Kallosh, A. Linde and D. Roest, Phys. Rev. Lett. **112**, no. 1, 011303 (2014) [arXiv:1310.3950 [hep-th]]. $\Longrightarrow$ $\Omega(\phi) = 1 + \xi f(\phi)$ and $V_J = \lambda f^2(\phi)$. All the models with $1 \ll \xi$ asymptote to a universal attractor!

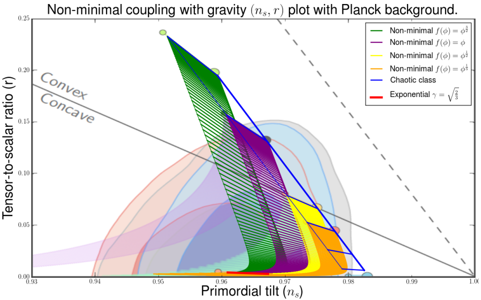
By redefining the metric and the scalar field as ($\kappa^2 = 1$):

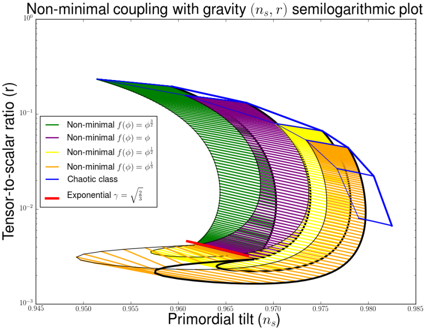
$$g_{\mu\nu} \to \Omega(\phi)^{-1} g_{\mu\nu} \qquad \left(\frac{d\varphi}{d\phi} \right)^2 = \Omega^{-1} + \frac{3}{2} \left(\frac{d \ln \Omega}{d\phi} \right)^2 , \quad (19)$$

we reduce to a minimally coupled field φ with canonical kinetic term.
M. P., arXiv:1510.03691 $\to$ $1 \ll \xi \Rightarrow$ the β-function for these models is:

$$\beta(\varphi) = -\exp \left\{ -\sqrt{\frac{2}{3}} (\varphi - \varphi_f) \right\} . \quad (20)$$

Predictions for n_s and r.





M. P., arXiv:1510.03691 [hep-ph].

Overview

Future Perspectives

- Generalized inflation:

$$S_I = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{\dot{\phi}^2}{2} - V(\phi)\right), \quad \longrightarrow \quad S_I = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}(X,\phi). \quad (21)$$

- Local reconstruction of $\beta(\phi)$ by using Planck data:

  Current constraints on $r, n_s, \alpha_s \Longrightarrow$ constraints on $\beta(\phi), \beta_{,\phi}(\phi), \beta_{,\phi\phi}(\phi)$.

- Quintessence and quintessential inflation:

  Models for quintessence can be interpreted in terms of $\beta(\phi)$.

The End

Thank you

Quintessence.

As discussed during the talk a nearly exponential expansion requires:

$$\frac{p+\rho}{\rho} = \frac{4}{3}\frac{W^2}{W^2_{,\phi}} = \frac{\beta^2(\phi)}{3} \to 0, \tag{22}$$

Let us remind the expressions for ρ and $W(\phi)$

$$\rho = \frac{3}{4}W^2 \tag{23}$$

$$W(\phi) = W_0 \exp\left\{-\frac{1}{2}\int_{\phi_f}^{\phi} \mathrm{d}\hat{\phi}\beta(\hat{\phi})\right\} \tag{24}$$

Combining (23) and (24) we can derive the following:

$$\rho \to 0 \qquad \Longleftrightarrow \qquad \int_{\phi_f}^{\phi} \mathrm{d}\phi'\beta(\phi') \to +\infty, \tag{25}$$

plus the condition that the fixed point is approached in the infinite future i.e.:

$$\int_{\phi_f}^{\phi} \frac{\mathrm{d}\phi'}{\beta(\phi')} \to +\infty. \tag{26}$$

Local reconstruction of $\beta(\phi)$ by using Planck data.

