General Boundary Formulation of Quantum Theory

and its relevance for the problem of Quantm Gravity

Daniele Colosi

Escuela Nacional de Estudios Superiores
Universidad Nacional Autónoma de México
Morelia, México

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Main difficulties

Lack of experimental data

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- Quantum theory requires a fixed, non-dynamical background metric
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Conceptual difficulties

- The problem of time
- Lack of a manifest local description of dynamics

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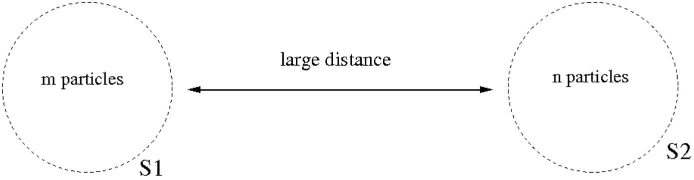
- In quantum field theory in Minkowski spacetime, causally separated systems are independent; cluster decomposition of the S-matrix: $S = S_1 S_2$



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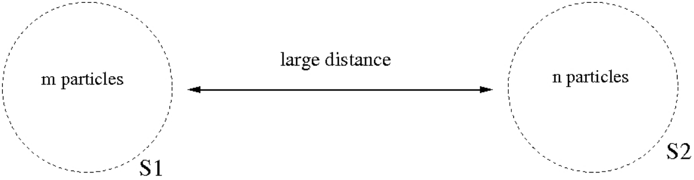

  We can separate the system from the rest of the universe.

- The notion of causal separation relies on the background metric. In a background independent context there is no way to isolate the system from the rest of the universe.

Conclusion

The standard formulation of QT has limitations that obstruct its application in a general relativistic context.

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Can we sufficiently extend the standard formulation of QT in order to render it compatible with the symmetries of GR?

- no explicit reference to a background (space)time
- description of physics in a manifestly local way
- ability to reproduce known physics

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The standard formulation of QT has limitations that obstruct its application in a general relativistic context.

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Can we sufficiently extend the standard formulation of QT in order to render it compatible with the symmetries of GR?

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- ability to reproduce known physics

YES, using:

- The mathematical framework of topological quantum field theory.
- A generalization of the Born rule.

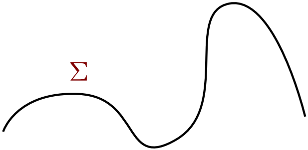
Outline

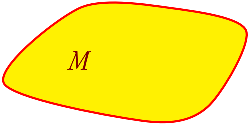
Basic structures

In the GBF algebraic structures are associated to geometric ones.

Geometric structures (representing pieces of spacetime):

- hypersurfaces: oriented manifolds of dimension $d-1$
- regions: oriented manifolds of dimension d with boundary





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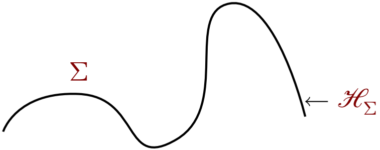

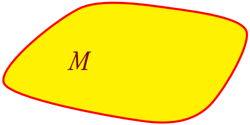

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- To Σ a Hilbert space $\mathcal{H}_\Sigma$

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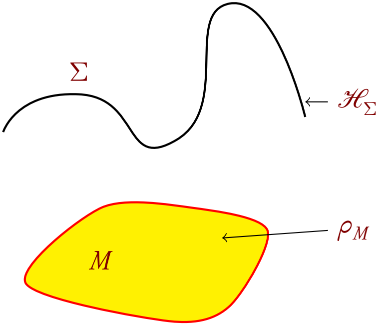

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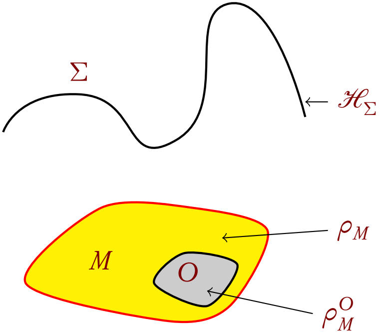


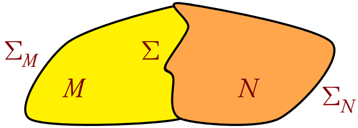
Algebraic structures:

- To Σ a Hilbert space $\mathcal{H}_\Sigma$
- To M a linear amplitude map $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$
- As in AQFT, observables are associated to spacetime regions: An observable O in a region M is a linear map $\rho_M^O : \mathcal{H}_{\partial M} \to \mathbb{C}$, called observable map.

Core axioms

These algebraic structures are subject to a number of axioms, in the spirit of TQFT.