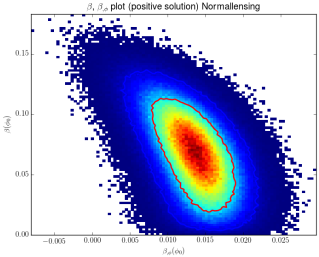
Using the definition of N number of e-foldings we define $\bar{\phi}$ as:

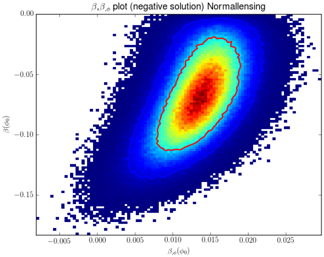
$$50 \div 60 \sim -\int_{\phi_f}^{\bar{\phi}} \frac{\mathrm{d}\hat{\phi}}{\beta(\hat{\phi})}. \tag{27}$$

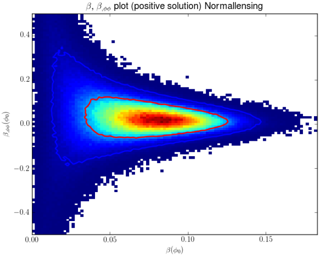
With Planck data we can fix constraints on $\beta(\phi), \beta_{,\phi}(\phi), \beta_{,\phi\phi}(\phi)$ for $\phi \sim \bar{\phi}$:

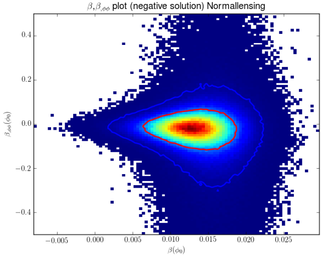
$$\begin{cases} \mathrm{r} = 8\beta^2 \\ n_s - 1 \simeq -\left[2\beta_{,\phi} + \beta^2\right] \\ \alpha_s \simeq -2\beta^2\beta_{,\phi} - 2\beta\beta_{,\phi\phi} \end{cases} \implies \begin{cases} \beta^2(\bar{\phi}) = \frac{\mathrm{r}}{8} \\ \beta_{,\phi}(\bar{\phi}) \simeq \frac{1}{2}\left[1 - n_s - \frac{\mathrm{r}}{8}\right] \\ \beta_{,\phi\phi}(\bar{\phi}) \simeq \mp\frac{\sqrt{2}\alpha_s}{\sqrt{\mathrm{r}}} \mp \sqrt{\frac{\mathrm{r}}{32}}\left[1 - n_s - \frac{\mathrm{r}}{8}\right] \end{cases}$$

Some numerical results.









Generalized Inflation 1.

A generalized action to describe inflation can be written as:

$$S = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}(X,\phi), \qquad X \equiv \frac{g^{\mu\nu}}{2}\partial_\mu\phi\partial_\nu\phi. \tag{28}$$

The energy density and pressure can be expressed as:

$$p(X,\phi) = \mathcal{L}(X,\phi), \qquad \rho(X,\phi) = 2Xp_{,X}(X,\phi) - p(X,\phi). \tag{29}$$

Using Hamilton-Jacobi formalism the system is described by:

$$\frac{\dot{a}}{a} = -\frac{W(\phi)}{2}, \qquad \dot{\phi} = W_{,\phi}\left(\frac{\partial p}{\partial X}\right)^{-1}, \qquad -p = \frac{3}{4}W^2 - W_{,\phi}^2\left(\frac{\partial p}{\partial X}\right)^{-1}. \tag{30}$$

If $\partial p/\partial X$ does not depend on X we define a new field φ:

$$\left(\frac{d\varphi}{d\phi}\right)^2 = \left(\frac{\partial p}{\partial X}\right) \tag{31}$$

As φ has a canonical kinetic term $\beta(\varphi) = d\varphi/d\ln a$ and thus:

$$\tilde{\beta}(\phi) \equiv \beta(\varphi(\phi)) = \frac{d\varphi(\phi)}{d\ln a} = \frac{d\varphi(\phi)}{d\phi}\frac{d\phi}{d\ln a} = \left(\frac{\partial p}{\partial X}\right)^{\frac{1}{2}}\frac{d\phi}{d\ln a} \tag{32}$$

Generalized Inflation 2.

By analogy we can generalize the definition of β using eq.(32):

$$\beta(\phi) \equiv \left(\frac{\partial p}{\partial X}\right)^{\frac{1}{2}} \frac{d\phi}{d\ln a} = -2\left(\frac{\partial p}{\partial X}\right)^{-\frac{1}{2}} \frac{W_{,\phi}}{W} \qquad (33)$$

The equation of state in terms of this function simply reads:

$$\frac{p+\rho}{\rho} = \frac{2Xp_{,X}(X,\phi)}{\rho} = \frac{4}{3}\frac{W_{,\phi}^2\left(\frac{\partial p}{\partial X}\right)^{-1}}{W^2} = \frac{\beta^2(\phi)}{3} \qquad (34)$$

And again inflation corresponds to $\beta(\phi) \to 0$.

Notice that in the standard case $p_{,X} = 1$ and thus we recover the results discussed previously.