- If $\overline{\Sigma}$ denote Σ with opposite orientation, then $\mathcal{H}_{\overline{\Sigma}} = \mathcal{H}_{\Sigma}^*$.
- (Decomposition rule) If $\Sigma = \Sigma_1 \cup \Sigma_2$, then $\mathcal{H}_{\Sigma} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\Sigma_2}$.
- (Gluing rule) If M and N are adjacent regions, then $\rho_{M \cup N} = \rho_M \circ \rho_N$. The composition $\circ$ involves a sum over a complete basis on the boundary hypersurface Σ shared by M and N.



$$\rho_{M\cup N}(\psi_M \otimes \psi_N) = \rho_M \circ \rho_N(\psi_M \otimes \psi_N) = \sum_i \rho_M(\psi_M \otimes \xi_i)\rho_N(\xi_i^* \otimes \psi_N)$$

where $\psi_M \in \mathcal{H}_M$, $\psi_N \in \mathcal{H}_N$ and $\{\xi_i\}$ is an ON-basis of $\mathcal{H}_{\Sigma}$.

Generalized probability interpretation

In quantum theory, probabilities are generally conditional probabilities: probability to observe a specific state given that some other specific state was prepared. Then probability depends on two type of data: *preparation* and *observation*.

In the GBF, both type of data encoded through closed subspaces of the state space $\mathcal{H}_{\partial M}$:

- $\mathcal{S} \subset \mathcal{H}_{\partial M}$ representing *preparation*
- $\mathcal{A} \subset \mathcal{H}_{\partial M}$ representing *observation*

The probability that the system is described by $\mathcal{A}$ given that it is described by $\mathcal{S}$ is:

$$P(\mathcal{A}|\mathcal{S}) = \frac{|\rho_M \circ P_{\mathcal{S}} \circ P_{\mathcal{A}}|^2}{|\rho_M \circ P_{\mathcal{S}}|^2} = \frac{\sum_{i\in J} |\rho_M(\xi_i)|^2}{\sum_{i\in I} |\rho_M(\xi_i)|^2}$$

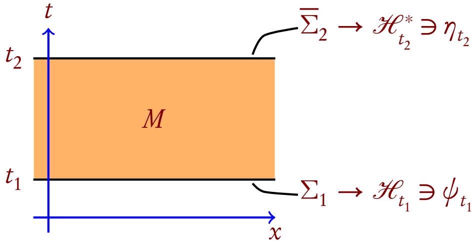
- $P_{\mathcal{S}}$ and $P_{\mathcal{A}}$ are the orthogonal projectors onto the subspaces.
- $\{\xi_i\}_{i\in I}$ is an ON-basis of $\mathcal{S}$, $\{\xi_i\}_{i\in J}$ is an ON-basis of $\mathcal{A}$.

Recovering of standard results (I)

A consistent probability interpretation exists and standard probabilities and expectation values can be recovered from the maps.

- Standard transition amplitudes of QFT can be recover from the GBF:

Spacetime region:
$M = [t_1, t_2] \times \mathbb{R}^3$
Boundary: $\partial M = \Sigma_1 \cup \overline{\Sigma}_2$
State space: $\mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}^*_{\Sigma_2}$



$$\rho_{[t_1,t_2]}(\psi_{t_1} \otimes \eta_{t_2}) = \langle \eta | U(t_1, t_2) | \psi \rangle$$

Recovering of standard results (II)

- The generalized probability reduces to a standard transition probability for a standard transition amplitude.
  The preparation corresponds to the subspace $\mathscr{S} = \psi \otimes \mathscr{H}_{t_2} \subset \mathscr{H}_{\partial M}$.
  The observation corresponds to the subspace $\mathscr{A} = \mathscr{H}_{t_1} \otimes \eta \subset \mathscr{H}_{\partial M}$.

$$P(\mathscr{A}|\mathscr{S}) = \frac{|\rho_M \circ P_{\mathscr{S}} \circ P_{\mathscr{A}}|^2}{|\rho_M \circ P_{\mathscr{S}}|^2} = \frac{|\rho_M(\psi \otimes \eta)|^2}{1} = |\langle \eta | U(t_1, t_2) | \psi \rangle|^2$$

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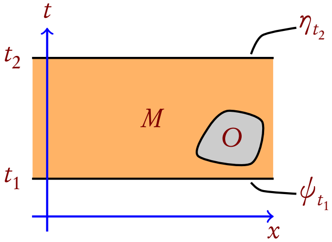
- Conventional expectation values of observable can be recovered



$$\lim_{t_2 \to t_1} \rho^{\mathrm{O}}_{[t_2,t_1]}(\psi \otimes \eta) = \rho^{\mathrm{O}}_{[t_1,t_1]}(\psi \otimes \eta) = \langle \eta | \hat{\mathrm{O}} | \psi \rangle$$

Advantages of the GBF

The general boundary formulation appears to be interesting for several reasons,

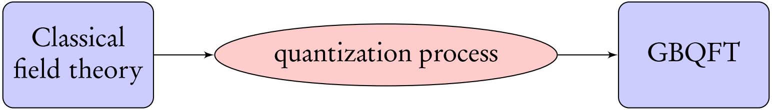
1. it is development of standard quantum theory compatible with known physics;
2. the versatility of the GBF, where general spacetime regions are considered, offers a new perspective on QT and its foundations and a better understanding of its geometrical aspects (holographic principle, boundaries, horizons);
3. it can treat situations where standard QT fails:
   - QFT in presence of a static black hole (rigorous treatment implementable with the hypercylinder geometry)
   - S-matrix in Anti-de Sitter spacetime;
4. it may solve some of the conceptual problems of background independent QFT (problem of time, local description of dynamics).
5. it is compatible with other approaches to quantum gravity (Spin Foam models, Group Field Theory)

Outline

GBF and QFT

Standard QFT can be formulated within the GBF

How to proceed?



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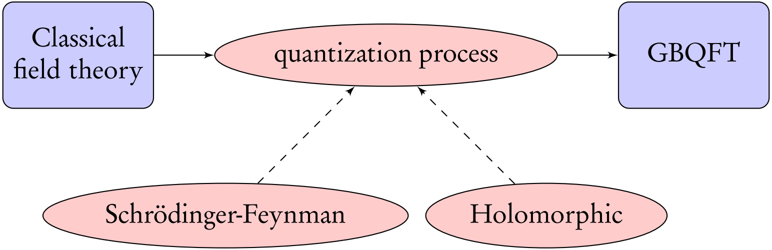

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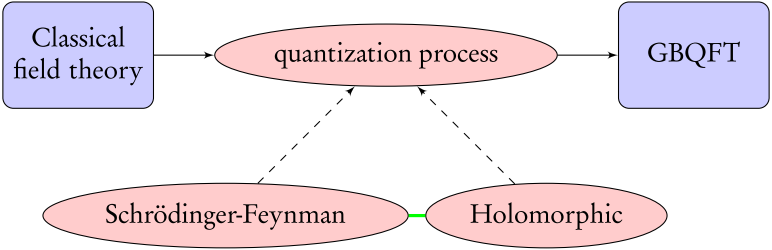

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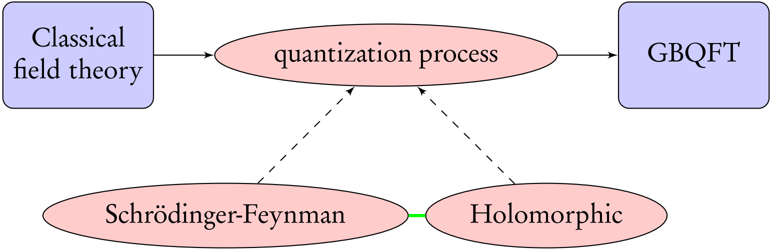


Results:

- An isomorphism can be constructed between the Hilbert spaces in the two representations
- The GBF axioms are satisfied by these quantization prescriptions
- Standard QFT fits into the GBF.

Schrödinger-Feynman quantization

- Schrödinger representation + Feynman path integral quantization
  The state space $\mathcal{H}_\Sigma$ for a hypersurface Σ is the space of functions on field configurations K_Σ on Σ.
- Inner product,

$$\langle \psi_2 | \psi_1 \rangle = \int_{K_\Sigma} \mathcal{D}\varphi \, \psi_1(\varphi) \overline{\psi_2(\varphi)}.$$

- Amplitude for a region M, $\psi \in \mathcal{H}_{\partial M}$,

$$\rho_M(\psi) = \int_{K_{\partial M}} \mathcal{D}\varphi \, \psi(\varphi) \int_{K_M, \phi|_{\partial M} = \varphi} \mathcal{D}\phi \, e^{iS_M(\phi)}.$$

- A classical observable F in M is modeled as a function on K_M. The quantization of F is the linear map $\rho_M^F : \mathcal{H}_{\partial M} \to \mathbb{C}$ defined as

$$\rho_M^F(\psi) = \int_{K_{\partial M}} \mathcal{D}\varphi \, \psi(\varphi) \int_{K_M, \phi|_{\partial M} = \varphi} \mathcal{D}\phi \, F(\phi) e^{iS_M(\phi)}.$$

Holomorphic quantization (I)

- Linear field theory: L_Σ is the vector space of solutions near the hypersurface Σ.
- L_Σ carries a non-degenerate symplectic structure ω_Σ and a complex structure $J_\Sigma : L_\Sigma \to L_\Sigma$ compatible with the symplectic structure:

$$J_\Sigma^2 = -\mathrm{id}_\Sigma \qquad \text{and} \qquad \omega_\Sigma(J_\Sigma(\cdot), J_\Sigma(\cdot)) = \omega_\Sigma(\cdot,\cdot).$$

- J_Σ and ω_Σ combine to a real inner product $g_\Sigma(\cdot,\cdot) = 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$ and to a complex inner product $\{\cdot,\cdot\}_\Sigma = g_\Sigma(\cdot,\cdot) + 2\mathrm{i}\omega_\Sigma(\cdot,\cdot)$ which makes L_Σ into a complex Hilbert space.
- The Hilbert space $\mathcal{H}_\Sigma$ associated with Σ is the space of holomorphic functions on L_Σ with the inner product

$$\langle \psi, \psi' \rangle_\Sigma = \int_{L_\Sigma} \overline{\psi(\phi)} \psi'(\phi) \exp\left(-\frac{1}{2} g_\Sigma(\phi,\phi)\right) \mathrm{d}\mu(\phi),$$

where μ is a (fictitious) translation-invariant measure on L_Σ.

Holomorphic quantization (II)

- The amplitude map $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$ associated with the spacetime region M for a state $\psi \in \mathcal{H}_{\partial M}$ is given by

$$\rho_M(\psi) = \int_{L_\Sigma} \psi(\phi) \exp\left(-\frac{1}{4} g_{\partial M}(\phi, \phi)\right) \mathrm{d}\mu_{\tilde{M}}(\phi).$$

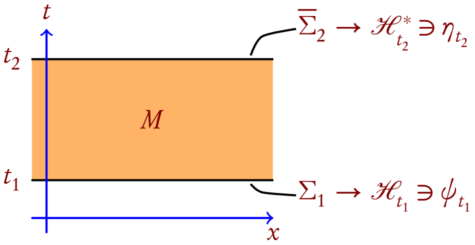
- The observable map associated to a classical observable F in a region M is

$$\rho_M^F(\psi) = \int_{L_\Sigma} \psi(\phi) F(\phi) \exp\left(-\frac{1}{4} g_{\partial M}(\phi, \phi)\right) \mathrm{d}\mu_{\tilde{M}}(\phi).$$

Klein-Gordon theory in Minkowski

The S-matrix technique is used to describe interacting QFT:

Spacetime region:
$M = [t_1, t_2] \times \mathbb{R}^3$
Boundary: $\partial M = \Sigma_1 \cup \overline{\Sigma}_2$
State space: $\mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}^*_{\Sigma_2}$



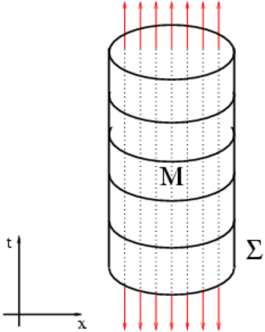
Assume interaction is relevant only between the initial time t_1 and the final time t_2.
The S-matrix is the asymptotic limit of the amplitude between free states at early and at late time:

$$\langle \psi_2 | \mathcal{S} | \psi_1 \rangle = \lim_{\substack{t_1 \to -\infty \\ t_2 \to +\infty}} \langle \psi_2 | U_{int}(t_1, t_2) | \psi_1 \rangle = \lim_{\substack{t_1 \to -\infty \\ t_2 \to +\infty}} \rho^U_{[t_1,t_2] \times \mathbb{R}^3}(\psi_1 \otimes \psi_2^*)$$

Spatially asymptotic S-matrix

Similarly, we can describe interacting QFT via a spatially asymptotic amplitude. Assume interaction is relevant only within a radius R from the origin in space (but at all times). Consider then the asymptotic limit of the amplitude of a free state on the hypercylinder when the radius goes to infinity:

$$\boxed{\mathcal{S}(\psi) = \lim_{R\to\infty} \rho_R(\psi)}$$



Result

The S-matrices are equivalent when both are valid.

Outline

Conclusions

- The general boundary formulation is a new powerful formulation of quantum theory.
- Many results have been obtained:
  - quantisation prescriptions (description of quantum states on timelike hypersurfaces) [Oeckl 2008, Oeckl 2012]
  - unitarity of evolution for QFT in curved space [DC, Oeckl 2011]
  - new S-matrices type asymptotic amplitudes [DC, Oeckl 2008; DC 2009; Dohse 2011; 2012]
  - GBQFT in Euclidean space [DC, Oeckl 2009]
  - de Sitter (derivation of the Polyakov propagator) [DC 2010; 2015]
  - Anti de Sitter (S-matrix for the hyper cylinder) [DC, Dohse, Oeckl 2012]
  - Rindler [DC, Rätzel 2014]
  - Unruh effect [DC, Rätzel 2012]
- It is a work in progress...